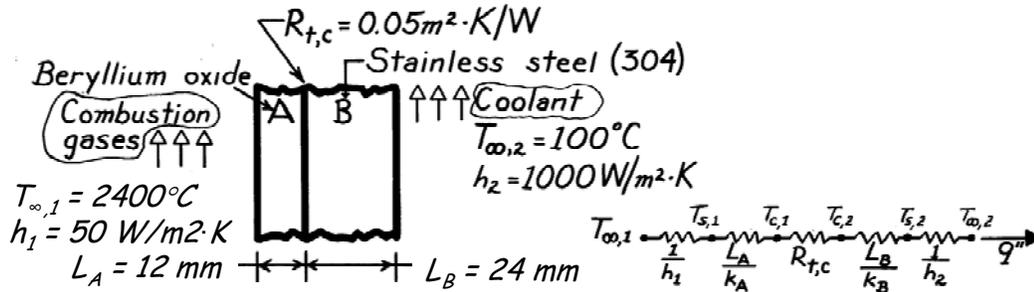


PROBLEM 3.26

KNOWN: Materials and dimensions of a composite wall separating a combustion gas from a liquid coolant.

FIND: (a) Heat loss per unit area, and (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: Table A-1, St. St. (304) ($\bar{T} \approx 1000\text{K}$): $k = 25.4 \text{ W/m}\cdot\text{K}$; Table A-2, Beryllium Oxide ($T \approx 1500\text{K}$): $k = 21.5 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) The desired heat flux may be expressed as

$$q'' = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_A}{k_A} + R_{t,c} + \frac{L_B}{k_B} + \frac{1}{h_2}} = \frac{(2400 - 100)^\circ\text{C}}{\left[\frac{1}{50} + \frac{0.012}{21.5} + 0.05 + \frac{0.024}{25.4} + \frac{1}{1000} \right] \frac{\text{m}^2 \cdot \text{K}}{\text{W}}}$$

$$q'' = 31,720 \text{ W/m}^2. \quad \leftarrow$$

(b) The composite surface temperatures may be obtained by applying appropriate rate equations. From the fact that $q'' = h_1 (T_{\infty,1} - T_{s,1})$, it follows that

$$T_{s,1} = T_{\infty,1} - \frac{q''}{h_1} = 2400^\circ\text{C} - \frac{31,720 \text{ W/m}^2}{50 \text{ W/m}^2 \cdot \text{K}} = 1766^\circ\text{C}.$$

With $q'' = (k_A / L_A)(T_{s,1} - T_{c,1})$, it also follows that

$$T_{c,1} = T_{s,1} - \frac{L_A q''}{k_A} = 1766^\circ\text{C} - \frac{0.012\text{m} \times 31,720 \text{ W/m}^2}{21.5 \text{ W/m}\cdot\text{K}} = 1748^\circ\text{C}.$$

Similarly, with $q'' = (T_{c,1} - T_{c,2}) / R_{t,c}$

$$T_{c,2} = T_{c,1} - R_{t,c} q'' = 1748^\circ\text{C} - 0.05 \frac{\text{m}^2 \cdot \text{K}}{\text{W}} \times 31,720 \frac{\text{W}}{\text{m}^2} = 162^\circ\text{C}$$

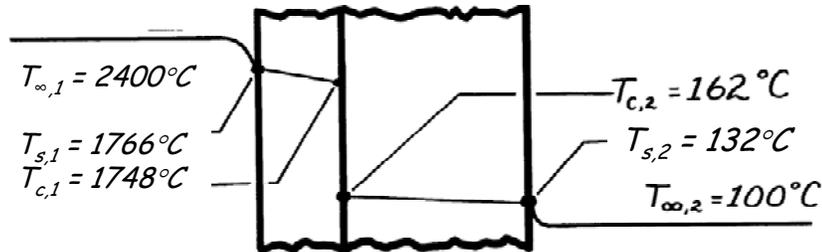
Continued ...

PROBLEM 3.26 (Cont.)

and with $q'' = (k_B / L_B)(T_{c,2} - T_{s,2})$,

$$T_{s,2} = T_{c,2} - \frac{L_B q''}{k_B} = 162^\circ\text{C} - \frac{0.024\text{m} \times 31,720 \text{ W/m}^2}{25.4 \text{ W/m} \cdot \text{K}} = 132^\circ\text{C}.$$

The temperature distribution is therefore of the following form:



COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q'' = h_2 (T_{s,2} - T_{\infty,2}) = 1000 \text{ W/m}^2 \cdot \text{K} (132 - 100)^\circ\text{C} = 31,700 \text{ W/m}^2$$

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to $T \approx 1750^\circ\text{C}$ for the oxide and $T \approx 150^\circ\text{C}$ for the steel.

(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.