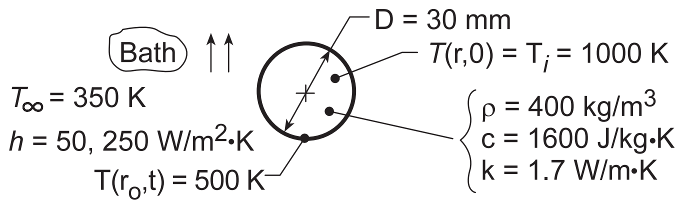
PROBLEM 5.61

KNOWN: A long cylinder, initially at a uniform temperature, is suddenly quenched in a large oil bath.

FIND: (a) Time required for the surface to reach 500 K, (b) Effect of convection coefficient on surface temperature history.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo > 0.2.

ANALYSIS: (a) Check first whether lumped capacitance method is applicable. For $h = 50$ W/m^2·K,

$$\mathrm{Bi_c} = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{50\,\mathrm{W/m^2\cdot K}(0.015\,\mathrm{m}/2)}{1.7\,\mathrm{W/m\cdot K}} = 0.221.$$

Since $\mathrm{Bi_c} > 0.1$, method is not suited. Using the approximate series solution for the infinite cylinder,

$$\theta^*(r^*,\mathrm{Fo}) = C_1 \exp\left(-\zeta_1^2 \mathrm{Fo}\right) \times J_0\left(\zeta_1 r^*\right) \qquad (1)$$

Solving for Fo and setting $r^* = 1$, find

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2} \ln\left[\frac{\theta^*}{C_1 J_0(\zeta_1)}\right]$$

where $\theta^* = (1,\mathrm{Fo}) = \dfrac{T(r_o,t_o) - T_\infty}{T_i - T_\infty} = \dfrac{(500-350)\,\mathrm{K}}{(1000-350)\,\mathrm{K}} = 0.231$.

From Table 5.1, with Bi = 0.441, find $\zeta_1 = 0.8882$ rad and $C_1 = 1.1019$. From Table B.4, find $J_0(\zeta_1) = 0.8121$. Substituting numerical values into Eq. (2),

$$\mathrm{Fo} = -\frac{1}{(0.8882)^2} \ln\left[0.231/1.1019 \times 0.8121\right] = 1.72.$$

From the definition of the Fourier number, $\mathrm{Fo} = \alpha t / r_o^2$, and $\alpha = k/\rho c$,

$$t = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo}\cdot r_o^2 \frac{\rho c}{k}$$

$$t = 1.72(0.015\,\mathrm{m})^2 \times 400\,\mathrm{kg/m^3} \times 1600\,\mathrm{J/kg\cdot K}/1.7\,\mathrm{W/m\cdot K} = 145\,\mathrm{s}. \qquad <$$

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following surface temperature histories were obtained.

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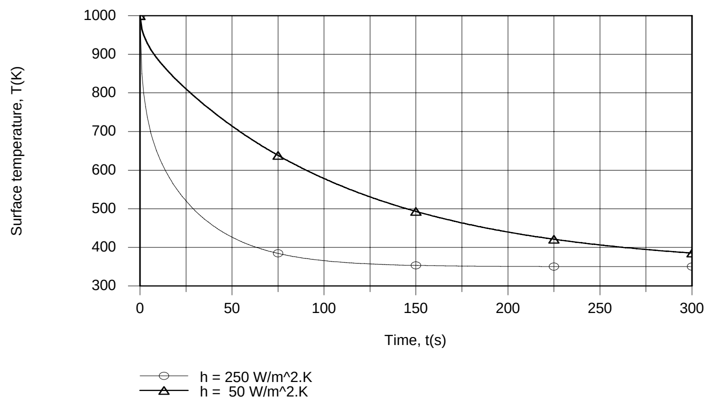
PROBLEM 5.61 (Cont.)



Increasing the convection coefficient by a factor of 5 has a significant effect on the surface temperature, greatly accelerating its approach to the oil temperature. However, even with h = 250 $W/m^2 \cdot K$, Bi = 1.1 and the convection resistance remains significant. Hence, in the interest of accelerated cooling, additional benefit could be achieved by further increasing the value of h.

COMMENTS: For Part (a), note that, since Fo = 1.72 > 0.2, the approximate series solution is appropriate.