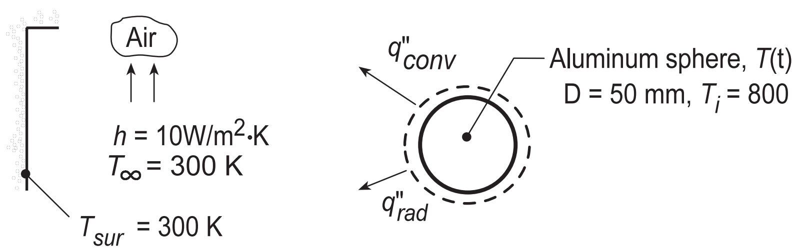
PROBLEM 5.34

KNOWN: Metal sphere, initially at a uniform temperature T_i, is suddenly removed from a furnace and suspended in a large room and subjected to a convection process (T_∞, h) and to radiation exchange with surroundings, T_{sur}.

FIND: (a) Time it takes for sphere to cool to some temperature T, neglecting radiation exchange, (b) Time it takes for sphere to cool to some temperature t, neglecting convection, (c) Procedure to obtain time required if both convection and radiation are considered, (d) Time to cool an anodized aluminum sphere to 400 K using results of Parts (a), (b) and (c).

SCHEMATIC:



ASSUMPTIONS: (1) Sphere is spacewise isothermal, (2) Constant properties, (3) Constant heat transfer convection coefficient, (4) Sphere is small compared to surroundings.

PROPERTIES: *Table A-1*, Aluminum, pure ($\bar{T}$ = [800 + 400] K/2 = 600 K): ρ = 2702 kg/m^3, c = 1033 J/kg·K, k = 231 W/m·K, $\alpha = k/\rho c = 8.276 \times 10^{-5}$ m^2/s; Aluminum, anodized finish: ε = 0.75, polished surface: ε = 0.1.

ANALYSIS: (a) Neglecting radiation, the time to cool is predicted by Eq. 5.5,

$$t = \frac{\rho V c}{h A_s} \ln \frac{\theta_i}{\theta} = \frac{\rho D c}{6h} \ln \frac{T_i - T_\infty}{T - T_\infty} \qquad (1) \quad <$$

where $V/A_s = (\pi D^3/6)/(\pi D^2) = D/6$ for the sphere.

(b) Neglecting convection, the time to cool is predicted by Eq. 5.18,

$$t = \frac{\rho D c}{24 \varepsilon \sigma T_{sur}^3} \left\{ \ln \left| \frac{T_{sur} + T}{T_{sur} - T} \right| - \ln \left| \frac{T_{sur} + T_i}{T_{sur} - T_i} \right| + 2 \left[\tan^{-1} \left(\frac{T}{T_{sur}} \right) - \tan^{-1} \left(\frac{T_i}{T_{sur}} \right) \right] \right\} \qquad (2)$$

where $V/A_{s,r} = D/6$ for the sphere.

(c) If convection and radiation exchange are considered, the energy balance requirement results in Eq. 5.15 (with $q''_s = \dot{E}_g = 0$). Hence

$$\frac{dT}{dt} = \frac{6}{\rho D c} \left[h \left(T - T_\infty \right) + \varepsilon \sigma \left(T^4 - T_{sur}^4 \right) \right] \qquad (3) \quad <$$

where $A_{s(c,r)} = A_s = \pi D^2$ and $V/A_{s(c,r)} = D/6$. This relation must be solved numerically in order to evaluate the time-to-cool.

(d) For the aluminum (pure) sphere with an anodized finish and the prescribed conditions, the times to cool from T_i = 800 K to T = 400 K are:

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Convection only, Eq. (1)

$$t = \frac{2702\,\text{kg}/\text{m}^3 \times 0.050\,\text{m} \times 1033\,\text{J/kg}\cdot\text{K}}{6 \times 10\,\text{W}/\text{m}^2\cdot\text{K}} \ln\frac{800-300}{400-300} = 3743\text{s} = 1.04\text{h} \qquad <$$

Radiation only, Eq. (2)

$$t = \frac{2702\,\text{kg}/\text{m}^3 \times 0.050\,\text{m} \times 1033\,\text{J/kg}\cdot\text{K}}{24 \times 0.75 \times 5.67 \times 10^{-8}\,\text{W}/\text{m}^2\cdot\text{K}^4 \times (300\,\text{K})^3} \cdot \left\{ \left(\ln\frac{400+300}{400-300} - \ln\frac{800+300}{800-300} \right) + \right.$$

$$\left. 2\left[\tan^{-1}\frac{400}{300} - \tan^{-1}\frac{800}{300} \right] \right\}$$

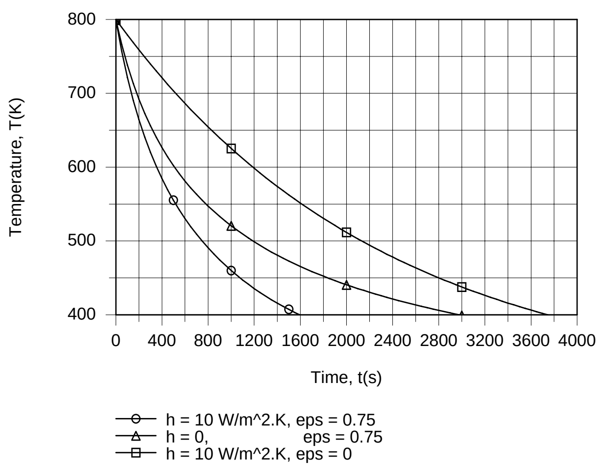
$$t = 5.065 \times 10^3 \{1.946 - 0.789 + 2(0.927 - 1.212)\} = 2973\text{s} = 0.826\text{h} \qquad <$$

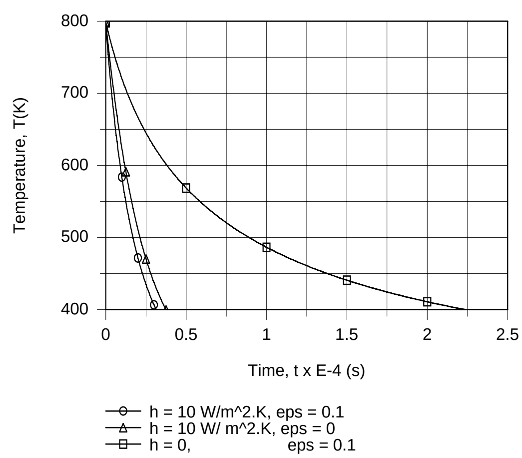
Radiation and convection, Eq. (3)

Using the IHT *Lumped Capacitance Model*, numerical integration yields

$$t \approx 1600\text{s} = 0.444\text{h}$$

In this case, heat loss by radiation exerts the stronger influence, although the effects of convection are by no means negligible. However, if the surface is polished (ε = 0.1), convection clearly dominates. For each surface finish and the three cases, the temperature histories are as follows.





COMMENTS: 1. A summary of the analyses shows the relative importance of the various modes of heat loss:

Active Modes	Time required to cool to 400 K (h)	
	ε = 0.75	ε = 0.1
Convection only	1.040	1.040
Radiation only	0.827	6.194
Both modes	0.444	0.889

2. Note that the spacewise isothermal assumption is justified since Be << 0.1. For the convection-only process,

$$Bi = h(r_o/3)/k = 10\ \text{W/m}^2\cdot\text{K}\ (0.025\ \text{m}/3)/231\ \text{W/m}\cdot\text{K} = 3.6 \times 10^{-4}$$