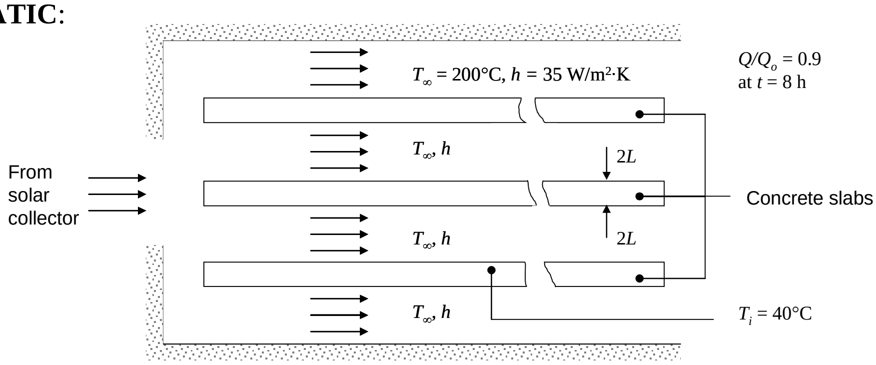
PROBLEM 5.53

KNOWN: Initial temperature of concrete slabs. Air temperature and convection heat transfer coefficient.

FIND: Slab thickness required so that $Q/Q_o = 0.90$ for $t = 8$ h.

SCHEMATIC:



ASSUMPTIONS: (1) Properties at 300 K are satisfactory at higher temperature, (2) One-dimensional conduction, (3) Constant convection heat transfer coefficient, (4) Negligible radiation because each concrete slab is surrounded by others at the same temperature.

PROPERTIES: *Table A.3*, Concrete (stone mix) (T = 300 K): $\rho = 2300$ kg/m^3, $c = 880$ J/kg·K, k = 1.4 W/m·K.

ANALYSIS: Without knowing the thickness, we cannot determine in advance whether the lumped capacitance approximation is valid. Considering the small thermal conductivity of concrete, we might anticipate that the Biot number will not be small. Considering the long time period, we may anticipate that the Fourier number is large. Therefore, we will begin by using the first-term approximation of the series solution and check its validity later. From Equation 5.49

$$\frac{Q}{Q_o} = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^* = 0.9 \qquad (1)$$

where from Equation 5.44

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right) \qquad (2)$$

In these equations, C_1 and ζ_1 are functions of Bi:

$$\zeta_1 \tan\zeta_1 = Bi\,, \qquad C_1 = \frac{4\sin\zeta_1}{2\zeta_1 + \sin(2\zeta_1)} \qquad (3,4)$$

where ζ_1 is the smallest root of Equation (3). Both Bi and Fo are unknown because L is unknown. They are given by

$$Bi = \frac{hL}{k}\,, \qquad Fo = \frac{\alpha t}{L^2} \qquad (5,6)$$

Continued…

where $\alpha = k/\rho c = 6.92 \times 10^{-7}$ W/m^2·K. These six simultaneous equations can be solved iteratively. One approach is to guess a value for ζ_1, from which C_1 can be calculated from Equation (4). Equation (1) can be used to find the required value of θ_o^*. Then Equation (2) can be used to determine *Fo* and a new value of *L* can be determined from Equation (6). Finally, *Bi* can be calculated from Equation (5) and a new value of ζ_1 can be found from Equation (3). Beginning this approach with a guessed value of $\zeta_1 = 1$, the iterations proceed as follows:

Guess:	$\zeta_1 = 1$
Equation (4):	$C_1 = 1.16$
Equation (1):	$\theta_o^* = 0.119$
Equation (2):	$Fo = 2.28$
Equation (6):	$L = 0.0936$ m
Equation (5):	$Bi = 2.34$
Equation (3):	$\zeta_1 = 1.1231$

Repeating with the new value of ζ_1 and iterating until *L* converges to two significant digits, we find

$$L = 0.11 \text{ m}$$ <

with $Bi = 2.7$, $Fo = 1.7$. Thus the lumped capacitance approximation is not appropriate, and the first-term approximation is valid.

COMMENTS: This hand-solution is time-consuming, especially since Equation (3) must itself be solved iteratively. A much faster approach would be to solve these six equations simultaneously using IHT or other software.