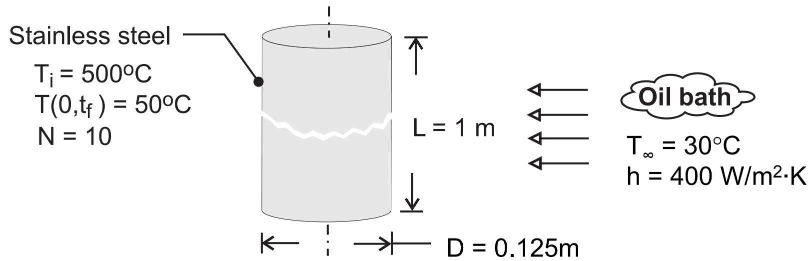
PROBLEM 5.67

KNOWN: Diameter and initial temperature of roller bearings. Temperature of oil bath and convection coefficient. Final centerline temperature. Number of bearings processed per hour.

FIND: Time required to reach centerline temperature. Cooling load.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction in rod, (2) Constant properties.

PROPERTIES: *Table A.1*, St. St. 304 $\left(\overline{T} = 548\,K\right)$: ρ=7900 kg/m^3, k = 19.0 W/m·K, c_p = 546 J/kg·K, α = 4.40 × 10^{-6} m^2/s.

ANALYSIS: With Bi = h (r_o/2)/k = 0.658, the lumped capacitance method can not be used. From the one-term approximation of Eq. 5.52c for the centerline temperature,

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{50-30}{500-30} = 0.0426 = C_1 \exp\left(-\zeta_1^2 Fo\right) = 1.2486 \exp\left[-(1.3643)^2\ Fo\right]$$

where, for Bi = hr_o/k = 1.316, C_1 = 1.2486 and ζ_1 = 1.3643 from Table 5.1.

$$Fo = 1.82$$

$$t_f = Fo\, r_o^2 / \alpha = 1.82 (0.0625\,m)^2 / 4.40 \times 10^{-6} = 1616\,s = 27\,min \qquad <$$

From Eqs. 5.47 and 5.54, the energy extracted from a single rod is

$$Q = \rho c V \left(T_i - T_\infty\right)\left[1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1)\right]$$

With J_1 (1.3643) = 0.535 from Table B.4,

$$Q = 7900\,kg/m^3 \times 546\,J/kg\cdot K\left[\pi (0.0625\,m)^2\, 1m\right] 470\,K\left[1 - \frac{0.0852 \times 0.535}{1.3643}\right] = 2.41 \times 10^7\ J$$

The nominal cooling load is

$$\overline{q} = \frac{N\,Q}{t_f} = \frac{10 \times 2.41 \times 10^7\ J}{1616\,s} = 1.49 \times 10^5\,W = 149\ kW \qquad <$$

COMMENTS: For a centerline temperature of 50°C, Eq. 5.52b yields a surface temperature of

$$T(r_o, t) = T_\infty + \left(T_i - T_\infty\right)\theta_o^*\ J_o(\zeta_1) = 30°C + 470°C \times 0.0426 \times 0.586 = 41.7°C$$