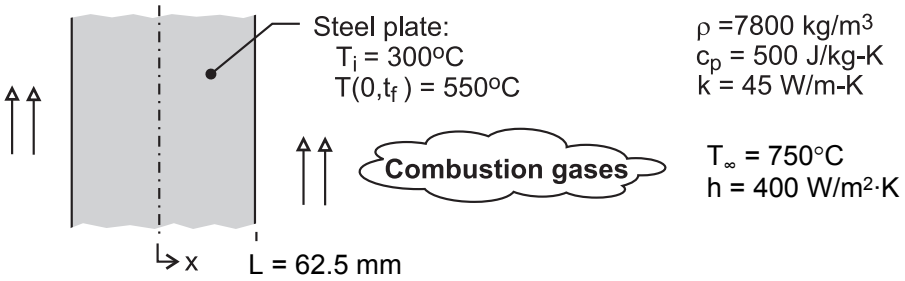
PROBLEM 5.52

KNOWN: Thickness, initial temperature and properties of steel plate. Convection conditions at both surfaces.

FIND: Time required to achieve a minimum temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in plate, (2) Symmetric heating on both sides, (3) Constant properties, (4) Negligible radiation from gases, (5) Fo > 0.2.

ANALYSIS: The smallest temperature exists at the midplane and, with Bi = hL/k = 400 W/m^2·K × 0.0625m/45 W/m·K = 0.556 and Fo > 0.2, may be determined from the one-term approximation of Eq. 5.41. From Table 5.1, C_1 = 1.076 and ζ_1 = 0.682. Hence, with θ_o^* = (T_o - T_∞)/(T_i - T_∞) = 0.444,

$$Fo = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = -\frac{\ln\left(0.444/1.076\right)}{\left(0.682\right)^2} = 1.901$$

$$t = \frac{Fo\ L^2}{\alpha} = \frac{1.901\left(0.0625\text{m}\right)^2}{\left(45\,\text{W/m}\cdot\text{K}/7800\ \text{kg/m}^3 \times 500\ \text{J/kg}\cdot\text{K}\right)} = 644\,\text{s} \quad <$$

COMMENTS: From Eq. 5.43b, the corresponding surface temperature is

$$T_s = T_\infty + \left(T_i - T_\infty\right)\theta_o^* \cos\left(\zeta_1\right) = 750°\text{C} - 450°\text{C} \times 0.444 \times 0.776 = 595°\text{C}$$

Because Bi is not much larger than 0.1, temperature gradients in the steel are moderate.