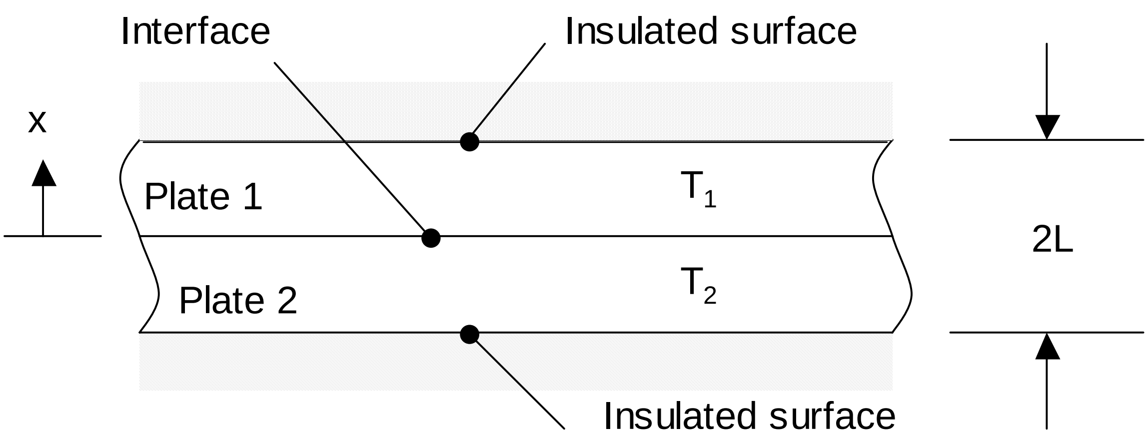
PROBLEM 5.56

KNOWN: Thickness and initial temperatures of two plates of the same material.

FIND: (a) Steady-state dimensionless temperatures of the two plates, $T^*_{ss,1}$ and $T^*_{ss,2}$, as well as the interface temperature, T^*_{int}, (b) Expression for the effective dimensionless overall heat transfer coefficient for the two-plate system, $U^*_{eff,2} \equiv q^*/\left(\overline{T}_2^* - \overline{T}_1^*\right)$ for $Fo > 0.2$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible thermal contact resistance.

ANALYSIS: (a) Since the two plates are of the same thickness and have the same properties, both plates will reach a steady-state temperature that is the average of T_1 and T_2. In terms of the dimensionless temperature, $T^*(Fo) \equiv (T - T_1)/(T_2 - T_1)$, this implies that $T^*_{ss,1} = T^*_{ss,2} = 0.5$. <

Accounting for the symmetry about $x = 0$, the dimensionless interface temperature will be $T^*_{int} = 0.5$ at any time. <

(b) Taking advantage of the geometrical symmetry about $x = 0$, we may simplify the problem by analyzing just one of the plates, accounting for one adiabatic surface and a second surface being held at a constant temperature. For the constant temperature boundary condition, $Bi = hL/k \to \infty$ and from Table 5.1 $\zeta_1 = \pi/2$, $C_1 = 1.2733$. Equations 5.44 and 5.49 may be combined to yield

$$\frac{Q}{Q_o} = 1 - \frac{\sin\zeta_1}{\zeta_1} C_1 \exp(-\zeta_1^2 Fo) \qquad (1)$$

The spatially-averaged dimensionless temperature for one plate is

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PROBLEM 5.56 (Cont.)

$$\overline{\theta *} = \frac{1}{V}\int \frac{T(x,t) - T_{\text{int}}}{T_i - T_{\text{int}}} dV$$

where T_{int} is the interface temperature. From Eq. 5.46b, $\overline{\theta *} = 1 - Q/Q_o$. (2)

From Eq. (1),

$$\frac{d(Q/Q_o)}{dFo} = q* = 2C_1\left(\sin\zeta_1\right)\exp(-\zeta_1^2 Fo)$$

and from Eq. (2)

$$\overline{\theta *} = \frac{C_1 \sin\zeta_1}{\zeta_1}\exp(-\zeta_1^2 Fo)$$

Therefore, for one plate, $U^*_{\text{eff},1} = \left.\frac{d(Q/Qo)}{dFo}\right/\overline{\theta *} = 2\zeta_1 = \pi$

The dimensionless temperature difference for the two-plate system is $T_2^* - T_1^* = 2\overline{\theta *}$.

Hence, for the two-plane system, $U^*_{\text{eff}} = U^*_{\text{eff, 2}}/2 = U^*_{\text{eff,1}}/2 = \pi/2$. <

COMMENTS: (1) For this case, the heat transfer rate between the two plates is proportional to the difference in the average temperatures of the plates. If $Fo < 0.2$, it may be shown that U^*_{eff} is initially infinite and decreases with time. This behavior becomes evident if one considers the situation immediately after the plates make contact when the heat transfer between the plates is very large, but the average plate temperatures have not been affected significantly by the heat transfer in the vicinity of the interface. (2) A proportionality between the dimensionless heat transfer rate and the difference in the average dimensionless plate temperatures also exists at large Fo, even if the plates are not identical.