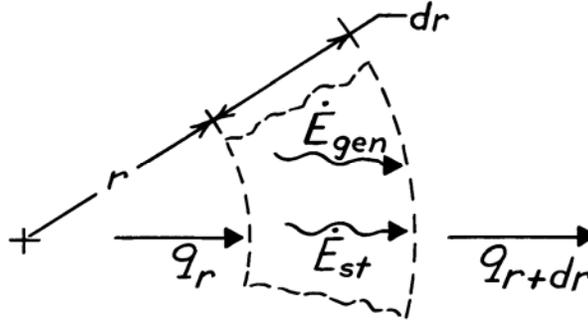


PROBLEM 2.44

KNOWN: Heat diffusion with internal heat generation for one-dimensional cylindrical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has volume, $V = A_r \cdot dr = 2\pi \cdot r \cdot dr \cdot 1$, with unit thickness normal to page. Using the conservation of energy requirement, Eq. 1.12c,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this one-dimensional coordinate system is

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \times 2\pi r \cdot 1 \times \frac{\partial T}{\partial r}.$$

At the outer surface, $r + dr$, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r} \left[-k \cdot 2\pi r \cdot \frac{\partial T}{\partial r} \right] dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r} \left[-k2\pi r \frac{\partial T}{\partial r} \right] dr \right] + \dot{q} \cdot 2\pi r dr = \rho \cdot 2\pi r dr \cdot c_p \frac{\partial T}{\partial t}$$

Dividing by the factor $2\pi r dr$, we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left[kr \frac{\partial T}{\partial r} \right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$

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COMMENTS: (1) Note how the result compares with Eq. 2.26 when the terms for the ϕ, z coordinates are eliminated. (2) Recognize that we did not require \dot{q} and k to be independent of r .