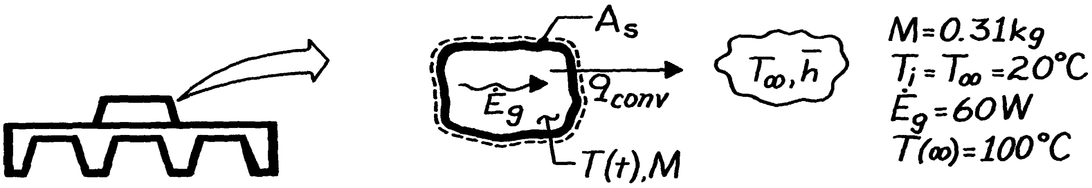
## PROBLEM 5.20

**KNOWN:** Electronic device on aluminum, finned heat sink modeled as spatially isothermal object with internal generation and convection from its surface.

**FIND:** (a) Temperature response after device is energized, (b) Temperature rise for prescribed conditions after 5 min.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Spatially isothermal object, (2) Object is primarily aluminum, (3) Initially, object is in equilibrium with surroundings at $T_\infty$.

**PROPERTIES:** *Table A-1*, Aluminum, pure $\left(\bar{T} = (20+100)^\circ \text{C}/2 \approx 333\text{K}\right)$: c = 918 J/kg·K.

**ANALYSIS:** (a) Following the general analysis of Section 5.3, apply the conservation of energy requirement to the object,

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \qquad \dot{E}_g - \bar{h}A_s\left(T - T_\infty\right) = Mc\frac{dT}{dt} \tag{1}$$

where T = T(t). Consider now steady-state conditions, in which case the storage term of Eq. (1) is zero. The temperature of the object will be $T(\infty)$ such that

$$\dot{E}_g = \bar{h}A_s\left(T(\infty) - T_\infty\right). \tag{2}$$

Substituting for $\dot{E}_g$ using Eq. (2) into Eq. (1), the differential equation is

$$\left[T(\infty) - T_\infty\right] - \left[T - T_\infty\right] = \frac{Mc}{\bar{h}A_s}\frac{dT}{dt} \qquad \text{or} \qquad \theta = -\frac{Mc}{\bar{h}A_s}\frac{d\theta}{dt} \tag{3,4}$$

with $\theta \equiv T - T(\infty)$ and noting that $d\theta = dT$. Identifying $R_t = 1/\bar{h}A_s$ and $C_t = Mc$, the differential equation is integrated with proper limits,

$$\frac{1}{R_tC_t}\int_0^t dt = -\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} \qquad \text{or} \qquad \frac{\theta}{\theta_i} = \exp\left[-\frac{t}{R_tC_t}\right] \tag{5}$$ <

where $\theta_i = \theta(0) = T_i - T(\infty)$ and $T_i$ is the initial temperature of the object.

(b) Using the information about steady-state conditions and Eq. (2), find first the thermal resistance and capacitance of the system,

$$R_t = \frac{1}{\bar{h}A_s} = \frac{T(\infty) - T_\infty}{\dot{E}_g} = \frac{(100-20)^\circ \text{C}}{60\ \text{W}} = 1.33\ \text{K/W} \qquad C_t = Mc = 0.31\ \text{kg} \times 918\ \text{J/kg}\cdot\text{K} = 285\ \text{J/K}.$$

Using Eq. (5), the temperature of the system after 5 minutes is

$$\frac{\theta(5\text{min})}{\theta_i} = \frac{T(5\text{min}) - T(\infty)}{T_i - T(\infty)} = \frac{T(5\text{min}) - 100^\circ\text{C}}{(20-100)^\circ \text{C}} = \exp\left[-\frac{5\times 60\text{s}}{1.33\ \text{K/W} \times 285\ \text{J/K}}\right] = 0.453$$

$$T(5\text{min}) = 100^\circ\text{C} + (20-100)^\circ \text{C} \times 0.453 = 63.8^\circ\text{C}$$ <

**COMMENTS:** Eq. 5.24 may be used directly for Part (b) with $a = hA_s/Mc$ and $b = \dot{E}_g/Mc$.