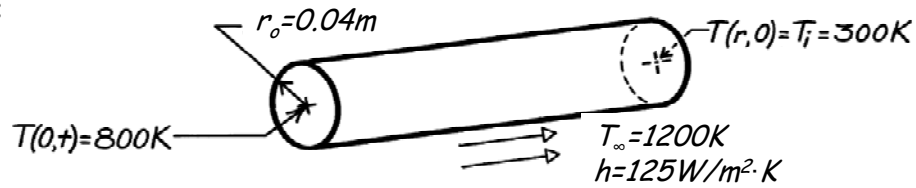


PROBLEM 5.15

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, *Table A.1* ($\bar{T} = 550 \text{ K}$): $\rho = 7832 \text{ kg/m}^3$, $k = 51.2 \text{ W/m}\cdot\text{K}$, $c = 541 \text{ J/kg}\cdot\text{K}$, $\alpha = 1.21 \times 10^{-5} \text{ m}^2/\text{s}$.

ANALYSIS: The Biot number is

$$\text{Bi} = \frac{hr_o/2}{k} = \frac{125 \text{ W/m}^2 \cdot \text{K} (0.04 \text{ m}/2)}{51.2 \text{ W/m}\cdot\text{K}} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\begin{aligned} \frac{T - T_\infty}{T_i - T_\infty} &= \exp \left[- \left(\frac{hAs}{\rho Vc} \right) t \right] = \exp \left[- \frac{4h}{\rho cD} t \right] \\ \ln \left(\frac{800 - 1200}{300 - 1200} \right) &= -0.811 = - \frac{4 \times 125 \text{ W/m}^2 \cdot \text{K}}{7832 \text{ kg/m}^3 (541 \text{ J/kg}\cdot\text{K}) 0.08 \text{ m}} t \\ t &= 550 \text{ s.} \end{aligned}$$

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.52c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1 \exp(-\zeta_1^2 \text{Fo})$$

For $\text{Bi} = hr_o/k = 0.0976$, Table 5.1 yields $\zeta_1 = 0.436$ and $C_1 = 1.024$. Hence

$$\begin{aligned} \frac{-(0.436)^2 (1.2 \times 10^{-5} \text{ m}^2/\text{s})}{(0.04 \text{ m})^2} t &= \ln(0.434) = -0.835 \\ t &= 585 \text{ s.} \end{aligned}$$

The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.