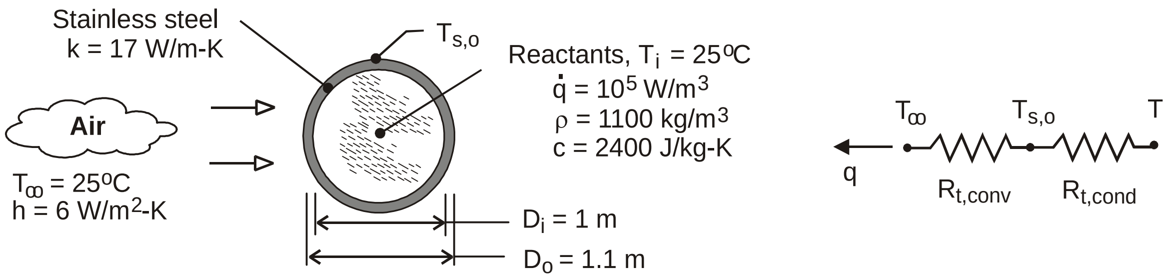
PROBLEM 5.18

KNOWN: Inner diameter and wall thickness of a spherical, stainless steel vessel. Initial temperature, density, specific heat and heat generation rate of reactants in vessel. Convection conditions at outer surface of vessel.

FIND: (a) Temperature of reactants after one hour of reaction time, (b) Effect of convection coefficient on thermal response of reactants.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature of well stirred reactants is uniform at any time and is equal to inner surface temperature of vessel ($T = T_{s,i}$), (2) Thermal capacitance of vessel may be neglected, (3) Negligible radiation exchange with surroundings, (4) Constant properties.

ANALYSIS: (a) Transient thermal conditions within the reactor may be determined from Eq. (5.25), which reduces to the following form for $T_i - T_\infty = 0$.

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

where $a = UA/\rho Vc$ and $b = \dot{E}_g / \rho Vc = \dot{q}/\rho c$. From Eq. (3.19) the product of the overall heat transfer coefficient and the surface area is $UA = (R_{cond} + R_{conv})^{-1}$, where from Eqs. (3.41) and (3.9),

$$R_{t,cond} = \frac{1}{2\pi k}\left(\frac{1}{D_i} - \frac{1}{D_o}\right) = \frac{1}{2\pi(17\,W/m\cdot K)}\left(\frac{1}{1.0m} - \frac{1}{1.1m}\right) = 8.51\times10^{-4}\,K/W$$

$$R_{t,conv} = \frac{1}{hA_o} = \frac{1}{\left(6\,W/m^2\cdot K\right)\pi\,(1.1m)^2} = 0.0438\,K/W$$

Hence, UA = 22.4 W/K. It follows that, with $V = \pi D_i^3/6$,

$$a = \frac{UA}{\rho Vc} = \frac{6(22.4\,W/K)}{1100\,kg/m^3 \times \pi\,(1m)^3\,2400\,J/kg\cdot K} = 1.620\times10^{-5}s^{-1}$$

$$b = \frac{\dot{q}}{\rho c} = \frac{10^4\,W/m^3}{1100\,kg/m^3 \times 2400\,J/kg\cdot K} = 3.788\times10^{-3}\,K/s$$

With (b/a) = 233.8°C and t = 18,000s,

$$T = 25°C + 233.8°C\left[1 - \exp\left(-1.62\times10^{-5}s^{-1}\times18,000s\right)\right] = 84.1°C \qquad <$$

Neglecting the thermal capacitance of the vessel wall, the heat rate by conduction through the wall is equal to the heat transfer by convection from the outer surface, and from the thermal circuit, we know that

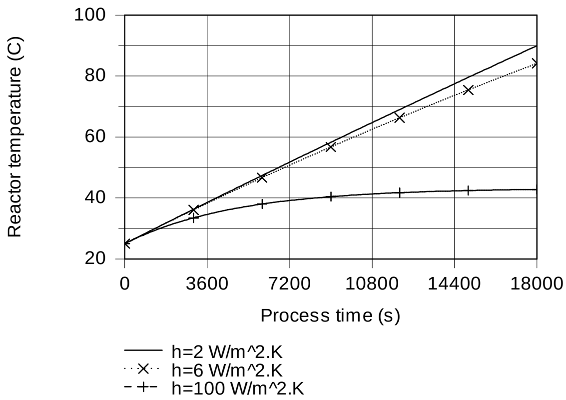
Continued …

PROBLEM 5.18 (Cont.)

$$\frac{T - T_{s,o}}{T_{s,o} - T_\infty} = \frac{R_{t,cond}}{R_{t,conv}} = \frac{8.51 \times 10^{-4}\,K/W}{0.0438\,K/W} = 0.0194$$

$$T_{s,o} = \frac{T + 0.0194\,T_\infty}{1.0194} = \frac{84.1°C + 0.0194(25°C)}{1.0194} = 83.0°C$$ <

(b) Representative low and high values of h could correspond to 2 $W/m^2 \cdot K$ and 100 $W/m^2 \cdot K$ for free and forced convection, respectively. Calculations based on Eq. (5.25) yield the following temperature histories.



Forced convection is clearly an effective means of reducing the temperature of the reactants and accelerating the approach to steady-state conditions.

COMMENTS: The validity of neglecting thermal energy storage effects for the vessel may be assessed by contrasting its thermal capacitance with that of the reactants. Selecting values of ρ = 8000 kg/m^3 and c = 475 $J/kg \cdot K$ for stainless steel from Table A-1, the thermal capacitance of the vessel is $C_{t,v} = (\rho Vc)_{st} = 6.57 \times 10^5$ J/K, where $V = (\pi/6)\left(D_o^3 - D_i^3\right)$. With $C_{t,r} = (\rho Vc)_r = 2.64 \times 10^6$ J/K for the reactants, $C_{t,r}/C_{t,v} \approx 4$. Hence, the capacitance of the vessel is not negligible and should be considered in a more refined analysis of the problem.