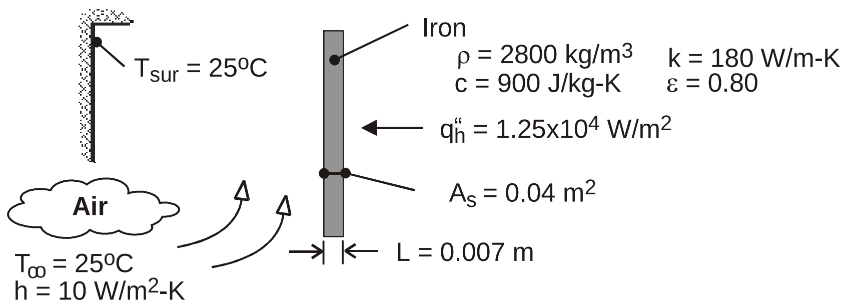
PROBLEM 5.11

KNOWN: Thickness, surface area, and properties of iron base plate. Heat flux at inner surface. Temperature of surroundings. Temperature and convection coefficient of air at outer surface.

FIND: Time required for plate to reach a temperature of 135°C. Operating efficiency of iron.

SCHEMATIC:



ASSUMPTIONS: (1) Radiation exchange is between a small surface and large surroundings, (2) Convection coefficient is independent of time, (3) Constant properties, (4) Iron is initially at room temperature ($T_i = T_\infty$).

ANALYSIS: Biot numbers may be based on convection heat transfer and/or the maximum heat transfer by radiation, which would occur when the plate reaches the desired temperature (T = 135°C). From Eq. (1.9) the corresponding radiation transfer coefficient is $h_r = \varepsilon\sigma(T + T_{sur})\left(T^2 + T_{sur}^2\right) = 0.8 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\ (408 + 298)\ \mathrm{K}\ (408^2 + 298^2)\ \mathrm{K^2} = 8.2\ \mathrm{W/m^2 \cdot K}$. Hence,

$$\mathrm{Bi} = \frac{hL}{k} = \frac{10\,\mathrm{W/m^2 \cdot K}\,(0.007\mathrm{m})}{180\,\mathrm{W/m \cdot K}} = 3.9 \times 10^{-4}$$

$$\mathrm{Bi}_r = \frac{h_r L}{k} = \frac{8.2\,\mathrm{W/m^2 \cdot K}\,(0.007\mathrm{m})}{180\,\mathrm{W/m \cdot K}} = 3.2 \times 10^{-4}$$

With convection and radiation considered independently or collectively, Bi, Bi_r, Bi + Bi_r << 1 and the lumped capacitance analysis may be used.

The energy balance, Eq. (5.15), associated with Figure 5.5 may be applied to this problem. With $\dot{E}g = 0$, the integral form of the equation is

$$T - T_i = \frac{A_s}{\rho V c}\int_0^t \left[q''_h - h\left(T - T_\infty\right) - \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]dt$$

Integrating numerically, we obtain, for T = 135°C,

$$t = 168\ \mathrm{s}$$ <

COMMENTS: Note that, if heat transfer is by natural convection, h, like h_r, will vary during the process from a value of 0 at t = 0 to a maximum at t = 168s.