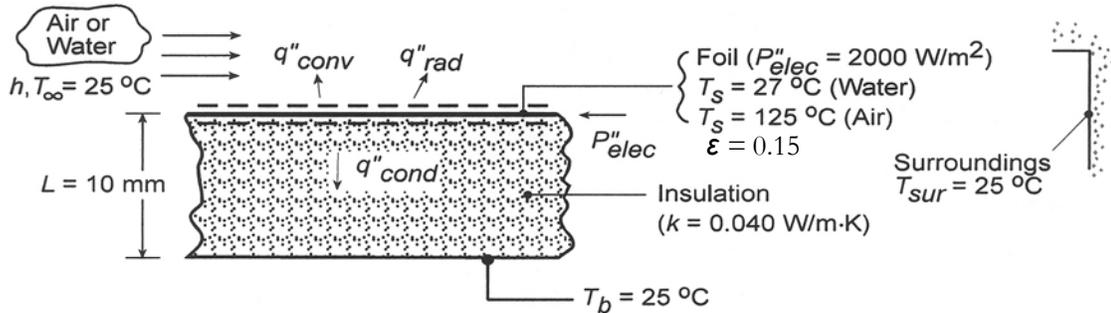


### PROBLEM 3.9

**KNOWN:** Design and operating conditions of a heat flux gage.

**FIND:** (a) Convection coefficient for water flow ( $T_s = 27^\circ\text{C}$ ) and error associated with neglecting conduction in the insulation, (b) Convection coefficient for air flow ( $T_s = 125^\circ\text{C}$ ) and error associated with neglecting conduction and radiation, (c) Effect of convection coefficient on error associated with neglecting conduction for  $T_s = 27^\circ\text{C}$ .

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state, (2) One-dimensional conduction, (3) Constant  $k$ .

**ANALYSIS:** (a) The electric power dissipation is balanced by convection to the water and conduction through the insulation. An energy balance applied to a control surface about the foil therefore yields

$$P''_{elec} = q''_{conv} + q''_{cond} = h(T_s - T_\infty) + k(T_s - T_b)/L$$

Hence,

$$h = \frac{P''_{elec} - k(T_s - T_b)/L}{T_s - T_\infty} = \frac{2000 \text{ W/m}^2 - 0.04 \text{ W/m} \cdot \text{K} (2 \text{ K})/0.01 \text{ m}}{2 \text{ K}}$$

$$h = \frac{(2000 - 8) \text{ W/m}^2}{2 \text{ K}} = 996 \text{ W/m}^2 \cdot \text{K} \quad <$$

If conduction is neglected, a value of  $h = 1000 \text{ W/m}^2 \cdot \text{K}$  is obtained, with an attendant error of  $(1000 - 996)/996 = 0.40\%$

(b) In air, energy may also be transferred from the foil surface by radiation, and the energy balance yields

$$P''_{elec} = q''_{conv} + q''_{rad} + q''_{cond} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{sur}^4) + k(T_s - T_b)/L$$

Hence,

$$h = \frac{P''_{elec} - \varepsilon\sigma(T_s^4 - T_{sur}^4) - k(T_s - T_\infty)/L}{T_s - T_\infty}$$

$$= \frac{2000 \text{ W/m}^2 - 0.15 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (398^4 - 298^4) - 0.04 \text{ W/m} \cdot \text{K} (100 \text{ K})/0.01 \text{ m}}{100 \text{ K}}$$

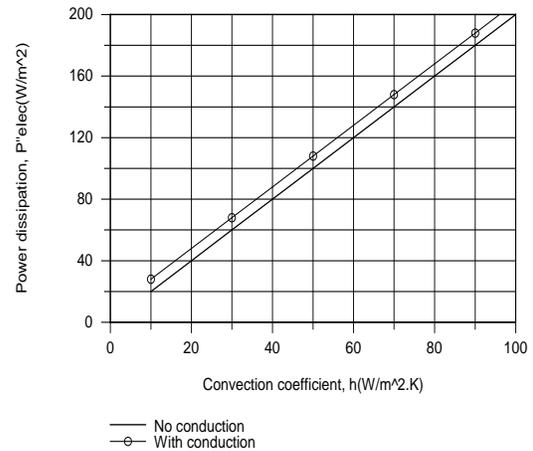
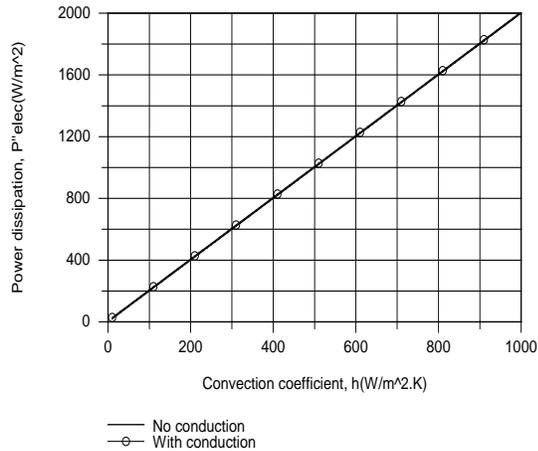
$$= \frac{(2000 - 146 - 400) \text{ W/m}^2}{100 \text{ K}} = 14.5 \text{ W/m}^2 \cdot \text{K} \quad <$$

Continued...

### PROBLEM 3.9 (Cont.)

If conduction, radiation, or conduction and radiation are neglected, the corresponding values of  $h$  and the percentage errors are  $18.5 \text{ W/m}^2\cdot\text{K}$  (27.6%),  $16 \text{ W/m}^2\cdot\text{K}$  (10.3%), and  $20 \text{ W/m}^2\cdot\text{K}$  (37.9%).

(c) For a fixed value of  $T_s = 27^\circ\text{C}$ , the conduction loss remains at  $q''_{\text{cond}} = 8 \text{ W/m}^2$ , which is also the fixed difference between  $P''_{\text{elec}}$  and  $q''_{\text{conv}}$ . Although this difference is not clearly shown in the plot for  $10 \leq h \leq 1000 \text{ W/m}^2\cdot\text{K}$ , it is revealed in the subplot for  $10 \leq 100 \text{ W/m}^2\cdot\text{K}$ .



Errors associated with neglecting conduction decrease with increasing  $h$  from values which are significant for small  $h$  ( $h < 100 \text{ W/m}^2\cdot\text{K}$ ) to values which are negligible for large  $h$ .

**COMMENTS:** In liquids (large  $h$ ), it is an excellent approximation to neglect conduction and assume that all of the dissipated power is transferred to the fluid.