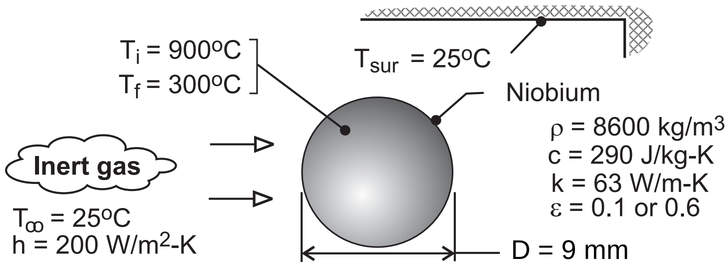
PROBLEM 5.24

KNOWN: Initial and final temperatures of a niobium sphere. Diameter and properties of the sphere. Temperature of surroundings and/or gas flow, and convection coefficient associated with the flow.

FIND: (a) Time required to cool the sphere exclusively by radiation, (b) Time required to cool the sphere exclusively by convection, (c) Combined effects of radiation and convection.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform temperature at any time, (2) Negligible effect of holding mechanism on heat transfer, (3) Constant properties, (4) Radiation exchange is between a small surface and large surroundings.

ANALYSIS: (a) If cooling is exclusively by radiation, the required time is determined from Eq. (5.18). With $V = \pi D^3/6$, $A_{s,r} = \pi D^2$, and $\varepsilon = 0.1$,

$$t = \frac{8600\ \mathrm{kg/m^3}\left(290\,\mathrm{J/kg\cdot K}\right)0.009\mathrm{m}}{24\left(0.1\right)5.67\times10^{-8}\mathrm{W/m^2\cdot K^4}\left(298\mathrm{K}\right)^3}\left\{\ln\left|\frac{298+573}{298-573}\right| - \ln\left|\frac{298+1173}{298-1173}\right|\right.$$

$$\left. +2\left[\tan^{-1}\left(\frac{573}{298}\right) - \tan^{-1}\left(\frac{1173}{298}\right)\right]\right\}$$

$$t = 6233\mathrm{s}\left\{1.153 - 0.519 + 2\left(1.091 - 1.322\right)\right\} = 1061\mathrm{s} \qquad \left(\varepsilon = 0.1\right)$$ <

If $\varepsilon = 0.6$, cooling is six times faster, in which case,

$$t = 177\mathrm{s} \qquad \left(\varepsilon = 0.6\right)$$ <

(b) If cooling is exclusively by convection, Eq. (5.5) yields

$$t = \frac{\rho c D}{6h}\ln\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right) = \frac{8600\ \mathrm{kg/m^3}\left(290\,\mathrm{J/kg\cdot K}\right)0.009\mathrm{m}}{1200\,\mathrm{W/m^2\cdot K}}\ln\left(\frac{875}{275}\right)$$

$$t = 21.6\mathrm{s}$$ <

(c) With both radiation and convection, the temperature history may be obtained from Eq. (5.15).

$$\rho\left(\pi D^3/6\right)c\frac{dT}{dt} = -\pi D^2\left[h\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Integrating numerically from $T_i = 1173$ K at $t = 0$ to $T = 573$K, we obtain

$$t = 18.9\mathrm{s}$$ <

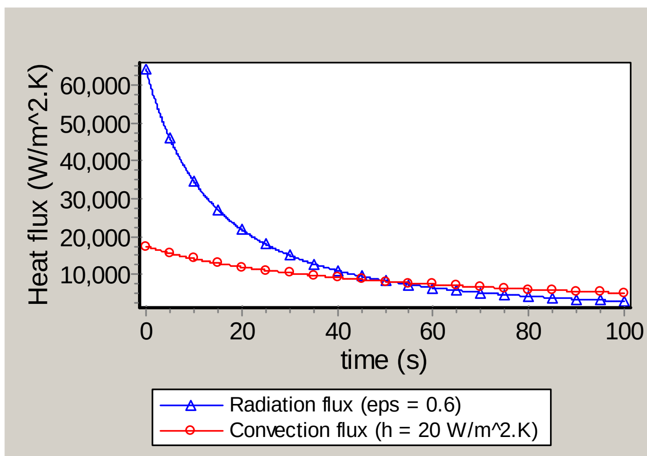
Continued …

PROBLEM 5.24 (Cont.)

Cooling times corresponding to representative changes in ε and h are tabulated as follows

$h(W/m^2\cdot K)$	200	200	20	500
ε	0.6	1.0	0.6	0.6
t(s)	18.9	17.5	92.4	8.2

For values of h representative of forced convection, the influence of radiation is secondary, even for a maximum possible emissivity of 1.0. Hence, to accelerate cooling, it is necessary to increase h. However, if cooling is by natural convection, radiation is significant. For a representative natural convection coefficient of $h = 20\ W/m^2\cdot K$, the radiation flux exceeds the convection flux at the surface of the sphere during early to intermediate stages of the transient.



COMMENTS: (1) Even for h as large as $500\ W/m^2\cdot K$, $Bi = h\,(D/6)/k = 500\ W/m^2\cdot K\ (0.009m/6)/63\ W/m\cdot K = 0.012 < 0.1$ and the lumped capacitance model is appropriate. (2) The largest value of h_r corresponds to $T_i = 1173$ K, and for $\varepsilon = 0.6$ Eq. (1.9) yields $h_r = 0.6 \times 5.67 \times 10^{-8}\ W/m^2\cdot K^4\ (1173 + 298)K\ (1173^2 + 298^2)K^2 = 73.3\ W/m^2\cdot K$.