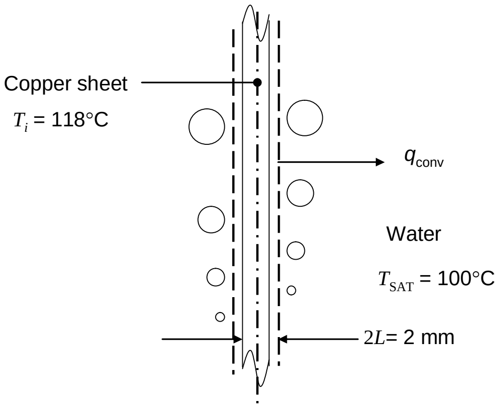
PROBLEM 5.14

KNOWN: Thickness and initial temperature of copper sheet. Dependence of the convection heat transfer coefficient on sheet temperature.

FIND: Time required to reach sheet temperature of $\overline{T}$ = 102°C.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties. (2) Lumped capacitance behavior.

PROPERTIES: Table A.1, copper (T = 383 K): ρ = 8933 kg/m^3, c = 394 J/kg·K, and k = 394 W/m·K.

ANALYSIS: Since h = 1010 W/m^2·K^3$(T - T_{sat})^2$ the values of C and n in Equation 5.26 are 1010 W/m^2·K^3 and 2, respectively. Equation 5.27 becomes

$$\frac{dT}{dt} = -\frac{CA_{s,c}(T - T_{sat})^3}{\rho Vc} = -\frac{C(T - T_{sat})^3}{L\rho c}$$

or

$$\int_{T_i}^{T} (T - T_{sat})^{-3} dT = -\int_0^t \frac{C}{L\rho c} dt$$

so that

$$\frac{L\rho c}{C}\left[\frac{\left(T - T_{sat}\right)^{-2}}{2} - \frac{\left(T_i - T_{sat}\right)^{-2}}{2}\right] = t$$

Substituting values,

$$t = \frac{1\times10^{-3}\ \text{m}\times8933\ \text{kg/m}^3\times394\ \text{J/kg}\cdot\text{K}}{1010\ \text{W/m}^2\cdot\text{K}^3}\left[\frac{\left(T - 100°\text{C}\right)^{-2}}{2} - \frac{\left(118°\text{C} - 100°\text{C}\right)^{-2}}{2}\right] \quad (1)$$

For T = 102°C, t = 0.43 s <

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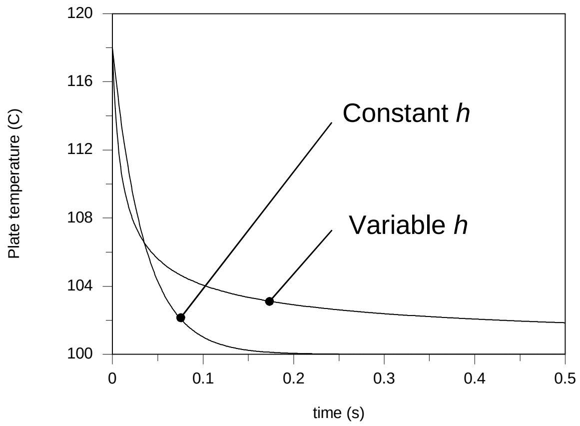
PROBLEM 5.14 (Cont.)

The heat transfer coefficient at $\overline{T}$ = 110°C is $\overline{h}$ = 1010 W/m^2·K^3×(10 K)^2 = 101,000 W/m^2·K. Hence, for the case where the heat transfer coefficient is constant Equation 5.6 becomes

$$t = \frac{\rho Lc}{\overline{h}} \ln\left[\frac{T_i - T_{\text{sat}}}{T - T_{\text{sat}}}\right] = \frac{8933 \text{ kg/m}^3 \times 1\times 10^{-3} \text{ m} \times 394 \text{ J/kg}\cdot\text{K}}{101{,}000 \text{ W/m}^2\cdot\text{K}} \ln\left[\frac{118°\text{C} - 100°\text{C}}{T - 100°\text{C}}\right] \quad (2)$$

Equations (1) and (2) may be solved for time-dependence of the plate temperature to yield



The convection heat transfer coefficient is initially relatively high and decays as the temperature difference between the plate and the water decreases. If the convection heat transfer coefficient is evaluated at the average plate temperature, the heat transfer coefficient is initially under-predicted, leading to a slower plate cooling rate at early times. However, the convection coefficient is over-predicted at later times, leading to an unrealistic high cooling rate as evident in the graph.

COMMENTS: (1) The time could also be calculated by solving Equation 5.28.

(2) The Biot number based upon the average heat transfer coefficient is

$Bi = \frac{\overline{h}L}{k} = \frac{101{,}000 \text{W/m}^2\cdot\text{K} \times 1\times 10^{-3} \text{ m}}{394 \text{ W/m}\cdot\text{K}} = 0.25$. The lumped capacitance approximation is not valid at early times. However, the trends evident in the comparison of the variable versus constant heat transfer coefficients would also occur if spatial temperature gradients were accounted for.