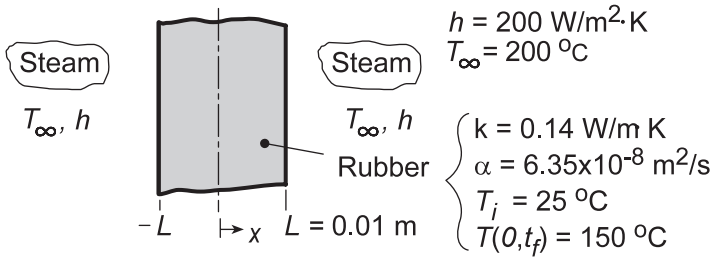
PROBLEM 5.58

KNOWN: Thickness and properties of rubber tire. Convection heating conditions. Initial and final midplane temperature.

FIND: (a) Time to reach final midplane temperature. (b) Effect of accelerated heating.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in a plane wall, (2) Constant properties, (3) Negligible radiation.

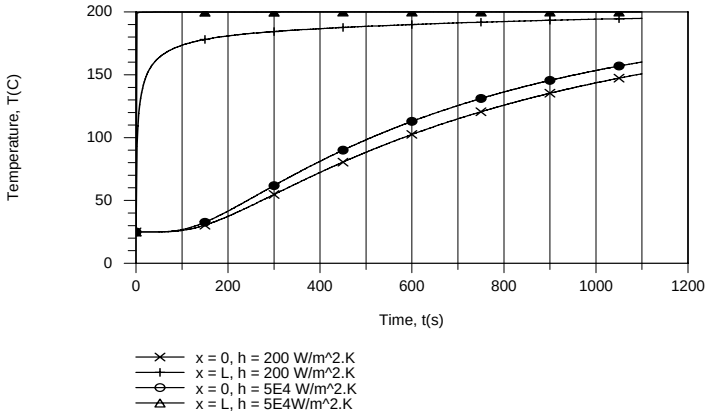
ANALYSIS: (a) With Bi = hL/k = 200 W/m^2·K(0.01 m)/0.14 W/m·K = 14.3, the lumped capacitance method is clearly inappropriate. Assuming Fo > 0.2, Eq. (5.44) may be used with $C_1 = 1.265$ and $\zeta_1 \approx 1.458$ rad from Table 5.1 to obtain

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right) = 1.265 \exp\left(-2.126\, Fo\right)$$

With $\theta_o^* = \left(T_o - T_\infty\right)/\left(T_i - T_\infty\right)$ = (-50)/(-175) = 0.286, $Fo = -\ln\left(0.286/1.265\right)/2.126 = 0.70 = \alpha t_f / L^2$

$$t_f = \frac{0.7\left(0.01\,m\right)^2}{6.35\times10^{-8}\,m^2/s} = 1100\,s$$ <

(b) The desired temperature histories were generated using the IHT *Transient Conduction Model* for a *Plane Wall*, with h = 5 × 10^4 W/m^2·K used to approximate imposition of a surface temperature of 200°C.



The fact that imposition of a constant surface temperature ($h \to \infty$) does not significantly accelerate the heating process should not be surprising. For h = 200 W/m^2·K, the Biot number is already quite large (Bi = 14.3), and limits to the heating rate are principally due to conduction in the rubber and not to convection at the surface. Any increase in h only serves to reduce what is already a small component of the total thermal resistance.

COMMENTS: The heating rate could be accelerated by increasing the steam temperature, but an upper limit would be associated with avoiding thermal damage to the rubber.