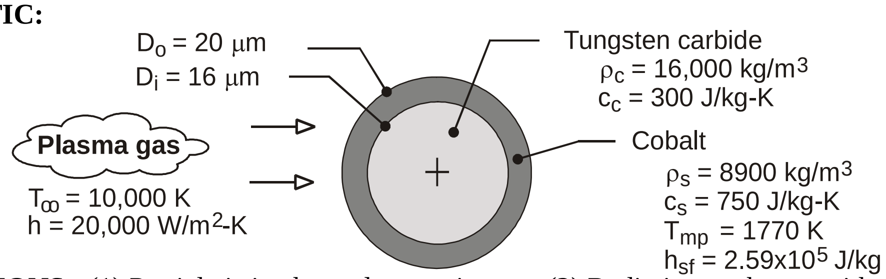
PROBLEM 5.37

KNOWN: Diameters, initial temperature and thermophysical properties of WC and Co in composite particle. Convection coefficient and freestream temperature of plasma gas. Melting point and latent heat of fusion of Co.

FIND: Times required to reach melting and to achieve complete melting of Co.

SCHEMATIC:



ASSUMPTIONS: (1) Particle is isothermal at any instant, (2) Radiation exchange with surroundings is negligible, (3) Negligible contact resistance at interface between WC and Co, (4) Constant properties.

ANALYSIS: From Eq. (5.5), the time required to reach the melting point is

$$t_1 = \frac{(\rho V c)_{tot}}{h \pi D_o^2} \ln \frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

where the total heat capacity of the composite particle is

$$(\rho Vc)_{tot} = (\rho Vc)_c + (\rho Vc)_s = 16,000\,\text{kg/m}^3 \left[\pi \left(1.6\times10^{-5}\,\text{m}\right)^3 / 6\right] 300\,\text{J/kg}\cdot\text{K}$$

$$+8900\,\text{kg/m}^3 \left\{\pi/6\left[\left(2.0\times10^{-5}\,\text{m}\right)^3 - \left(1.6\times10^{-5}\,\text{m}\right)^3\right]\right\} 750\,\text{J/kg}\cdot\text{K}$$

$$= \left(1.03\times10^{-8} + 1.36\times10^{-8}\right)\text{J/K} = 2.39\times10^{-8}\,\text{J/K}$$

$$t_1 = \frac{2.39\times10^{-8}\,\text{J/K}}{\left(20,000\,\text{W/m}^2\cdot\text{K}\right)\pi\left(2.0\times10^{-5}\,\text{m}\right)^2} \ln \frac{(300 - 10,000)\,\text{K}}{(1770 - 10,000)\,\text{K}} = 1.56\times10^{-4}\,\text{s} \qquad <$$

The time required to melt the Co may be obtained by applying the first law, Eq. (1.12b) to a control surface about the particle. It follows that

$$E_{in} = h\pi D_o^2 \left(T_\infty - T_{mp}\right) t_2 = \Delta E_{st} = \rho_s (\pi/6)\left(D_o^3 - D_i^3\right) h_{sf}$$

$$t_2 = \frac{8900\,\text{kg/m}^3 (\pi/6)\left[\left(2\times10^{-5}\,\text{m}\right)^3 - \left(1.6\times10^{-5}\,\text{m}\right)^3\right] 2.59\times10^5\,\text{J/kg}}{\left(20,000\,\text{W/m}^2\cdot\text{K}\right)\pi\left(2\times10^{-5}\,\text{m}\right)^2 (10,000 - 1770)\,\text{K}} = 2.28\times10^{-5}\,\text{s} \qquad <$$

COMMENTS: (1) The largest value of the radiation coefficient corresponds to $h_r = \varepsilon\sigma (T_{mp} + T_{sur})\left(T_{mp}^2 + T_{sur}^2\right)$. For the maximum possible value of ε = 1 and T_{sur} = 300K, h_r = 378 W/m$^2\cdot$K << h = 20,000 W/m$^2\cdot$K. Hence, the assumption of negligible radiation exchange is excellent. (2) Despite the large value of h, the small values of D_o and D_i and the large thermal conductivities (~ 40 W/m·K and 70 W/m·K for WC and Co, respectively) render the lumped capacitance approximation a good one. (3) A detailed treatment of plasma heating of a composite powder particle is provided by Demetriou, Lavine and Ghoniem (Proc. 5th ASME/JSME Joint Thermal Engineering Conf., March, 1999).