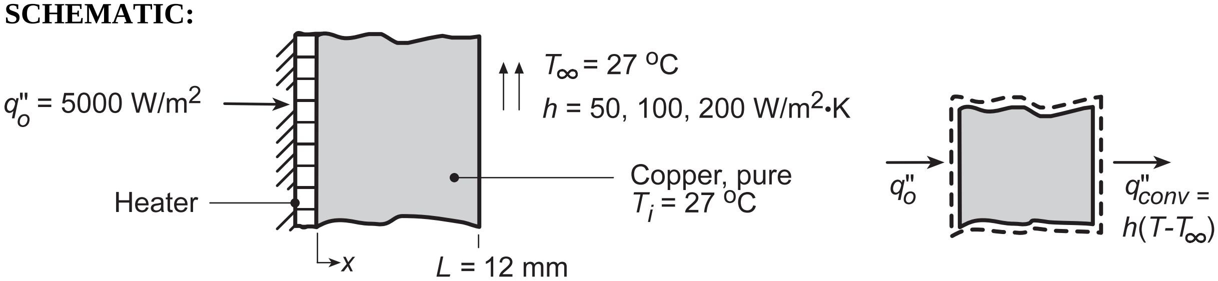
PROBLEM 5.30

KNOWN: Electrical heater attached to backside of plate while front is exposed to a convection process (T_∞, h); initially plate is at uniform temperature T_∞ before heater power is switched on.

FIND: (a) Expression for temperature of plate as a function of time assuming plate is spacewise isothermal, (b) Approximate time to reach steady-state and T(∞) for prescribed T_∞, h and q''_o when wall material is pure copper, (c) Effect of h on thermal response.

SCHEMATIC:



ASSUMPTIONS: (1) Plate behaves as lumped capacitance, (2) Negligible loss out backside of heater, (3) Negligible radiation, (4) Constant properties.

PROPERTIES: *Table A-1*, Copper, pure (350 K): k = 397 W/m·K, c_p = 385 J/kg·K, ρ = 8933 kg/m³.

ANALYSIS: (a) Following the analysis of Section 5.3, the energy conservation requirement for the system is $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$ or $q''_o - h(T - T_\infty) = \rho L c_p \, dT/dt$. Rearranging, and with R''_t = 1/h and C''_t = $\rho L c_p$,

$$T - T_\infty - q''_o/h = -R''_t \cdot C''_t \, dT/dt \tag{1}$$

Defining $\theta(t) \equiv T - T_\infty - q''_o/h$ with dθ = dT, the differential equation is

$$\theta = -R''_t C''_t \frac{d\theta}{dt}. \tag{2}$$

Separating variables and integrating,

$$\int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t \frac{dt}{R''_t C''_t}$$

it follows that

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{R''_t C''_t}\right) \tag{3}$$ <

where $\theta_i = \theta(0) = T_i - T_\infty - (q''_o/h)$ (4)

(b) For h = 50 W/m² ·K, the steady-state temperature can be determined from Eq. (3) with t → ∞; that is,

$$\theta(\infty) = 0 = T(\infty) - T_\infty - q''_o/h \qquad \text{or} \qquad T(\infty) = T_\infty + q''_o/h\,,$$

giving T(∞) = 27°C + 5000 W/m² /50 W/m²·K = 127°C. To estimate the time to reach steady-state, first determine the thermal time constant of the system,

$$\tau_t = R''_t C''_t = \left(\frac{1}{h}\right)(\rho c_p L) = \left(\frac{1}{50\,\mathrm{W/m^2 \cdot K}}\right)\left(8933\,\mathrm{kg/m^3} \times 385\,\mathrm{J/kg \cdot K} \times 12 \times 10^{-3}\,\mathrm{m}\right) = 825\,\mathrm{s}$$

Continued...

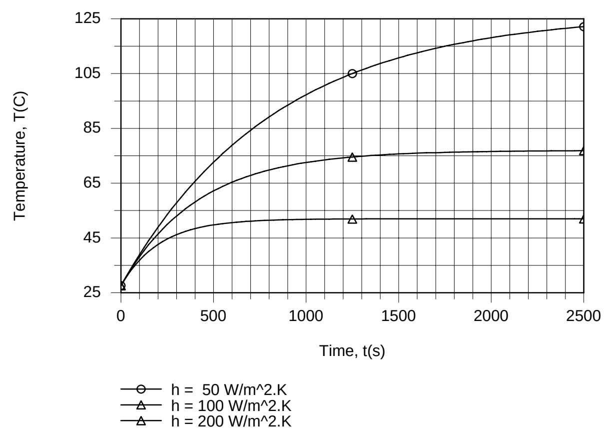
PROBLEM 5.30 (Cont.)

When $t = 3\tau_t = 3\times825\text{s} = 2475\text{s}$, Eqs. (3) and (4) yield

$$\theta(3\tau_t) = T(3\tau_t) - 27^\circ\text{C} - \frac{5000\,\text{W}/\text{m}^2}{50\,\text{W}/\text{m}^2\cdot\text{K}} = e^{-3}\left[27^\circ\text{C} - 27^\circ\text{C} - \frac{5000\,\text{W}/\text{m}^2}{50\,\text{W}/\text{m}^2\cdot\text{K}}\right]$$

$T(3\tau_t) = 122°\text{C}$ <

(c) As shown by the following graphical results, which were generated using the IHT *Lumped Capacitance Model*, the steady-state temperature and the time to reach steady-state both decrease with increasing h.



COMMENTS: Note that, even for h = 200 $\text{W/m}^2\cdot\text{K}$, $Bi = hL/k << 0.1$ and assumption (1) is reasonable.