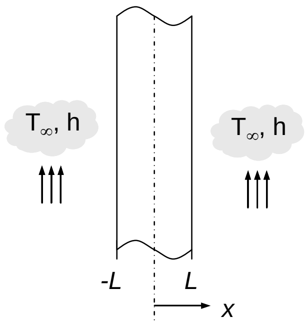
PROBLEM 5.47

KNOWN: One-dimensional convective heating of a plane slab with $Bi = 1$ for a dimensionless time of Fo_1.

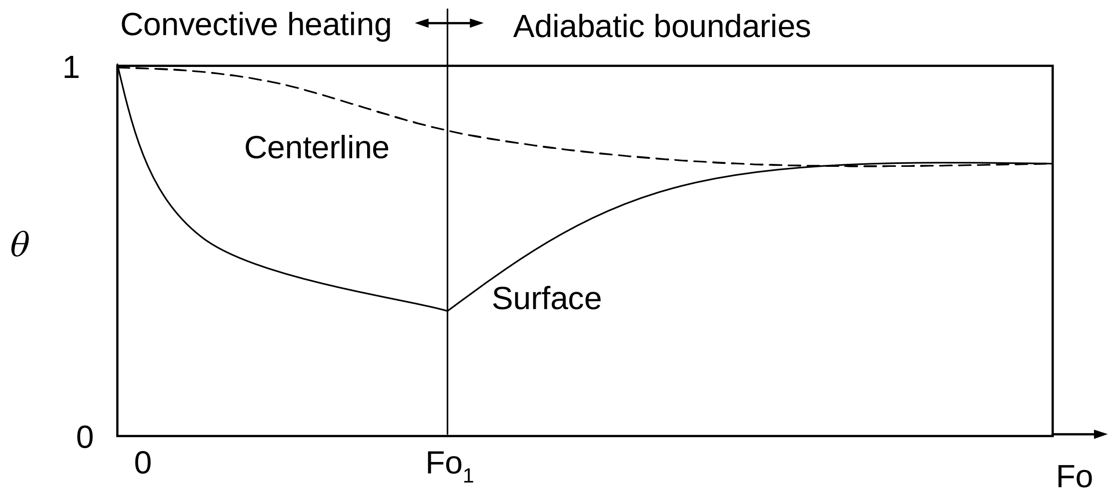
FIND: (a) Sketch of the dimensionless midplane and surface temperatures of the slab as a function of dimensionless time over the range $0 < Fo_1 < Fo < \infty$. Relative value of Fo_2, needed to achieve a steady-state midplane temperature equal to the midplane temperature at Fo_1. (b) Analytical expression for, and value of $\Delta Fo = Fo_2 - Fo_1$ for $Bi = 1$, $Fo_1 > 0.2$, $Fo_2 > 0.2$. (c) Value of ΔFo for Bi = 0.01, 0.1, 10, 100 and ∞.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Approximate, one-term solutions are valid.

ANALYSIS: (a) A sketch of the dimensionless midplane and surface temperatures is shown below. Note that, at Fo_1, the surface of the slab will be warm (small θ) relative to the midplane since temperature gradients within the slab are significant ($Bi = 1$). At the curtailment of heating (Fo_1), the surface temperature cools rapidly while warm temperatures continue to propagate toward the midplane, slowly heating the midplane until a steady-state, isothermal condition is eventually reached.



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Based on the sketch above, one could achieve a steady-state midplane temperature equal to the midplane temperature at Fo_1 by reducing the duration of convective heating to Fo_2, as shown in the sketch below.

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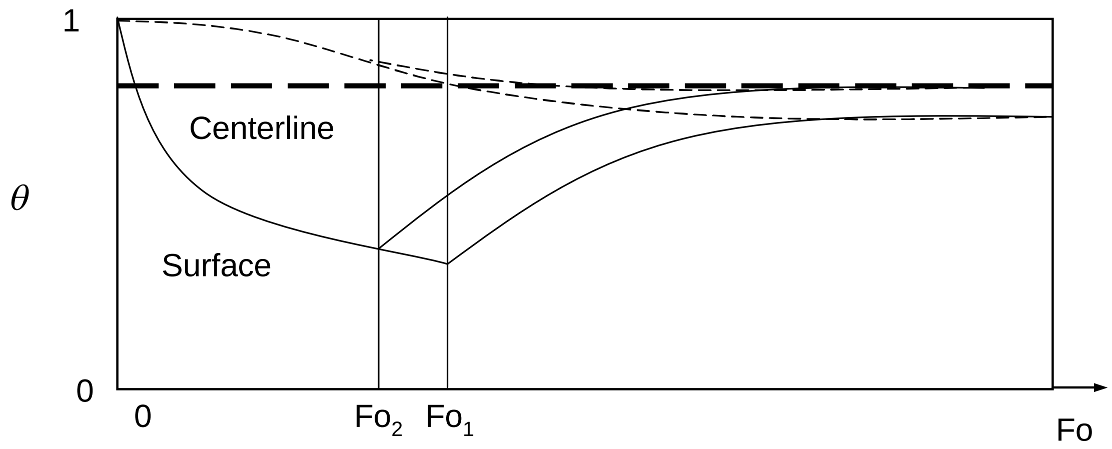

Hence, $Fo_2 < Fo_1$. <

(b) Using the approximate solutions of Section 5.5.2, and noting that the steady-state temperature of the slab is uniform and related to the energy transferred to the slab,

$$\theta_o^*(Fo_1) = 1 - \frac{Q}{Q_o}(Fo_2)$$

or,

$$1 - \theta_o^*(Fo_1) = \frac{Q}{Q_o}(Fo_1 + \Delta Fo_1) \qquad (1)$$

Substituting Eqs. 5.44 and 5.49 into Eq. (1) yields

$$1 - C_1 \exp(-\zeta_1^2 Fo_1) = 1 - \frac{\sin\zeta_1}{\zeta_1} C_1 \exp\left[-\zeta_1^2 \left(Fo_1 + \Delta Fo\right)\right]$$

which may be simplified to

$$\Delta Fo = -\frac{1}{\zeta_1^2} \ln\left(\frac{\zeta_1}{\sin\zeta_1}\right)$$ <

From Table 5.1, $\zeta_1 = 0.8603$ rad at $Bi = 1$. Hence,

$$\Delta Fo = -\frac{1}{0.8603^2} \ln\left(\frac{0.8603}{\sin 0.8603}\right) = -0.1709$$ <

(c) The expression for ΔFo may be evaluated for a range of Bi, resulting in the following.

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PROBLEM 5.47 (Cont.)

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Bi	ζ_1	ΔFo
0.01	0.0998	-0.1667
0.1	0.3111	-0.1672
1	0.8603	-0.1709
10	1.4289	-0.1847
100	1.5552	-0.1826
∞	1.5708	-0.1830

COMMENTS: (1) Note that the dimensionless temperature, $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$, is defined in a manner such that for slab heating, increases in actual temperature correspond to decreases in the dimensionless temperature. (2) The dimensionless time lag, ΔFo, is weakly-dependent on the value of the Biot number and is independent of the heating time. Hence, a general rule-of-thumb is that a time lag of $\Delta Fo \approx$ - 0.17 should be specified in order to achieve an ultimate midplane temperature equal to that predicted at Fo_1 for convective heating or cooling. (3) For applications such as materials or food processing, where a certain minimum midplane temperature is desired, assuming that Fo_1 (as determined by Eq. 5.44 is the appropriate processing or cooking time can result in significant over-heating of the material or food, especially at small Fourier numbers. (4) Significant energy and time savings can be realized by reducing the processing or cooking time from Fo_1 to Fo_2.