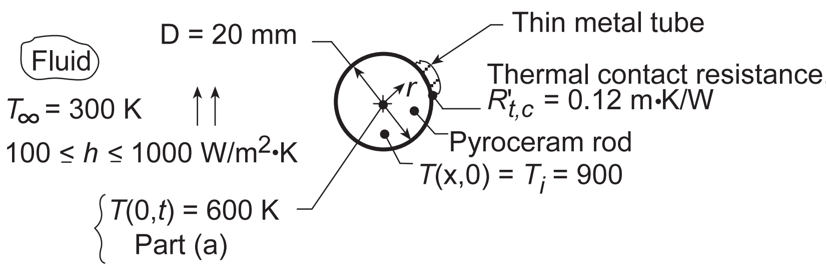
PROBLEM 5.63

KNOWN: Long pyroceram rod, initially at a uniform temperature of 900 K, and clad with a thin metallic tube giving rise to a thermal contact resistance, is suddenly cooled by convection.

FIND: (a) Time required for rod centerline to reach 600 K, (b) Effect of convection coefficient on cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Thermal resistance and capacitance of metal tube are negligible, (3) Constant properties, (4) Fo ≥ 0.2.

PROPERTIES: *Table A-2*, Pyroceram ($\bar{T}$ = (600 + 900)K/2 = 750 K): ρ = 2600 kg/m^3, c = 1100 J/kg·K, k = 3.13 W/m·K.

ANALYSIS: (a) The thermal contact and convection resistances can be combined to give an overall heat transfer coefficient. Note that $R'_{t,c}$ [m·K/W] is expressed per unit length for the outer surface. Hence, for h = 100 W/m^2·K,

$$U = \frac{1}{1/h + R'_{t,c}(\pi D)} = \frac{1}{1/100\,W/m^2\cdot K + 0.12\,m\cdot K/W(\pi \times 0.020\,m)} = 57.0\,W/m^2\cdot K\,.$$

Using the approximate series solution, Eq. 5.52c, the Fourier number can be expressed as

$$Fo = -\left(1/\zeta_1^2\right)\ln\left(\theta_o^*/C_1\right).$$

From Table 5.1, find ζ_1 = 0.5884 rad and C_1 = 1.0441 for

$$Bi = Ur_o/k = 57.0\,W/m^2\cdot K\left(0.020\,m/2\right)/3.13\,W/m\cdot K = 0.182\,.$$

The dimensionless temperature is

$$\theta_o^*(0, Fo) = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = \frac{(600-300)K}{(900-300)K} = 0.5.$$

Substituting numerical values to find Fo and then the time t,

$$Fo = \frac{-1}{(0.5884)^2}\ln\frac{0.5}{1.0441} = 2.127$$

$$t = Fo\frac{r_o^2}{\alpha} = Fo\cdot r_o^2\,\frac{\rho c}{k}$$

$$t = 2.127\left(0.020\,m/2\right)^2 2600\,kg/m^3 \times 1100\,J/kg\cdot K/3.13\,W/m\cdot K = 194s\,.$$ <

(b) The following temperature histories were generated using the IHT *Transient conduction Model* for a *Cylinder*.

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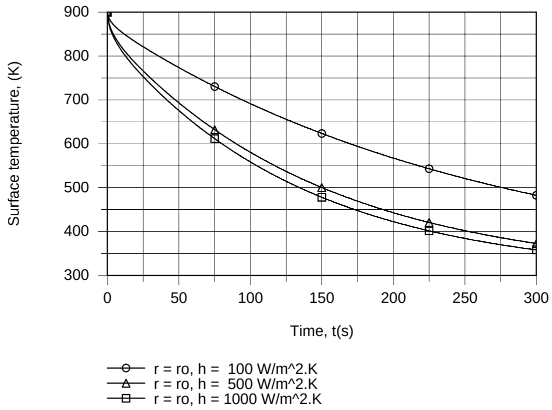
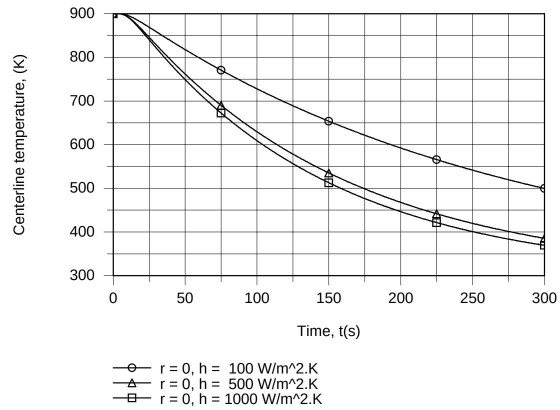
PROBLEM 5.63 (Cont.)





While enhanced cooling is achieved by increasing h from 100 to 500 $W/m^2 \cdot K$, there is little benefit associated with increasing h from 500 to 1000 $W/m^2 \cdot K$. The reason is that for h much above 500 $W/m^2 \cdot K$, the contact resistance becomes the dominant contribution to the total resistance between the fluid and the rod, rendering the effect of further reductions in the convection resistance negligible. Note that, for h = 100, 500 and 1000 $W/m^2 \cdot K$, the corresponding values of U are 57.0, 104.8 and 117.1 $W/m^2 \cdot K$, respectively.

COMMENTS: For Part (a), note that, since Fo = 2.127 > 0.2, Assumption (4) is satisfied.