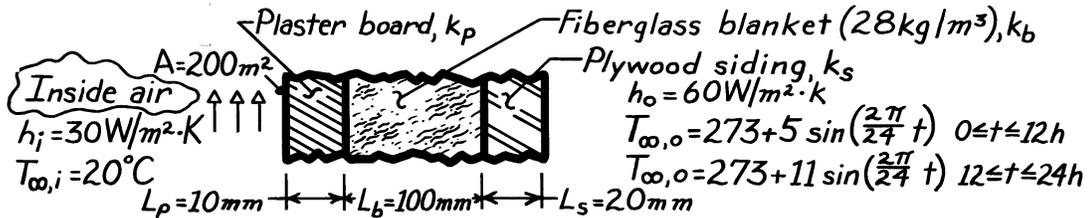


PROBLEM 3.14

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: Daily heat loss for prescribed diurnal variation in ambient air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction (negligible change in wall thermal energy storage over 24h period), (2) Negligible contact resistance.

PROPERTIES: Table A-3, $T \approx 300$ K: Fiberglass blanket (28 kg/m^3), $k_b = 0.038 \text{ W/m}\cdot\text{K}$; Plywood, $k_s = 0.12 \text{ W/m}\cdot\text{K}$; Plasterboard, $k_p = 0.17 \text{ W/m}\cdot\text{K}$.

ANALYSIS: The heat loss may be approximated as $Q = \int_0^{24\text{h}} \frac{T_{\infty,i} - T_{\infty,o}}{R_{\text{tot}}} dt$ where

$$R_{\text{tot}} = \frac{1}{A} \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_b}{k_b} + \frac{L_s}{k_s} + \frac{1}{h_o} \right]$$

$$R_{\text{tot}} = \frac{1}{200\text{m}^2} \left[\frac{1}{30 \text{ W/m}^2 \cdot \text{K}} + \frac{0.01\text{m}}{0.17 \text{ W/m}\cdot\text{K}} + \frac{0.1\text{m}}{0.038 \text{ W/m}\cdot\text{K}} + \frac{0.02\text{m}}{0.12 \text{ W/m}\cdot\text{K}} + \frac{1}{60 \text{ W/m}^2 \cdot \text{K}} \right]$$

$$R_{\text{tot}} = 0.01454 \text{ K/W.}$$

Hence the heat rate is

$$Q = \frac{1}{R_{\text{tot}}} \left\{ \int_0^{12\text{h}} \left[293 - \left[273 + 5 \sin \frac{2\pi}{24} t \right] \right] dt + \int_{12}^{24\text{h}} \left[293 - \left[273 + 11 \sin \frac{2\pi}{24} t \right] \right] dt \right\}$$

$$Q = 68.8 \frac{\text{W}}{\text{K}} \left\{ \left[20t + 5 \left[\frac{24}{2\pi} \right] \cos \frac{2\pi t}{24} \right] \Big|_0^{12} + \left[20t + 11 \left[\frac{24}{2\pi} \right] \cos \frac{2\pi t}{24} \right] \Big|_{12}^{24} \right\} \text{K}\cdot\text{h}$$

$$Q = 68.8 \left\{ \left[240 + \frac{60}{\pi} (-1 - 1) \right] + \left[480 - 240 + \frac{132}{\pi} (1 + 1) \right] \right\} \text{W}\cdot\text{h}$$

$$Q = 68.8 \{ 480 - 38.2 + 84.03 \} \text{W}\cdot\text{h}$$

$$Q = 36.18 \text{ kW}\cdot\text{h} = 1.302 \times 10^8 \text{ J.}$$

COMMENTS: From knowledge of the fuel cost, the total daily heating bill could be determined. For example, at a cost of 0.18\$/kW·h, the heating bill would be \$6.52/day.