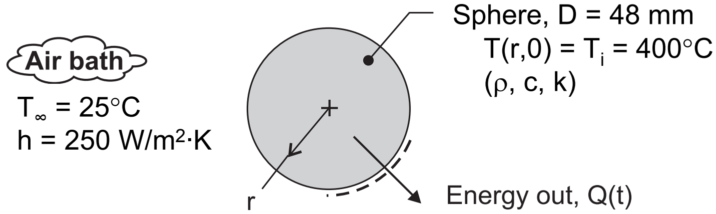
PROBLEM 5.78

KNOWN: Spheres of 48-mm diameter heated to a uniform temperature of 400°C are suddenly removed from an oven and placed in a forced-air bath operating at 25°C with a convection coefficient of 250 W/m^2·K.

FIND: (a) Time the spheres must remain in the bath for 80% of the thermal energy to be removed, and (b) Uniform temperature the spheres will reach when removed from the bath at this condition and placed in a carton that prevents further heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in the spheres, (2) Constant properties, and (3) No heat loss from sphere after removed from the bath and placed into the packing carton.

PROPERTIES: Sphere (*given*): ρ = 3000 kg/m^3, c = 850 J/kg·K, k = 15 W/m·K.

ANALYSIS: (a) From Eq. 5.55, the fraction of thermal energy removed during the time interval $\Delta t = t_o$ is

$$\frac{Q}{Q_o} = 1 - 3\theta_o^* / \zeta_1^3 \left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right] \qquad (1)$$

where Q/Q_o = 0.8. The Biot number is

$$\mathrm{Bi} = hr_o / k = 250\ \mathrm{W/m^2 \cdot K} \times 0.024\,\mathrm{m} / 15\ \mathrm{W/m \cdot K} = 0.40$$

and for the one-term series approximation, from Table 5.1,

$$\zeta_1 = 1.0528\ \mathrm{rad} \qquad C_1 = 1.1164 \qquad (2)$$

The dimensionless temperature θ_o^*, follows from Eq. 5.53b.

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 \mathrm{Fo}\right) \qquad (3)$$

where $\mathrm{Fo} = \alpha\, t_o / r_o^2$. Substituting Eq. (3) into Eq. (1), solve for Fo and t_o.

$$\frac{Q}{Q_o} = 0.8 = 1 - 3\, C_1 \exp\left(-\zeta_1^2 \mathrm{Fo}\right) / \zeta_1^3 \left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right] \qquad (4)$$

$$\mathrm{Fo} = 1.45 \qquad t_o = 142\ \mathrm{s}$$ <

(b) Performing an overall energy balance on the sphere during the interval of time $t_o \le t \le \infty$,

$$E_{in} - E_{out} = \Delta E = E_f - E_i = 0 \qquad (5)$$

where E_i represents the thermal energy in the sphere at t_o,

$$E_i = (1 - 0.8) Q_o = (1 - 0.8)\rho c V \left(T_i - T_\infty\right) \qquad (6)$$

and E_f represents the thermal energy in the sphere at t = ∞,

$$E_f = \rho c V \left(T_{avg} - T_\infty\right) \qquad (7)$$

Combining the relations, find the average temperature

$$\rho c V \left[\left(T_{avg} - T_\infty\right) - (1 - 0.8)\left(T_i - T_\infty\right)\right] = 0$$

$$T_{avg} = 100°\mathrm{C}$$ <