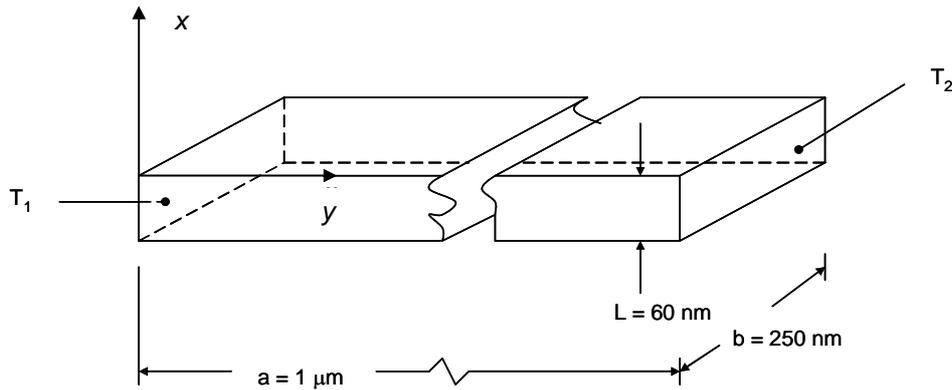


PROBLEM 3.178

KNOWN: Dimensions of and temperature difference applied across thin gold film.

FIND: (a) Energy conducted along the film, (b) Plot the thermal conductivity along and across the thin dimension of the film, for film thicknesses $30 \leq L \leq 140$ nm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x- and y-directions, (2) Steady-state conditions, (3) Constant properties, (4) Thermal conductivity not affected by nanoscale effects associated with 250 nm dimension.

PROPERTIES: Table A.1, gold (bulk, 300 K): $k = 317$ W/m·K.

ANALYSIS:

a) From Fourier's law,

$$q_y = -kA \frac{dT}{dy} = k_y L b \left[\frac{T_1 - T_2}{a} \right] \quad (1)$$

From Eq. 2.9b,

$$k_y = k \left[1 - \frac{2\lambda_{\text{mfp}}}{3\pi L} \right] \quad (2)$$

Combining Eqs. (1) and (2), and using the value of $\lambda_{\text{mfp}} = 31$ nm from Table 2.1 yields

$$\begin{aligned} q_y &= k \left[1 - \frac{2\lambda_{\text{mfp}}}{3\pi L} \right] L b \left[\frac{T_1 - T_2}{a} \right] \\ &= 317 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \left[1 - \frac{2 \times 31 \times 10^{-9} \text{ m}}{3 \times \pi \times 60 \times 10^{-9} \text{ m}} \right] \times 60 \times 10^{-9} \text{ m} \times 250 \times 10^{-9} \text{ m} \times \frac{20^\circ\text{C}}{1 \times 10^{-6} \text{ m}} \\ &= 85 \times 10^{-6} \text{ W} = 85 \mu\text{W} \quad < \end{aligned}$$

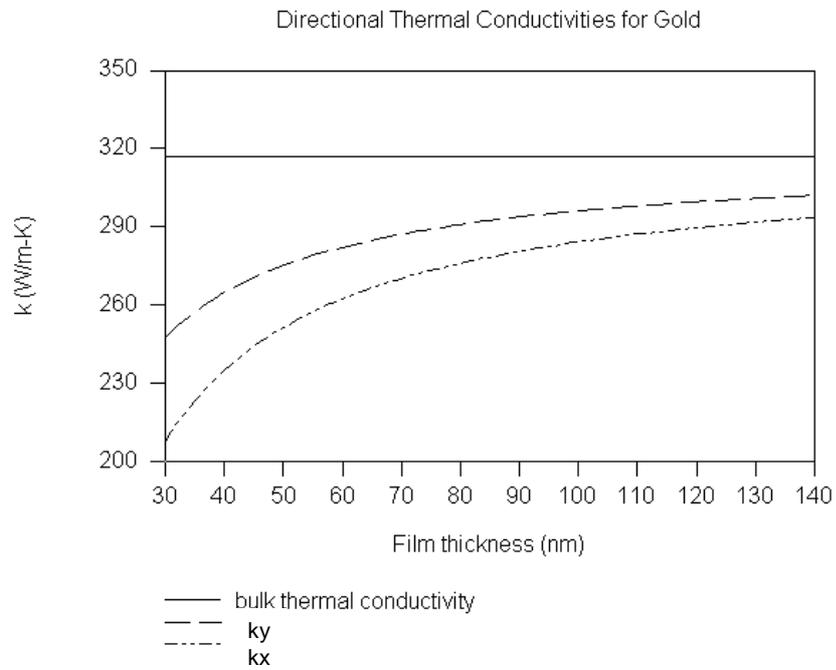
(b) The spanwise thermal conductivity may be found from Eq. 2.9a,

$$k_x = k \left[1 - \frac{\lambda_{\text{mfp}}}{3L} \right] \quad (3)$$

Continued...

PROBLEM 3.178 (Cont.)

The plot is shown below.



COMMENT: Nanoscale effects become less significant as the thickness of the film is increased.