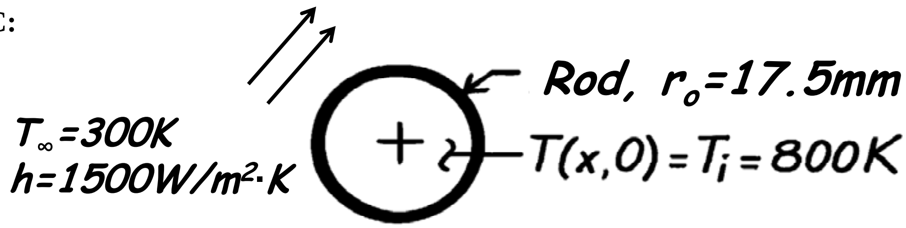
PROBLEM 5.64

KNOWN: Sapphire rod, initially at a uniform temperature of 800 K is suddenly cooled by a convection process; after 30 s, the rod is wrapped in insulation.

FIND: Temperature rod reaches after a long time following the insulation wrap.

SCHEMATIC:



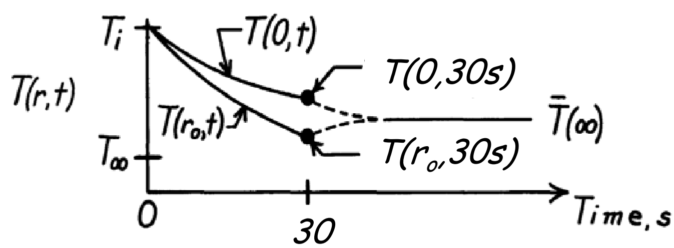
ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) No heat losses from the rod when insulation is applied.

PROPERTIES: *Table A-2*, Aluminum oxide, sapphire (550K): ρ = 3970 kg/m^3, c = 1068 J/kg·K, k = 22.3 W/m·K, $\alpha = 5.259\times10^{-6}$ m^2/s.

ANALYSIS: First calculate the Biot number with $L_c = r_o/2$,

$$\mathrm{Bi} = \frac{h\,L_c}{k} = \frac{h\,(r_o/2)}{k} = \frac{1500\ \mathrm{W/m^2\cdot K}\,(0.0175\ \mathrm{m}/2)}{22.3\ \mathrm{W/m\cdot K}} = 0.59.$$

Since Bi > 0.1, the rod cannot be approximated as a lumped capacitance system. The temperature distribution during the cooling process, $0 \le t \le 30$ s, and for the time following the application of insulation, t > 30 s, will appear as



Eventually ($t \to \infty$), the temperature of the rod will be uniform at $\bar{T}(\infty)$.

We begin by determining the energy transferred from the rod at t = 30 s. We have

$$\mathrm{Bi} = \frac{hr_o}{k} = \frac{1500\ \mathrm{W/m^2\cdot K} \times 0.0175\ \mathrm{m}}{22.3\ \mathrm{W/m\cdot K}} = 1.18$$

$$\mathrm{Fo} = \alpha t/r_o^2 = 5.259\times10^{-6}\ \mathrm{m^2/s}\times 30\ \mathrm{s}/(0.0175\ \mathrm{m})^2 = 0.515$$

Since Fo > 0.2, we can use the one-term approximation. From Table 5.1, ζ_1 = 1.3177 rad, C_1 = 1.2307. Then from Equation 5.49c,

$$\theta_o^* = C_1\exp(-\zeta_1^2\mathrm{Fo}) = 1.2307\exp(-1.3177^2\times0.515) = 0.5033$$

and from Equation 5.54

Continued…

$$\frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1) = 1 - \frac{2 \times 0.5033}{1.3177} 0.5258 = 0.598$$

where $J_1(\zeta_1)$ was found from App. B.4. Since the rod is well insulated after t = 30 s, the energy transferred from the rod remains unchanged. To find $\overline{T}(\infty)$, write the conservation of energy requirement for the rod on a *time interval* basis, $E_{in} - E_{out} = \Delta E \equiv E_{final} - E_{initial}$. Using the nomenclature of Section 5.5.3 and basing energy relative to T_∞, the energy balance becomes

$$-Q = \rho\, cV\left(\overline{T}(\infty) - T_\infty\right) - Q_o$$

where $Q_o = \rho cV(T_i - T_\infty)$. Dividing through by Q_o and solving for $\overline{T}(\infty)$, find

$$\overline{T}(\infty) = T_\infty + \left(T_i - T_\infty\right)\left(1 - Q/Q_o\right).$$

Hence,

$$\overline{T}(\infty) = 300\text{K} + \left(800 - 300\right)\text{K}\ \left(1\text{-}0.598\right) = 500\ \text{K}. \qquad <$$