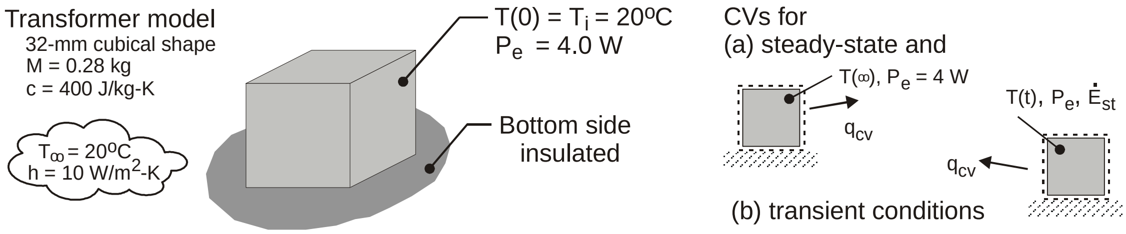
PROBLEM 5.40

KNOWN: Electrical transformer of approximate cubical shape, 32 mm to a side, dissipates 4.0 W when operating in ambient air at 20°C with a convection coefficient of 10 $W/m^2 \cdot K$.

FIND: (a) Develop a model for estimating the steady-state temperature of the transformer, $T(\infty)$, and evaluate $T(\infty)$, for the operating conditions, and (b) Develop a model for estimating the temperature-time history of the transformer if initially the temperature is $T_i = T_\infty$ and suddenly power is applied. Determine the time required to reach within 5°C of its steady-state operating temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Transformer is spatially isothermal object, (2) Initially object is in equilibrium with its surroundings, (3) Bottom surface is adiabatic.

ANALYSIS: (a) Under steady-state conditions, for the control volume shown in the schematic above, the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad 0 - q_{cv} + P_e = -h\,A_s\left[T(\infty) - T_\infty\right] + P_e = 0 \qquad (1)$$

where $A_s = 5 \times L^2 = 5 \times 0.032\text{m} \times 0.032\text{m} = 5.12 \times 10^{-3}\ \text{m}^2$, find

$$T(\infty) = T_\infty + P_e / h\,A_s = 20°\text{C} + 4\,\text{W} / \left(10\ \text{W/m}^2 \cdot \text{K} \times 5.12 \times 10^{-3}\text{m}^2\right) = 98.1°\text{C} \qquad <$$

(b) Under transient conditions, for the control volume shown above, the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \qquad 0 - q_{cv} + P_e = Mc\frac{dT}{dt} \qquad (2)$$

Substitute from Eq. (1) for P_e, separate variables, and define the limits of integration.

$$-h\left[T(t) - T_\infty\right] + h\left[T(\infty) - T_\infty\right] = Mc\frac{dT}{dt}$$

$$-h\left[T(t) - T(\infty)\right] = Mc\frac{d}{dt}\left(T - T(\infty)\right) \qquad \frac{h}{Mc}\int_0^{t_o} dt = -\int_{\theta_i}^{\theta_o}\frac{d\theta}{\theta}$$

where $\theta = T(t) - T(\infty)$; $\theta_i = T_i - T(\infty) = T_\infty - T(\infty)$; and $\theta_o = T(t_o) - T(\infty)$ with t_o as the time when θ_o = - 5°C. Integrating and rearranging find (see Eq. 5.5),

$$t_o = \frac{Mc}{h\,A_s}\ell n\frac{\theta_i}{\theta_o}$$

$$t_o = \frac{0.28\ \text{kg} \times 400\ \text{J/kg} \cdot \text{K}}{10\ \text{W/m}^2 \cdot \text{K} \times 5.12 \times 10^{-3}\text{m}^2}\ell n\frac{(20 - 98.1)°\text{C}}{-5°\text{C}} = 1.67\ \text{hour} \qquad <$$

COMMENTS: The spacewise isothermal assumption may not be a gross over simplification since most of the material is copper and iron, and the external resistance by free convection is high. However, by ignoring internal resistance, our estimate for t_o is optimistic.