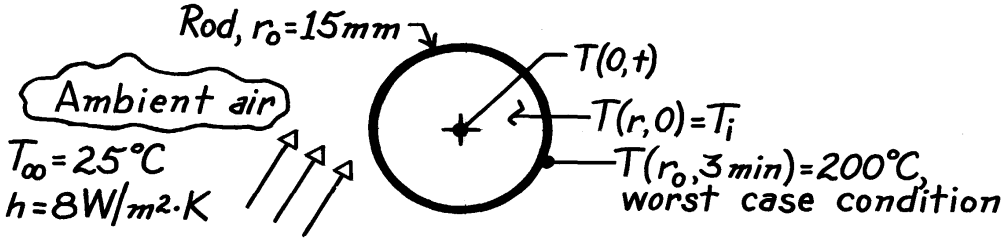
PROBLEM 5.66

KNOWN: Long plastic rod of diameter D heated uniformly in an oven to T_i and then allowed to convectively cool in ambient air (T_∞, h) for a 3 minute period. Minimum temperature of rod should not be less than 200°C and the maximum-minimum temperature within the rod should not exceed 10°C.

FIND: Initial uniform temperature T_i to which rod should be heated. Whether the 10°C internal temperature difference is exceeded.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Uniform and constant convection coefficients.

PROPERTIES: Plastic rod (given): k = 0.3 W/m·K, ρc_p = 1040 kJ/m^3·K.

ANALYSIS: For the worst case condition, the rod cools for 3 minutes and its outer surface is at least 200°C in order that the subsequent pressing operation will be satisfactory. Hence,

$$\mathrm{Bi} = \frac{h r_o}{k} = \frac{8\ \mathrm{W/m^2\cdot K} \times 0.015\ \mathrm{m}}{0.3\ \mathrm{W/m\cdot K}} = 0.40$$

$$\mathrm{Fo} = \frac{\alpha\, t}{r_o^2} = \frac{k}{\rho\, c_p} \cdot \frac{t}{r_o^2} = \frac{0.3\ \mathrm{W/m\cdot K}}{1040\times 10^3\ \mathrm{J/m^3\cdot K}} \times \frac{3\times 60\mathrm{s}}{(0.015\ \mathrm{m})^2} = 0.2308.$$

Using Eq. 5.52a and $\zeta_1 = 0.8516$ rad and C_1 = 1.0932 from Table 5.1,

$$\theta^* = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = C_1 J_0\left(\zeta_1 r_o^*\right)\exp\left(-\zeta_1^2 \mathrm{Fo}\right).$$

With $r_o^* = 1$, from Table B.4, $J_0(\zeta_1 \times 1) = J_o(0.8516) = 0.8263$, giving

$$\frac{200-25}{T_i - 25} = 1.0932 \times 0.8263 \exp\left(-0.8516^2 \times 0.2308\right) \qquad T_i = 254^\circ\mathrm{C}. \quad <$$

At this time (3 minutes) what is the difference between the center and surface temperatures of the rod? From Eq. 5.52b,

$$\frac{\theta^*}{\theta_o} = \frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = \frac{200-25}{T(0,t) - 25} = J_0\left(\zeta_1 r_o^*\right) = 0.8263$$

which gives T(0,t) = 237°C. Hence,

$$\Delta T = T(0,180\mathrm{s}) - T(r_o,180\mathrm{s}) = (237-200)^\circ\,\mathrm{C} = 37^\circ\mathrm{C}. \quad <$$

Hence, the desired max-min temperature difference sought (10°C) is not achieved.

COMMENTS: ΔT could be reduced by decreasing the cooling rate; however, h can not be made much smaller. Two solutions are (a) increase ambient air temperature and (b) non-uniformly heat rod in oven by controlling its residence time.