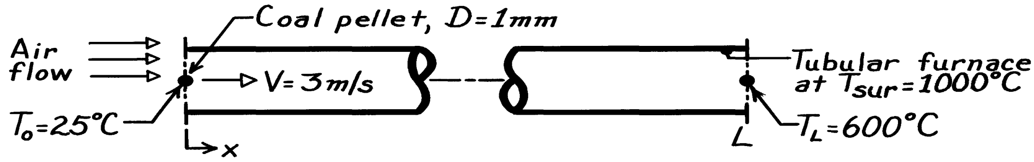
PROBLEM 5.32

KNOWN: Spherical coal pellet at 25°C is heated by radiation while flowing through a furnace maintained at 1000°C.

FIND: Length of tube required to heat pellet to 600°C.

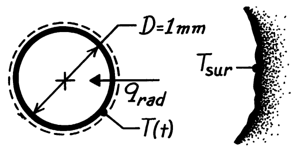
SCHEMATIC:



ASSUMPTIONS: (1) Pellet is suspended in air flow and subjected to only radiative exchange with furnace, (2) Pellet is small compared to furnace surface area, (3) Coal pellet has emissivity, ε = 1.

PROPERTIES: *Table A-3*, Coal $\left(\bar{T} = (600 + 25)\,°C/2 = 585K,\ \text{however, only 300K data available}\right)$: ρ = 1350 kg/m^3, c_p = 1260 J/kg·K, k = 0.26 W/m·K.

ANALYSIS: Considering the pellet as spatially isothermal, use the lumped capacitance method of Section 5.3 to find the time required to heat the pellet from T_o = 25°C to T_L = 600°C. From an energy balance on the pellet $\dot{E}_{in} = \dot{E}_{st}$ where



$$\dot{E}_{in} = q_{rad} = \sigma A_s\left(T_{sur}^4 - T_s^4\right) \qquad \dot{E}_{st} = \rho \forall c_p \frac{dT}{dt}$$

giving
$$A_s \sigma\left(T_{sur}^4 - T_s^4\right) = \rho \forall c_p \frac{dT}{dt}.$$

Separating variables and integrating with limits shown, the temperature-time relation becomes

$$\frac{A_s\sigma}{\rho\forall c_p}\int_0^t dt = \int_{T_o}^{T_L}\frac{dT}{T_{sur}^4 - T^4}.$$

The integrals are evaluated in Eq. 5.18 giving

$$t = \frac{\rho\forall c_p}{4A_s\sigma T_{sur}^3}\left\{\ln\left|\frac{T_{sur}+T}{T_{sur}-T}\right| - \ln\left|\frac{T_{sur}+T_i}{T_{sur}-T_i}\right| + 2\left[\tan^{-1}\left[\frac{T}{T_{sur}}\right] - \tan^{-1}\left[\frac{T_i}{T_{sur}}\right]\right]\right\}.$$

Recognizing that $A_s = \pi D^2$ and $\forall = \pi D^3/6$ or $A_s/\forall = 6/D$ and substituting values,

$$t = \frac{1350\ \text{kg/m}^3(0.001\ \text{m})1260\ \text{J/kg}\cdot\text{K}}{24\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4(1273\ \text{K})^3}\left\{\ln\frac{1273+873}{1273-873} - \ln\frac{1273+298}{1273-298}\right.$$

$$\left.+2\left[\tan^{-1}\left(\frac{873}{1273}\right) - \tan^{-1}\left(\frac{298}{1273}\right)\right]\right\} = 1.18\text{s}.$$

Hence, $L = V\cdot t = 3\text{m/s}\times 1.18\text{s} = 3.54\text{m}.$ <

The validity of the lumped capacitance method requires $Bi = h(\forall/A_s)/k < 0.1$. Using Eq. (1.9) for $h = h_r$ and $\forall/A_s = D/6$, find that when T = 600°C, Bi = 0.19; but when T = 25°C, Bi = 0.10. At early times, when the pellet is cooler, the assumption is reasonable but becomes less appropriate as the pellet heats.