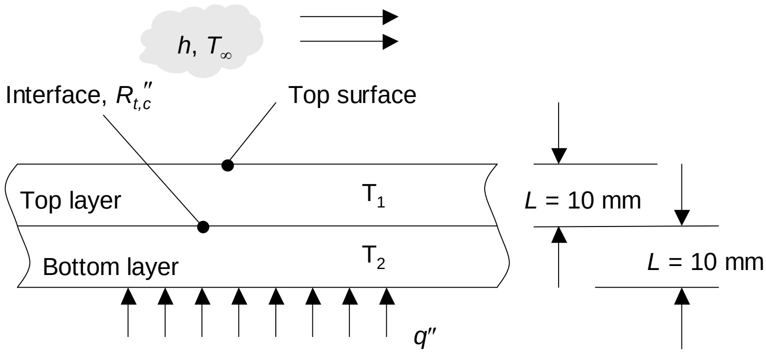
PROBLEM 5.35

KNOWN: Thickness and initial temperatures of two layers of copper and aluminum. Contact resistance at the interface between the layers, applied heat flux, and convective conditions on the upper surface of the top layer.

FIND: (a) Times at which the copper (bottom) and aluminum (top) reach a temperature of T_f = 90°C. (b) Times at which the copper (top) and aluminum (bottom) reach a temperature of T_f = 90°C.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance behavior, (2) Constant properties, (3) Negligible radiation.

PROPERTIES: Table A.1; copper (T = 300 K): ρ_A = 8933 kg/m^3, c_A = 385 J/kg·K, k_A = 401 W/m·K; aluminum (T = 300 K): ρ_B = 2702 kg/m^3, c_B = 903 J/kg·K, k_B = 237 W/m·K.

ANALYSIS: (a) For copper on the bottom, a modified form of Eq. 5.15 may be applied to both materials, resulting in

Copper: $$q_s'' - \frac{1}{R_{t,c}''}(T_A - T_B) = \rho_A c_A L_A \frac{dT_A}{dt} \qquad (1)$$

Aluminum: $$\frac{1}{R_{t,c}''}(T_A - T_B) - h(T_B - T_\infty) = \rho_B c_B L_B \frac{dT_B}{dt} \qquad (2)$$

Equations 1 and 2 are coupled differential equations with the initial conditions $T_{i,A} = T_{i,B}$ = 25°C. Hence, a numerical solution is required, yielding $t_{f,A}$ = 34.4 s and $t_{f,B}$ = 44.6 s. <

(b) For copper on the top, modified Eq. 5.15 is written as

Copper: $$\frac{1}{R_{t,c}''}(T_B - T_A) - h\left(T_A - T_\infty\right) = \rho_A c_A L_A \frac{dT_A}{dt}$$

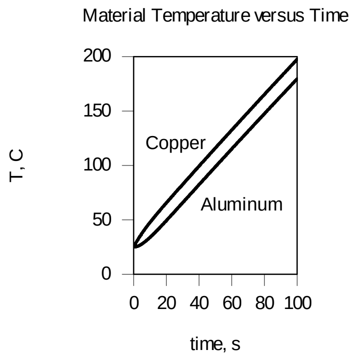
Aluminum: $$q_s'' - \frac{1}{R_{t,c}''}(T_B - T_A) = \rho_B c_B L_B \frac{dT_B}{dt}$$

The numerical solution yields $t_{f,A}$ = 44.6 s and $t_{f,B}$ = 30.4 s. <

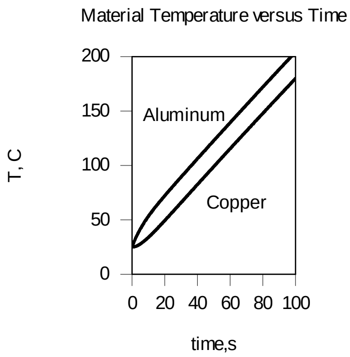
Continued…

PROBLEM 5.35 (Cont.)

COMMENTS: (1) For the aluminum on the top, the Biot number associated with the aluminum is $Bi_B = hL_B/k_B = (40\ W/m^2\cdot K \times 10 \times 10^{-3}\ m)/237\ W/m\cdot K = 0.0017$. The lumped capacitance approach is valid for the aluminum. For the copper, the contact resistance and the conduction resistance through the top aluminum layer pose thermal resistances in series with the convective resistance. In addition, the thermal conductivity of copper is greater than that of the aluminum, so lumped capacitance behavior is also expected for the copper. For the copper on top, the Biot number for the copper is $Bi_A = 0.0010$. Lumped capacitance behavior will also exist for this configuration. (2) The thermal responses for the two cases are shown below. Can you explain the differences?



Copper on Bottom



Copper on Top

(3) At $t = 44.6$ s, the temperature of the copper in part (a) is $T_A(t = 44.6s) = 107.1°C$. At $t = 44.6$ s, the temperature of the bottom aluminum in part (b) is $T_B = 113.9°C$. Hence, the increase in thermal energy of both materials per unit area during the first 44.6 s of heating for part (a) is $\Delta E_{st}'' = L_A c_A[T_A(t = 44.6\ s) - T_i] + L_B c_B[T_B(t = 44.6\ s) - T_i] = 10 \times 10^{-3} m \times 385\ J/kg\cdot K \times [107.1 - 25]°C + 10 \times 10^{-3} m \times 903\ J/kg\cdot K \times [90 - 25]°C = 903\ J/m^2$. For part (b) the increase in thermal energy is $\Delta E_{st}'' = = 10 \times 10^{-3} m \times 385\ J/kg\cdot K \times [90 - 25]°C + 10 \times 10^{-3} m \times 903\ J/kg\cdot K \times [113.9 - 25]°C = 1053\ J/m^2$. The difference between the two values is due to differences in the cumulative convective losses from the top surface for the two configurations. Why are convective heat losses smaller in part (b) than in part (a)? (4) The IHT Code used to solve Equations 1 and 2 is shown below. A time step of $\Delta t = 0.1$ s was specified.

```
//Dimensions
LA = 10/1000                //m
LB = 10/1000                //m

//Properties
cA = 385                    //Copper specific heat, J/kgK
rhoA = 8933                 //Copper density, kg/m^3
cB = 903                    //Aluminum specific heat, J/kgK
rhoB = 2702                 //Aluminum density, kg/m^3

//Thermal Conditions
Tinf = 25                   //Ambient temperature, C
h = 40                      //Convection coefficient, W/m^2K
Rcont = 400*10^-6           //Contact resistance, m^2K/W
qflux = 100*10^3            //Heat flux, W/m^2

//Copper on Bottom Case
//Energy Balance on A
//rhoA*LA*cA*Der(TA,t) = qflux - (TA - TB)/Rcont
//Energy Balance on B
//rhoB*LB*cB*Der(TB,t) = (TA - TB)/Rcont - h*(TB - Tinf)

//Copper on Top Case
//Energy Balance on A
rhoA*LA*cA*Der(TA,t) = (TB - TA)/Rcont - h*(TA - Tinf)
//Energy Balance on B
rhoB*LB*cB*Der(TB,t) = qflux - (TB - TA)/Rcont
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