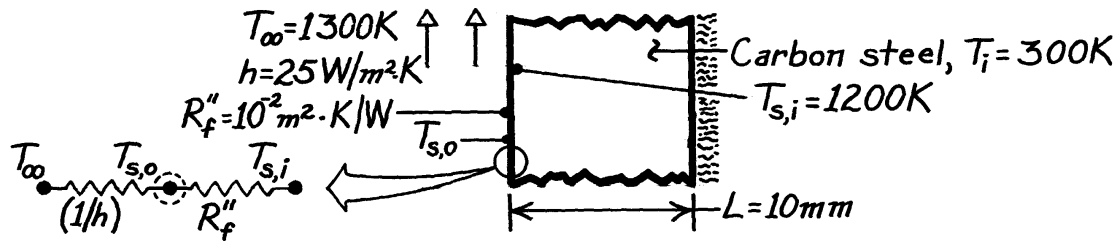


PROBLEM 5.22

KNOWN: Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel (given): $\rho = 7850 \text{ kg/m}^3$, $c = 430 \text{ J/kg}\cdot\text{K}$, $k = 60 \text{ W/m}\cdot\text{K}$.

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

$$U = (R_{\text{tot}}'')^{-1} = \left(\frac{1}{h} + R_f'' \right)^{-1} = \left(\frac{1}{25 \text{ W/m}^2 \cdot \text{K}} + 10^{-2} \text{ m}^2 \cdot \text{K/W} \right)^{-1} = 20 \text{ W/m}^2 \cdot \text{K}.$$

Hence,

$$\text{Bi} = \frac{UL}{k} = \frac{20 \text{ W/m}^2 \cdot \text{K} \times 0.01 \text{ m}}{60 \text{ W/m} \cdot \text{K}} = 0.0033$$

and the lumped capacitance method can be used.

(a) It follows that

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = \exp(-t/\tau_t) = \exp(-t/RC) = \exp(-Ut/\rho Lc)$$

$$t = -\frac{\rho Lc}{U} \ln \frac{T - T_{\infty}}{T_i - T_{\infty}} = -\frac{7850 \text{ kg/m}^3 (0.01 \text{ m}) 430 \text{ J/kg}\cdot\text{K}}{20 \text{ W/m}^2 \cdot \text{K}} \ln \frac{1200 - 1300}{300 - 1300}$$

$$t = 3886 \text{ s} = 1.08 \text{ h.} \quad <$$

(b) Performing an energy balance at the outer surface (s,o),

$$h(T_{\infty} - T_{s,o}) = (T_{s,o} - T_{s,i})/R_f''$$

$$T_{s,o} = \frac{hT_{\infty} + T_{s,i}/R_f''}{h + (1/R_f'')} = \frac{25 \text{ W/m}^2 \cdot \text{K} \times 1300 \text{ K} + 1200 \text{ K}/10^{-2} \text{ m}^2 \cdot \text{K/W}}{(25 + 100) \text{ W/m}^2 \cdot \text{K}}$$

$$T_{s,o} = 1220 \text{ K.} \quad <$$

COMMENTS: The film increases τ_t by increasing R_t but not C_t .