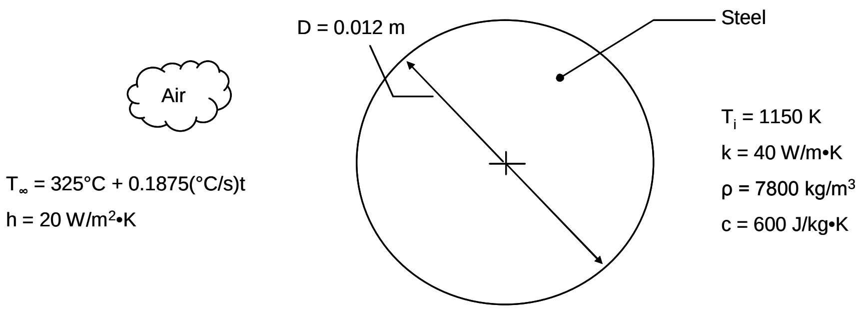
PROBLEM 5.7

KNOWN: Diameter and initial temperature of steel balls in air. Expression for the air temperature versus time.

FIND: (a) Expression for the sphere temperature, T(t), (b) Graph of T(t) and explanation of special features.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation heat transfer.

PROPERTIES: Given: k = 40 W/m·K, ρ = 7800 kg/m^3, c = 600 J/kg·K.

ANALYSIS:
(a) Applying Equation 5.10 to a sphere ($L_c = r_o/3$),

$$B_i = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{20\ \text{W/m}^2\cdot\text{K}\ (0.002\ \text{m})}{40\ \text{W/m}\cdot\text{K}} = 0.001$$

Hence, the temperature of the steel sphere remains approximately uniform during the cooling process. Equation 5.2 is written, with $T_\infty = T_o + at$, as

$$-hA_s(T - T_o - at) = \rho\forall c\frac{dT}{dt}$$

Letting $\theta = T - T_o$, $dT = d\theta$ and $-hA_s(\theta - at) = \rho\forall c\frac{d\theta}{dt}$ or $\frac{d\theta}{dt} = -C(\theta - at)$ where $C = \frac{hA_s}{\rho\forall c}$

The solution may be written as the sum of the homogeneous and particular solutions,

$$\theta = \theta_h + \theta_p \quad \text{where} \quad \theta_h = c_1\exp(-Ct).$$

Assuming $\theta_p = f(t)\theta_h$, we substitute into the differential equation to find

$$\frac{df}{dt} = \text{Cat}\exp(Ct)/c_1 \text{ from which } f = a(t - 1/C)\exp(Ct)/c_1.$$

Thus, the complete solution is

$\theta = c_1\exp(-Ct) + a(t - 1/C)$ and applying the initial condition we find

$$T = (T_i - T_o + a/C)\exp(-Ct) + a(t - 1/C) + T_o$$ <

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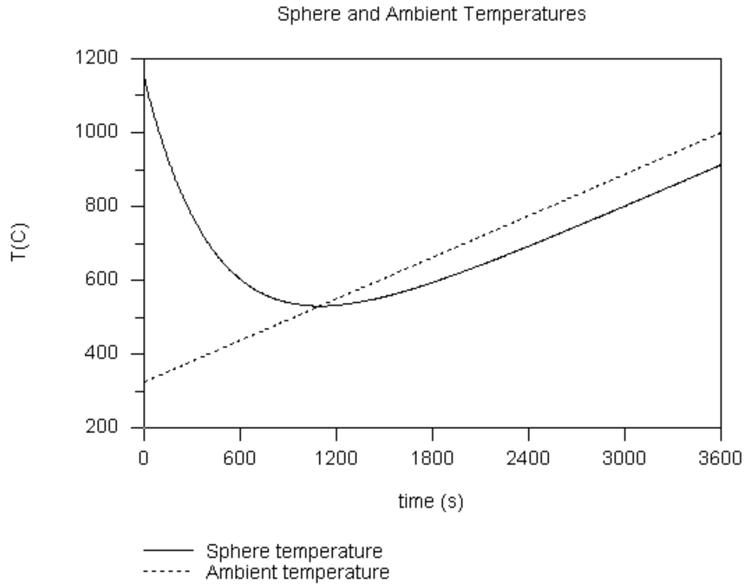
PROBLEM 5.7 (Cont.)

(b) The ambient and sphere temperatures for $0 \leq t \leq 3600$ s are shown in the plot below.



Note that:

(1) For small times ($t \leq 600$s) the sphere temperature decreases rapidly,

(2) at $t \approx 1100$ s, $T = T_\infty$ and, from Equation 5.2, $dT/dt = 0$,

(3) at $t \geq 1100$ s, $T < T_\infty$,

(4) at large time, $T - T_\infty$ and dT/dt are constant.

COMMENTS: Unless the air environment of Problem 5.6 is cooled, the air temperature will increase in temperature as energy is transferred from the balls. However, the actual air temperature versus time may not be linear.