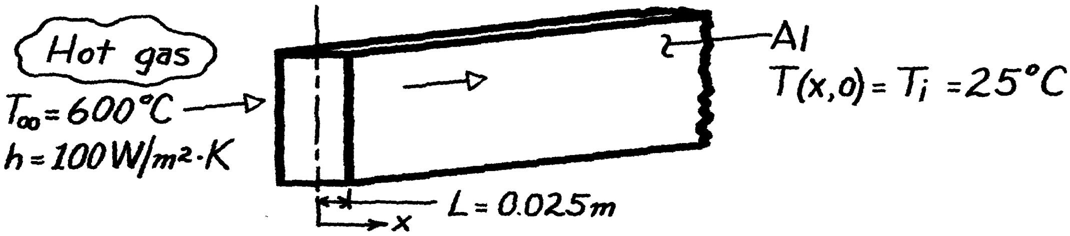
PROBLEM 5.16

KNOWN: Configuration, initial temperature and charging conditions of a thermal energy storage unit.

FIND: Time required to achieve 75% of maximum possible energy storage. Temperature of storage medium at this time.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation exchange with surroundings.

PROPERTIES: *Table A-1*, Aluminum, pure $\left(\overline{T} \approx 600\text{K} = 327°\text{C}\right)$: k = 231 W/m·K, c = 1033 J/kg·K, ρ = 2702 kg/m^3.

ANALYSIS: Recognizing the characteristic length is the half thickness, find

$$\text{Bi} = \frac{\text{hL}}{\text{k}} = \frac{100\ \text{W/m}^2 \cdot \text{K} \times 0.025\text{m}}{231\ \text{W/m} \cdot \text{K}} = 0.011.$$

Hence, the lumped capacitance method may be used. From Eq. 5.8,

$$\text{Q} = (\rho \text{Vc})\theta_\text{i}\ \left[1 - \exp\left(-\text{t}/\tau_\text{t}\right)\right] = -\Delta\text{E}_\text{st} \qquad (1)$$

$$-\Delta\text{E}_{\text{st,max}} = (\rho \text{Vc})\theta_\text{i}. \qquad (2)$$

Dividing Eq. (1) by (2),

$$\Delta\text{E}_\text{st} / \Delta\text{E}_{\text{st,max}} = 1\ - \exp\left(-\text{t}/\tau_\text{th}\right) = 0.75.$$

Solving for $\tau_\text{th} = \dfrac{\rho \text{Vc}}{\text{hA}_\text{s}} = \dfrac{\rho \text{Lc}}{\text{h}} = \dfrac{2702\ \text{kg/m}^3 \times 0.025\text{m} \times 1033\ \text{J/kg} \cdot \text{K}}{100\ \text{W/m}^2 \cdot \text{K}} = 698\text{s}.$

Hence, the required time is

$$-\exp(-\text{t}/698\text{s}) = -0.25 \qquad \text{or} \qquad \text{t} = 968\text{s}. \qquad <$$

From Eq. 5.6,

$$\frac{\text{T} - \text{T}_\infty}{\text{T}_\text{i} - \text{T}_\infty} = \exp\left(-\text{t}/\tau_\text{th}\right)$$

$$\text{T} = \text{T}_\infty + \left(\text{T}_\text{i} - \text{T}_\infty\right)\exp\left(-\text{t}/\tau_\text{th}\right) = 600°\text{C} - \left(575°\text{C}\right)\exp(-968/698)$$

$$\text{T} = 456°\text{C}. \qquad <$$

COMMENTS: For the prescribed temperatures, the property temperature dependence is significant and some error is incurred by assuming constant properties. However, selecting properties at 600K was reasonable for this estimate.