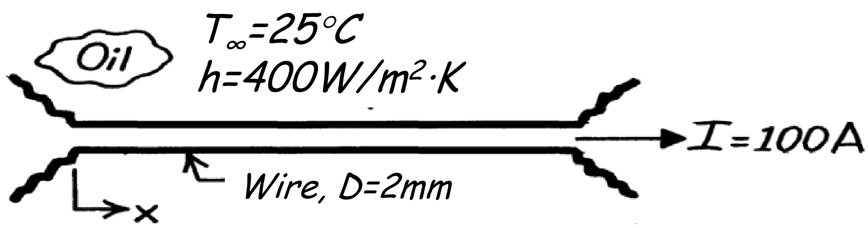
PROBLEM 5.29

KNOWN: Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.

FIND: Steady-state temperature of the wire. Time for the wire temperature to come within 1°C of its steady-state value.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Wire temperature is independent of x.

PROPERTIES: Wire (given): $\rho = 8000\ \text{kg/m}^3$, $c_p = 500\ \text{J/kg·K}$, $k = 20\ \text{W/m·K}$, $R'_e = 0.01\ \Omega\text{/m}$.

ANALYSIS: Since

$$Bi = \frac{h(r_o/2)}{k} = \frac{400\ \text{W/m}^2\cdot\text{K}\left(5.0\times10^{-4}\text{m}\right)}{20\ \text{W/m}\cdot\text{K}} = 0.01 < 0.1$$

the lumped capacitance method can be used. The problem has been analyzed in Example 1.4, and without radiation the steady-state temperature is given by

$$\pi\ Dh\left(T - T_\infty\right) = I^2R'_e.$$

Hence

$$T = T_\infty + \frac{I^2R'_e}{\pi\ Dh} = 25^\circ\text{C} + \frac{(100\text{A})^2\ 0.01\Omega/\text{m}}{\pi(0.002\ \text{m})400\ \text{W/m}^2\cdot\text{K}} = 64.8^\circ\text{C}. \qquad <$$

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.4)

$$\frac{dT}{dt} = \frac{I^2R'_e}{\rho c_p\left(\pi D^2/4\right)} - \frac{4h}{\rho\ c_pD}\left(T - T_\infty\right).$$

With $T = T_i = 25°C$ at $t = 0$, the solution is

$$\frac{T - T_\infty - \left(I^2R'_e/\pi\ Dh\right)}{T_i - T_\infty - \left(I^2R'_e/\pi\ Dh\right)} = \exp\left(-\frac{4h}{\rho\ c_pD}t\right).$$

Substituting numerical values, find

$$\frac{63.8 - 25 - 39.8}{25 - 25 - 39.8} = \exp\left(-\frac{4\times400\ \text{W/m}^2\cdot\text{K}}{8000\ \text{kg/m}^3\times500\ \text{J/kg}\cdot\text{K}\times0.002\ \text{m}}t\right)$$

$$t = 18.4\text{s}. \qquad <$$

COMMENTS: The time to reach steady state increases with increasing ρ, c_p and D and with decreasing h.