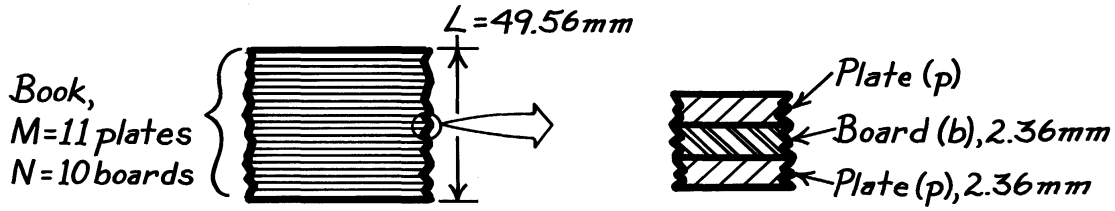
PROBLEM 5.45

KNOWN: Stack or book comprised of 11 metal plates (p) and 10 boards (b) each of 2.36 mm thickness and prescribed thermophysical properties.

FIND: Effective thermal conductivity, k, and effective thermal capacitance, (ρc_p).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible contact resistance between plates and boards.

PROPERTIES: Metal plate (p, given): $\rho_p = 8000$ kg/m^3, $c_{p,p} = 480$ J/kg·K, $k_p = 12$ W/m·K; Circuit boards (b, given): $\rho_b = 1000$ kg/m^3, $c_{p,b} = 1500$ J/kg·K, $k_b = 0.30$ W/m·K.

ANALYSIS: The thermal resistance of the book is determined as the sum of the resistance of the boards and plates,

$$R''_{tot} = NR''_b + MR''_p$$

where M,N are the number of plates and boards in the book, respectively, and $R''_i = L_i / k_i$ where L_i and k_i are the thickness and thermal conductivities, respectively.

$$R''_{tot} = M\left(L_p / k_p\right) + N\left(L_b / k_b\right)$$
$$R''_{tot} = 11\left(0.00236 \text{ m}/12 \text{ W/m}\cdot\text{K}\right) + 10\left(0.00236 \text{ m}/0.30 \text{ W/m}\cdot\text{K}\right)$$
$$R''_{tot} = 2.163\times10^{-3} + 7.867\times10^{-2} = 8.083\times10^{-2} \text{ K/W}.$$

The effective thermal conductivity of the book of thickness (10 + 11) 2.36 mm is

$$k = L/R''_{tot} = \frac{0.04956 \text{ m}}{8.083\times10^{-2} \text{ K/W}} = 0.613 \text{ W/m}\cdot\text{K}. \quad <$$

The thermal capacitance of the stack is

$$C''_{tot} = M\left(\rho_p L_p c_p\right) + N\left(\rho_b L_b c_b\right)$$
$$C''_{tot} = 11\left(8000 \text{ kg/m}^3 \times 0.00236 \text{ m} \times 480 \text{ J/kg}\cdot\text{K}\right) + 10\left(1000 \text{ kg/m}^3 \times 0.00236 \text{ m} \times 1500 \text{ J/kg}\cdot\text{K}\right)$$
$$C''_{tot} = 9.969\times10^4 + 3.540\times10^4 = 1.35\times10^5 \text{ J/m}^2\cdot\text{K}.$$

The effective thermal capacitance of the book is

$$\left(\rho c_p\right) = C''_{tot} / L = 1.351\times10^5 \text{ J/m}^2\cdot\text{K}/0.04956 \text{ m} = 2.726\times10^6 \text{ J/m}^3\cdot\text{K}. \quad <$$

COMMENTS: The results of the analysis allow for representing the stack as a homogeneous medium with *effective* properties: k = 0.613 W/m·K and $\alpha = (k/\rho c_p) = 2.249\times10^{-7}$ m^2/s.