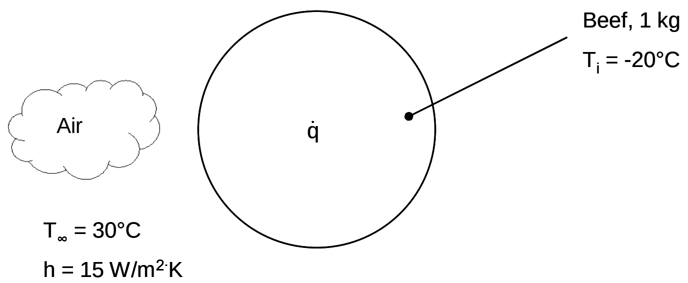
PROBLEM 5.33

KNOWN: Mass and initial temperature of frozen ground beef. Temperature and convection coefficient of air. Rate of microwave power absorbed in beef.

FIND: (a) Time for beef to reach 0°C, (b) Time for beef to be heated from liquid at 0°C to 80°C, and (c) Explain nonuniform heating in microwave and reason for low power setting for thawing.

SCHEMATIC:



ASSUMPTIONS: (1) Beef is nearly isothermal, (2) Beef has properties of water (ice or liquid), (3) Radiation is negligible, (4) Constant properties (different for ice and liquid water).

PROPERTIES: Table A.3, Ice (≈ 273 K): ρ = 920 kg/m^3, c = 2040 J/kg·K, Table A.6, Water (≈ 315 K): c = 4179 J/kg·K.

ANALYSIS: (a) We apply conservation of energy to the beef

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{st}$$

$$hA_s(T_\infty - T) + \dot{q} = mc\frac{dT}{dt} \quad (1)$$

The initial condition is $T(0) = T_i$. This differential equation can be solved by defining

$$\theta = T - T_\infty - \frac{\dot{q}}{hA_s}$$

Then Eq.(1) becomes $\frac{d\theta}{dt} = -\frac{hA_s}{mc}\theta$

Separating variables and integrating,

$$\int_{\theta(0)}^{\theta(t)}\frac{d\theta}{\theta} = -\frac{hA_s}{mc}\int_0^t dt$$

$$\ln\left[\frac{\theta(t)}{\theta(0)}\right] = -\frac{hA_st}{mc}$$

$$\ln\left[\frac{T - T_\infty - \dot{q}/hA_s}{T_i - T_\infty - \dot{q}/hA_s}\right] = -\frac{hA_st}{mc} \quad (2)$$

The heat generation rate is given by $\dot{q} = 0.03P = 0.03(1000\text{ W}) = 30$ W. The radius of the sphere can be found from knowledge of the mass and density:

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PROBLEM 5.33 (Cont.)

$$m = \rho V = \rho \frac{4}{3} \pi r_o^3$$

$$r_o = \left(\frac{3}{4\pi} \frac{m}{\rho} \right)^{1/3} = \left(\frac{3}{4\pi} \frac{1\ \text{kg}}{920\ \text{kg/m}^3} \right)^{1/3} = 0.0638\ \text{m}$$

Thus $\quad A_s = 4\pi r_o^2 = 4\pi(0.0638\ \text{m})^2 = 0.0511\ \text{m}^2$

Substituting numerical values into Eq.(2), we can find the time at which the temperature reaches 0°C:

$$\ln\left[\frac{0°\text{C} - 30°\text{C} - 30\ \text{W}/(15\ \text{W/m}^2 \cdot \text{K} \times 0.0511\ \text{m}^2)}{-20°\text{C} - 30°\text{C} - 30\ \text{W}/(15\ \text{W/m}^2 \cdot \text{K} \times 0.0511\ \text{m}^2)} \right] = -\frac{15\ \text{W/m}^2 \cdot \text{K} \times 0.0511\ \text{m}^2}{1\ \text{kg} \times 2040\ \text{J/kg} \cdot \text{K}} t$$

Thus t = 676 s = 11.3 min $\quad <$

(b) After all the ice is converted to liquid, the absorbed power is $\dot{q} = 0.95P = 950$ W. The time for the beef to reach 80°C can again be found from Eq.(2):

$$\ln\left[\frac{80°\text{C} - 30°\text{C} - 950\ \text{W}/(15\ \text{W/m}^2 \cdot \text{K} \times 0.0511\ \text{m}^2)}{0°\text{C} - 30°\text{C} - 950\ \text{W}/(15\ \text{W/m}^2 \cdot \text{K} \times 0.0511\ \text{m}^2)} \right] = -\frac{15\ \text{W/m}^2 \cdot \text{K} \times 0.0511\ \text{m}^2}{1\ \text{kg} \times 4179\ \text{J/kg} \cdot \text{K}} t$$

Thus t = 355 s = 5.9 min $\quad <$

(c) Microwave power is more efficiently absorbed in regions of liquid water. Therefore, if food or the microwave irradiation is not homogeneous or uniform, the power will be absorbed nonuniformly, resulting in a nonuniform temperature rise. Thawed regions will absorb more energy per unit volume than frozen regions. If food is of low thermal conductivity, there will be insufficient time for heat conduction to make the temperature more uniform. Use of low power allows more time for conduction to occur.

COMMENTS: (1) The time needed to turn the ice at 0°C into liquid water at 0°C was not calculated. The required energy is $Q = mh_{fg}$ = 1 kg × 2502 kJ/kg = 2502 kJ. The required time depends on how the fraction of microwave power absorbed changes during the thawing process. The minimum possible time would be t_{min} = 2502 kJ/950 W = 2600 s = 44 min. Therefore, the time to thaw is significant.

(2) Radiation may not be negligible. It depends on the temperature of the oven walls and the emissivity of the beef. Radiation would contribute to heating the beef.