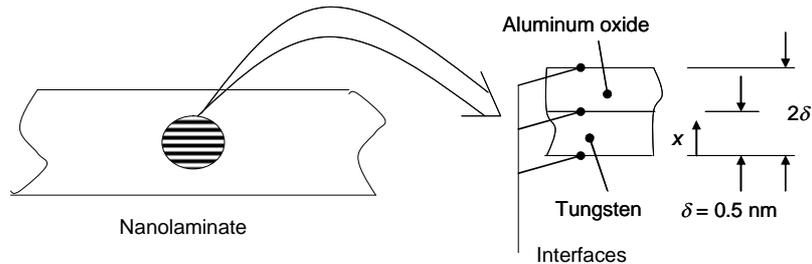


### PROBLEM 3.177

**KNOWN:** Thickness of alternating tungsten and aluminum oxide layers, interface thermal resistance, thermal conductivities of tungsten and aluminum oxide thin films.

**FIND:** (a) Effective thermal conductivity of the nanolaminate. Comparison with bulk thermal conductivities of aluminum oxide and tungsten, (b) Effective thermal conductivity of the nanolaminate using bulk values of the thermal conductivity of aluminum oxide and tungsten.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state, one-dimensional conditions, (2) Constant properties.

**PROPERTIES:** Table A.1, tungsten (300 K):  $k_T = 174 \text{ W/m}\cdot\text{K}$ . Table A.2, aluminum oxide (300 K):  $k_A = 36 \text{ W/m}\cdot\text{K}$ .

**ANALYSIS:** (a) Consider a unit cell consisting of one layer of aluminum oxide, one layer of tungsten, and two interfaces of unit cell thickness  $2\delta = 1.0 \text{ nm}$  as shown in the schematic. The sum of the thermal resistances is

$$\sum R_t'' = 2(R_{t,i}'') + \frac{\delta}{k_A} + \frac{\delta}{k_T} = 2 \times 3.85 \times 10^{-9} \frac{\text{m}^2 \cdot \text{K}}{\text{W}} + \frac{0.5 \times 10^{-9} \text{ m}}{1.65 \text{ W/m}\cdot\text{K}} + \frac{0.5 \times 10^{-9} \text{ m}}{6.10 \text{ W/m}\cdot\text{K}} = 8.08 \times 10^{-9} \frac{\text{m}^2 \cdot \text{K}}{\text{W}}$$

The effective thermal conductivity is

$$k_{\text{eff}} = \frac{L}{\sum R_t''} = \frac{2\delta}{\sum R_t''} = \frac{2 \times 0.5 \times 10^{-9} \text{ m}}{8.08 \times 10^{-9} \frac{\text{m}^2 \cdot \text{K}}{\text{W}}} = 0.123 \frac{\text{W}}{\text{m}\cdot\text{K}} <$$

The value of the effective thermal conductivity is  $0.123/174 \times 100 = 0.07\%$  that of bulk tungsten and  $0.123/36 \times 100 = 0.34\%$  that of bulk aluminum oxide.

(b) Repeating part (a) using  $k_T = 174 \text{ W/m}\cdot\text{K}$  and  $k_A = 36 \text{ W/m}\cdot\text{K}$  yields  $k_{\text{eff}} = 0.129 \frac{\text{W}}{\text{m}\cdot\text{K}} <$

**COMMENTS:** (1) The effective thermal conductivity is dominated by the interface resistances and is relatively insensitive to the thermal conductivity of the two materials. Although the interface resistance is very small compared to typical contact resistance values (see Table 3.2), by using extremely thin layer thicknesses, many such interfaces may be packed into the laminated structure, resulting in very small values of the effective or bulk thermal conductivity. The material service temperature would be limited to values less than approximately  $1000^\circ\text{C}$ , due to the tendency of the material to lose thermal stability at high temperatures and, in turn, lose its nanolaminated structure. (2) See Costescu, Cahill, Fabreguette, Sechrist, and George, "Ultra-Low Thermal Conductivity in  $\text{W}/\text{Al}_2\text{O}_3$  Nanolaminates," *Science*, Vol. 303, pp. 989 – 990, 2004, for additional information.