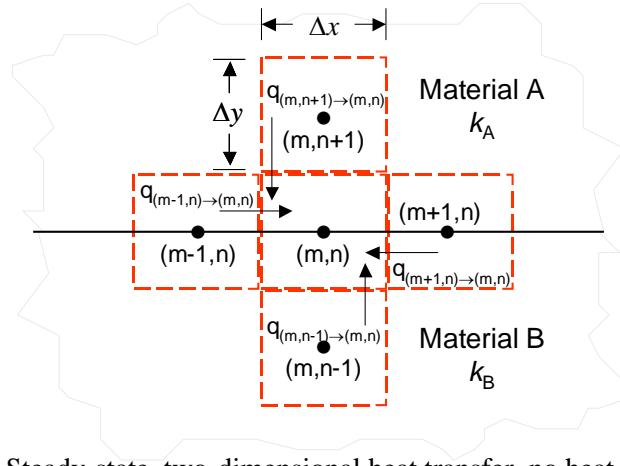


PROBLEM 4.42

KNOWN: Control volume and nodal configuration in the vicinity of the interface between two materials.

FIND: Expressions for control surface heat rates. Finite difference equation at node m,n.

SCHEMATIC:



ASSUMPTIONS: Steady-state, two-dimensional heat transfer, no heat generation, negligible contact resistance.

ANALYSIS: Conduction from Node (m,n+1) to Node (m,n) occurs exclusively in Material A. Therefore,

$$q_{(m,n+1) \rightarrow (m,n)} = k_A L \frac{\Delta x}{\Delta y} [T_{m,n+1} - T_{m,n}] \quad <$$

Likewise for conduction from Node (m,n-1) to Node (m,n),

$$q_{(m,n-1) \rightarrow (m,n)} = k_B L \frac{\Delta x}{\Delta y} [T_{m,n-1} - T_{m,n}] \quad <$$

Conduction from Node (m-1,n) to Node (m,n) occurs in both Material A and Material B. In Material A,

$$q_{A(m-1,n) \rightarrow (m,n)} = k_A L \frac{\Delta y/2}{\Delta x} [T_{m-1,n} - T_{m,n}]$$

Likewise for conduction in Material B,

$$q_{B(m-1,n) \rightarrow (m,n)} = k_B L \frac{\Delta y/2}{\Delta x} [T_{m-1,n} - T_{m,n}]$$

For both materials,

$$\begin{aligned} q_{(m-1,n) \rightarrow (m,n)} &= q_{A(m-1,n) \rightarrow (m,n)} + q_{B(m-1,n) \rightarrow (m,n)} \\ &= k_A L \frac{\Delta y/2}{\Delta x} [T_{m-1,n} - T_{m,n}] + k_B L \frac{\Delta y/2}{\Delta x} [T_{m-1,n} - T_{m,n}] \\ &= (k_A + k_B) L \frac{\Delta y/2}{\Delta x} [T_{m-1,n} - T_{m,n}] \quad < \end{aligned}$$

Similarly for conduction from Node (m+1,n) to (m,n),

$$q_{(m+1,n) \rightarrow (m,n)} = (k_A + k_B) L \frac{\Delta y/2}{\Delta x} [T_{m+1,n} - T_{m,n}] \quad <$$

Continued...

PROBLEM 4.42 (Cont.)

An energy balance on node m,n yields

$$q_{(m-1,n) \rightarrow (m,n)} + q_{(m+1,n) \rightarrow (m,n)} + q_{(m,n-1) \rightarrow (m,n)} + q_{(m,n+1) \rightarrow (m,n)} = 0$$

or

$$k_A \frac{\Delta x}{\Delta y} [T_{m,n+1} - T_{m,n}] + k_B \frac{\Delta x}{\Delta y} [T_{m,n-1} - T_{m,n}] + (k_A + k_B) \frac{\Delta y/2}{\Delta x} [T_{m-1,n} + T_{m+1,n} - 2T_{m,n}] = 0 \quad <$$

COMMENTS: How would you modify the analysis if the contact resistance is significant?