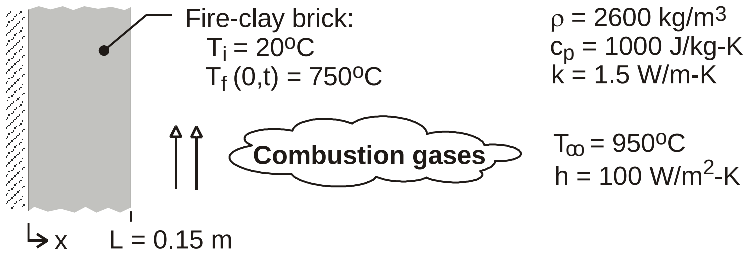
PROBLEM 5.51

KNOWN: Thickness, initial temperature and properties of furnace wall. Convection conditions at inner surface.

FIND: Time required for outer surface to reach a prescribed temperature. Corresponding temperature distribution in wall and at intermediate times.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in a plane wall, (2) Constant properties, (3) Adiabatic outer surface, (4) Fo > 0.2, (5) Negligible radiation from combustion gases.

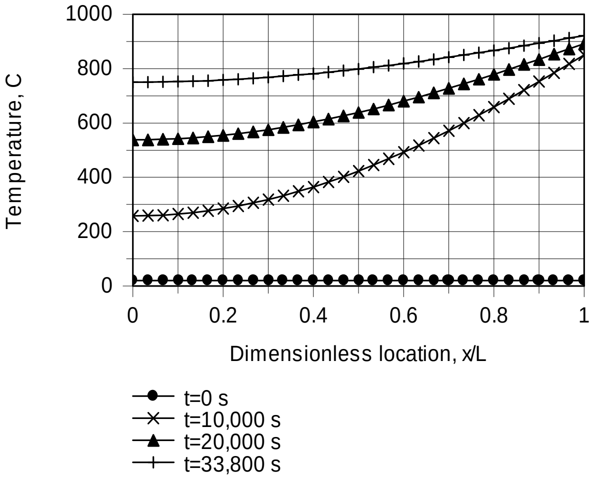
ANALYSIS: The wall is equivalent to one-half of a wall of thickness 2L with symmetric convection conditions at its two surfaces. With Bi = hL/k = 100 W/m^2·K × 0.15m/1.5 W/m·K = 10 and Fo > 0.2, the one-term approximation, Eq. 5.44 may be used to compute the desired time, where

$\theta_o^* = (T_o - T_\infty)/(T_i - T_\infty) = 0.215$. From Table 5.1, C_1 = 1.262 and ζ_1 = 1.4289. Hence,

$$\text{Fo} = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = -\frac{\ln(0.215/1.262)}{(1.4289)^2} = 0.867$$

$$t = \frac{\text{Fo }L^2}{\alpha} = \frac{0.867(0.15\text{m})^2}{\left(1.5\,\text{W/m}\cdot\text{K}/2600\text{ kg/m}^3 \times 1000\text{ J/kg}\cdot\text{K}\right)} = 33{,}800\,\text{s}$$ <

The corresponding temperature distribution, as well as distributions at t = 0, 10,000, and 20,000 s are plotted below



COMMENTS: Because Bi >>1, the temperature at the inner surface of the wall increases much more rapidly than at locations within the wall, where temperature gradients are large. The temperature gradients decrease as the wall approaches a steady-state for which there is a uniform temperature of 950°C.