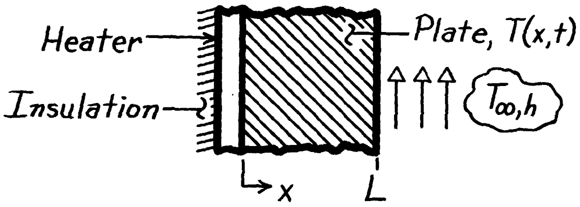
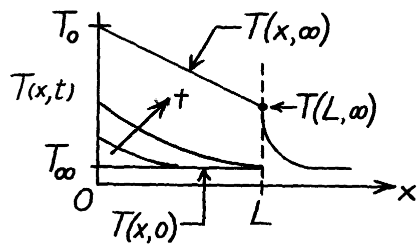
PROBLEM 5.1

KNOWN: Electrical heater attached to backside of plate while front surface is exposed to convection process (T_∞,h); initially plate is at a uniform temperature of the ambient air and suddenly heater power is switched on providing a constant q''_o.

FIND: (a) Sketch temperature distribution, T(x,t), (b) Sketch the heat flux at the outer surface, $q''_x(L,t)$ as a function of time.

SCHEMATIC:





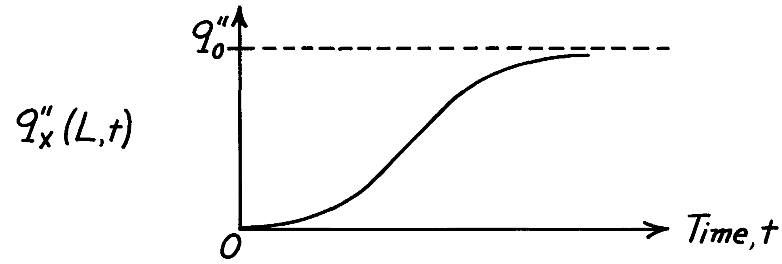
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible heat loss from heater through insulation.

ANALYSIS: (a) The temperature distributions for four time conditions including the initial distribution, T(x,0), and the steady-state distribution, T(x,∞), are as shown above.

Note that the temperature gradient at x = 0, $-dT/dx)_{x=0}$, for t > 0 will be a constant since the flux, $q''_x(0)$, is a constant. Noting that $T_o = T(0,\infty)$, the steady-state temperature distribution will be linear such that

$$q''_o = k\frac{T_o - T(L,\infty)}{L} = h\left[T(L,\infty) - T_\infty\right].$$

(b) The heat flux at the front surface, x = L, is given by $q''_x(L,t) = -k(dT/dx)_{x=L}$. From the temperature distribution, we can construct the heat flux-time plot.



COMMENTS: At early times, the temperature and heat flux at x = L will not change from their initial values. Hence, we show a zero slope for $q''_x(L,t)$ at early times. Eventually, the value of $q''_x(L,t)$ will reach the steady-state value which is q''_o.