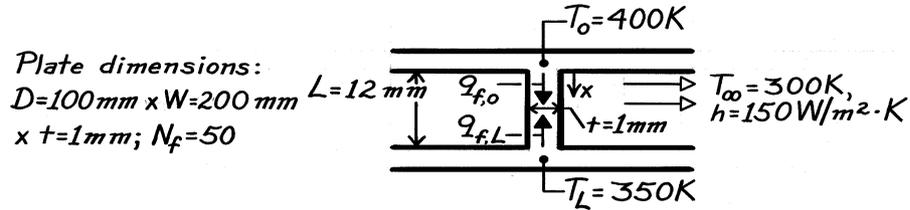


### PROBLEM 3.142

**KNOWN:** Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

**FIND:** (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Uniform  $h$ , (6) Negligible variation in  $T_\infty$ , (7) Negligible contact resistance.

**PROPERTIES:** Table A.1, Aluminum (pure), 375 K:  $k = 240\text{ W/m}\cdot\text{K}$ .

**ANALYSIS:** (a) The general solution for the temperature distribution in a fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions:  $\theta(0) = \theta_o = T_o - T_\infty$ ,  $\theta(L) = \theta_L = T_L - T_\infty$ .

Hence  $\theta_o = C_1 + C_2$   $\theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + (\theta_o - C_1) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \quad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}$$

Hence 
$$\theta(x) = \frac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \left[ e^{m(L-x)} - e^{-m(L-x)} \right] + \theta_L \left( e^{mx} - e^{-mx} \right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}$$

The fin heat transfer rate is then

$$q_f = -kA_c \frac{dT}{dx} = -kDt \left[ -\frac{\theta_o m}{\sinh mL} \cosh m(L-x) + \frac{\theta_L m}{\sinh mL} \cosh mx \right]$$

Hence 
$$q_{f,o} = kDt \left( \frac{\theta_o m}{\tanh mL} - \frac{\theta_L m}{\sinh mL} \right) <$$

$$q_{f,L} = kDt \left( \frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL} \right) <$$

Continued ...

**PROBLEM 3.142 (Cont.)**

$$(b) \quad m = \left( \frac{hP}{kA_c} \right)^{1/2} = \left( \frac{150 \text{ W/m}^2 \cdot \text{K} (2 \times 0.1 \text{ m} + 2 \times 0.001 \text{ m})}{240 \text{ W/m} \cdot \text{K} \times 0.1 \text{ m} \times 0.001 \text{ m}} \right)^{1/2} = 35.5 \text{ m}^{-1}$$

$$mL = 35.5 \text{ m}^{-1} \times 0.012 \text{ m} = 0.43$$

$$\sinh mL = 0.439 \quad \tanh mL = 0.401 \quad \theta_o = 100 \text{ K} \quad \theta_L = 50 \text{ K}$$

$$q_{f,o} = 240 \text{ W/m} \cdot \text{K} \times 0.1 \text{ m} \times 0.001 \text{ m} \left( \frac{100 \text{ K} \times 35.5 \text{ m}^{-1}}{0.401} - \frac{50 \text{ K} \times 35.5 \text{ m}^{-1}}{0.439} \right)$$

$$q_{f,o} = 115.4 \text{ W} \quad (\text{from the top plate})$$

$$q_{f,L} = 240 \text{ W/m} \cdot \text{K} \times 0.1 \text{ m} \times 0.001 \text{ m} \left( \frac{100 \text{ K} \times 35.5 \text{ m}^{-1}}{0.439} - \frac{50 \text{ K} \times 35.5 \text{ m}^{-1}}{0.401} \right)$$

$$q_{f,L} = 87.8 \text{ W}. \quad (\text{into the bottom plate})$$

Maximum power dissipations are therefore

$$q_{o,\max} = N_f q_{f,o} + (W - N_f t) Dh \theta_o$$

$$q_{o,\max} = 50 \times 115.4 \text{ W} + (0.200 - 50 \times 0.001) \text{ m} \times 0.1 \text{ m} \times 150 \text{ W/m}^2 \cdot \text{K} \times 100 \text{ K}$$

$$q_{o,\max} = 5770 \text{ W} + 225 \text{ W} = 5995 \text{ W} \quad <$$

$$q_{L,\max} = -N_f q_{f,L} + (W - N_f t) Dh \theta_o$$

$$q_{L,\max} = -50 \times 87.8 \text{ W} + (0.200 - 50 \times 0.001) \text{ m} \times 0.1 \text{ m} \times 150 \text{ W/m}^2 \cdot \text{K} \times 50 \text{ K}$$

$$q_{L,\max} = -4390 \text{ W} + 112 \text{ W} = -4278 \text{ W}. \quad <$$

**COMMENTS:** (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to  $\Delta T_\infty = 5 \text{ K}$ , its flowrate must be

$$\dot{m}_{\text{air}} = \frac{q_{\text{tot}}}{c_p \Delta T_\infty} = \frac{1717 \text{ W}}{1007 \text{ J/kg} \cdot \text{K} \times 5 \text{ K}} = 0.34 \text{ kg/s}.$$

Its mean velocity is then

$$V_{\text{air}} = \frac{\dot{m}_{\text{air}}}{\rho_{\text{air}} A_c} = \frac{0.34 \text{ kg/s}}{1.16 \text{ kg/m}^3 \times 0.012 \text{ m} (0.2 - 50 \times 0.001) \text{ m}} = 163 \text{ m/s}.$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s,  $A_c$  would have to be increased substantially by increasing  $W$  (and hence the space between fins) and by increasing  $L$ . The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

(2) A negative value of  $q_{L,\max}$  implies that the bottom plate must be cooled externally to maintain the plate at 350 K.