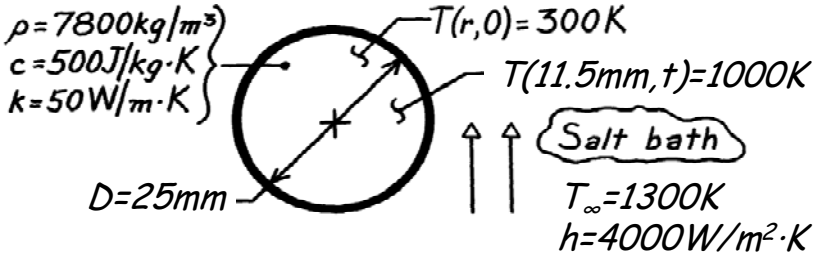
PROBLEM 5.71

KNOWN: A ball bearing is suddenly immersed in a molten salt bath; heat treatment to harden occurs at locations with T > 1000 K.

FIND: Time required to harden outer layer of 1mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo ≥ 0.2.

ANALYSIS: Since any location within the ball whose temperature exceeds 1000 K will be hardened, the problem is to find the time when the location r = 11.5 mm reaches 1000 K. Then a 1 mm outer layer will be hardened. Begin by finding the Biot number.

$$\mathrm{Bi} = \frac{h\, r_o}{k} = \frac{4000\ \mathrm{W/m^2 \cdot K}\left(0.025\ \mathrm{m}/2\right)}{50\ \mathrm{W/m \cdot K}} = 1.00.$$

Using the one-term approximate solution for a sphere, find

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2}\ln\left[\theta^* / C_1 \frac{1}{\zeta_1 r^*}\sin\left(\zeta_1 r^*\right)\right].$$

From Table 5.1 with Bi = 1.00, for the sphere find $\zeta_1 = 1.5708$ rad and $C_1 = 1.2732$. With r* = r/r_o = (11.5 mm/12.5 mm) = 0.92, substitute numerical values.

$$\mathrm{Fo} = \frac{-1}{(1.5708)^2}\ln\left[\frac{(1000-1300)\,\mathrm{K}}{(300-1300)\,\mathrm{K}} / 1.2732 \frac{1}{1.5708 \times 0.92}\sin\left(1.5708 \times 0.92\ \mathrm{rad}\right)\right] = 0.433.$$

From the definition of the Fourier number with $\alpha = k/\rho c$,

$$t = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo} \cdot r_o^2 \frac{\rho\, c}{k} = 0.433 \times \left[\frac{0.025\ \mathrm{m}}{2}\right]^2 7800 \frac{\mathrm{kg}}{\mathrm{m}^3} \times 500 \frac{\mathrm{J}}{\mathrm{kg \cdot K}} / 50\ \mathrm{W/m \cdot K} = 5.3\ \mathrm{s}. \quad <$$

COMMENTS: (1) Note the very short time required to harden the ball. At this time it can be easily shown the center temperature is T(0, 5.3 s) = 863 K.