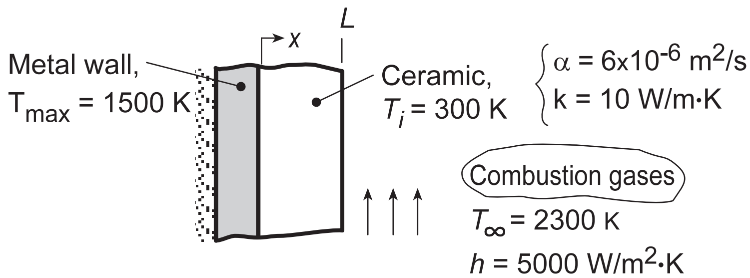
PROBLEM 5.55

KNOWN: Properties and thickness L of ceramic coating on rocket nozzle wall. Convection conditions. Initial temperature and maximum allowable wall temperature.

FIND: (a) Maximum allowable engine operating time, t_{max}, for L = 10 mm, (b) Coating inner and outer surface temperature histories for L = 10 and 40 mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in a plane wall, (2) Constant properties, (3) Negligible thermal capacitance of metal wall and heat loss through back surface, (4) Negligible contact resistance at wall/ceramic interface, (5) Negligible radiation.

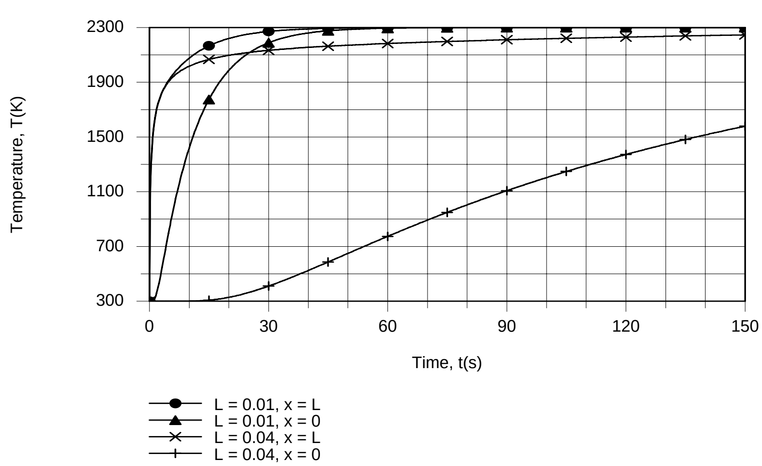
ANALYSIS: (a) Subject to assumptions (3) and (4), the maximum wall temperature corresponds to the ceramic temperature at x = 0. Hence, for the ceramic, we wish to determine the time t_{max} at which T(0,t) = T_o(t) = 1500 K. With Bi = hL/k = 5000 W/m^2·K(0.01 m)/10 W/m·K = 5, the lumped capacitance method cannot be used. Assuming Fo > 0.2, obtaining ζ_1 = 1.3138 and C_1 = 1.2402 from Table 5.1, and evaluating $\theta_o^* = \left(T_o - T_\infty\right)/\left(T_i - T_\infty\right)$ = 0.4, Equation 5.44 yields

$$Fo = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = -\frac{\ln\left(0.4/1.2402\right)}{\left(1.3138\right)^2} = 0.656$$

confirming the assumption of Fo > 0.2. Hence,

$$t_{max} = \frac{Fo\left(L^2\right)}{\alpha} = \frac{0.656\left(0.01\,m\right)^2}{6\times10^{-6}\,m^2/s} = 10.9\,s$$ <

(b) Using the IHT *Lumped Capacitance Model* for a *Plane Wall*, the inner and outer surface temperature histories were computed and are as follows:



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The increase in the inner ($x = 0$) surface temperature lags that of the outer surface, but within $t \approx 45s$ both temperatures are within a few degrees of the gas temperature for $L = 0.01$ m. For $L = 0.04$ m, the increased thermal capacitance of the ceramic slows the approach to steady-state conditions. The thermal response of the inner surface significantly lags that of the outer surface, and it is not until $t \approx 137s$ that the inner surface reaches 1500 K. At this time there is still a significant temperature difference across the ceramic, with $T(L,t_{max}) = 2240$ K.

COMMENTS: The allowable engine operating time increases with increasing thermal capacitance of the ceramic and hence with increasing L.