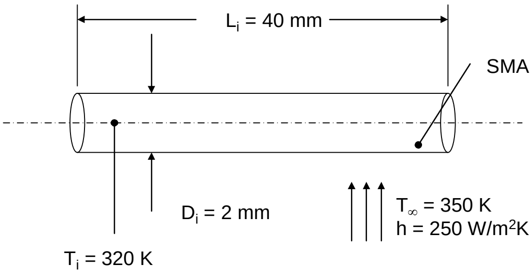
PROBLEM 5.31

KNOWN: Initial dimensions and temperature of SMA rod, ambient temperature and convection heat transfer coefficient. Properties of SMA.

FIND: Thermal response of the rod assuming constant and variable specific heats, time for rod temperature to experience 95% of the maximum temperature change.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance behavior, (2) Effect of change in density and dimensions is negligible, (3) Negligible radiation.

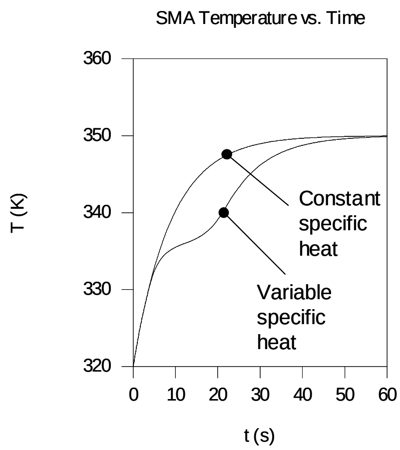
PROPERTIES: *Given*: $c = 500\dfrac{\text{J}}{\text{kg}\cdot\text{K}} + 3630\ \text{J/kg}\cdot\text{K} \times e^{\left(-0.808\text{K}^{-1}\times|T-336\text{K}|\right)}$, ρ = 8900 kg/m³, k = 23 W/m·K.

ANALYSIS: The Biot number associated with the rod (evaluating dimensions at the initial temperature) is $Bi = h(D_i/4)/k$ = 250 W/m²·K × 2 × 10^{-3} m/4/23W/m·K = 0.005 << 0.1. Therefore, the lumped capacitance approach is valid. Neglecting the change in the surface area as the rod is heated,

$$\frac{dT}{dt} = -\frac{hA_s(T-T_\infty)}{\rho Vc} = -\frac{4h}{\rho D_i c}(T-T_\infty)$$

$$= -\frac{4\times 250\text{W/m}^2\cdot\text{K}\times(T-T_\infty)}{8900\text{kg/m}^3\times 0.002\text{m}\times\left[500\dfrac{\text{J}}{\text{kg}\cdot\text{K}}+3630\ \text{J/kg}\cdot\text{K}\times e^{\left(-0.808\text{K}^{-1}\times|T-336\text{K}|\right)}\right]}$$

Because of the absolute value function, the preceding expression is most readily integrated numerically to determine $T(t)$. Based upon an initial temperature of T_i = 320 K, the following results are found using the IHT code included in the Comments.



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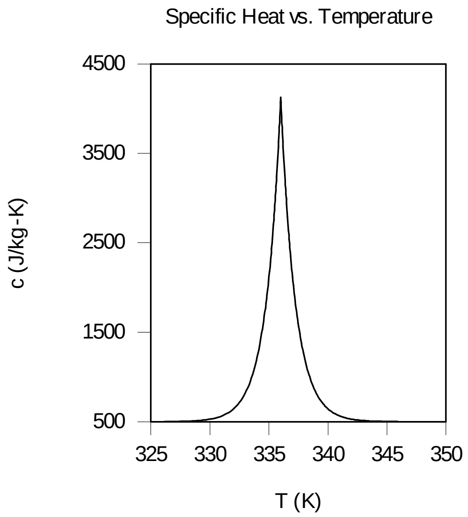
PROBLEM 5.31 (Cont.)

Inspection of the predicted response shows the time needed for the rod to experience 95% of its total temperature change (to $T_{95} = T_i + 0.95\times (T_f - T_i) = 320\text{ K} + 0.95 \times (350 - 320)\text{ K} = 348.5\text{ K}$) is 38.2 s and 27.1 s for the variable and constant specific heat cases, respectively. $<$

COMMENTS: (1) A plot of the specific heat is shown below. The onset of large specific heat values associated with the crystalline transformation slows the thermal response, as evident in the preceding plot. (2) The IHT code is listed below.

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// Solid Properties and Geometry

rho = 8900                      //kg/m^3
A= pi*D*L                       //m^2
D= 2/1000                       //m
L = 40/1000                     //m
V = L*pi*D*D/4                  //m^3
cpnot = 500                     //J/kgK

//Convective Conditions

Tinf = 350                      //K
h = 250                         //W/m^2K

//Variable Property Solution

der(Tvp,t) = h*A*(Tinf - Tvp)/rho/cpstar/V
cpstar = cpnot +( 9*10^3)*0.403*exp(-abs(0.808*(Tair-336)))


//Constant Property Solution

der(Tcp,t) = h*A*(Tinf - Tcp)/rho/cpnot/V
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