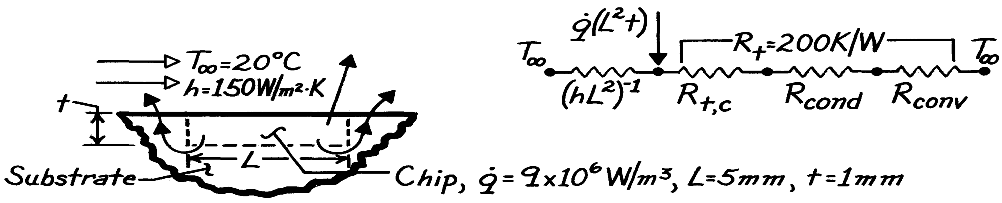
PROBLEM 5.28

KNOWN: Dimensions and operating conditions of an integrated circuit.

FIND: Steady-state temperature and time to come within 1°C of steady-state.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties.

PROPERTIES: Chip material (given): ρ = 2000 kg/m^3, c_p = 700 J/kg·K.

ANALYSIS: The direct and indirect paths for heat transfer from the chip to the coolant are in parallel, and the equivalent resistance is

$$R_{equiv} = \left[hL^2 + R_t^{-1}\right]^{-1} = \left[\left(3.75\times10^{-3} + 5\times10^{-3}\right)\text{W/K}\right]^{-1} = 114.3\ \text{K/W}.$$

The corresponding overall heat transfer coefficient is

$$U = \frac{\left(R_{equiv}\right)^{-1}}{L^2} = \frac{0.00875\ \text{W/K}}{(0.005\ \text{m})^2} = 350\ \text{W/m}^2\cdot\text{K}.$$

To obtain the steady-state temperature, apply conservation of energy to a control surface about the chip.

$$-\dot{E}_{out} + \dot{E}_g = 0 \qquad -UL^2\left(T_f - T_\infty\right) + \dot{q}\left(L^2\cdot t\right) = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{U} = 20^\circ\text{C} + \frac{9\times10^6\ \text{W/m}^3 \times 0.001\ \text{m}}{350\ \text{W/m}^2\cdot\text{K}} = 45.7^\circ\text{C}.$$ <

From the general lumped capacitance analysis, Equation 5.15 yields

$$\rho\left(L^2 t\right)c\frac{dT}{dt} = \dot{q}\left(L^2 t\right) - U\left(T - T_\infty\right)L^2.$$

With

$$a \equiv \frac{U}{\rho\ tc} = \frac{350\ \text{W/m}^2\cdot\text{K}}{\left(2000\ \text{kg/m}^3\right)(0.001\ \text{m})(700\ \text{J/kg}\cdot\text{K})} = 0.250\ \text{s}^{-1}$$

$$b = \frac{\dot{q}}{\rho\ c} = \frac{9\times10^6\ \text{W/m}^3}{\left(2000\ \text{kg/m}^3\right)(700\ \text{J/kg}\cdot\text{K})} = 6.429\ \text{K/s}$$

Equation 5.24 yields

$$\exp(-at) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{(44.7 - 20 - 25.7)\text{K}}{(20 - 20 - 25.7)\text{K}} = 0.0389$$

$$t = -\ln(0.0389)/0.250\ \text{s}^{-1} = 13.0\ \text{s}.$$ <

COMMENTS: Heat transfer through the substrate is comparable to that associated with direct convection to the coolant.