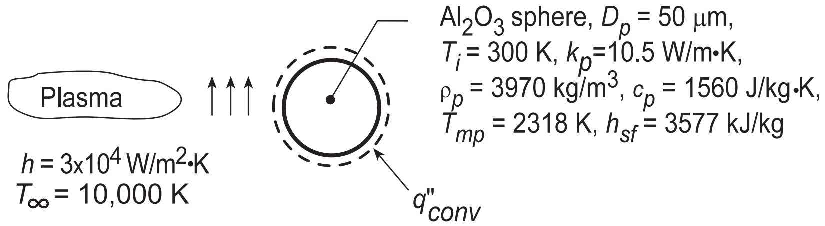
PROBLEM 5.25

KNOWN: Diameter and thermophysical properties of alumina particles. Convection conditions associated with a two-step heating process.

FIND: (a) Time-in-flight ($t_{i\text{-}f}$) required for complete melting, (b) Validity of assuming negligible radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Particle behaves as a lumped capacitance, (2) Negligible radiation, (3) Constant properties.

ANALYSIS: (a) The two-step process involves (i) the time t_1 to heat the particle to its melting point and (ii) the time t_2 required to achieve complete melting. Hence, $t_{i\text{-}f} = t_1 + t_2$, where from Eq. (5.5),

$$t_1 = \frac{\rho_p V c_p}{hA_s} \ln\frac{\theta_i}{\theta} = \frac{\rho_p D_p c_p}{6h} \ln\frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

$$t_1 = \frac{3970\,\text{kg/m}^3\left(50\times10^{-6}\,\text{m}\right)1560\,\text{J/kg}\cdot\text{K}}{6\left(30,000\,\text{W/m}^2\cdot\text{K}\right)} \ln\frac{(300 - 10,000)}{(2318 - 10,000)} = 4\times10^{-4}\,\text{s}$$

Performing an energy balance for the second step, we obtain

$$\int_{t_1}^{t_1+t_2} q_{conv}\,dt = \Delta E_{st}$$

where $q_{conv} = hA_s(T_\infty - T_{mp})$ and $\Delta E_{st} = \rho_p V h_{sf}$. Hence,

$$t_2 = \frac{\rho_p D_p}{6h}\frac{h_{sf}}{\left(T_\infty - T_{mp}\right)} = \frac{3970\,\text{kg/m}^3\left(50\times10^{-6}\,\text{m}\right)}{6\left(30,000\,\text{W/m}^2\cdot\text{K}\right)} \times \frac{3.577\times10^6\,\text{J/kg}}{(10,000 - 2318)\,\text{K}} = 5\times10^{-4}\,\text{s}$$

Hence $t_{i-f} = 9\times10^{-4}\,\text{s} \approx 1\,\text{ms}$ <

(b) Contrasting the smallest value of the convection heat flux, $q''_{conv,min} = h\left(T_\infty - T_{mp}\right) = 2.3\times10^8\,\text{W/m}^2$ to the largest radiation flux, $q''_{rad,max} = \varepsilon\sigma\left(T_{mp}^4 - T_{sur}^4\right)$ = 6.7 × 10^5 W/m^2, with $\varepsilon = 0.41$ from Table A.8 for aluminum oxide at 1500 K, and Tsur = 300 K we conclude that radiation is, in fact, negligible.

COMMENTS: (1) Since Bi = $(hr_p/3)/k \approx 0.02$, the lumped capacitance assumption is good. (2) In an actual application, the droplet should impact the substrate in a superheated condition ($T > T_{mp}$), which would require a slightly larger $t_{i\text{-}f}$.