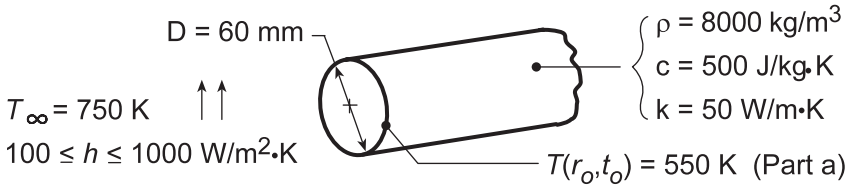
PROBLEM 5.60

KNOWN: Long rod with prescribed diameter and properties, initially at a uniform temperature, is heated in a forced convection furnace maintained at 750 K with a convection coefficient of h = 1000 W/m^2·K.

FIND: (a) The corresponding center temperature of the rod, T(0, t_o), when the surface temperature T(r_o, t_o) is measured as 550 K, (b) Effect of h on centerline temperature history.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction in rod, (2) Constant properties, (3) Rod, when initially placed in furnace, had a uniform (but unknown) temperature, (4) Fo ≥ 0.2.

ANALYSIS: (a) Since the rod was initially at a uniform temperature and Fo ≥ 0.2, the approximate solution for the infinite cylinder is appropriate. From Eq. 5.52b,

$$\theta^*\left(r^*, Fo\right) = \theta_o^*\left(Fo\right) J_0\left(\zeta_1 r^*\right) \tag{1}$$

where, for $r^* = 1$, the dimensionless temperatures are, from Eq. 5.34,

$$\theta^*\left(1, Fo\right) = \frac{T\left(r_o, t_o\right) - T_\infty}{T_i - T_\infty} \qquad \theta_o^*\left(Fo\right) = \frac{T\left(0, t_o\right) - T_\infty}{T_i - T_\infty} \tag{2,3}$$

Combining Eqs. (2) and (3) with Eq. (1) and rearranging,

$$\frac{T\left(r_o, t_o\right) - T_\infty}{T_i - T_\infty} = \frac{T\left(0, t_o\right) - T_\infty}{T_i - T_\infty} J_0\left(\zeta_1 \cdot 1\right)$$

$$T\left(0, t_o\right) = T_\infty + \frac{1}{J_0\left(\zeta_1\right)}\left[T\left(r_o, t_o\right) - T_\infty\right] \tag{4}$$

The eigenvalue, $\zeta_1 = 1.0185$ rad, follows from Table 5.1 for the Biot number

$$Bi = \frac{h r_o}{k} = \frac{1000\,W/m^2 \cdot K\left(0.060\,m/2\right)}{50\,W/m \cdot K} = 0.60\,.$$

From Table B-4, with $\zeta_1 = 1.0185$ rad, $J_0(1.0185) = 0.7568$. Hence, from Eq. (4)

$$T\left(0, t_o\right) = 750\,K + \frac{1}{0.7568}\left[550 - 750\right]K = 486\,K \qquad <$$

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following temperature histories were generated.

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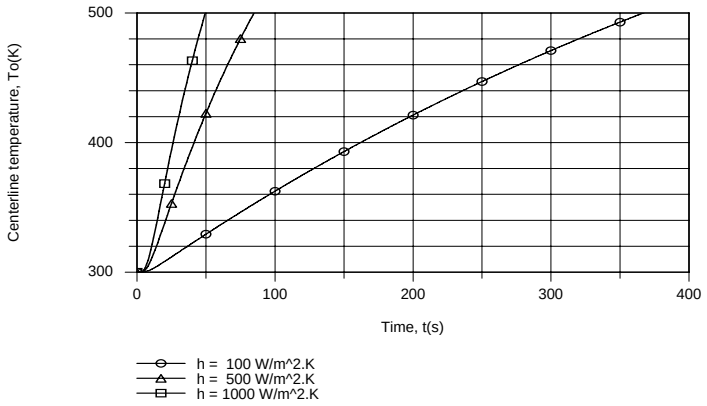

The times required to reach a centerline temperature of 500 K are 367, 85 and 51s, respectively, for h = 100, 500 and 1000 W/m^2·K. The corresponding values of the Biot number are 0.06, 0.30 and 0.60. Hence, even for h = 1000 W/m^2·K, the convection resistance is not negligible relative to the conduction resistance and significant reductions in the heating time could still be effected by increasing h to values considerably in excess of 1000 W/m^2·K.

COMMENTS: For Part (a), recognize why it is not necessary to know T_i or the time t_o. We require that Fo ≥ 0.2, which for this sphere corresponds to t ≥ 14s. For this situation, the time dependence of the surface and center are the same.