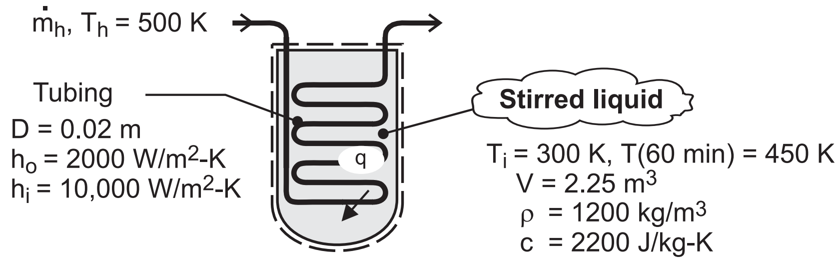
PROBLEM 5.19

KNOWN: Volume, density and specific heat of chemical in a stirred reactor. Temperature and convection coefficient associated with saturated steam flowing through submerged coil. Tube diameter and outer convection coefficient of coil. Initial and final temperatures of chemical and time span of heating process.

FIND: Required length of submerged tubing. Minimum allowable steam flowrate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible heat loss from vessel to surroundings, (3) Chemical is isothermal, (4) Negligible work due to stirring, (5) Negligible thermal energy generation (or absorption) due to chemical reactions associated with the batch process, (6) Negligible tube wall conduction resistance, (7) Negligible kinetic energy, potential energy, and flow work changes for steam.

ANALYSIS: Heating of the chemical can be treated as a transient, lumped capacitance problem, wherein heat transfer from the coil is balanced by the increase in thermal energy of the chemical. Hence, conservation of energy yields

$$\frac{dU}{dt} = \rho Vc\frac{dT}{dt} = UA_s\left(T_h - T\right)$$

Integrating, $$\int_{T_i}^{T}\frac{dT}{T_h - T} = \frac{UA_s}{\rho Vc}\int_o^t dt$$

$$-\ln\frac{T_h - T}{T_h - T_i} = \frac{UA_s t}{\rho Vc}$$

$$A_s = -\frac{\rho Vc}{Ut}\ln\frac{T_h - T}{T_h - T_i} \qquad (1)$$

$$U = \left(h_i^{-1} + h_o^{-1}\right)^{-1} = \left[\left(1/10{,}000\right) + \left(1/2000\right)\right]^{-1} \mathrm{W/m^2\cdot K}$$

$$U = 1670\,\mathrm{W/m^2\cdot K}$$

$$A_s = -\frac{\left(1200\,\mathrm{kg/m^3}\right)\left(2.25\mathrm{m^3}\right)\left(2200\,\mathrm{J/kg\cdot K}\right)}{\left(1670\,\mathrm{W/m^2\cdot K}\right)\left(3600\mathrm{s}\right)}\ln\frac{500-450}{500-300} = 1.37\mathrm{m^2}$$

$$L = \frac{A_s}{\pi D} = \frac{1.37\mathrm{m^2}}{\pi\left(0.02\mathrm{m}\right)} = 21.8\mathrm{m}$$ <

COMMENTS: Eq. (1) could also have been obtained by adapting Eq. (5.5) to the conditions of this problem, with T_∞ and h replaced by T_h and U, respectively.