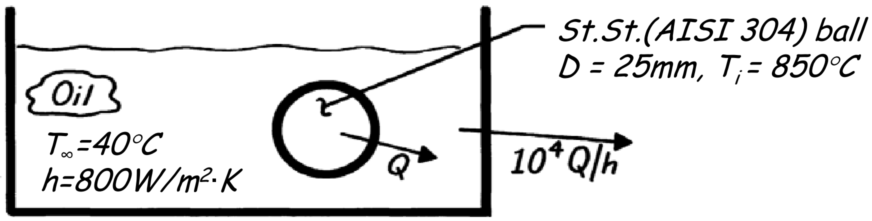
PROBLEM 5.74

KNOWN: Diameter and initial temperature of ball bearings to be quenched in an oil bath.

FIND: (a) Time required for surface to cool to 100°C and the corresponding center temperature, (b) Oil bath cooling requirements.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in ball bearings, (2) Constant properties.

PROPERTIES: *Table A-1,* St. St., AISI 304, (T ≈ 500°C): k = 22.2 W/m·K, c_p = 579 J/kg·K, ρ = 7900 kg/m^3, α = 4.85×10^{-6} m^2/s.

ANALYSIS: (a) To determine whether use of the lumped capacitance method is suitable, first compute

$$\mathrm{Bi} = \frac{h\left(r_o/3\right)}{k} = \frac{800\ \mathrm{W/m^2 \cdot K}\left(0.0125\ \mathrm{m}/3\right)}{22.2\ \mathrm{W/m \cdot K}} = 0.15.$$

We conclude that, although the lumped capacitance method could be used as a first approximation, the exact solution should be used in the interest of improving accuracy. We assume that the one-term approximation is valid and check later. Hence, with

$$\mathrm{Bi} = \frac{hr_o}{k} = \frac{800\ \mathrm{W/m^2 \cdot K}\left(0.0125\ \mathrm{m}\right)}{22.2\ \mathrm{W/m \cdot K}} = 0.450$$

from Table 5.1, ζ_1 = 1.1092, C_1 = 1.1301. Then

$$\theta^*(r^* = 1, \mathrm{Fo}) = \frac{T(r_o, t) - T_\infty}{T_i - T_\infty} = \frac{100°\mathrm{C} - 40°\mathrm{C}}{850°\mathrm{C} - 40°\mathrm{C}} = 0.0741$$

and Equation 5.53b can be solved for θ_o^*:

$$\theta_o^* = \theta^* \zeta_1 r^* / \sin(\zeta_1 r^*) = 0.0741 \times 1.1092 \times 1/\sin(1.1092) = 0.0918$$

Then Equation 5.53c can be solved for Fo:

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2} \ln\left(\theta_o^*/C_1\right) = -\frac{1}{1.1092^2} \ln\left(0.0918/1.1301\right) = 2.04$$

$$t = \frac{r_o^2 \mathrm{Fo}}{\alpha} = \frac{\left(0.0125\ \mathrm{m}\right)^2 \left(2.04\right)}{4.85 \times 10^{-6} \mathrm{m^2/s}} = 66\ \mathrm{s}. \qquad <$$

Note that the one-term approximation is accurate, since Fo > 0.2.

Continued …

PROBLEM 5.74 (Cont.)

Also,

$$\theta_o = T_o - T_\infty = 0.0918\left(T_i - T_\infty\right) = 0.0918\left(850 - 40\right) = 74^\circ C$$

$$T_o = 114^\circ C \qquad <$$

(b) Equation 5.55 can be used to calculate the heat loss from a single ball:

$$\frac{Q}{Q_o} = 1 - \frac{3\theta_o^*}{\zeta_1^3}\left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right] = 1 - \frac{3 \times 0.0918}{1.1092^3}\left[\sin(1.1092) - 1.1092 \cos(1.1092)\right] = 0.919$$

Hence, from Equation 5.47,

$$Q = 0.919\ \rho c_p V\left(T_i - T_\infty\right)$$

$$Q = 0.919 \times 7900\ \text{kg/m}^3 \times 579\ \text{J/kg} \cdot \text{K} \times \frac{\pi}{6}\left(0.025\ \text{m}\right)^3 \times 810^\circ C$$

$$Q = 2.79 \times 10^4\ \text{J}$$

is the amount of energy transferred from a single ball during the cooling process. Hence, the oil bath cooling rate must be

$$q = 10^4 Q/3600\ \text{s}$$

$$q = 7.74 \times 10^4 \text{W} = 77.4\ \text{kW}. \qquad <$$

COMMENTS: If the lumped capacitance method is used, the cooling time, obtained from Equation 5.5, would be t = 57 s, where the ball is assumed to be uniformly cooled to 100°C. This result, and the fact that T_o - $T(r_o)$ = 14°C at the conclusion, suggests that use of the lumped capacitance method would have been reasonable.