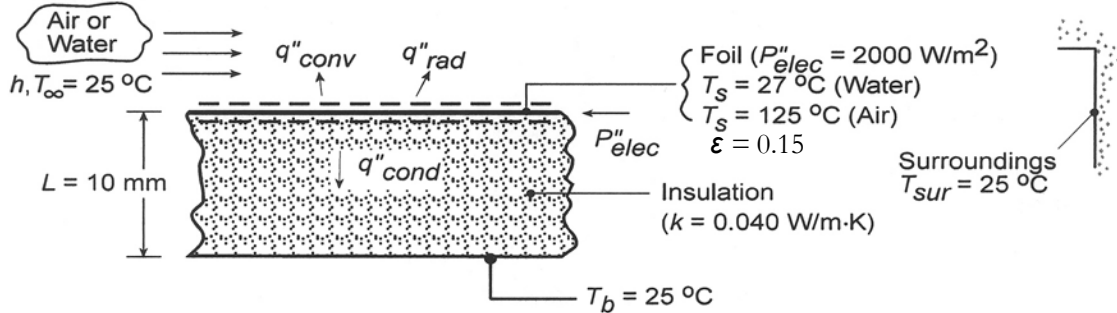


PROBLEM 3.9

KNOWN: Design and operating conditions of a heat flux gage.

FIND: (a) Convection coefficient for water flow ($T_s = 27^\circ\text{C}$) and error associated with neglecting conduction in the insulation, (b) Convection coefficient for air flow ($T_s = 125^\circ\text{C}$) and error associated with neglecting conduction and radiation, (c) Effect of convection coefficient on error associated with neglecting conduction for $T_s = 27^\circ\text{C}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant k .

ANALYSIS: (a) The electric power dissipation is balanced by convection to the water and conduction through the insulation. An energy balance applied to a control surface about the foil therefore yields

$$P''_{elec} = q''_{conv} + q''_{cond} = h(T_s - T_\infty) + k(T_s - T_b)/L$$

Hence,

$$h = \frac{P''_{elec} - k(T_s - T_b)/L}{T_s - T_\infty} = \frac{2000 \text{ W/m}^2 - 0.04 \text{ W/m} \cdot \text{K} (2 \text{ K}) / 0.01 \text{ m}}{2 \text{ K}}$$

$$h = \frac{(2000 - 8) \text{ W/m}^2}{2 \text{ K}} = 996 \text{ W/m}^2 \cdot \text{K}$$

If conduction is neglected, a value of $h = 1000 \text{ W/m}^2 \cdot \text{K}$ is obtained, with an attendant error of $(1000 - 996)/996 = 0.40\%$

(b) In air, energy may also be transferred from the foil surface by radiation, and the energy balance yields

$$P''_{elec} = q''_{conv} + q''_{rad} + q''_{cond} = h(T_s - T_\infty) + \epsilon \sigma (T_s^4 - T_{sur}^4) + k(T_s - T_b)/L$$

Hence,

$$h = \frac{P''_{elec} - \epsilon \sigma (T_s^4 - T_{sur}^4) - k(T_s - T_\infty)/L}{T_s - T_\infty}$$

$$= \frac{2000 \text{ W/m}^2 - 0.15 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (398^4 - 298^4) \text{ K}^4 - 0.04 \text{ W/m} \cdot \text{K} (100 \text{ K}) / 0.01 \text{ m}}{100 \text{ K}}$$

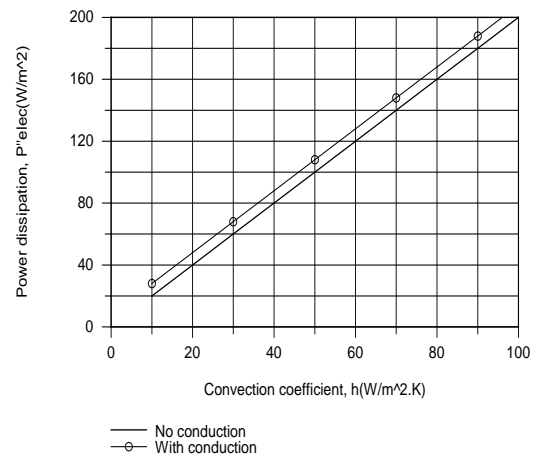
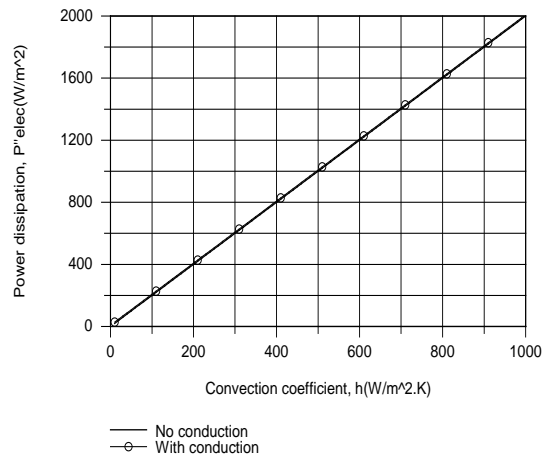
$$= \frac{(2000 - 146 - 400) \text{ W/m}^2}{100 \text{ K}} = 14.5 \text{ W/m}^2 \cdot \text{K}$$

Continued...

PROBLEM 3.9 (Cont.)

If conduction, radiation, or conduction and radiation are neglected, the corresponding values of h and the percentage errors are $18.5 \text{ W/m}^2\cdot\text{K}$ (27.6%), $16 \text{ W/m}^2\cdot\text{K}$ (10.3%), and $20 \text{ W/m}^2\cdot\text{K}$ (37.9%).

(c) For a fixed value of $T_s = 27^\circ\text{C}$, the conduction loss remains at $q''_{\text{cond}} = 8 \text{ W/m}^2$, which is also the fixed difference between P''_{elec} and q''_{conv} . Although this difference is not clearly shown in the plot for $10 \leq h \leq 1000 \text{ W/m}^2\cdot\text{K}$, it is revealed in the subplot for $10 \leq 100 \text{ W/m}^2\cdot\text{K}$.



Errors associated with neglecting conduction decrease with increasing h from values which are significant for small h ($h < 100 \text{ W/m}^2\cdot\text{K}$) to values which are negligible for large h .

COMMENTS: In liquids (large h), it is an excellent approximation to neglect conduction and assume that all of the dissipated power is transferred to the fluid.