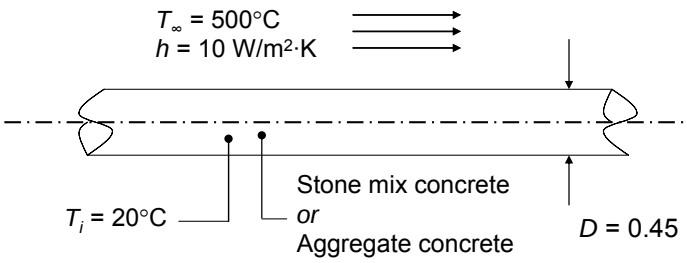
PROBLEM 5.65

KNOWN: Dimensions and initial temperature of stone mix concrete beam. Ambient temperature and convection heat transfer coefficient. Properties of aggregate beam.

FIND: Centerline temperature after t = 5 hours for each beam.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties. (2) Negligible radiation. (3) Beam is an infinite cylinder.

PROPERTIES: Table A.3: Stone mix concrete; k = 1.4 W/m·K, ρ = 2300 kg/m^3, c_p = 880 J/kg·K. Problem statement: Aggregate concrete: k = 0.789 W/m·K, ρ = 1495 kg/m^3, c_p = 880 J/kg·K.

ANALYSIS: (a) To determine whether spatial effects are important, the Biot number is calculated in the conservative fashion

$$Bi = \frac{hr_o}{k} = \frac{hD}{2k} = \frac{10\ \text{W/m}^2\cdot\text{K}\times 0.45\ \text{m}}{2\times 1.4\ \text{W/m}\cdot\text{K}} = 1.61$$

The dimensionless time is

$$Fo = \frac{\alpha t}{r_o^2} = \frac{4kt}{\rho c D^2} = \frac{4\times 1.4\ \text{W/m}\cdot\text{K}\times 5\text{h}\times 60\text{min/h}\times 60\text{s/min}}{2300\ \text{kg/m}^3\times 880\ \text{J/kg}\cdot\text{K}\times(0.45\ \text{m})^2} = 0.25$$

Since Bi > 0.1, spatial effects are important. Because Fo > 0.2, the approximate solution of Section 5.6 is valid. From Table 5.1 ζ_1 = 1.47 and C_1 = 1.29. Therefore,

$$\begin{aligned} T &= (T_i - T_\infty)C_1\exp\left(-\zeta_1^2 Fo\right) + T_\infty \\ &= (20°\text{C} - 500°\text{C})\times 1.29\times\exp\left(-1.47^2\times 0.25\right) + 500°\text{C} \\ &= 139°\text{C} \end{aligned}$$ <

(b) The preceding calculations may be repeated for the aggregate concrete, yielding

Bi = 2.85, Fo = 0.21, ζ_1 = 1.76, C_1 = 1.41, T = 147°C <

COMMENTS: (1) Because both its thermal conductivity and density are small relative to the stone mix beam, the thermal diffusivity of the aggregate beam is approximately the same as that of the stone mix beam. Hence the Fourier numbers associated with the two materials are approximately equal. (2) Aggregate concrete is often preferred over a more dense concrete for fire protection purposes.