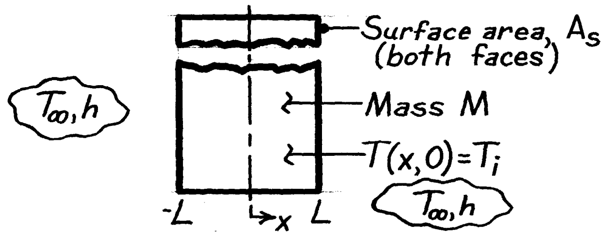
PROBLEM 5.4

KNOWN: Plate initially at a uniform temperature T_i is suddenly subjected to convection process (T_∞, h) on both surfaces. After elapsed time t_o, plate is insulated on both surfaces.

FIND: (a) Assuming Bi >> 1, sketch on T - x coordinates: initial and steady-state $(t \to \infty)$ temperature distributions, $T(x,t_o)$ and distributions for two intermediate times $t_o < t < \infty$, (b) Sketch on T - t coordinates midplane and surface temperature histories, (c) Repeat parts (a) and (b) assuming Bi << 1, and (d) Obtain expression for $T(x,\infty) = T_f$ in terms of plate parameters (M, c_p), thermal conditions (T_i, T_∞, h), surface temperature $T(L,t)$ and heating time t_o.

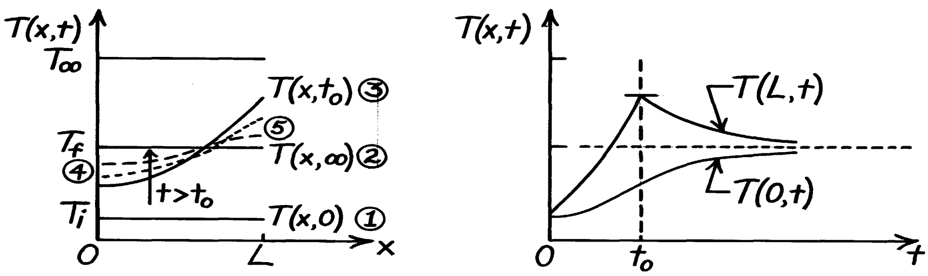
SCHEMATIC:

Time, t	Process
$t = 0$	Uniform T_i
$0 - t_o$	Heating, (T_∞, h)
$t > t_o$	Insulated



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal generation, (4) Plate is perfectly insulated for $t > t_o$, (5) $T(0, t < t_o) < T_\infty$.

ANALYSIS: (a,b) With Bi >> 1, appreciable temperature gradients exist in the plate following exposure to the heating process.



On *T-x* coordinates: (1) initial, uniform temperature, (2) steady-state conditions when $t \to \infty$, (3) distribution at t_o just before plate is covered with insulation, (4) gradients are always zero (symmetry), and (5) when $t > t_o$ (dashed lines) gradients approach zero everywhere.

(c) If Bi << 1, plate is space-wise isothermal (no gradients). On *T-x* coordinates, the temperature distributions are flat; on T-t coordinates, $T(L,t) = T(0,t)$.

(d) The conservation of energy requirement for the interval of time $\Delta t = t_o$ is

$$E_{in} - E_{out} = \Delta E = E_{final} - E_{initial} \qquad 2\int_0^{t_o} hA_s\left[T_\infty - T(L,t)\right]dt - 0 = Mc_p\left(T_f - T_i\right)$$

where E_{in} is due to convection heating over the period of time $t = 0 \to t_o$. With knowledge of $T(L,t)$, this expression can be integrated and a value for T_f determined.