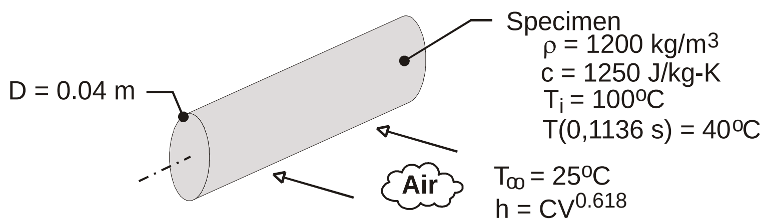
PROBLEM 5.69

KNOWN: Initial temperature, density, specific heat and diameter of cylindrical rod. Convection coefficient and temperature of air flow. Time for centerline to reach a prescribed temperature. Dependence of convection coefficient on flow velocity.

FIND: (a) Thermal conductivity of material, (b) Effect of velocity and centerline temperature and temperature histories for selected velocities.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance analysis can not be used but one-term approximation for an infinite cylinder is appropriate, (2) One-dimensional conduction in r, (3) Constant properties, (4) Negligible radiation, (5) Negligible effect of thermocouple hole on conduction.

ANALYSIS: (a) With $\theta_o^* = [T_o(0,1136s) - T_\infty]/(T_i - T_\infty) = (40 - 25)/(100 - 25) = 0.20$, Eq. 5.52c yields

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\ t}{\rho c_p r_o^2} = \frac{k(1136\,s)}{1200\ kg/m^3 \times 1250\,J/kg\cdot K \times (0.02\,m)^2} = -\ln\left(0.2/C_1\right)/\zeta_1^2 \qquad (1)$$

Because C_1 and ζ_1 depend on $Bi = hr_o/k$, a trial-and-error procedure must be used. For example, a value of k may be assumed and used to calculate Bi, which may then be used to obtain C_1 and ζ_1 from Table 5.1. Substituting C_1 and ζ_1 into Eq. (1), k may be computed and compared with the assumed value. Iteration continues until satisfactory convergence is obtained, with

$$k \approx 0.30\ W/m\cdot K$$ <

and, hence, $Bi = 3.67$, $C_1 = 1.45$, $\zeta_1 = 1.87$ and $Fo = 0.568$. For the above value of k, $-\ln\left(0.2/C_1\right)/\zeta_1^2 = 0.567$, which equals the Fourier number, as prescribed by Eq. (1).

(b) With $h = 55\ W/m^2\cdot K$ for $V = 6.8$ m/s, $h = CV^{0.618}$ yields a value of $C = 16.8\ W\cdot s^{0.618}/m^{2.618}\cdot K$. The desired variations of the centerline temperature with velocity (for t = 1136 s) and time (for V = 3, 10 and 20 m/s) are as follows:

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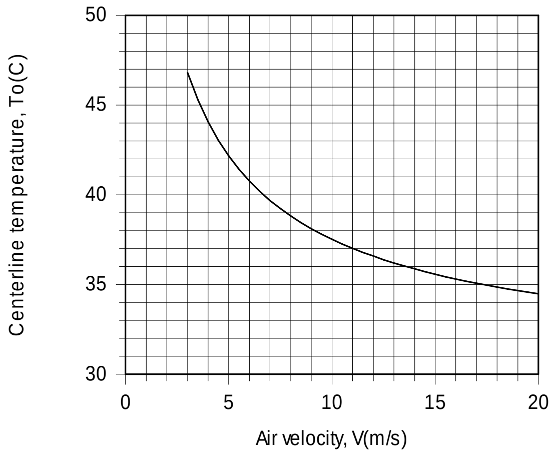
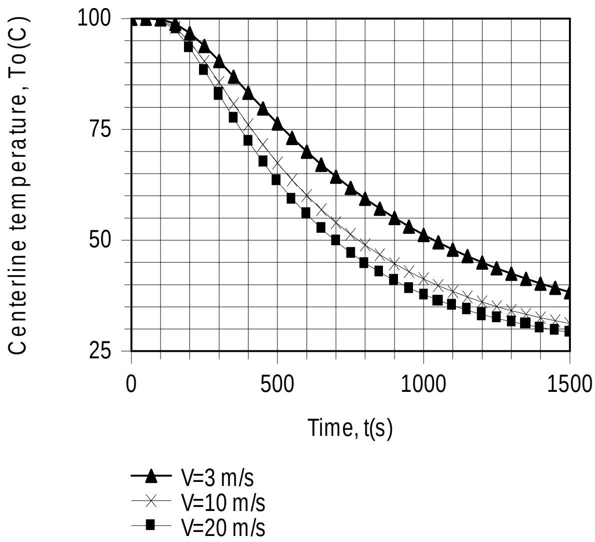
PROBLEM 5.69 (Cont.)





With increasing V from 3 to 20 m/s, h increases from 33 to 107 $W/m^2 \cdot K$, and the enhanced cooling reduces the centerline temperature at the prescribed time. The accelerated cooling associated with increasing V is also revealed by the temperature histories, and the time required to achieve thermal equilibrium between the air and the cylinder decreases with increasing V.

COMMENTS: (1) For the smallest value of h = 33 $W/m^2 \cdot K$, $Bi \equiv h\,(r_o/2)/k = 1.1 >> 0.1$, and use of the lumped capacitance method is clearly inappropriate.

(2) The *IHT* Transient Conduction Model for a cylinder was used to perform the calculations of Part (b). Because the model is based on the exact solution, Eq. 5.50a, it is accurate for values of $Fo < 0.2$, as well as $Fo > 0.2$. Although in principle, the model may be used to calculate the thermal conductivity for the conditions of Part (a), convergence is elusive and may only be achieved if the initial guesses are close to the correct results.