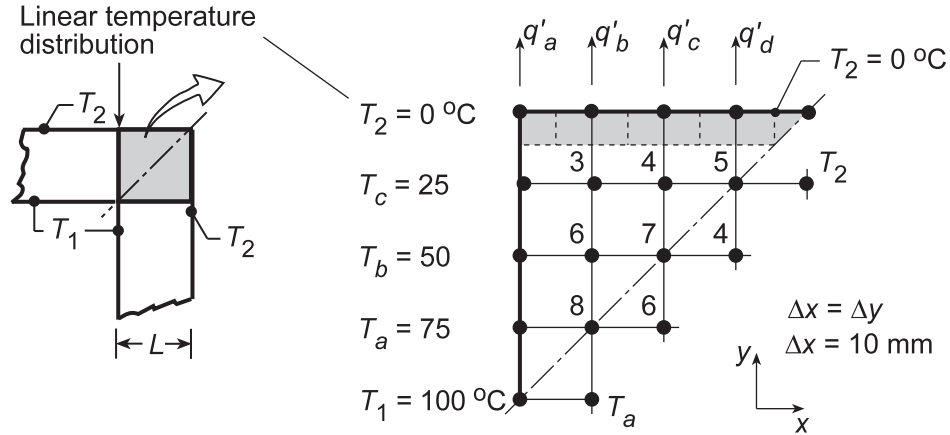


PROBLEM 4.76

KNOWN: Edge of adjoining walls ($k = 1 \text{ W/m}\cdot\text{K}$) represented by symmetrical element bounded by the diagonal symmetry adiabat and a section of the wall thickness over which the temperature distribution is assumed to be linear.

FIND: (a) Temperature distribution, heat rate and shape factor for the edge using the nodal network with $\Delta x = \Delta y = 10 \text{ mm}$; compare shape factor result with that from Table 4.1; (b) Assess the validity of assuming linear temperature distributions across sections at various distances from the edge.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Constant properties, and (3) Linear temperature distribution at specified locations across the section.

ANALYSIS: (a) Taking advantage of symmetry along the adiabat diagonal, all the nodes may be treated as interior nodes. Across the left-hand boundary, the temperature distribution is specified as linear. The finite-difference equations required to determine the temperature distribution, and hence the heat rate, can be written by inspection.

$$T_3 = 0.25(T_2 + T_4 + T_6 + T_c)$$

$$T_4 = 0.25(T_2 + T_5 + T_7 + T_3)$$

$$T_5 = 0.25(T_2 + T_2 + T_4 + T_4)$$

$$T_6 = 0.25(T_3 + T_7 + T_8 + T_b)$$

$$T_7 = 0.25(T_4 + T_4 + T_6 + T_6)$$

$$T_8 = 0.25(T_6 + T_6 + T_a + T_a)$$

The heat rate for both surfaces of the edge is

$$q'_{\text{tot}} = 2[q'_a + q'_b + q'_c + q'_d]$$

$$q'_{\text{tot}} = 2\left[k(\Delta x/2)(T_c - T_2)/\Delta y + k\Delta x(T_3 - T_2)/\Delta y + k\Delta x(T_4 - T_2)/\Delta y + k\Delta x(T_5 - T_2)/\Delta x\right]$$

The shape factor for the full edge is defined as

$$q'_{\text{tot}} = kS'(T_1 - T_2)$$

Solving the above equation set in IHT, the temperature ($^{\circ}\text{C}$) distribution is

Continued...

PROBLEM 4.76 (Cont.)

| | | | | |
|-----|-------|------|------|---|
| 0 | 0 | 0 | 0 | 0 |
| 25 | 18.75 | 12.5 | 6.25 | |
| 50 | 37.5 | 25.0 | | |
| 75 | 56.25 | | | |
| 100 | | | | |

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and the heat rate and shape factor are

$$q'_{\text{tot}} = 100 \text{ W/m} \quad S = 1$$

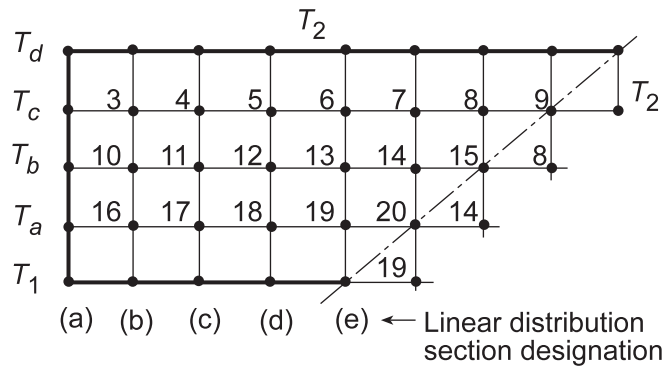
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From Table 4.1, the edge shape factor is 0.54, considerably below our estimate from this coarse grid analysis.

(b) The effect of the linear temperature distribution on the shape factor estimate can be explored using a more extensive grid as shown below. The FDE analysis was performed with the linear distribution imposed as the different sections a, b, c, d, e. Following the same approach as above, find

| | | | | | |
|--|-------|-------|-------|-------|------|
| <i>Location of linear distribution</i> | (a) | (b) | (c) | (d) | (e) |
| <i>Shape factor, S</i> | 0.797 | 0.799 | 0.809 | 0.857 | 1.00 |

The shape factor estimate decreases as the imposed linear temperature distribution section is located further from the edge. We conclude that assuming the temperature distribution across the section directly at the edge is a poor-one.



COMMENTS: The grid spacing for this analysis is quite coarse making the estimates in poor agreement with the Table 4.1 result. However, the analysis does show the effect of positioning the linear temperature distribution condition.