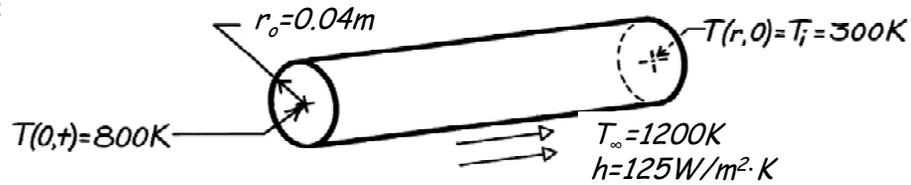


### PROBLEM 5.15

**KNOWN:** Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

**FIND:** Time required for shaft centerline to reach a prescribed temperature.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional, radial conduction, (2) Constant properties.

**PROPERTIES:** AISI 1010 carbon steel, *Table A.1* ( $\bar{T} = 550 \text{ K}$ ):  $\rho = 7832 \text{ kg/m}^3$ ,  $k = 51.2 \text{ W/m} \cdot \text{K}$ ,  $c = 541 \text{ J/kg} \cdot \text{K}$ ,  $\alpha = 1.21 \times 10^{-5} \text{ m}^2/\text{s}$ .

**ANALYSIS:** The Biot number is

$$Bi = \frac{hr_o/2}{k} = \frac{125 \text{ W/m}^2 \cdot \text{K} (0.04 \text{ m}/2)}{51.2 \text{ W/m} \cdot \text{K}} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hAs}{\rho Vc}\right)t\right] = \exp\left[-\frac{4h}{\rho cD}t\right]$$

$$\ln\left(\frac{800 - 1200}{300 - 1200}\right) = -0.811 = -\frac{4 \times 125 \text{ W/m}^2 \cdot \text{K}}{7832 \text{ kg/m}^3 (541 \text{ J/kg} \cdot \text{K}) 0.08 \text{ m}} t$$

$$t = 550 \text{ s.} \quad \leftarrow$$

**COMMENTS:** To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.52c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1 \exp(-\zeta_1^2 Fo)$$

For  $Bi = hr_o/k = 0.0976$ , Table 5.1 yields  $\zeta_1 = 0.436$  and  $C_1 = 1.024$ . Hence

$$\frac{-(0.436)^2 (1.2 \times 10^{-5} \text{ m}^2/\text{s})}{(0.04 \text{ m})^2} t = \ln(0.434) = -0.835$$

$$t = 585 \text{ s.}$$

The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.