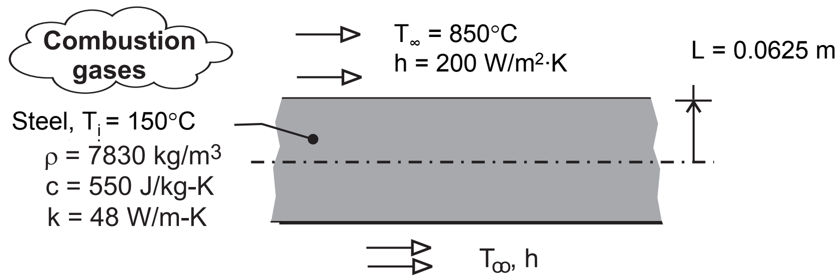
# PROBLEM 5.49

**KNOWN:** Thickness, properties and initial temperature of steel slab. Convection conditions.

**FIND:** Heating time required to achieve a minimum temperature of 500°C in the slab.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Negligible radiation effects, (3) Constant properties.

**ANALYSIS:** With a Biot number of hL/k = (200 W/m$^2$·K × 0.0625m)/48 W/m·K = 0.260, a lumped capacitance analysis should not be performed. At any time during heating, the lowest temperature in the slab is at the midplane, and from the one-term approximation to the transient thermal response of a plane wall, Eq. (5.44), we obtain

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{(500-850)°C}{(150-850)°C} = 0.50 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

With $\zeta_1 \approx 0.488$ rad and $C_1 \approx 1.0396$ from Table 5.1 and $\alpha = k/\rho c = 1.115 \times 10^{-5} m^2/s$,

$$-\zeta_1^2\left(\alpha t/L^2\right) = \ln(0.481) = -0.732$$

$$t = \frac{0.732\, L^2}{\zeta_1^2 \alpha} = \frac{0.732(0.0625m)^2}{(0.488)^2 \, 1.115 \times 10^{-5} m^2/s} = 1080s$$ <

**COMMENTS:** The surface temperature at t = 1080s may be obtained from Eq. (5.43b), where $\theta^* = \theta_o^* \cos\left(\zeta_1 x^*\right) = 0.50 \cos(0.488 \text{ rad}) = 0.442$. Hence, $T(L, 1080s) \equiv T_s = T_\infty + 0.442(T_i - T_\infty)$ $= 850°C - 309°C = 541°C$. Assuming a surface emissivity of ε = 1 and surroundings that are at $T_{sur} = T_\infty = 850°C$, the radiation heat transfer coefficient corresponding to this surface temperature is $h_r = \varepsilon\sigma(T_s + T_{sur})\left(T_s^2 + T_{sur}^2\right) = 211 W/m^2 \cdot K$. Since this value is comparable to the convection coefficient, radiation is not negligible and the desired heating will occur well before t = 1080s.