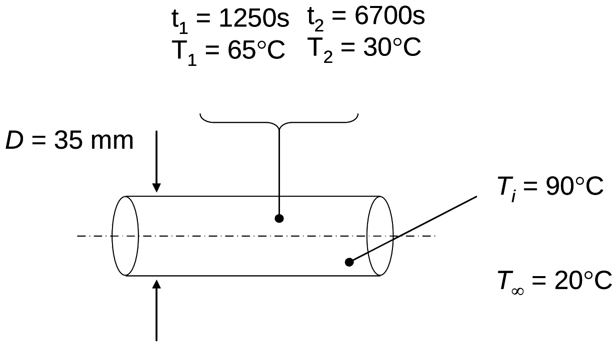
PROBLEM 5.38

KNOWN: Diameter of highly polished aluminum rod. Temperature of rod initially and at two later times. Room air temperature.

FIND: Values of constants C and n in Equation 5.26. Plot rod temperature vs. time for varying and constant heat transfer coefficients.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Radiation negligible because rod is highly polished, (3) Lumped capacitance approximation is valid.

PROPERTIES: *Table A.1*, Aluminum (T = 328 K): c = 916 J/kg·K, ρ = 2702 kg/m^3, k = 238 W/m·K.

ANALYSIS: If the heat transfer coefficient is given by Equation 5.26, then the temperature as a function of time is given by Equation 5.28:

$$\frac{\theta}{\theta_i} = \left[\frac{nCA_{s,c}\theta_i^n}{\rho Vc}t + 1\right]^{-1/n} \tag{1}$$

where $\theta = T - T_\infty$ and $A_{s,c}$ is the area exposed to convection, $A_{s,c} = \pi DL$. Since the rod temperature is known at two different times, Equation (1) can be evaluated at these two times, making it possible to solve for the two unknowns, C and n. The two equations are

$$\frac{\theta_1}{\theta_i} = \left[\frac{nCA_{s,c}\theta_i^n}{\rho Vc}t_1 + 1\right]^{-1/n} \qquad \frac{\theta_2}{\theta_i} = \left[\frac{nCA_{s,c}\theta_i^n}{\rho Vc}t_2 + 1\right]^{-1/n} \tag{2a,b}$$

These equations cannot be explicitly solved for C and n. They can be numerically solved in this form, using IHT or some other software, or they can be further manipulated to solve for the times:

$$t_1 = \left[\left(\frac{\theta_1}{\theta_i}\right)^{-n} - 1\right]\frac{\rho cD}{4nC\theta_i^n} \qquad t_2 = \left[\left(\frac{\theta_2}{\theta_i}\right)^{-n} - 1\right]\frac{\rho cD}{4nC\theta_i^n} \tag{3a,b}$$

where we have used $V/A_{s,c} = D/4$. Taking the ratio of Equations (3a) and (3b) yields

$$\frac{t_2}{t_1} = \frac{\left(\frac{\theta_2}{\theta_i}\right)^{-n} - 1}{\left(\frac{\theta_1}{\theta_i}\right)^{-n} - 1} \qquad \frac{6700\text{ s}}{1250\text{ s}} = \frac{\left(\frac{30°\text{C} - 20°\text{C}}{90°\text{C} - 20°\text{C}}\right)^{-n} - 1}{\left(\frac{65°\text{C} - 20°\text{C}}{90°\text{C} - 20°\text{C}}\right)^{-n} - 1} \qquad 5.36 = \frac{(0.143)^{-n} - 1}{(0.643)^{-n} - 1} \tag{4a,b,c}$$

Continued...

PROBLEM 5.38 (Cont.)

This can be iteratively or numerical solved for n, to find n = 0.25. Then C can be determined from Equation (3a) or (3b):

$$C = \left[\left(\frac{\theta_1}{\theta_i}\right)^{-n} - 1\right]\frac{\rho c D}{4n\theta_i^n t_1} = \left[\left(\frac{45°\text{C}}{70°\text{C}}\right)^{-0.25} - 1\right]\frac{2702\ \text{kg/m}^3 \times 916\ \text{J/kg}\cdot\text{K} \times 0.035\ \text{m}}{4 \times 0.25 \times (70°\text{C})^{0.25} \times 1250\ \text{s}} = 2.8\ \text{W/m}^2\cdot\text{K}^{1.25}$$

$$C = 2.8\ \text{W/m}^2\cdot\text{K}^{1.25},\ n = 0.25$$ <

Now that these constants are known, the validity of the lumped capacitance approximation can checked. The maximum heat transfer coefficient occurs at the initial time,

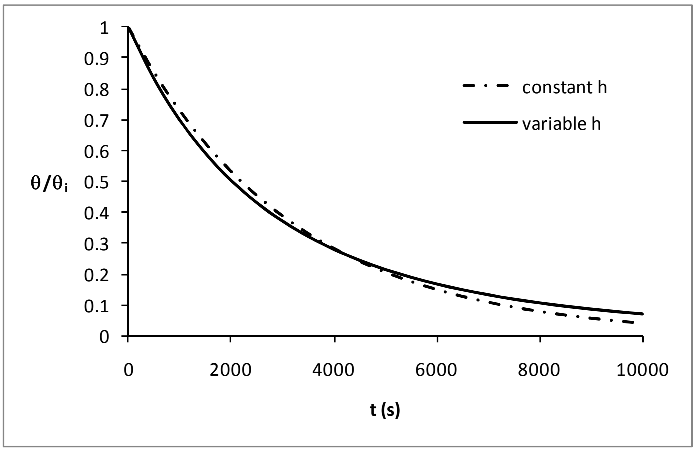
$$h = C(T - T_\infty)^n = 2.8\ \text{W/m}^2\cdot\text{K}^{1.25}\left[(90-20)\text{K}\right]^{0.25} = 8.1\ \text{W/m}^2\cdot\text{K}$$

Thus, using the conservative definition, $Bi = hD/2k = 6 \times 10^{-4}$. The lumped capacitance approximation is valid.

The heat transfer coefficient corresponding to a rod temperature of $\overline{T} = (T_i + T_\infty)/2 = 55°\text{C}$ is

$$h = C(T - T_\infty)^n = 2.8\ \text{W/m}^2\cdot\text{K}^{1.25}\left[(55-20)\text{K}\right]^{0.25} = 6.8\ \text{W/m}^2\cdot\text{K}$$

The plot below shows the rod temperature as a function of time using Equation (1) above for variable heat transfer coefficient, as well as the rod temperature assuming the constant value of h = 6.8 W/m^2·K, using text Equation 5.6.



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COMMENTS: (1) Since the heat transfer coefficient is temperature difference-dependent (variable h), the initial cooling rates are larger when this dependence is accounted for. As the temperature difference decreases, the variable h case cools slower relative to the constant h case. (2) The discrepancy between the variable and constant heat transfer coefficient cases is not large under these conditions. The difference would be greater if n were larger.