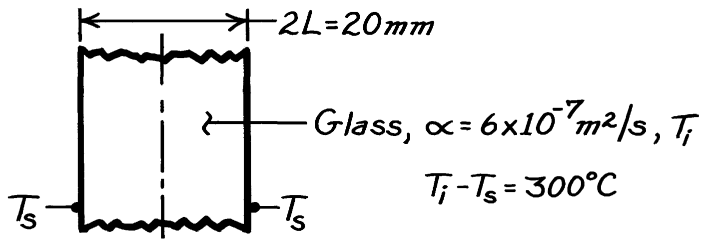
PROBLEM 5.57

KNOWN: Initial temperature, thickness and thermal diffusivity of glass plate. Prescribed surface temperature.

FIND: (a) Time to achieve 50% reduction in midplane temperature, (b) Maximum temperature gradient at that time.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: Prescribed surface temperature is analogous to $h \to \infty$ and $T_\infty = T_s$. Hence, Bi = ∞. Assume validity of one-term approximation to series solution for T (x,t).

(a) At the midplane,

$$\theta_o^* = \frac{T_o - T_s}{T_i - T_s} = 0.50 = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)$$

$$\zeta_1 \tan\zeta_1 = \text{Bi} = \infty \rightarrow \zeta_1 = \pi/2.$$

Hence

$$C_1 = \frac{4\sin\zeta_1}{2\zeta_1 + \sin(2\zeta_1)} = \frac{4}{\pi} = 1.273$$

$$\text{Fo} = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = 0.379$$

$$t = \frac{\text{Fo}L^2}{\alpha} = \frac{0.379(0.01\text{ m})^2}{6\times10^{-7}\text{ m}^2/\text{s}} = 63\text{ s.} \qquad <$$

(b) With $\theta^* = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)\cos\zeta_1 x^*$

$$\frac{\partial T}{\partial x} = \frac{(T_i - T_s)}{L}\frac{\partial\theta^*}{\partial x^*} = -\frac{(T_i - T_s)}{L}\zeta_1 C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)\sin\zeta_1 x^*$$

$$\partial T/\partial x\Big|_{max} = \partial T/\partial x\Big|_{x^*=1} = -\frac{300^\circ\text{C}}{0.01\text{ m}}\frac{\pi}{2}0.5 = -2.36\times10^4\ ^\circ\text{C/m.} \qquad <$$

COMMENTS: Validity of one-term approximation is confirmed by Fo > 0.2.