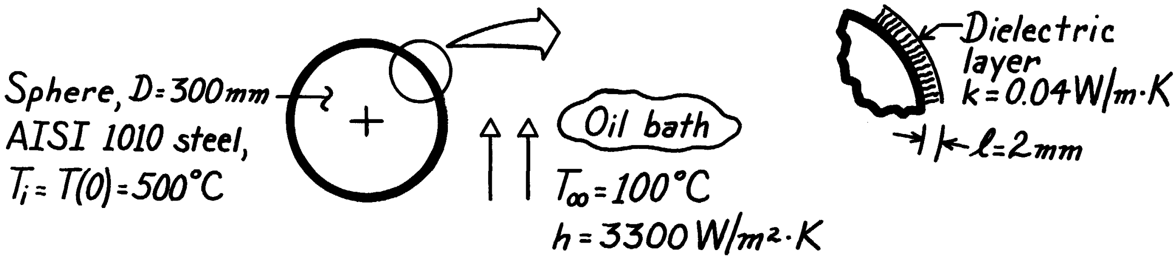
PROBLEM 5.9

KNOWN: Solid steel sphere (AISI 1010), coated with dielectric layer of prescribed thickness and thermal conductivity. Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath.

FIND: Time required for sphere to reach 140°C.

SCHEMATIC:



PROPERTIES: *Table A-1*, AISI 1010 Steel $\left(\bar{T} = [500+140]^\circ C/2 = 320^\circ C \approx 600K\right)$:

$\rho = 7832\ kg/m^3,\ c = 559\ J/kg\cdot K,\ k = 48.8\ W/m\cdot K.$

ASSUMPTIONS: (1) Steel sphere is space-wise isothermal, (2) Dielectric layer has negligible thermal capacitance compared to steel sphere, (3) Layer is thin compared to radius of sphere, (4) Constant properties, (5) Neglect contact resistance between steel and coating.

ANALYSIS: The thermal resistance to heat transfer from the sphere is due to the dielectric layer and the convection coefficient. That is,

$$R'' = \frac{\ell}{k} + \frac{1}{h} = \frac{0.002m}{0.04\ W/m\cdot K} + \frac{1}{3300\ W/m^2\cdot K} = (0.050 + 0.0003) = 0.0503\frac{m^2\cdot K}{W},$$

or in terms of an overall coefficient, $U = 1/R'' = 19.88\ W/m^2\cdot K$. The effective Biot number is

$$Bi_e = \frac{UL_c}{k} = \frac{U(r_o/3)}{k} = \frac{19.88\ W/m^2\cdot K \times (0.300/6)m}{48.8\ W/m\cdot K} = 0.0204$$

where the characteristic length is $L_c = r_o/3$ for the sphere. Since $Bi_e < 0.1$, the lumped capacitance approach is applicable. Hence, Eq. 5.5 is appropriate with h replaced by U,

$$t = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{\theta_i}{\theta_o} = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{T(0) - T_\infty}{T(t) - T_\infty}.$$

Substituting numerical values with $(V/A_s) = r_o/3 = D/6$,

$$t = \frac{7832\ kg/m^3 \times 559\ J/kg\cdot K}{19.88\ W/m^2\cdot K}\left[\frac{0.300m}{6}\right] \ln\frac{(500-100)^\circ C}{(140-100)^\circ C}$$

$t = 25{,}358s = 7.04h.$ <

COMMENTS: (1) Note from calculation of R'' that the resistance of the dielectric layer dominates and therefore nearly all the temperature drop occurs across the layer.