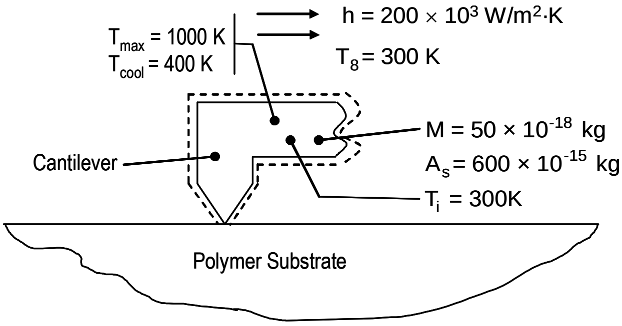
PROBLEM 5.41

KNOWN: Mass and exposed surface area of a silicon cantilever, convection heat transfer coefficient, initial and ambient temperatures.

FIND: (a) The ohmic heating needed to raise the cantilever temperature from T_i = 300 K to T = 1000 K in t_h = 1μs, (b) The time required to cool the cantilever from T = 1000 K to T = 400 K, t_c and the thermal processing time ($t_p = t_h + t_c$), (c) The number of bits that can be written onto a 1 mm × 1 mm surface area and time needed to write the data for a processing head equipped with M cantilevers.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance behavior, (2) Negligible radiation heat transfer, (3) Constant properties, (4) Negligible heat transfer to polymer substrate.

PROPERTIES: Table A.1, silicon ($\overline{T}$ = 650 K): c_p = 878.5 J/kg·K.

ANALYSIS:

(a) From Problem 5.20 we note that

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right) \quad (1)$$

where $\theta \equiv T - T(\infty)$ and $T(\infty)$ is the steady-state temperature corresponding to $t \rightarrow \infty$;

$\theta_i = T_i - T(\infty)$, $R = \dfrac{1}{hA_s}$, and $C = Mc_p$. For this problem,

$$R = \frac{1}{200 \times 10^3 \text{ W/m}^2 \cdot \text{K} \times 600 \times 10^{-15} \text{ m}^2} = 8.33 \times 10^6 \text{ K/W}$$

$$C = 50 \times 10^{-18} \text{ kg} \times 878.5 \text{ J/kg} \cdot \text{K} = 43.9 \times 10^{-15} \text{ J/K}$$

$$R \times C = 8.33 \times 10^6 \text{ K/W} \times 43.9 \times 10^{-15} \text{ J/K} = 366 \times 10^{-9} \text{ s}$$

Therefore, Equation 1 may be evaluated as

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PROBLEM 5.41 (Cont.)

$$\frac{1000 - T(\infty)}{300 - T(\infty)} = \exp\left(-\frac{1 \times 10^{-6}\ \text{s}}{366 \times 10^{-9}\ \text{s}}\right) = 0.0651$$

hence, $T(\infty) = 1049\text{K}$.

At steady-state, Equation 1.12b yields

$$\dot{E}_g = hA_s(T(\infty) - T_\infty) = 200 \times 10^3\ \text{W/m}^2 \cdot \text{K} \times 600 \times 10^{-15}\ \text{m}^2 (1049 - 300)\ \text{K}$$

$$= 90 \times 10^{-6}\ \text{W} = 90\ \mu\text{W}$$ <

(b) Equation 5.6 may be used. Hence,

$$\frac{\theta}{\theta_i} = \exp\left[-\left(\frac{hA_s}{Mc}\right)t_c\right] \quad \text{where } \theta = T - T_\infty. \text{ Therefore}$$

$$\frac{400 - 300}{1000 - 300} = 0.143 = \exp\left[-\left(\frac{200 \times 10^3\ \text{W/m}^2 \cdot \text{K} \times 600 \times 10^{-15}\ \text{m}^2}{50 \times 10^{-18}\ \text{kg} \times 878.5\ \text{J/kg} \cdot \text{K}}\right)t_c\right]$$

or $t_c = 0.71 \times 10^{-6}$ s = 0.71 μs <

and $t_p = t_h + t_c = 1.0\ \mu\text{s} + 0.71\ \mu\text{s} = 1.71\ \mu\text{s}$ <

(c) Each bit occupies $A_b = 50 \times 10^{-9}\ \text{m} \times 50 \times 10^{-9}\ \text{m} = 2.5 \times 10^{-15}\ \text{m}^2$

Therefore, the number of bits on a 1 mm × 1 mm substrate is

$$N = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}\ \text{m}^2}{2.5 \times 10^{-15}\ \text{m}^2} = 400 \times 10^6\ \text{bits}$$ <

The total time needed to write the data (t_t) is,

$$t_t = \frac{N \times t_p}{M} = \frac{400 \times 10^6\ \text{bits} \times 1.71 \times 10^{-6}\ \text{s/bit}}{100} = 6.84\ \text{s}$$ <

COMMENTS: (1) Lumped thermal capacitance behavior is an excellent approximation for such a small device (2) Each cantilever writes N/M = 400×10^6 bits/100 cantilevers = 400×10^4 bits/cantilever. With a separation distance of 50×10^{-9} m, the total distance traveled is 50×10^{-9} m $\times 400 \times 10^4 = 200 \times 10^{-3}$ m = 200 mm. If the head travels at 200 mm/s, it will take 1 second to move the head, providing a total writing and moving time of 6.84 s + 1 s = 7.84 s. The speed of the process is heat transfer-limited.