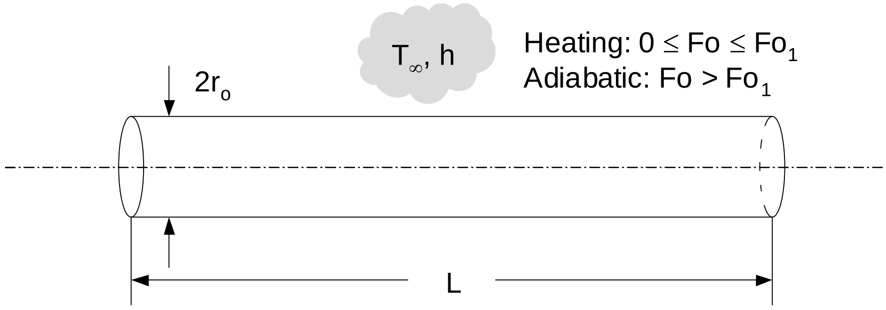
PROBLEM 5.62

KNOWN: One-dimensional convective heating of an $L/r_o = 20$ cylinder with $Bi = 1$ for a dimensionless time of Fo_1.

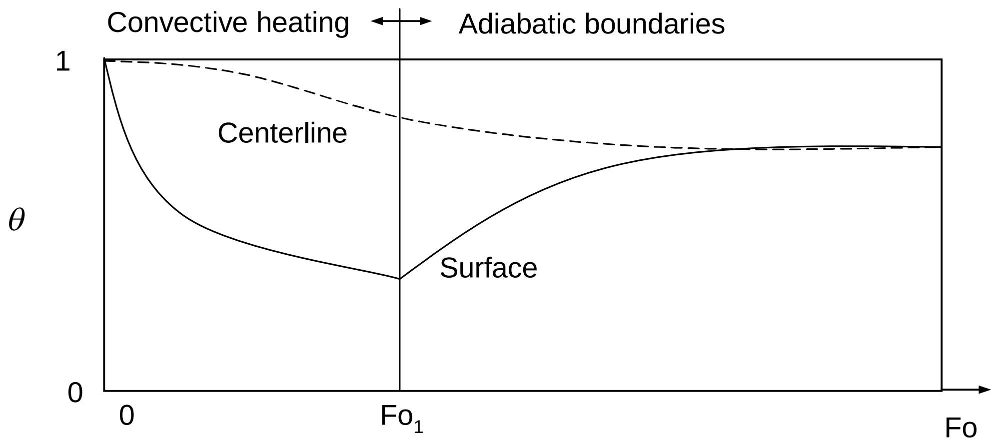
FIND: (a) Sketch of the dimensionless centerline and surface temperatures of the cylinder as a function of dimensionless time over the range $0 < Fo_1 < Fo < \infty$. Relative value of Fo_2 needed to achieve a steady-state centerline temperature equal to the centerline temperature at Fo_1. (b) Analytical expression for, and value of $\Delta Fo = Fo_2 - Fo_1$ for $Bi = 1$, $Fo_1 > 0.2$, $Fo_2 > 0.2$. (c) Value of ΔFo for $Bi = 0.01$, 0.1, 10, 100 and ∞.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Approximate, one-term solutions are valid.

ANALYSIS: (a) A sketch of the dimensionless centerline and surface temperatures is shown below. Note that, at Fo_1, the surface of the cylinder will be warm (smaller θ) relative to the centerline since temperature gradients within the cylinder are significant ($Bi = 1$). At the curtailment of heating (Fo_1), the surface temperature cools rapidly while warm temperatures continue to propagate toward the centerline, slowly heating the centerline until a steady-state, isothermal condition is eventually reached.



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Based on the sketch above, one could achieve a steady-state centerline temperature equal to the centerline temperature at Fo_1 by reducing the duration of convective heating to Fo_2, as shown in the sketch below.

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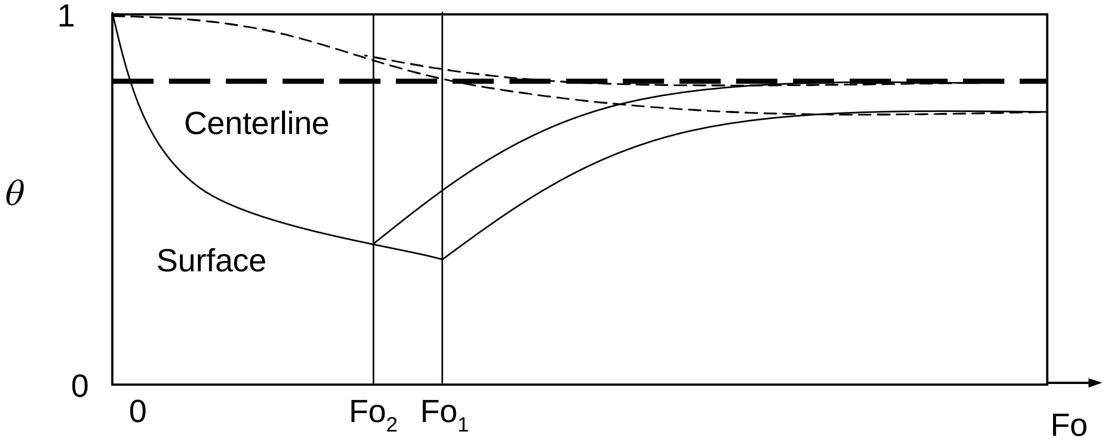
PROBLEM 5.62 (Cont.)



Hence, $Fo_2 < Fo_1$. <

(b) Using the approximate solutions of Sections 5.6.2 and 5.6.3, and noting that the steady-state temperature of the cylinder is uniform and related to the energy transferred to the cylinder,

$$\theta_o^*(Fo_1) = 1 - \frac{Q}{Q_o}(Fo_2)$$

or,

$$1 - \theta_o^*(Fo_1) = \frac{Q}{Q_o}(Fo_1 + \Delta Fo_1) \qquad (1)$$

Substituting Eqs. 5.52c and 5.54 into Eq. (1) yields

$$1 - C_1 \exp(-\zeta_1^2 Fo_1) = 1 - \frac{2C_1 \exp(-\zeta_1^2(Fo_1 + \Delta Fo))}{\zeta_1} J_1(\zeta_1)$$

which may be simplified to

$$\Delta Fo = -\frac{1}{\zeta_1^2} \ln\left(\frac{\zeta_1}{2J_1(\zeta_1)}\right)$$ <

From Table 5.1, ζ_1 = 1.2558 rad at Bi = 1, and from Table B.4, $J_1(\zeta_1)$ = 0.512. Hence,

$$\Delta Fo = -\frac{1}{1.2558^2} \ln\left(\frac{1.2558}{2J_1(1.2558)}\right) = -\frac{1}{1.2558^2} \ln\left(\frac{1.2558}{2 \times 0.512}\right) = -0.1294$$ <

(c) The expression for ΔFo may be evaluated for a range of Bi, resulting in the following.

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PROBLEM 5.62 (Cont.)

Bi	ζ_1	ΔFo
0.01	0.1412	-0.1250
0.1	0.4417	-0.1255
1	1.2558	-0.1294
10	2.1795	-0.1406
100	2.3809	-0.1447
∞	2.4050	-0.1452

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COMMENTS: (1) Note that the dimensionless temperature, $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$, is defined in a manner such that for cylinder heating, increases in actual temperature correspond to decreases in the dimensionless temperature. (2) The dimensionless time lag, ΔFo, is weakly-dependent on the value of the Biot number and is independent of the heating time. Hence, a general rule-of-thumb is that a time lag of $\Delta Fo \approx -0.13$ should be specified in order to achieve an ultimate centerline temperature equal to that predicted at Fo_1 for convective heating or cooling. (3) For applications such as materials or food processing, where a certain minimum centerline temperature is desired, assuming that Fo_1 (as determined by Eq. 5.52c is the appropriate processing or cooking time can result in significant over-heating of the material or food, especially at small Fourier numbers. (4) Significant energy and time savings can be realized by reducing the processing or cooking time from Fo_1 to Fo_2.