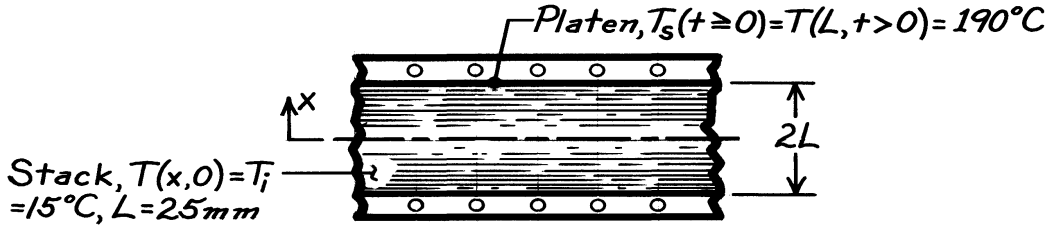
PROBLEM 5.46

KNOWN: Stack of circuit board-pressing plates, initially at a uniform temperature, is subjected by upper/lower platens to a higher temperature.

FIND: (a) Elapsed time, t_e, required for the mid-plane to reach cure temperature when platens are suddenly changed to $T_s = 190°C$, (b) Energy removal from the stack needed to return its temperature to T_i.

SCHEMATIC:



PROPERTIES: Stack (given): $k = 0.613$ W/m·K, $\rho c_p = 2.73\times10^6$ J/m^3·K; $\alpha = k/\rho c_p = 2.245\times10^{-7}$ m^2/s.

ANALYSIS: (a) Recognize that sudden application of surface temperature corresponds to $h \to \infty$, or $Bi \to \infty$. With $T_s = T_\infty$,

$$\theta_o^* = \frac{T(0,t) - T_s}{T_i - T_s} = \frac{(170-190)^\circ C}{(15-190)^\circ C} = 0.114.$$

Using Eq. 5.44 with values of $\zeta_1 = 1.5707$ and $C_1 = 1.2733$ for $Bi \to \infty$ (Table 5.1), find Fo

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

$$Fo = -\frac{1}{\zeta_1^2}\ln\left(\theta_o^*/C_1\right) = -\frac{1}{(1.5707)^2}\ln\left(0.114/1.2733\right) = 0.977$$

where $Fo = \alpha t/L^2$,

$$t = \frac{FoL^2}{\alpha} = \frac{0.977\left(25\times10^{-3}\ m\right)^2}{2.245\times10^{-7}\ m^2/s} = 2.720\times10^3 s = 45.3\ min. \quad <$$

(b) The energy removal is equivalent to the energy gained by the stack per unit area for the time interval $0 \to t_e$. With Q_o'' corresponding to the maximum amount of energy that could be transferred,

$$Q_o'' = \rho c(2L)(T_i - T_\infty) = 2.73\times10^6\ J/m^3\cdot K\left(2\times25\times10^{-3}\ m\right)(15-190)K = -2.389\times10^7\ J/m^2.$$

Q'' may be determined from Eq. 5.49,

$$\frac{Q''}{Q_o''} = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^* = 1 - \frac{\sin(1.5707\ rad)}{1.5707\ rad}\times0.114 = 0.927$$

We conclude that the energy to be removed from the stack per unit area to return it to T_i is

$$Q'' = 0.927Q_o'' = 0.927\times2.389\times10^7\ J/m^2 = 2.21\times10^7\ J/m^2. \quad <$$