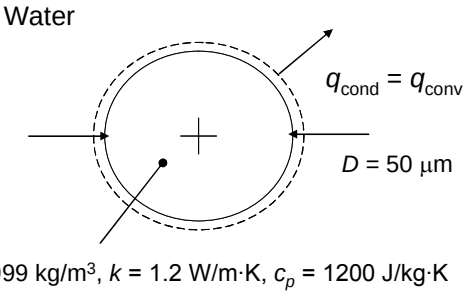
PROBLEM 5.17

KNOWN: Diameter and properties of neutrally-buoyant spherical particles.

FIND: Time constant of the particles.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties. (2) Infinite medium. (3) Lumped capacitance behavior.

PROPERTIES: Particle: $\rho = 999$ kg/m^3, $k_p = 1.2$ W/m·K, and $c_p = 1200$ J/kg·K. Table A.6 water ($T = 300$ K): $k = 0.613$ W/m·K.

ANALYSIS: Treating heat transfer between the particle and water as conduction, we may use the shape factor corresponding to Case1 of Table 4.1 with $z \to \infty$. Hence

$$S = 2\pi D \tag{1}$$

The surface energy balance may be expressed as

$$q = Sk(T - T_\infty) = hA(T - T_\infty) = h\pi D^2(T - T_\infty)$$

where h is an effective heat transfer coefficient from which

$$h = \frac{Sk}{\pi D^2} = 2\frac{k}{D} \tag{2}$$

The thermal time constant is

$$\tau_t = \left(\frac{1}{hA_s}\right)\left(\rho V c_p\right) \tag{3}$$

Combining Equations (1) through (3) yields

$$\tau_t = \frac{\rho c D^2}{12k} = \frac{999 \text{ kg/m}^3 \times 1200 \text{ J/kg}\cdot\text{K} \times \left(50\times10^{-6} \text{ m}\right)^2}{12 \times 0.613 \text{ W/m}\cdot\text{K}} = 407\times10^{-6} \text{ s} \quad <$$

COMMENTS: (1) The Biot number is $Bi = hL_c/k_p = hD/6k_p = k/3k_p = 0.613$ W/m·K/(3 × 1.2 W/m·K) = 0.17. Lumped capacitance behavior will not exist in the particle and the analysis must be viewed as approximate. (2) Regardless of whether the lumped capacitance approximation is valid, the thermal time constant is relatively small. Hence an assumption that the particle temperature is the same as that of the surrounding water may be valid.