**Solutions Manual for**

# Fluid Mechanics

**Seventh Edition in SI Units**

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## Chapter 3

## Integral Relations for a

## Control Volume

**P3.1** Discuss Newton's second law (the linear momentum relation) in these three forms:

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum \mathbf{F} = \frac{d}{dt}(m\mathbf{V}) \qquad \sum \mathbf{F} = \frac{d}{dt}\left( \int_{system} \mathbf{V}\rho\, d\upsilon \right)$$

**Solution:** These questions are just to get the students thinking about the basic laws of mechanics. They are valid and equivalent for constant-mass systems, and we can make use of all of them in certain fluids problems, e.g. the #1 form for small elements, #2 form for rocket propulsion, but the #3 form is control-volume related and thus the most popular in this chapter.

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**P3.2** Consider the angular-momentum relation in the form

$$\sum \mathbf{M}_O = \frac{d}{dt}\left[ \int_{system} (\mathbf{r} \times \mathbf{V})\rho\, d\upsilon \right]$$

What does **r** mean in this relation? Is this relation valid in both solid and fluid mechanics? Is it related to the *linear*-momentum equation (Prob. 3.1)? In what manner?

**Solution:** These questions are just to get the students thinking about angular momentum versus linear momentum. One might forget that **r** is the position vector from the moment-center O to the elements $\rho \; d\upsilon$ where momentum is being summed. Perhaps $\mathbf{r}_O$ is a better notation.

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**P3.3** For steady laminar flow through a long tube (see Prob. 1.14), the axial velocity distribution is given by $u = C(R^2 - r^2)$, where R is the tube outer radius and C is a constant. Integrate $u(r)$ to find the total volume flow $Q$ through the tube.

**Solution:** The area element for this axisymmetric flow is $dA = 2\pi r\, dr$. From Eq. (3.7),

$$Q = \int u \; dA = \int_0^R C(R^2 - r^2)2\pi r\, dr = \frac{\boldsymbol{\pi}}{\mathbf{2}}\mathbf{CR^4} \quad \textit{Ans.}$$

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**P3.4** A fire hose has a 12.5-cm inside diameter and is flowing at 2.27 m$^3$/min. The flow exits through a nozzle contraction at a diameter $D_n$. For steady flow, what should $D_n$ be, in cm, to create an exit velocity of 25 m/s?

**Solution**: This is a straightforward one-dimensional steady-flow continuity problem.

$$2.27 \text{ m}^3/\text{min} = 0.038 \text{ m}^3/\text{s};$$

The hose diameter (12.5 cm) would establish a hose average velocity of 9.8 ft/s, but we don't really need this. Go directly to the volume flow:

$$Q = 0.038 \text{ m}^3/\text{s} = A_n V_n = \frac{\pi}{4} D_n^2 (25\frac{\text{m}}{\text{s}}) \;;\quad \text{Solve for} \quad D_n \approx \mathbf{4.4\ cm} \quad Ans.$$

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**P3.5** Water at 20°C flows through a 12.5-cm-diameter smooth pipe at a high Reynolds number, for which the velocity profile is given by $u \approx U_o(y/R)^{1/8}$, where $U_o$ is the centerline velocity, $R$ is the pipe radius, and $y$ is the distance measured from the wall toward the centerline. If the centerline velocity is 7.62 m/s, estimate the volume flow rate in m$^3$ per minute.

**Solution**: The formula for average velocity in this power-law case was given in Example 3.4:

$$V_{av} = U_o \frac{2}{(1+m)(2+m)} = U_o \frac{2}{(1+1/8)(2+1/8)} = 0.837U_o = 0.837(7.62) = 6.38 \frac{\text{m}}{\text{s}}$$

$$\textit{Thus} \quad Q = V_{av} A_{pipe} = [6.38 \frac{\text{m}}{\text{s}}]\, \pi(6.25\times 10^{-2} \text{ m})^2 = 0.078 \frac{\text{m}^3}{\text{s}} \approx \mathbf{4.7} \frac{\text{m}^3}{\text{min}} \quad Ans.$$

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**P3.6** Given the simplified firefighting nozzle model in P3.6, find the position of the nozzle $\vec{R}$, exit velocity $\vec{V}_{\text{nozzle}}$, and $\vec{R}\times\vec{V}_{\text{nozzle}}$ when considering with respect to given coordinates.

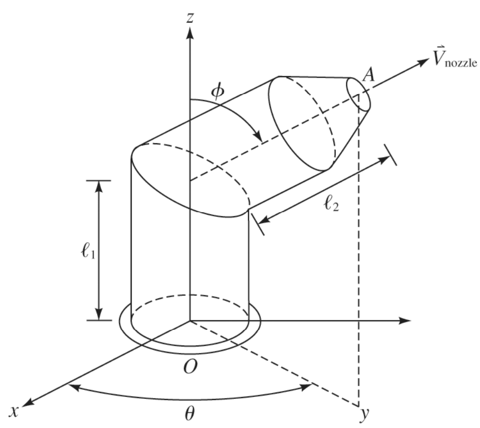


Fig. P3.6

**Solution:** From the given figure, $\vec{R}$ is position vector from point O to A.

$$\vec{R} = l_2 \sin\phi\cos\theta\ \vec{\mathrm{i}} + l_2 \sin\phi\sin\theta\ \vec{\mathrm{j}} + (l_1 + l_2\cos\phi)\vec{k}$$
$$\vec{V}_{\text{nozzle}} = V_{\text{nozzle}}(\sin\phi\cos\theta\ \vec{\mathrm{i}} + \sin\phi\sin\theta\ \vec{\mathrm{j}} + \cos\phi\vec{k})$$

where $V_{\text{nozzle}}$ is equal to exit speed of outflow from the nozzle.

$$\vec{R}\times\vec{V}_{\text{nozzle}} = \begin{vmatrix} \vec{\mathrm{i}} & \vec{\mathrm{j}} & \vec{k} \\ l_2\, \mathrm{s}\phi\mathrm{c}\theta & l_2\, \mathrm{s}\phi s\theta & l_1 + l_2\,\mathrm{c}\phi \\ V_{\text{nozzle}}\, \mathrm{s}\phi\mathrm{c}\theta & V_{\text{nozzle}}\, \mathrm{s}\phi s\theta & V_{\text{nozzle}}\, \mathrm{c}\phi \end{vmatrix}$$
$$= -V_{\mathrm{n}} l_1\, \mathrm{s}\phi s\theta\ \vec{\mathrm{i}} + V_{\mathrm{n}} l_1\, \mathrm{s}\phi c\theta\ \vec{\mathrm{j}}$$

$$\text{where}\ \left.\begin{aligned} \mathrm{s}\phi &= \sin\phi \\ \mathrm{c}\phi &= \cos\phi \\ \mathrm{s}\theta &= \sin\theta \\ \mathrm{c}\theta &= \cos\theta \end{aligned}\right\}\ \text{for simplification.}$$

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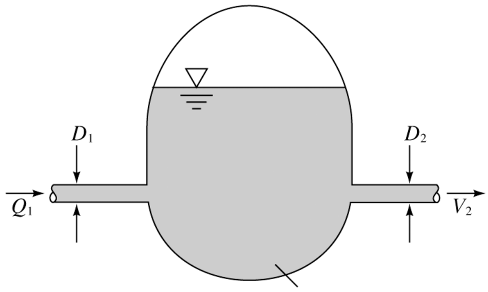
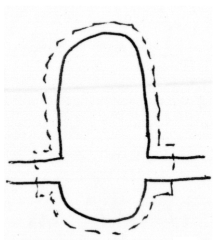
**P3.7** To calculate the loss or gain of hydraulic oil in Fig. P3.7, we need to select the problem's control volume. Select a control volume and show how to obtain your solution.



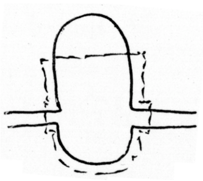

**Solution:** This problem is about conservation of mass under unsteady flow. Control volume selected will definitely affect solutions for the problem. However, the question is about loss or gain hydraulic fluid in the accumulator. Suppose we select the control volume as follows:



This selection would include air trapped in the control volume. But if we assume that density of the air in the control volume does not change as much, we would have

$$\frac{\partial}{\partial t}\int_{cv}\rho\, dV = \frac{\partial}{\partial t}(\rho V) = \dot{m} = \dot{m}_{in} - \dot{m}_{out}$$

Suppose we select the control volume as follows.



We would not have to consider the air; then the solution would be

$$\frac{\partial}{\partial t}\int_{cv}\rho\, dV = \rho\frac{\partial V}{\partial t} = \frac{\partial V}{\partial t} = Q_{in} - Q_{out}$$

Since the hydraulic fluid density does not change.

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**P3.8** When a gravity-driven liquid jet issues from a slot in a tank, as in Fig. P3.8, an approximation for the exit velocity distribution is $u \approx \sqrt{2g(h-z)}$, where $h$ is the depth of the jet centerline. Near the slot, the jet is horizontal, two-dimensional, and of thickness $2L$, as shown. Find a general expression for the total volume flow $Q$ issuing from the slot; then take the limit of your result if $L \ll h$.

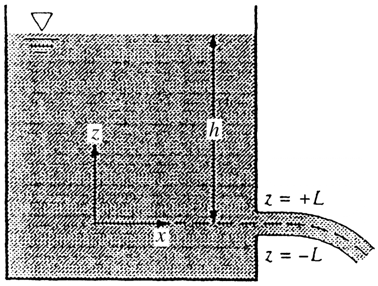


**Fig. P3.8**

**Solution:** Let the slot width be $b$ into the paper. Then the volume flow from Eq. (3.7) is

$$Q = \int u\, dA = \int_{-L}^{+L} [2g(h-z)]^{1/2}\, b\, dz = \mathbf{\frac{2b}{3}\sqrt{(2g)}[(h+L)^{3/2} - (h-L)^{3/2}]} \quad \textit{Ans.}$$

In the limit of $L \ll h$, this formula reduces to $\mathbf{Q \approx (2Lb)\sqrt{(2gh)}}$ *Ans.*

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**P3.9** A spherical tank, of diameter 35 cm, is leaking air through a 5-mm-diameter hole in its side. The air exits the hole at 360 m/s and a density of 2.5 kg/m$^3$. Assuming uniform mixing, (*a*) find a formula for the rate of change of average density in the tank; and (*b*) calculate a numerical value for ($d\rho/dt$) in the tank for the given data.

**Solution**: If the control volume surrounds the tank and cuts through the exit flow,

$$\frac{dm}{dt}\Big|_{system} = 0 = \frac{d}{dt}(\rho_{\text{tank}}\upsilon_{\text{tank}}) + \dot{m}_{out} = \upsilon_{\text{tank}}\frac{d}{dt}(\rho_{\text{tank}}) + (\rho AV)_{out}$$

Solve for $\quad \frac{d}{dt}(\rho_{\text{tank}}) = -\frac{(\rho AV)_{out}}{\upsilon_{\text{tank}}}$ *Ans.*(*a*)

(b) For the given data, we calculate

$$\frac{d\rho_{\text{tank}}}{dt} = -\frac{(2.5\, kg/m^3)[(\pi/4)(0.005m)^2](360\, m/s)}{(\pi/6)(0.35m)^3} = \mathbf{-0.79\ \frac{kg/m^3}{s}} \quad \textit{Ans.}(b)$$

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**P3.10** Three pipes steadily deliver water at 20°C to a large exit pipe in Fig. P3.10. The velocity $V_2$ = 5 m/s, and the exit flow rate $Q_4$ = 120 m$^3$/h. Find (a) $V_1$; (b) $V_3$; and (c) $V_4$ if it is known that increasing $Q_3$ by 20% would increase $Q_4$ by 10%.

**Solution:** (a) For steady flow we have $Q_1 + Q_2 + Q_3 = Q_4$, or

$$V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \qquad (1)$$

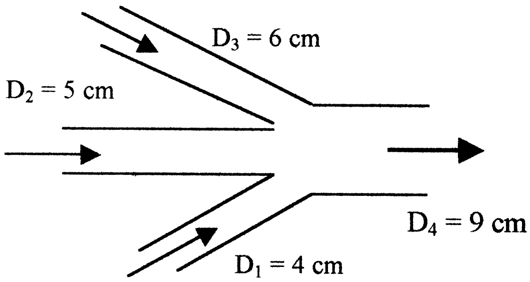


**Fig. P3.10**

Since $0.2Q_3 = 0.1Q_4$, and $Q_4$ = (120 m$^3$/h)(1 h/3600 s) = 0.0333 m$^3$/s,

$$V_3 = \frac{Q_4}{2A_3} = \frac{(0.0333 \text{ m}^3/\text{s})}{\frac{\pi}{2}(0.06^2)} = \mathbf{5.89\ m/s} \quad \textit{Ans.}\text{ (b)}$$

Substituting into (1),

$$V_1\left(\frac{\pi}{4}\right)(0.04^2) + (5)\left(\frac{\pi}{4}\right)(0.05^2) + (5.89)\left(\frac{\pi}{4}\right)(0.06^2) = 0.0333 \qquad \mathbf{V_1 = 5.45\ m/s} \quad \textit{Ans.}\text{ (a)}.$$

From mass conservation, $Q_4 = V_4A_4$

$$(0.0333 \text{ m}^3/\text{s}) = V_4(\pi)(0.06^2)/4 \qquad \mathbf{V_4 = 5.24\ m/s} \quad \textit{Ans.}\text{ (c)}$$

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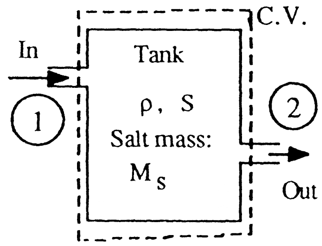
**P3.11** A laboratory test tank contains seawater of salinity $S$ and density $\rho$. Water enters the tank at conditions ($S_1$, $\rho_1$, $A_1$, $V_1$) and is assumed to mix immediately in the tank. Tank water leaves through an outlet $A_2$ at velocity $V_2$. If salt is a "conservative" property (neither created nor destroyed), use the Reynolds transport theorem to find an expression for the rate of change of salt mass $M_{salt}$ within the tank.




**Solution:** By definition, salinity S = $\rho_{salt}/\rho$. Since salt is a "conservative" substance (not consumed or created in this problem), the appropriate control volume relation is

$$\left.\frac{dM_{salt}}{dt}\right|_{system} = \frac{d}{dt}\left(\int_{CV} \rho_s\, d\upsilon\right) + S\dot{m}_2 - S_1\dot{m}_1 = 0$$

$$\text{or:}\quad \left.\frac{\mathbf{dM_s}}{\mathbf{dt}}\right|_{CV} = \mathbf{S_1\rho_1A_1V_1 - S\rho A_2V_2} \quad \textit{Ans.}$$

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**3.12** Water flowing through an 8-cm-diameter pipe enters a porous section, as in Fig. P3.12, which allows a uniform radial velocity $v_w$ through the wall surfaces for a distance of 1.2 m. If the entrance average velocity $V_1$ is 12 m/s, find the exit velocity $V_2$ if (a) $v_w$ = 15 cm/s out of the pipe walls; (b) $v_w$ = 10 cm/s into the pipe. (c) What value of $v_w$ will make $V_2$ = 9 m/s?

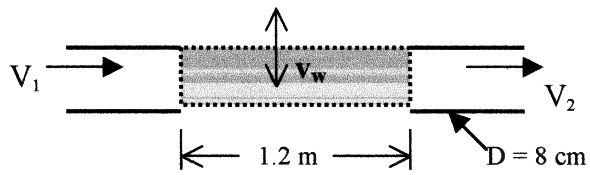


**Fig. P3.12**

**Solution:** (a) For a suction velocity of $v_w$ = 0.15 m/s, and a cylindrical suction surface area,

$$A_w = 2\pi(0.04)(1.2) = 0.3016\ m^2$$

$$Q_1 = Q_w + Q_2$$

$$(12)(\pi)(0.08^2)/4 = (0.15)(0.3016) + V_2(\pi)(0.08^2)/4 \qquad \mathbf{V_2 = 3\ m/s} \quad \textit{Ans. (a)}$$

(b) For an injection velocity into the pipe, $v_w$ = 0.10 m/s, $Q_1 + Q_w = Q_2$, or:

$$(12)(\pi)(0.08^2)/4 + (0.10)(0.3016) = V_2(\pi)(0.08^2)/4 \qquad \mathbf{V_2 = 18\ m/s} \quad \textit{Ans. (b)}$$

(c) Setting the outflow $V_2$ to 9 m/s, the wall suction velocity is,

$$(12)(\pi)(0.08^2)/4 = (v_w)(0.3016) + (9)(\pi)(0.08^2)/4 \qquad \mathbf{v_w = 0.05\ m/s = 5\ cm/s}\ \textbf{\textit{out}}$$

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**P3.13** The inlet section of a vacuum cleaner is a rectangle, 2.5 cm by 12.5 cm. The blower is able to provide suction at 700 L/min. (*a*) What is the average velocity at the inlet, in m/s? (*b*) If conditions are sea level standard, what is the mass flow of air, in kg/s?

**Solution**: (*a*) Convert 700 L/min to $1.167\times10^{-2}\ \mathrm{m^3/s}$. Then the inlet velocity is

$$V_{inlet} = \frac{Q}{A_{inlet}} = \frac{1.167\times10^{-2}\ \mathrm{m^3/s}}{(2.5\times10^{-2})(12.5\times10^{-2})} = \mathbf{3.73}\ \frac{\mathrm{m}}{\mathrm{s}} \qquad \textit{Ans.}(a)$$

(*b*) At sea level, $\rho_{\mathrm{air}} = 1.2255\ \mathrm{kg/m^3}$. Then

$$\dot{m}_{air} = \rho_{air}Q = (1.2255\frac{\mathrm{kg}}{\mathrm{m^3}})(0.01167\frac{\mathrm{m^3}}{\mathrm{s}}) = \mathbf{0.0143}\ \frac{\mathrm{kg}}{\mathrm{s}} \qquad \textit{Ans.}(b)$$

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**P3.14** The pipe flow in Fig. P3.14 fills a cylindrical tank as shown. At time t = 0, the water depth in the tank is 30 cm. Estimate the time required to fill the remainder of the tank.

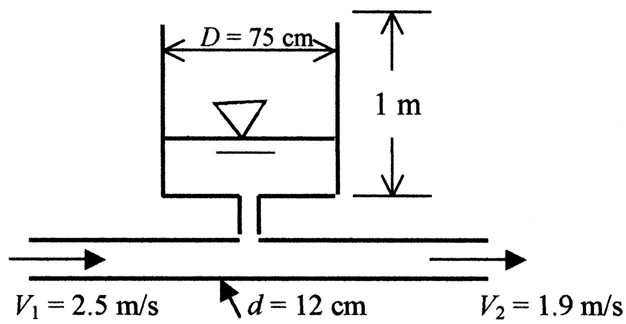


**Fig. P3.14**

**Solution:** For a control volume enclosing the tank and the portion of the pipe below the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} - \dot{m}_{in} = 0$$

$$\rho\pi R^2\frac{dh}{dt} + (\rho AV)_{out} - (\rho AV)_{in} = 0$$

$$\frac{dh}{dt} = \frac{4}{998(\pi)(0.75^2)}\left[998\left(\frac{\pi}{4}\right)(0.12^2)(2.5-1.9)\right] = 0.0153\ m/s,$$

$$\Delta t = 0.7/0.0153 = \mathbf{46}\ s \quad \textit{Ans.}$$

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**P3.15** The cylindrical container in Fig. P3.15 is 20 cm in diameter and has a conical contraction at the bottom with an exit hole 3 cm in diameter. The tank contains fresh water at standard sea-level conditions. If the water surface is falling at the nearly steady rate $dh/dt \approx -0.072$ m/s, estimate the average velocity $V$ from the bottom exit.

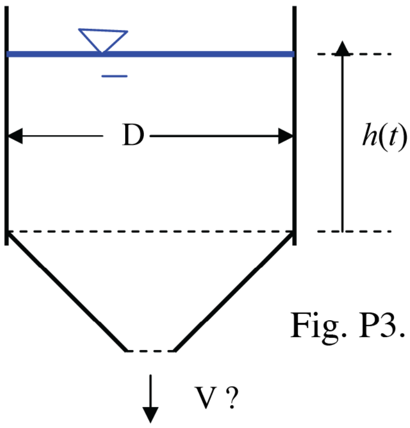


Fig. P3.15

**Solution**: We could simply note that *dh/dt* is the same as the water *velocity* at the surface and use $Q_1 = Q_2$, or, more instructive, approach it as a control volume problem. Let the control volume encompass the entire container. Then the mass relation is

$$\frac{dm}{dt}\Big|_{system} = 0 = \frac{d}{dt}\Big(\int_{CV} \rho\, d\upsilon\Big) + \dot{m}_{out} = \frac{d}{dt}\Big(\cancel{\upsilon_{cone}} + \rho\frac{\pi}{4}D^2 h\Big) \Big| + \rho\frac{\pi}{4}D_{exit}^2 V \,,$$

$$\text{or}: \rho\frac{\pi}{4}D^2\frac{dh}{dt} + \rho\frac{\pi}{4}D_{exit}^2 V = 0 \qquad \text{Cancel } \rho\frac{\pi}{4}: \; V = \Big(\frac{D}{D_{exit}}\Big)^2\Big(-\frac{dh}{dt}\Big)$$

$$\text{Introduce the data}: \; V = \Big(\frac{20\,cm}{3\,cm}\Big)^2\Big[-\Big(-0.072\frac{m}{s}\Big)\Big] = \mathbf{3.2\frac{m}{s}} \qquad \textit{Ans.}$$

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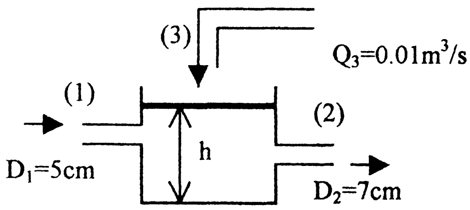
**P3.16** The open tank in the figure contains water at 20°C. For incompressible flow, (a) derive an analytic expression for *dh/dt* in terms of ($Q_1$, $Q_2$, $Q_3$). (b) If $h$ is constant, determine $V_2$ for the given data if $V_1 = 3$ m/s and $Q_3 = 0.01$ m$^3$/s.




**Solution:** For a control volume enclosing the tank,

$$\frac{d}{dt}\left(\int_{CV}\rho\, d\upsilon\right) + \rho(Q_2 - Q_1 - Q_3) = \rho\frac{\pi d^2}{4}\frac{dh}{dt} + \rho(Q_2 - Q_1 - Q_3),$$

$$\textit{solve} \quad \mathbf{\frac{dh}{dt} = \frac{Q_1 + Q_3 - Q_2}{(\pi d^2/4)}} \qquad \textit{Ans.}\text{ (a)}$$

If $h$ is constant, then

$$Q_2 = Q_1 + Q_3 = 0.01 + \frac{\pi}{4}(0.05)^2(3.0) = 0.0159 = \frac{\pi}{4}(0.07)^2 V_2,$$

$$\textit{solve} \quad \mathbf{V_2 = 4.13\ m/s} \qquad \textit{Ans.}\text{ (b)}$$

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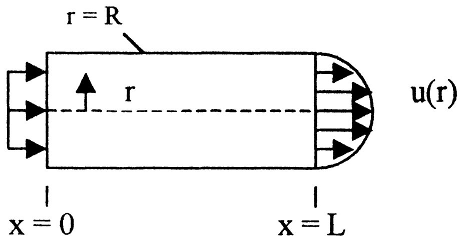
**P3.17** Water flows steadily through the round pipe in the figure. The entrance velocity is $V_o$. The exit velocity approximates turbulent flow, $u = u_{max}(1 - r/R)^{1/7}$. Determine the ratio $U_o/u_{max}$ for this incompressible flow.

**Solution:** Inlet and outlet flow must balance:




$$Q_1 = Q_2, \quad or: \quad \int_0^R U_o 2\pi r\, dr = \int_0^R u_{max}\left(1 - \frac{r}{R}\right)^{1/7} 2\pi r\, dr, \quad or: \quad U_o \pi R^2 = u_{max}\frac{49\pi}{60}R^2$$

Cancel and rearrange for this assumed incompressible pipe flow:

$$\frac{U_o}{u_{max}} = \mathbf{\frac{49}{60}} \quad Ans.$$

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**P3.18** An incompressible fluid flows past an impermeable flat plate, as in Fig. P3.18, with a uniform inlet profile $u = U_o$ and a cubic polynomial exit profile

$$u \approx U_o\left(\frac{3\eta - \eta^3}{2}\right) \quad \text{where } \eta = \frac{y}{\delta}$$

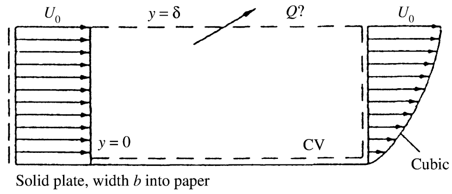


**Fig. P3.18**

Compute the volume flow $Q$ across the top surface of the control volume.

**Solution:** For the given control volume and incompressible flow, we obtain

$$0 = Q_{top} + Q_{right} - Q_{left} = Q + \int_0^\delta U_o\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right) b\, dy - \int_0^\delta U_o b\, dy$$

$$= Q + \frac{5}{8}U_o b\delta - U_o b\delta, \quad \text{solve for } \mathbf{Q = \frac{3}{8}U_o b\delta} \quad Ans.$$

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**P3.19** Incompressible steady flow in the inlet between parallel plates in Fig. P3.19 is uniform, $u = U_o = 8$ cm/s, while downstream the flow develops into the parabolic laminar profile $u = az(z_o - z)$, where $a$ is a constant. If $z_o = 4$ cm and the fluid is SAE 30 oil at 20°C, what is the value of $u_{max}$ in cm/s?

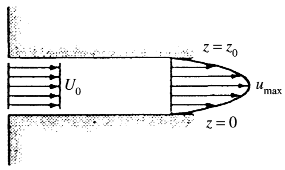


**Fig. P3.19**

**Solution:** Let *b* be the plate width into the paper. Let the control volume enclose the inlet and outlet. The walls are solid, so no flow through the wall. For incompressible flow,

$$0 = Q_{out} - Q_{in} = \int_0^{z_o} az(z_o - z)b\,dz - \int_0^{z_o} U_o b\,dz = abz_o^3/6 - U_o b z_o = 0, \quad \text{or:} \quad a = 6U_o/z_o^2$$

Thus continuity forces the constant *a* to have a particular value. Meanwhile, *a* is also related to the maximum velocity, which occurs at the center of the parabolic profile:

$$\text{At } z = z_o/2: \quad u = u_{max} = a\left(\frac{z_o}{2}\right)\left(z_o - \frac{z_o}{2}\right) = az_o^2/4 = (6U_o/z_o^2)(z_o^2/4)$$

$$\text{or:} \quad u_{max} = \frac{3}{2}U_o = \frac{3}{2}(8\text{ cm/s}) = \mathbf{12\ \frac{cm}{s}} \quad \textit{Ans.}$$

Note that the result is independent of $z_o$ or of the particular fluid, which is SAE 30 oil.

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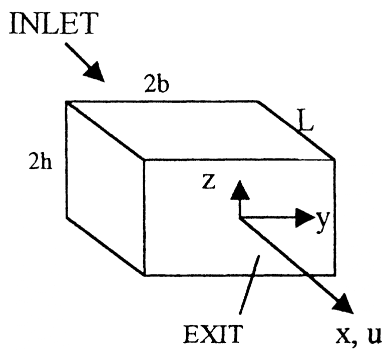
**P3.20** An incompressible fluid flows steadily through the rectangular duct in the figure. The exit velocity profile is given by $u \approx u_{max}(1 - y^2/b^2)(1 - z^2/h^2)$. (a) Does this profile satisfy the correct boundary conditions for viscous fluid flow? (b) Find an analytical expression for the volume flow *Q* at the exit. (c) If the inlet flow is 8.5 $m^3$/min, estimate $u_{max}$ in m/s.




**Solution:** (a) The *fluid should not slip* at any of the duct surfaces, which are defined by $y = \pm b$ and $z = \pm h$. From our formula, we see **u ≡ 0 at all duct surfaces**, OK. *Ans.* (a)
(b) The exit volume flow *Q* is defined by the integral of *u* over the exit plane area:

$$Q = \int\int u\,dA = \int_{-h}^{+h}\int_{-b}^{+b} u_{max}\left(1 - \frac{y^2}{b^2}\right)\left(1 - \frac{z^2}{h^2}\right) dy\ dz = u_{max}\left(\frac{4b}{3}\right)\left(\frac{4h}{3}\right)$$

$$= \mathbf{\frac{16bhu_{max}}{9}} \quad \textit{Ans.}\text{ (b)}$$

(c) Given $Q = 8.5\text{ m}^3/\text{min} = 0.1417\text{ m}^3/\text{s}$ and $b = h = 10$ cm, the maximum exit velocity is

$$Q = 0.1416\ \frac{m^3}{s} = \frac{16}{9}(0.1\text{ m})(0.1\text{ m})u_{max}, \quad \textit{solve for}\ \mathbf{u_{max} = 7.97\ m/s} \quad \textit{Ans.}\text{ (c)}$$

---

**P3.21** Water from a storm drain flows over an outfall onto a porous bed which absorbs the water at a uniform vertical velocity of 8 mm/s, as shown in Fig. P3.21. The system is 5 m deep into the paper. Find the length *L* of bed which will completely absorb the storm water.

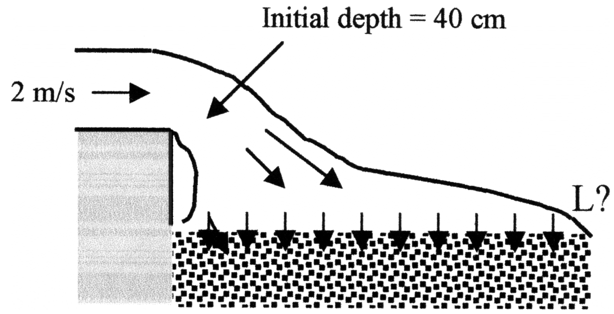



**Fig. P3.21**

**Solution:** For the bed to completely absorb the water, the flow rate over the outfall must equal that into the porous bed,

$$Q_1 = Q_{PB}; \quad \text{or} \quad (2\text{ m/s})(0.2\text{ m})(5\text{ m}) = (0.008\text{ m/s})(5\text{ m})L \quad \mathbf{L \approx 50\ m} \quad \textit{Ans.}$$

---

**P3.22** Oil (SG-0.91) enters the thrust bearing at 250 N/hr and exits radially through the narrow clearance between thrust plates. Compute (a) the outlet volume flow in mL/s, and (b) the average outlet velocity in cm/s.

**Solution:** The specific weight of the oil is (0.91)(9790) = 8909 N/m$^3$. Then

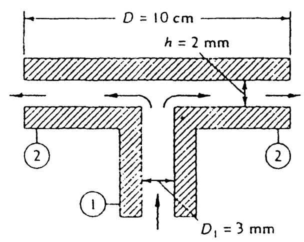



**Fig. P3.22**

$$Q_2 = Q_1 = \frac{250/3600\text{ N/s}}{8909\text{ N/m}^3} = 7.8\times10^{-6}\ \frac{\text{m}^3}{\text{s}} = \mathbf{7.8\ \frac{mL}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{But also} \quad Q_2 = V_2\,\pi(0.1\text{ m})(0.002\text{ m}) = 7.8\times10^{-6}, \quad \text{solve for } V_2 = \mathbf{1.24\ \frac{cm}{s}} \quad \textit{Ans.}\text{ (b)}$$

---

**P3.23** For the two-port tank in Fig. E3.5, let the dimensions remain the same, but assume $V_2$ = 1 m/s and that $V_1$ is unknown. If the water surface is rising at a rate of 2.5 cm/s, (*a*) determine the average velocity at section 1. (*b*) Is the flow at section 1 in or out?

**Solution**: Simply modify the calculations of Ex. 3.5 to match the new data. Assuming the water density is constant, the mass balance reduces to a set of volume flows:

$$A_1V_1 + A_2V_2 = A_t\frac{dh}{dt}\ ,\ \text{or}: \frac{\pi}{4}(2.5\times10^{-2}\ \text{m})^2 V_1 + \frac{\pi}{4}(7.5\times10^{-2}\ \text{m})^2(1\ \text{m/s})$$
$$= (0.186\ \text{m}^2)(2.5\times10^{-2}\ \text{m/s})$$
$$(4.91\times10^{-4}\text{m}^2)V_1 + (4.42\times10^{-3}\text{m}^3/\text{s}) = (4.65\times10^{-3}\text{m}^3/\text{s})$$
$$\textit{Solve for}\quad V_1 = \mathbf{0.47}\ \text{m/s}\ (\textit{flow}\ \mathbf{in})\quad \textit{Ans.}$$

---

**P3.24** The converging-diverging nozzle shown in Fig. P3.24 expands and accelerates dry air to supersonic speeds at the exit, where $p_2$ = 8 kPa and $T_2$ = 240 K. At the throat, $p_1$ = 284 kPa, $T_1$ = 665 K, and $V_1$ = 517 m/s. For steady compressible flow of an ideal gas, estimate (a) the mass flow in kg/h, (b) the velocity $V_2$, and (c) the Mach number $Ma_2$.

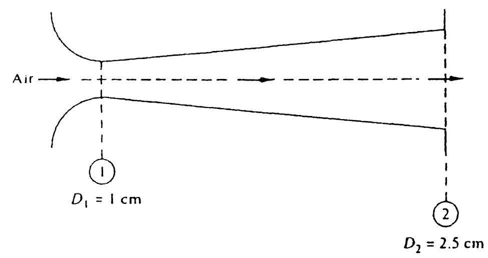



**Fig. P3.24**

**Solution:** The mass flow is given by the throat conditions:

$$\dot{m} = \rho_1 A_1 V_1 = \left[\frac{284000}{(287)(665)}\frac{\text{kg}}{\text{m}^3}\right]\frac{\pi}{4}(0.01\ \text{m})^2\left(517\ \frac{\text{m}}{\text{s}}\right) = \mathbf{0.0604}\ \frac{\mathbf{kg}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(a)}$$

For steady flow, this must equal the mass flow at the exit:

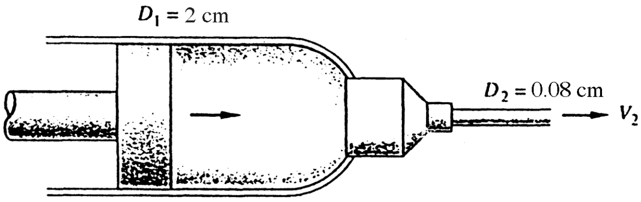
$$0.0604\ \frac{\text{kg}}{\text{s}} = \rho_2 A_2 V_2 = \left[\frac{8000}{287(240)}\right]\frac{\pi}{4}(0.025)^2 V_2,\quad \text{or}\quad V_2 \approx \mathbf{1060}\ \frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(b)}$$

Recall from Eq. (1.39) that the speed of sound of an ideal gas = $(kRT)^{1/2}$. Then

$$\text{Mach number at exit:}\quad Ma = V_2/a_2 = \frac{1060}{[1.4(287)(240)]^{1/2}} \approx \mathbf{3.41}\quad \textit{Ans.}\ \text{(c)}$$

---

**P3.25** The hypodermic needle in the figure contains a liquid (SG = 1.05). If the serum is to be injected steadily at 6 cm$^3$/s, how fast should the plunger be advanced (a) if leakage in the plunger clearance is neglected; and (b) if leakage is 10 percent of the needle flow?




**Solution:** (a) For incompressible flow, the volume flow is the same at piston and exit:

$$Q = 6\ \frac{\text{cm}^3}{\text{s}} = A_1V_1 = \frac{\pi}{4}(2\text{ cm})^2V_1, \quad solve\ V_{piston} = \mathbf{1.91\ \frac{cm}{s}} \quad Ans.\text{ (a)}$$

(b) If there is 10% leakage, the piston must deliver both needle flow and leakage:

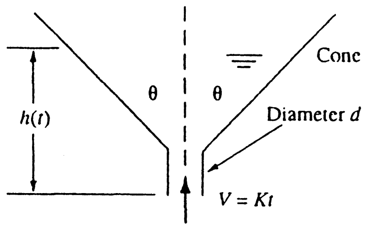
$$A_1V_1 = Q_{needle} + Q_{clearance} = 6 + 0.1(6) = 6.6\ \frac{cm^3}{s} = \frac{\pi}{4}(2)^2V_1,$$

$$V_1 = \mathbf{2.1\ \frac{cm}{s}} \quad Ans.\text{ (b)}$$

---

**P3.26** Water enters the bottom of the cone in the figure at a uniformly increasing average velocity $V = Kt$. If $d$ is very small, derive an analytic formula for the water surface rise $h(t)$, assuming $h = 0$ at $t = 0$.

**Solution:** For a control volume around the cone, the mass relation becomes




$$\frac{d}{dt}\left(\int \rho\, d\upsilon\right) - \dot{m}_{in} = 0 = \frac{d}{dt}\left[\rho\frac{\pi}{3}(h\tan\theta)^2h\right] - \rho\frac{\pi}{4}d^2Kt$$

$$Integrate:\quad \rho\frac{\pi}{3}h^3\tan^2\theta = \rho\frac{\pi}{8}d^2Kt^2$$

$$Solve\ for\quad \mathbf{h(t) = \left[\frac{3}{8}Kt^2d^2\cot^2\theta\right]^{1/3}} \quad Ans.$$

---

**P3.27** As will be discussed in Chaps. 7 and 8, the flow of a stream $U_o$ past a blunt flat plate creates a broad low-velocity wake behind the plate. A simple model is given in Fig. P3.27, with only half of the flow shown due to symmetry. The velocity profile behind the plate is idealized as "dead air" (near-zero velocity) behind the plate, plus a higher velocity, decaying vertically above the wake according to the variation $u \approx U_o + \Delta U e^{-z/L}$, where $L$ is the plate height and $z = 0$ is the top of the wake. Find $\Delta U$ as a function of stream speed $U_o$.

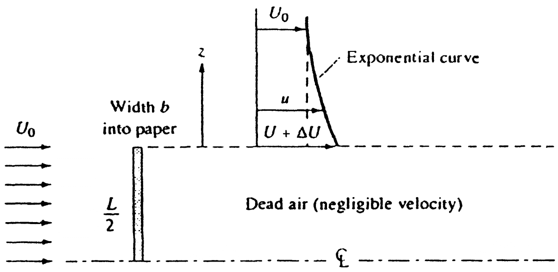



**Fig. P3.27**

**Solution:** For a control volume enclosing the upper half of the plate and the section where the exponential profile applies, extending upward to a large distance H such that exp(–H/L) ≈ 0, we must have inlet and outlet volume flows the same:

$$Q_{in} = \int_{-L/2}^{H} U_o dz = Q_{out} = \int_{0}^{H} (U_o + \Delta U e^{-z/L}) dz, \quad \text{or:} \quad U_o\left(H + \frac{L}{2}\right) = U_o H + \Delta U L$$

$$\text{Cancel } U_o H \text{ and solve for} \quad \Delta U \approx \mathbf{\frac{1}{2} U_o} \quad \textit{Ans.}$$

---

**P3.28** A thin layer of liquid, draining from an inclined plane, as in the figure, will have a laminar velocity profile $u = U_o(2y/h - y^2/h^2)$, where $U_o$ is the surface velocity. If the plane has width $b$ into the paper, (a) deter-mine the volume rate of flow of the film. (b) Suppose that $h$ = 1.25 cm and the flow rate per foot of channel width is 5 L/min. Estimate $U_o$ in m/s.

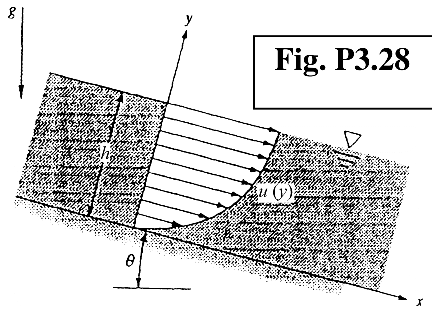



**Fig. P3.28**

**Solution:** (a) The total volume flow is computed by integration over the flow area:

$$Q = \int V_n \, dA = \int_0^h U_o\left(\frac{2y}{h} - \frac{y^2}{h^2}\right) b \, dy = \mathbf{\frac{2}{3} U_o bh} \quad \textit{Ans.}\text{ (a)}$$

(b) Evaluate the above expression for the given data:

$$Q = 5 \, \frac{\text{L}}{\text{min}} = 8.33 \times 10^{-5} \, \frac{\text{m}^3}{\text{s}} = \frac{2}{3} U_o bh = \frac{2}{3} U_o (0.3 \text{ m})(1.25 \times 10^{-2} \text{ m}),$$

$$\textit{solve for} \quad U_o = \mathbf{0.033 \ m/s} \quad \textit{Ans.}\text{ (b)}$$

---

**P3.29** Consider a highly pressurized air tank at conditions ($p_o$, $\rho_o$, $T_o$) and volume $\upsilon_o$. In Chap. 9 we will learn that, if the tank is allowed to exhaust to the atmosphere through a well-designed converging nozzle of exit area *A*, the outgoing mass flow rate will be

$$\dot{m} = \frac{\alpha p_o A}{\sqrt{RT_o}} \quad , \qquad \text{where} \quad \alpha \approx 0.685 \quad \text{for air}$$

This rate persists as long as $p_o$ is at least twice as large as the atmospheric pressure. Assuming constant $T_o$ and an ideal gas, (a) derive a formula for the change of density $\rho_o(t)$ within the tank. (*b*) Analyze the time $\Delta t$ required for the density to decrease by 25%.

**Solution**: First convert the formula to reflect tank *density* instead of pressure:

$$\dot{m} = \frac{\alpha p_o A}{\sqrt{RT_o}} = \frac{\alpha(\rho_o R T_o)A}{\sqrt{RT_o}} = \alpha \rho_o A \sqrt{RT_o}$$

(a) Now apply a mass balance to a control volume surrounding the tank:

$$\frac{dm}{dt}\Big|_{system} = 0 = \frac{d}{dt}(\rho_o \upsilon_o) + \dot{m}_{out} = \upsilon_o \frac{d\rho_o}{dt} + \alpha\rho_o A\sqrt{RT_o}$$

$$\text{Separate variables:} \quad \frac{d\rho_o}{\rho_o} = -\alpha A\sqrt{RT_o}\, dt$$

$$\text{Integrate from state 1 to state 2:} \quad \frac{\rho_{o2}}{\rho_{o1}} = \exp\left[\frac{-\alpha A\sqrt{RT_o}}{\upsilon_o}(t_2 - t_1)\right] \qquad Ans.(a)$$

(*b*) If the density drops by 25%, then we compute

$$\frac{\alpha A\sqrt{RT_o}}{\upsilon_o}(t_2 - t_1) = -\ln(0.75) = 0.288 \ ; \quad \text{Thus } \Delta t = \frac{0.288\,\upsilon_o}{\alpha A\sqrt{RT_o}} \qquad Ans.(b)$$

---

**P3.30** Air, assumed to be a perfect gas from Table A.4, flows through a long, 2-cm-diameter insulated tube. At section 1, the pressure is 1.1 MPa and the temperature is 345 K. At section 2, 67 meters further downstream, the density is 1.34 kg/m$^3$, the temperature 298 K, and the Mach number is 0.90. For one-dimensional flow, calculate (*a*) the mass flow; (*b*) $p_2$; (c) $V_2$; and (*d*) the change in entropy between 1 and 2. (*e*) How do you explain the entropy change?

**Solution**: For air, $k = 1.40$ and $R = 287$ m$^2$/s$^2$-K, hence $c_p = kR/(k-1) = 1005$ m$^2$/s$^2$-K. (*a, c*) We have enough information at section 2 to calculate the velocity, hence the mass flow:

$$a_2 = \sqrt{kRT_2} = \sqrt{1.4(287)(298K)} = 346\frac{m}{s}, \text{ thus } V_2 = Ma_2\, a_2 = (0.9)(346) = \mathbf{311}\frac{m}{s} \quad Ans.(c)$$

$$\text{Then } \dot{m} = \rho_2 A_2 V_2 = (1.34\frac{kg}{m^3})[\frac{\pi}{4}(0.02\,m)^2](311\frac{m}{s}) = \mathbf{0.131}\,\frac{kg}{s} \quad Ans.(a)$$

(*b*) The pressure at section 2 follows from the perfect gas law:

$$p_2 = \rho_2 RT_2 = (1.34\frac{kg}{m^3})(287\frac{N-m}{kg\ K})(298K) = 115{,}000\frac{N}{m^2} = \mathbf{115{,}000}\ Pa \quad Ans.(b)$$

(*d*) For a perfect gas with constant specific heats, the entropy change is

$$s_2 - s_1 = c_p\ln(\frac{T_2}{T_1}) - R\ln(\frac{p_2}{p_1}) = (1005)\ln(\frac{298K}{345K}) - (287)\ln(\frac{115kPa}{1100kPa})$$

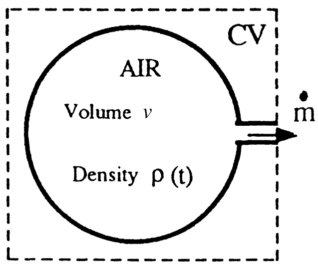
$$= -147 - (-648) = +\mathbf{501}\ \frac{J}{kg-K} \qquad Ans.(d)$$

(*e*) The entropy has *increased*, yet there is no heat transfer (insulated pipe). The answer is *irreversibility*. ***Friction*** in the long pipe has caused viscous dissipation in the fluid. NOTE: These numbers <u>are not just made up</u>. They represent a typical case of *compressible flow of air in a long pipe with friction*, to be studied in Chapter 9.

---

**P3.31** In elementary compressible-flow theory (Chap. 9), compressed air will exhaust from a small hole in a tank at the mass flow rate $\dot{m} \approx C\rho$, where $\rho$ is the air density in the tank and *C* is a constant.

If $\rho_o$ is the initial density in a tank of volume *v*, derive a formula for the density change $\rho(t)$ after the hole is opened. Apply your formula to the following case: a spherical tank of diameter 50 cm, with initial pressure 300 kPa and temperature 100°C, and a hole whose initial exhaust rate is 0.01 kg/s. Find the time required for the tank density to drop by 50 percent.




**Solution:** For a control volume enclosing the tank and the exit jet, we obtain

$$0 = \frac{d}{dt}\left(\int \rho\, dv\right) + \dot{m}_{out}, \quad \text{or:} \quad v\frac{d\rho}{dt} = -\dot{m}_{out} = -C\rho,$$

$$\text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\frac{C}{v}\int_0^t dt, \quad \text{or:} \quad \frac{\rho}{\rho_o} \approx \mathbf{exp}\left[-\frac{\mathbf{C}}{v}\mathbf{t}\right] \quad Ans.$$

Now apply this formula to the given data. If $p_o$ = 300 kPa and $T_o$ = 100°C = 373°K, then $\rho_o$ = p/RT = (300,000)/[287(373)] ≈ 2.80 kg/m$^3$. This establishes the constant "C":

$$\dot{m}_o = C\rho_o = 0.01\ \frac{kg}{s} = C\left(2.80\ \frac{kg}{m^3}\right), \quad \text{or} \quad C \approx 0.00357\ \frac{m^3}{s} \text{ for this hole.}$$

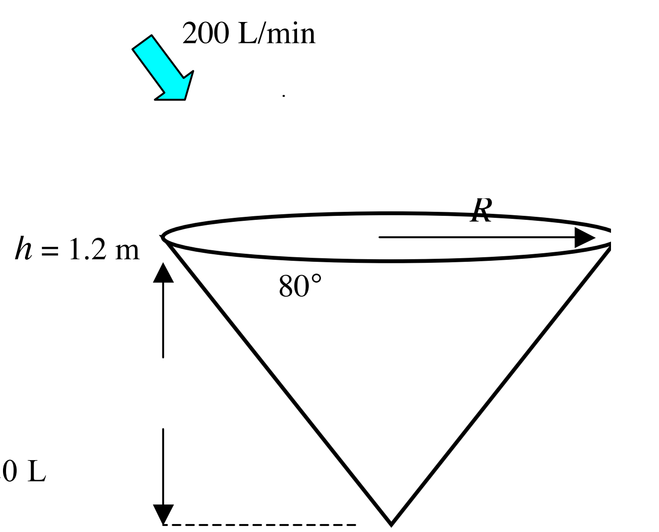
The tank volume is $\upsilon = (\pi/6)D^3 = (\pi/6)(0.5\text{ m})^3 \approx 0.00654\text{ m}^3$. Then we require

$$\rho/\rho_o = 0.5 = \exp\left[-\frac{0.00357}{0.00654}t\right] \quad \text{if } t \approx \mathbf{1.3\ s} \quad \textit{Ans.}$$

---

**P3.32** A hollow conical container, standing point-down, is 1.2 m high and has a total included cone angle of 80°. It is being filled with water from a hose at 200 L per minute. How long will it take to fill the cone?

**Solution**: The control volume, of course, surrounds the cone with one inlet, no exits. We don't need any complicated integral mass relations, for the flow rate is known, as is the cone volume. The radius of the upper "base" of the cone is




$$R = h\tan(40^\circ) = (1.2\,m)(0.839) = 1.007\,m$$

The volume of the cone is

$$\upsilon = \frac{\pi}{3}R^2 h = \frac{\pi}{3}(1.007\text{m})^2(1.2\text{m}) = 3.82\,\text{m}^3 = 3820\text{ L}$$

Clearly, then, the time to fill the cone is (3820 L)/(200 L/min) = **19.1 minutes**. *Ans.*

---

**P3.33** A bellows may be modeled as a deforming wedge-shaped volume as in Fig. P3.33. The check valve on the left (pleated) end is closed during the stroke. If $b$ is the bellows width into the paper, derive an expression for outlet mass flow $\dot{m}_o$ as a function of stroke $\theta(t)$.

**Solution:** For a control volume enclosing the bellows and the outlet flow, we obtain

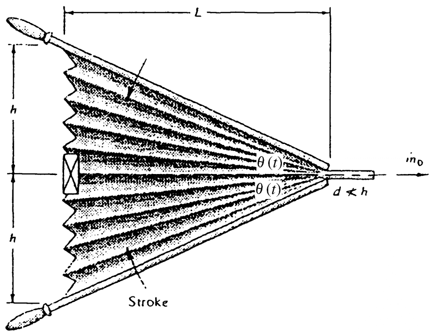


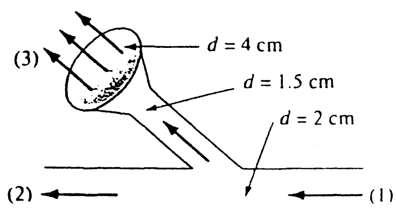

**Fig. P3.33**

$$\frac{d}{dt}(\rho \upsilon) + \dot{m}_{out} = 0, \quad \text{where } \upsilon = bhL = bL^2 \tan\theta$$

$$\text{since L is constant, solve for} \quad \dot{m}_o = -\frac{d}{dt}(\rho b L^2 \tan\theta) = \mathbf{-\rho b L^2 \sec^2\theta \frac{d\theta}{dt}} \quad \textit{Ans.}$$

---

**P3.34** Water at 20°C flows through the piping junction in the figure, entering section 1 at 75 L/min. The average velocity at section 2 is 2.5 m/s. A portion of the flow is diverted through the showerhead, which contains 100 holes of 1-mm diameter. Assuming uniform shower flow, estimate the exit velocity from the showerhead jets.




**Solution:** A control volume around sections (1, 2, 3) yields

$$Q_1 = Q_2 + Q_3 = 1.25 \times 10^{-3} \text{ m}^3/\text{s}.$$

Meanwhile, with $V_2$ = 2.5 m/s known, we can calculate $Q_2$ and then $Q_3$:

$$Q_2 = V_2 A_2 = (2.5 \text{ m})\frac{\pi}{4}(0.02 \text{ m})^2 = 0.000785 \ \frac{\text{m}^3}{\text{s}},$$

$$\textit{hence} \quad Q_3 = Q_1 - Q_2 = 0.00125 - 0.000785 = 0.000465 \ \frac{\text{m}^3}{\text{s}}$$

$$\textit{Each hole carries} \quad Q_3/100 = 0.00000465 \ \frac{\text{m}^3}{\text{s}} = \frac{\pi}{4}(0.001)^2 V_{jet},$$

$$\textit{solve} \quad V_{jet} = \mathbf{5.92 \ \frac{m}{s}} \quad \textit{Ans.}$$

---

**P3.35** In some wind tunnels the test section is perforated to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig. P3.35 contains 1200 holes of 5-mm diameter each per square meter of wall area. The suction velocity through each hole is $V_r = 8$ m/s, and the test-section entrance velocity is $V_1 = 35$ m/s. Assuming incompressible steady flow of air at 20°C, compute (a) $V_o$, (b) $V_2$, and (c) $V_f$, in m/s.

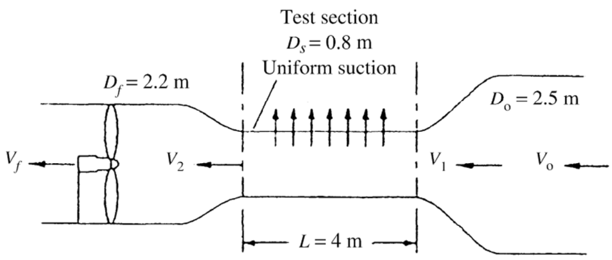



**Fig. P3.35**

**Solution:** The test section wall area is ($\pi$)(0.8 m)(4 m) = 10.053 m$^2$, hence the total number of holes is (1200)(10.053) = 12064 holes. The total suction flow leaving is

$$Q_{suction} = NQ_{hole} = (12064)(\pi/4)(0.005\text{ m})^2(8\text{ m/s}) \approx 1.895\text{ m}^3/\text{s}$$

$$\text{(a) Find } V_o\text{:} \quad Q_o = Q_1 \quad \text{or} \quad V_o\frac{\pi}{4}(2.5)^2 = (35)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for } \quad V_o \approx \mathbf{3.58}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)} \quad Q_2 = Q_1 - Q_{suction} = (35)\frac{\pi}{4}(0.8)^2 - 1.895 = V_2\frac{\pi}{4}(0.8)^2,$$

$$\text{or:} \quad V_2 \approx \mathbf{31.2}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\text{ (b)}$$

$$\text{(c) Find } V_f\text{:} \quad Q_f = Q_2 \quad \text{or} \quad V_f\frac{\pi}{4}(2.2)^2 = (31.2)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for } \quad V_f \approx \mathbf{4.13}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\text{ (c)}$$

**P3.36** A rocket motor is operating steadily, as shown in Fig. P3.36. The products of combustion flowing out the exhaust nozzle approximate a perfect gas with a molecular weight of 28. For the given conditions calculate $V_2$ in m/s.

**Solution:** Exit gas: Molecular weight = 28, thus $R_{gas} = 297\ m^2/(s^2\cdot K)$. Then,

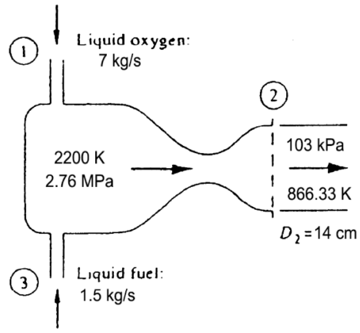



**Fig. P3.36**

$$\rho_{\text{exit gas}} = \frac{p}{RT} = \frac{103\ \text{kPa}}{(297)(866.33)} \approx 0.4\ \text{kg/m}^3$$

For mass conservation, the exit mass flow must equal <u>fuel + oxygen entering</u> = 8.5 kg/s:

$$\dot{m}_{\text{exit}} = 8.5\ \text{kg/s} = \rho_e A_e V_e = (0.4)\frac{\pi}{4}(14\times 10^{-2})^2 V_e, \quad \text{solve for } V_e \approx \mathbf{1380.4\ \ m/s} \quad \textit{Ans.}$$

---

**P3.37** In contrast to the liquid rocket in Fig. P3.36, the solid-propellant rocket in Fig. P3.37 is self-contained and has no entrance ducts. Using a control-volume analysis for the conditions shown in Fig. P3.37, compute the rate of mass loss of the propellant, assuming that the exit gas has a molecular weight of 28.

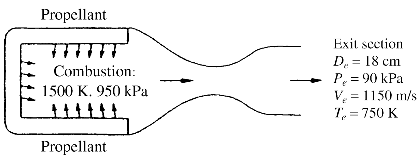



**Fig. P3.37**

**Solution:** With M = 28, R = 8313/28 =297 $m^2/(s^2\cdot K)$, hence the exit gas density is

$$\rho_{\text{exit}} = \frac{p}{RT} = \frac{90{,}000\ \text{Pa}}{(297)(750\ \text{K})} = 0.404\ \text{kg/m}^3$$

For a control volume enclosing the rocket engine and the outlet flow, we obtain

$$\frac{d}{dt}(m_{CV}) + \dot{m}_{out} = 0,$$

$$\text{or:} \quad \frac{d}{dt}(m_{\text{propellant}}) = -\dot{m}_{\text{exit}} = -\rho_e A_e V_e = -(0.404)(\pi/4)(0.18)^2(1150) \approx \mathbf{-11.8\ \frac{kg}{s}} \quad \textit{Ans.}$$

---

**P3.38** The jet pump in Fig. P3.38 injects water at $U_1$ = 40 m/s through a 7.5-cm pipe and entrains a secondary flow of water $U_2$ = 3 m/s in the annular region around the small pipe. The two flows become fully mixed down-stream, where $U_3$ is approximately constant. For steady incompressible flow, compute $U_3$ in m/s.

**Solution:** First modify the units: $D_1$ = 0.075 m, $D_2$ = 0.25 m. For incompressible flow, the volume flows at inlet and exit must match:

$$Q_1 + Q_2 = Q_3, \quad \text{or:} \quad \underbrace{\frac{\pi}{4}(0.075)^2(40)}_{0.1767} + \underbrace{\frac{\pi}{4}[(0.25)^2 - (0.075)^2](3)}_{0.134} = \frac{\pi}{4}(0.25)^2 U_3$$

Solve for $\mathbf{U_3 \approx 6.33\ m/s}$ (OK) *Ans.*

---

**P3.39** If the rectangular tank full of water in Fig. P3.39 has its right-hand wall lowered by an amount $\delta$, as shown, water will flow out as it would over a weir or dam. In Prob. P1.16 we deduced that the outflow $Q$ would be given by

$$Q = C b g^{1/2} \delta^{3/2}$$

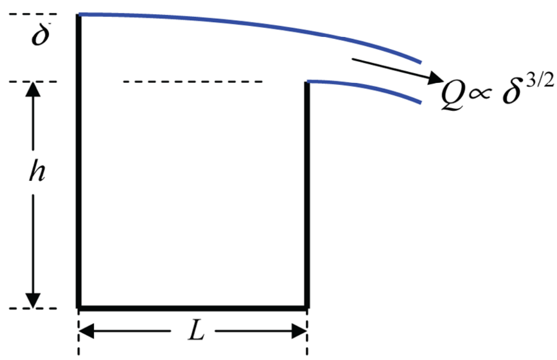



**Fig. P3.39**

where $b$ is the tank width into the paper, $g$ is the acceleration of gravity, and $C$ is a dimensionless constant. Assume that the water surface is horizontal, not slightly curved as in the figure. Let the initial excess water level be $\delta_o$. Derive a formula for the time required to reduce the excess water level to (*a*) $\delta_o/10$; and (*b*) to zero.

**Solution**: The control volume encloses the tank and cuts through the outlet flow. From Eq. (3.20),

$$\frac{d}{dt}\left(\int \rho\, d\upsilon\right) + \rho Q_{out} = \frac{d}{dt}[\rho L b(h+\delta)] + \rho C b g^{1/2} \delta^{3/2}\ , \text{ cancel } \rho \text{ and } b;$$

$$L\frac{d\delta}{dt} = -C g^{1/2} \delta^{3/2}\ . \text{ Separate variables: } \int_{\delta_o}^{\delta} \frac{d\delta}{\delta^{3/2}} = -\int_0^t \frac{C g^{1/2}}{L} dt$$

where $\delta$ is the instantaneous excess water level. The integrated result for water level $\delta(t)$ is

$$\frac{1}{\delta^{1/2}} = \frac{1}{\delta_o^{1/2}} + \frac{C g^{1/2}}{2L} t$$

The two specific results requested are:

$$(a)\ \delta = \frac{\delta_o}{10}:\ t = \frac{4.32L}{C g^{1/2} \delta_o^{1/2}} \quad Ans.(a) \quad ; \quad (b)\ \delta = 0:\ t = \infty \quad Ans.(b)$$

It doesn't really take infinitely long to reach the final level, because surface tension comes into play, at the lip of the dam, as $\delta$ becomes very small.

---

**P3.40** An incompressible fluid is squeezed between two disks by downward motion $V_o$ of the upper disk. Assuming 1-dimensional radial outflow, find the velocity V(r).

**Solution:** Let the CV enclose the disks and have an upper surface moving down at speed $V_o$. There is no inflow. Thus

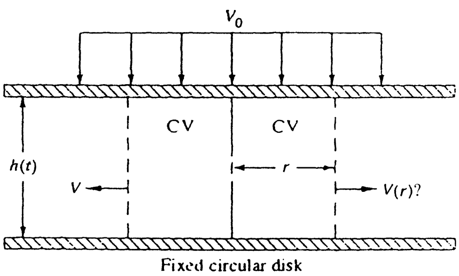



**Fig. P3.40**

$$\frac{d}{dt}\left(\int_{CV} \rho\, d\upsilon\right) + \int_{CS} \rho V_{out}\, dA = 0 = \frac{d}{dt}(\rho\pi r^2 h) + \rho 2\pi r h\, V,$$

$$\text{or:}\quad r^2 \frac{dh}{dt} + 2rhV = 0, \quad \text{but} \quad \frac{dh}{dt} = -V_o(\text{the disk velocity})$$

As the disk spacing drops, $h(t) \approx h_o - V_o t$, the outlet velocity is $\mathbf{V = V_o\, r/(2h)}$. *Ans.*

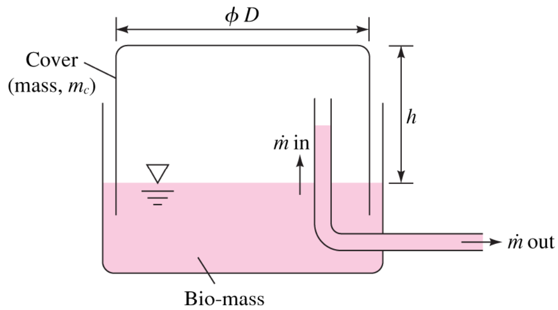
---

**P3.41** The average rainfall for 3 hours in a 15-km$^2$ area is 25 mm/h. The average soil absorption is about 2.5 mm/h. Is there any water left after 24 hours? If there is some water left, how deep is it?

**Solution:** This problem is a simple conservation of mass with steady flow process. Therefore, the rainfall causes water to rise after three hours of raining by 3(25 – 2.5) = 67.5 mm. Within 24 hours, the soil can absorb water up to 24(2.5) = 60.0 mm. There is 7.5 mm depth of water left.

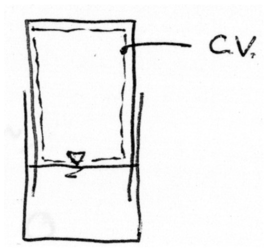
---

**P3.42** A simple bio-gas well in Fig. P3.42 operates in batches. Bio-mass in the well produces bio-gas to fill the cover, which is a cylinder with one end closed. Before the gas is released to customers, the cover's end must be at $h_{\max}$ from the bio-mass surface. Then a batch ends when the cover's end reaches $h_{\min}$. With the given information, how long does it take for each batch of bio-gas to be produced?




**P3.42**

**Solution:** consider control volume as follows:




From conservation of mass

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (1)$$

Assume that the friction between bio-mass and the cover is low and gas temperature is constant. Normally the cover moves up and down slow enough so the pressure is maintained by weight of the cover. Then (1) becomes

$$\rho\frac{\partial}{\partial t}[\pi D^2\cdot h] + \dot{m}_{\text{out}} - \dot{m}_{\text{in}} = 0$$

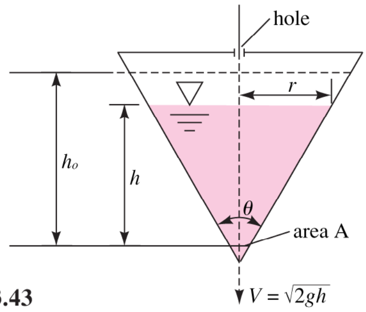
$$\pi\rho D^2\frac{\partial h}{\partial t} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}$$

$$\int_{h_{\max}}^{h_{\min}} dh = \frac{1}{\pi\rho D^2}\int_0^t(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})dt$$

$$\therefore \qquad t = \frac{\pi\rho D^2(h_{\min} - h_{\max})}{(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})}$$

---

**P3.43** A cone-shaped tank was filled with liquid as shown in Fig. P3.43. The tank's apex was cut open allowing the liquid to drain. The tank's base was punched to keep pressure inside the tank equal to atmospheric pressure. Assume that draining speed is a function of depth of the liquid measured from the free surface to the hole, that is, $V = \sqrt{2gh}$. At the beginning, the liquid's depth was $h_o$. Determine the time to drain the liquid in terms of initial volume, $V_0$, and flow rate, $Q_0$,

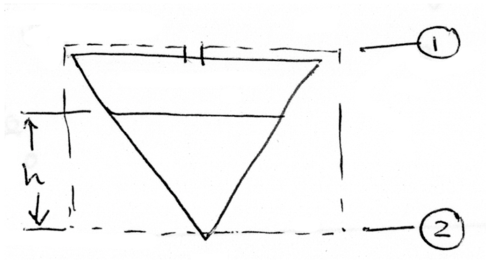



**P3.43**

**Solution:** Consider conservation of mass equation

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$$

Given control volume




In the control volume, there are both liquid and air. Therefore, we can write the equation above as

$$\frac{\partial}{\partial t}\int_{CV_{\text{liq}}}\rho_{\text{liq}}\, dV_{\text{liq}} + \frac{\partial}{\partial t}\int_{CV_{\text{air}}}\rho_{\text{air}}\, dV_{\text{air}} - \rho_{\text{air}}V_1A_1 + \rho_{\text{liq}}VA = 0$$

Since $\rho_{\text{liq}} \gg \rho_{\text{air}}$, then effect of any changes from air would be negligible, so that

$$\frac{\partial}{\partial t}\int_{CV_{\text{liq}}}\rho_{\text{liq}}\, dV_{\text{liq}} + \rho_{\text{liq}}VA = 0$$

$$\frac{\partial}{\partial t}(\rho_{\text{liq}}V_{\text{liq}}) + \rho_{\text{liq}}VA = 0$$

Find $V_{\text{liq}} = \int_{CV_{\text{liq}}} dV = \int_0^h \pi r^2\, dh = \int_0^h \pi h^2 \tan^2\left(\frac{\theta}{2}\right) dh = \frac{1}{3}\pi\tan^2\left(\frac{\theta}{2}\right)h^3$

Substitute $V = \sqrt{2gh}$ into equation above, we have

$$\frac{\partial}{\partial t}\left(\frac{1}{3}\rho_{\text{liq}}\pi\tan^2\left(\tfrac{\theta}{2}\right)h^3\right) + \rho_{\text{liq}}\sqrt{2gh}\ \text{A} = 0$$

$$\rho_{\text{liq}}\ \pi\tan^2\left(\frac{\theta}{2}\right)h^2\frac{dh}{dt} + \rho_{\text{liq}}\sqrt{2gh}\ \text{A} = 0$$

$$\therefore\quad h^{3/2}dh = \frac{-\sqrt{2g}\ \text{A}}{\pi\tan^2\left(\frac{\theta}{2}\right)}dt$$

Integrate from $h = h_0$ at $t = 0$ to $h = 0$ at $t = t$.

$$\int_{h_0}^{0} h^{3/2}dh = \frac{-\sqrt{2g}\ \text{A}}{\pi\tan^2\left(\frac{\theta}{2}\right)}t$$

$$\therefore\quad t = \frac{2}{5}\frac{\pi\tan^2\left(\frac{\theta}{2}\right)h_0^{5/2}}{\sqrt{2g}\ \text{A}}\quad,\quad V_0 = \frac{1}{3}\pi\tan^2\left(\frac{\theta}{2}\right)h_0^3$$

$$\theta_0 = \sqrt{2gh_0}\ \text{A}$$

$$t = \frac{6}{5}\frac{V_0}{Q_0}$$

*Ans.*

**P3.44** A wedge splits a sheet of 20°C water, as shown in Fig. P3.44. Both wedge and sheet are very long into the paper. If the force required to hold the wedge stationary is $F$ = 124 N per meter of depth into the paper, what is the angle $\theta$ of the wedge?

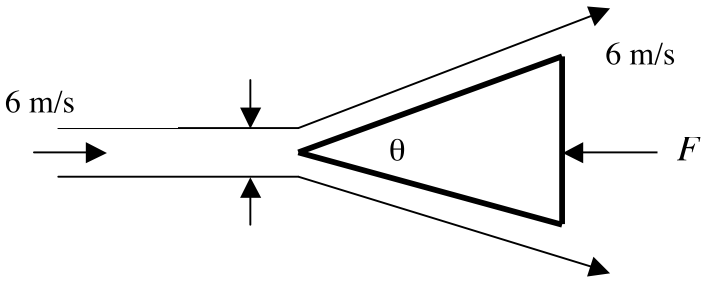



**Fig. P3.44** 4 cm 6 m/s

**Solution**: For water take $\rho$ = 998 kg/m$^3$. First compute the mass flow per unit depth:

$$\dot{m}/b = \rho V h = (998\,kg/m^3)(6\,m/s)(0.04\,m) = 239.5\,kg/s-m$$

The mass flow (and velocity) are the same entering and leaving. Let the control volume surround the wedge. Then the $x$-momentum integral relation becomes

$$\Sigma F_x = -F = \dot{m}(u_{out} - u_{in}) = \dot{m}(V\cos\frac{\theta}{2} - V) = \dot{m}V(\cos\frac{\theta}{2} - 1)$$

$$or:\ -124\,N/m = (239.5\,kg/s-m)(6\,m/s)(\cos\frac{\theta}{2} - 1)$$

$$\text{Solve } \cos\frac{\theta}{2} = 0.9137\,,\ \frac{\theta}{2} = 24^o\,,\ \theta = \mathbf{48^o} \quad \textit{Ans.}$$

---

**P3.45** The water jet in Fig. P3.45 strikes normal to a fixed plate. Neglect gravity and friction, and compute the force $F$ in newtons required to hold the plate fixed.

**Solution:** For a CV enclosing the plate and the impinging jet, we obtain:

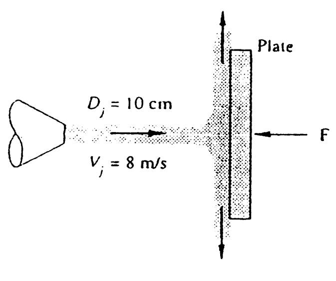



**Fig. P3.45**

$$\sum F_x = -F = \dot{m}_{up}u_{up} + \dot{m}_{down}u_{down} - \dot{m}_j u_j$$
$$= -\dot{m}_j u_j,\quad \dot{m}_j = \rho A_j V_j$$

$$\text{Thus } F = \rho A_j V_j^2 = (998)\pi(0.05)^2(8)^2 \approx \mathbf{500\ N} \leftarrow \quad \textit{Ans.}$$

---

**P3.46** In Fig. P3.46 the vane turns the water jet completely around. Find the maximum jet velocity $V_o$ for a force $F_o$.

**Solution:** For a CV enclosing the vane and the inlet and outlet jets,

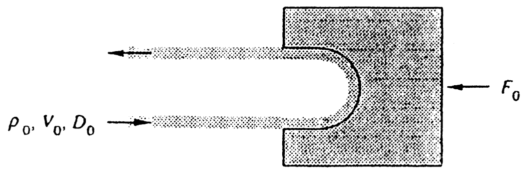



**Fig. P3.46**

$$\sum F_x = -F_o = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}_{jet}(-V_o) - \dot{m}_{jet}(+V_o)$$

$$\text{or:}\quad F_o = 2\rho_o A_o V_o^2,\quad \text{solve for } V_o = \sqrt{\frac{\mathbf{F_o}}{\mathbf{2\rho_o(\pi/4)D_o^2}}}\quad \textit{Ans.}$$

---

**P3.47** A liquid of density $\rho$ flows through the sudden contraction in Fig. P3.47 and exits to the atmosphere. Assume uniform conditions ($p_1$, $V_1$, $D_1$) at section 1 and ($p_2$, $V_2$, $D_2$) at section 2. Find an expression for the force $F$ exerted by the fluid on the contraction.

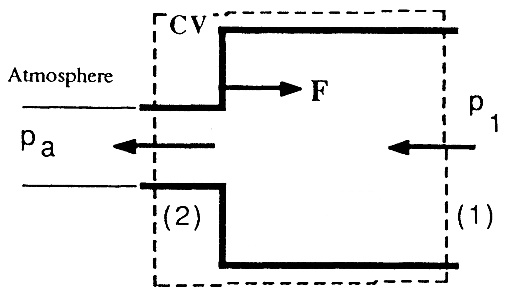



**Fig. P3.47**

**Solution:** Since the flow exits directly to the atmosphere, the exit pressure equals atmospheric: $p_2 = p_a$. Let the CV enclose sections 1 and 2, as shown. Use our trick (page 129 of the text) of subtracting $p_a$ everywhere, so that the only non-zero pressure on the CS is at section 1, $p = p_1 - p_a$. Then write the linear momentum relation with *x* to the right:

$$\sum F_x = F - (p_1 - p_a)A_1 = \dot{m}_2 u_2 - \dot{m}_1 u_1,\quad \text{where } \dot{m}_2 = \dot{m}_1 = \rho_1 A_1 V_1$$

$$\text{But } u_2 = -V_2 \text{ and } u_1 = -V_1.\quad \text{Solve for } F_{\underline{on}\text{ fluid}} = (p_1 - p_a)A_1 + \rho_1 A_1 V_1(-V_2 + V_1)$$

Meanwhile, from continuity, we can relate the two velocities:

$$Q_1 = Q_2,\quad \text{or}\quad (\pi/4)D_1^2 V_1 = (\pi/4)D_2^2 V_2,\quad \text{or:}\quad V_2 = V_1(D_1^2/D_2^2)$$

Finally, the force of the fluid <u>on the wall</u> is equal and opposite to $F_{\text{on fluid}}$, to the *left*:

$$F_{\text{fluid on wall}} = (p_1 - p_a)A_1 - \rho_1 A_1 V_1^2\left[\left(D_1^2/D_2^2\right) - 1\right],\quad A_1 = \frac{\pi}{4}D_1^2\quad \textit{Ans.}$$

The pressure term is larger than the momentum term, thus F > 0 and acts to the left.

---

**P3.48** Water at 20°C flows through a 5-cm-diameter pipe which has a 180° vertical bend, as in Fig. P3.48. The total length of pipe between flanges 1 and 2 is 75 cm. When the weight flow rate is 230 N/s, $p_1$ = 165 kPa, and $p_2$ = 134 kPa. Neglecting pipe weight, determine the total force which the flanges must withstand for this flow.

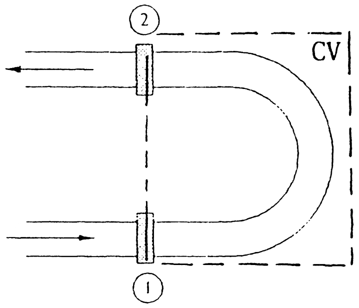



**Fig. P3.48**

**Solution:** Let the CV cut through the flanges and surround the pipe bend. The mass flow rate is $(230\text{ N/s})/(9.81\text{ m/s}^2) = 23.45$ kg/s. The volume flow rate is Q = 230/9790 = 0.0235 $m^3/s$. Then the pipe inlet and exit velocities are the same magnitude:

$$V_1 = V_2 = V = Q/A = \frac{0.0235\text{ m}^3/\text{s}}{(\pi/4)(0.05\text{ m})^2} \approx 12.0\ \frac{\text{m}}{\text{s}}$$

Subtract $p_a$ everywhere, so only $p_1$ and $p_2$ are non-zero. The horizontal force balance is:

$$\sum F_x = F_{x,\text{flange}} + (p_1 - p_a)A_1 + (p_2 - p_a)A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1$$

$$= F_{x,\text{fl}} + (64000)\frac{\pi}{4}(0.05)^2 + (33000)\frac{\pi}{4}(0.05)^2 = (23.45)(-12.0 - 12.0\text{ m/s})$$

$$\text{or:}\quad F_{x,\text{flange}} = -126 - 65 - 561 \approx \mathbf{-750\ N}\quad \textit{Ans.}$$

The total *x*-directed force on the flanges acts <u>to the left</u>. The vertical force balance is

$$\sum F_y = F_{y,\text{flange}} = W_{\text{pipe}} + W_{\text{fluid}} = 0 + (9790)\frac{\pi}{4}(0.05)^2(0.75) \approx \mathbf{14\ N}\quad \textit{Ans.}$$

Clearly the fluid weight is pretty small. The largest force is due to the 180° turn.

---

**P3.49** Consider uniform flow past a cylinder with a V-shaped *wake*, as shown. Pressures at (1) and (2) are equal. Let *b* be the width into the paper. Find a formula for the force *F* on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 L b)$.

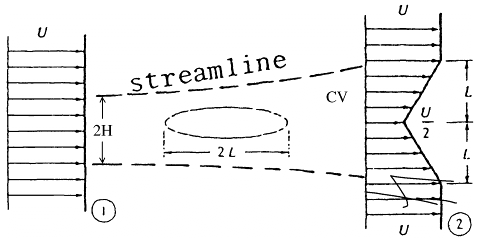



**Fig. P3.49**

**Solution:** The proper CV is the entrance (1) and exit (2) plus *streamlines* above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

$$0 = \int_2 \rho u \; dA - \int_1 \rho u \; dA = 2\int_0^L \rho \frac{U}{2}\left(1+\frac{y}{L}\right) b \; dy - 2\rho UbH,$$

where 2H is the inlet height. Solve for H = 3L/4.

Now the linear momentum relation is used. Note that the drag force F is to the right (force of the fluid on the body) thus the force F *of the body on fluid is to the left.* We obtain,

$$\sum F_x = 0 = \int_2 u\rho u \; dA - \int_1 u\rho u \; dA = 2\int_0^L \frac{U}{2}\left(1+\frac{y}{L}\right)\rho\frac{U}{2}\left(1+\frac{y}{L}\right) b\,dy - 2H\rho U^2 b = -F_{drag}$$

$$\text{Use} \quad H = \frac{3L}{4}, \quad \text{then } F_{drag} = \frac{3}{2}\rho U^2 Lb - \frac{7}{6}\rho U^2 Lb \approx \mathbf{\frac{1}{3}\rho U^2 Lb} \quad \textit{Ans.}$$

The dimensionless force, or drag coefficient $F/(\rho U^2 Lb)$, equals $\mathbf{C_D = 1/3}$. *Ans.*

---

**P3.50** A 12-cm-diameter pipe, containing water flowing at 200 N/s, is capped by an orifice plate, as in Fig. P3.50. The exit jet is 25 mm in diameter. The pressure in the pipe at section 1 is 800 kPa-gage. Calculate the force *F* required to hold the orifice plate.

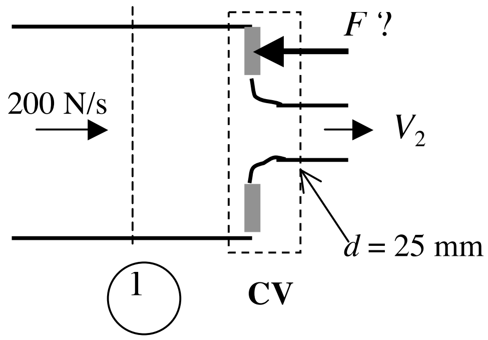



Fig. P3.50

*Solution*: For water take $\rho$ = 998 kg/m³. This is a straightforward *x*-momentum problem. First evaluate the mass flow and the two velocities:

$$\dot{m} = \frac{\dot{w}}{g} = \frac{200\,N/s}{9.81\,m/s^2} = 20.4\frac{kg}{s}\;;\; V_1 = \frac{\dot{m}}{\rho A_1} = \frac{20.4\,kg/s}{(998\,kg/m^3)(\pi/4)(0.12\,m)^2} = 1.81\frac{m}{s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{20.4\,kg/s}{(998\,kg/m^3)(\pi/4)(0.025\,m)^2} = 41.6\frac{m}{s}$$

Now apply the *x*-momentum relation for a control volume surrounding the plate:

$$\Sigma F_x = -F + p_{1,gage}A_1 = \dot{m}(V_2 - V_1)\;, \quad \text{or:}$$

$$F = (800000\,Pa)\frac{\pi}{4}(0.12\,m)^2 - (20.4\frac{kg}{s})(41.6 - 1.81\frac{m}{s}) = 9048 - 812 = \mathbf{8240\,N} \;\; \textit{Ans.}$$

---

**P3.51** When a jet strikes an inclined plate, it breaks into two jets of equal velocity V but unequal fluxes $\alpha$Q at (2) and (1 – $\alpha$)Q at (3), as shown. Find $\alpha$, assuming that the tangential force on the plate is zero. Why doesn't the result depend upon the properties of the jet flow?

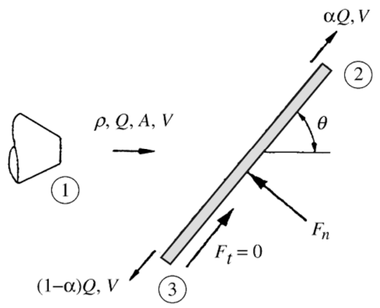



**Fig. P3.51**

**Solution:** Let the CV enclose all three jets and the surface of the plate. Analyze the force and momentum balance *tangential* to the plate:

$$\sum F_t = F_t = 0 = \dot{m}_2 V + \dot{m}_3(-V) - \dot{m}_1 V\cos\theta$$
$$= \alpha\dot{m}V - (1-\alpha)\dot{m}V - \dot{m}V\cos\theta = 0, \quad \text{solve for } \boldsymbol{\alpha = \frac{1}{2}(1+\cos\theta)} \quad \textit{Ans.}$$

The jet mass flow cancels out. Jet (3) has a fractional flow (1 – $\alpha$) = (1/2)(1 – cos$\theta$).

---

**P3.52** A liquid jet $V_j$ of diameter $D_j$ strikes a fixed cone and deflects back as a conical sheet at the same velocity. Find the cone angle $\theta$ for which the restraining force $F = (3/2)\rho A_j V_j^2$.

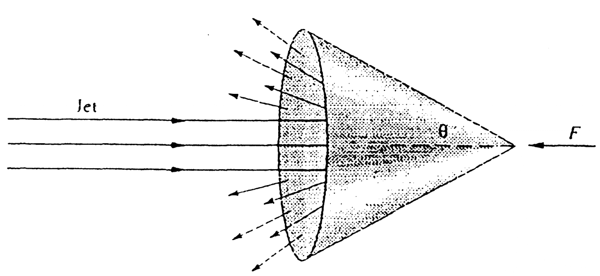



**Fig. P3.52**

**Solution:** Let the CV enclose the cone, the jet, and the sheet. Then,

$$\sum F_x = -F = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}(-V_j\cos\theta) - \dot{m}V_j, \quad \text{where } \dot{m} = \rho A_j V_j$$

$$\text{Solve for} \quad F = \rho A_j V_j^2(1+\cos\theta) = \frac{3}{2}\rho A_j V_j^2 \quad \text{if } \cos\theta = \frac{1}{2} \quad \text{or} \quad \theta = \mathbf{60^\circ} \quad \textit{Ans.}$$

---

**P3.53** The small boat is driven at steady speed $V_o$ by compressed air issuing from a 3-cm-diameter hole at $V_e$ = 343 m/s and $p_e$ = 1 atm, $T_e$ = 30°C. Neglect air drag. The hull drag is $kV_o^2$, where k = 19 N · s²/m². Estimate the boat speed $V_o$.

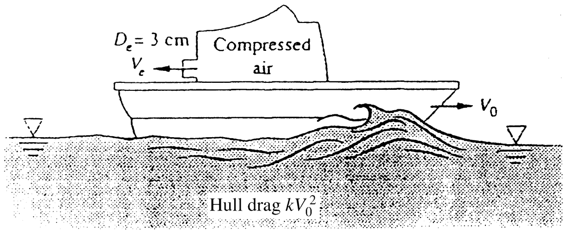



**Fig. P3.53**

**Solution:** For a CV enclosing the boat and *moving to the right at boat speed* $V_o$, the air appears to leave the left side at speed ($V_o$ + $V_e$). The air density is $p_e/RT_e \approx 1.165$ kg/m³. The only mass flow across the CS is the air moving to the left. The force balance is

$$\sum F_x = -\text{Drag} = -kV_o^2 = \dot{m}_{out}u_{out} = [\rho_e A_e(V_o + V_e)](-V_o - V_e),$$

$$\text{or:}\quad \rho_e A_e(V_o + V_e)^2 = kV_o^2,\quad (1.165)(\pi/4)(0.03)^2(V_o + 343)^2 = 19V_o^2$$

work out the numbers: $(V_o + 343) = V_o\sqrt{}(23060)$, solve for $\mathbf{V_o = 2.27\ m/s}$ *Ans.*

---

**P3.54** The horizontal nozzle in Fig. P3.54 has $D_1$ = 30 cm, $D_2$ = 15 cm, with $p_1$ = 262 kPa (absolute) and $V_2$ = 17 m/s. For water at 20°C, find the force provided by the flange bolts to hold the nozzle fixed.

**Solution:** For an open jet, $p_2 = p_a$ = 103.42 kPa(abs). Subtract $p_a$ everywhere so the only nonzero pressure is $p_1$ = 262 – 103.42 = 158.58 kPa (gage).

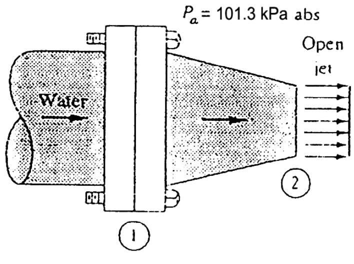
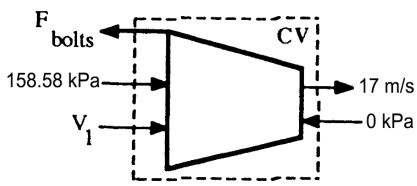



**Fig. P3.54**

The mass balance yields the inlet velocity:

$$V_1\frac{\pi}{4}(0.3)^2 = (17)\frac{\pi}{4}(0.15)^2,\quad V_1 = 4.25\ \text{m/s}$$




The density of water is 999.8 kg/m³. Then the horizontal force balance is

$$\sum F_x = -F_{bolts} + (158.58\ \text{kPa})\frac{\pi}{4}(0.3)^2 = \dot{m}_2u_2 - \dot{m}_1u_1 = \dot{m}(V_2 - V_1)$$

$$\text{Compute}\quad F_{bolts} = 11209.36 - (999.8)\frac{\pi}{4}(0.3)^2(4.25\ \text{m/s})(17 - 4.25\ \text{m/s}) \approx \mathbf{7379.84\ N}\quad \textit{Ans.}$$

---

**P3.55** The jet engine in Fig. P3.55 admits air at 20°C and 1 atm at (1), where $A_1 = 0.5$ $m^2$ and $V_1 = 250$ m/s. The fuel-air ratio is 1:30. The air leaves section (2) at 1 atm, $V_2 = 900$ m/s, and $A_2 = 0.4\ m^2$. Compute the test stand support reaction $R_x$ needed.

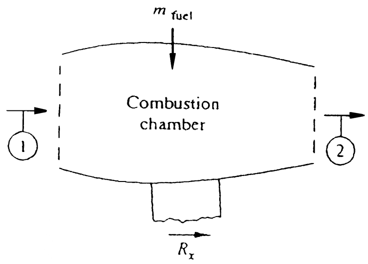


**Fig. P3.55**

**Solution:** $\rho_1 = p/RT = 101350/[287(293)] = 1.205\ kg/m^3$. For a CV enclosing the engine,

$$\dot{m}_1 = \rho_1 A_1 V_1 = (1.205)(0.5)(250) = 151\ \text{kg/s}, \quad \dot{m}_2 = 151\left(1+\frac{1}{30}\right) = 156\ \text{kg/s}$$

$$\sum F_x = R_x = \dot{m}_2 u_2 - \dot{m}_1 u_1 - \dot{m}_{fuel} u_{fuel} = 156(900) - 151(250) - 0 \approx \mathbf{102{,}000\ N} \quad \textit{Ans.}$$

---

**P3.56** A liquid jet of velocity $V_j$ and area $A_j$ strikes a single 180° bucket on a turbine wheel rotating at angular velocity $\Omega$. Find an expression for the power *P* delivered. At what $\Omega$ is the power a maximum? How does the analysis differ if there are many buckets, so the jet continually strikes at least one?

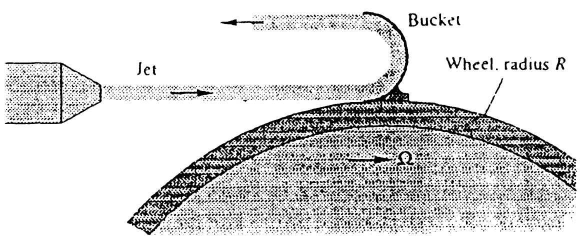
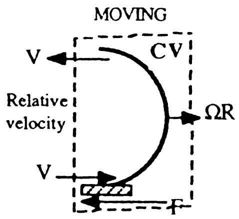


**Fig. P3.56**

**Solution:** Let the CV enclose the bucket and jet and let it move to the right at bucket velocity $V = \Omega R$, so that the jet enters the CV at relative speed $(V_j - \Omega R)$. Then,

$$\sum F_x = -F_{bucket} = \dot{m}u_{out} - \dot{m}u_{in}$$
$$= \dot{m}[-(V_j - \Omega R)] - \dot{m}[V_j - \Omega R]$$




$$\text{or:} \quad F_{bucket} = 2\dot{m}(V_j - \Omega R) = 2\rho A_j (V_j - \Omega R)^2,$$

$$\text{and the power is} \quad \mathbf{P} = \Omega R F_{bucket} = \mathbf{2\rho A_j \Omega R (V_j - \Omega R)^2} \quad \textit{Ans.}$$

Maximum power is found by differentiating this expression:

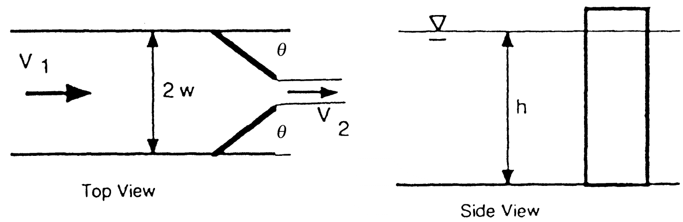
$$\frac{dP}{d\Omega} = 0 \quad \text{if} \quad \mathbf{\Omega R = \frac{V_j}{3}} \quad \textit{Ans.} \quad \left(\text{whence } P_{max} = \frac{8}{27}\rho A_j V_j^3\right)$$

If there were many buckets, then the *full* jet mass flow would be available for work:

$$\dot{m}_{available} = \rho A_j V_j, \quad P = 2\rho A_j V_j \Omega R(V_j - \Omega R), \quad \mathbf{P_{max} = \frac{1}{2}\rho A_j V_j^3} \quad \textbf{at } \mathbf{\Omega R = \frac{V_j}{2}} \quad \textit{Ans.}$$

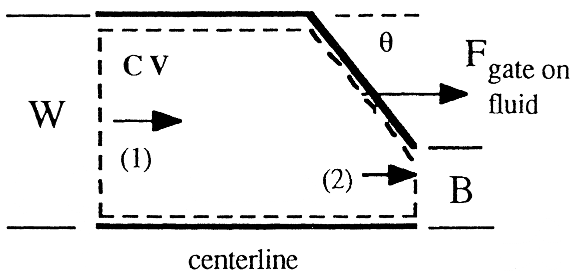
---

**P3.57** The vertical gate in a water channel is partially open, as in Fig. P3.57. Assuming no change in water level and a hydrostatic pressure distribution, derive an expression for the streamwise force $F_x$ on one-half of the gate as a function of ($\rho$, $h$, $w$, $\theta$, $V_1$). Apply your result to the case of water at 20°C, $V_1$ = 0.8 m/s, $h$ = 2 m, $w$ = 1.5 m, and $\theta$ = 50°.




**Solution:** Let the CV enclose sections (1) and (2), the centerline, and the inside of the gate, as shown. The volume flows are

$$V_1 Wh = V_2 Bh, \quad \text{or:} \quad V_2 = V_1\frac{W}{B} = V_1\frac{1}{1 - \sin\theta}$$




since B = W − W sin$\theta$. The problem is unrealistically idealized by letting the water depth remain *constant*, whereas actually the depth would decrease at section 2. Thus we have no net hydrostatic pressure force on the CV in this model! The force balance reduces to

$$\sum F_x = F_{gate\ on\ fluid} = \dot{m}V_2 - \dot{m}V_1, \quad \text{where } \dot{m} = \rho WhV_1 \quad \text{and} \quad V_2 = V_1/(1 - \sin\theta)$$

$$\text{Solve for} \quad F_{\underline{fluid\ on\ gate}} = \mathbf{-\rho WhV_1^2\left[\frac{1}{(1 - \sin\theta)} - 1\right]} \quad \textbf{\textit{(to the left)}} \quad \textit{Ans.}$$

This is unrealistic—the pressure force would turn this gate force around to the right. For the particular data given, W = 1.5 m, $\theta$ = 50°, B = W(1 − sin$\theta$ ) = 0.351 m, $V_1$ = 0.8 m/s, thus $V_2 = V_1/(1 - \sin 50°)$ = 3.42 m/s, $\rho$ = 998 kg/m$^3$, h = 2 m. Thus compute

$$F_{fluid\ on\ gate} = (998)(2)(1.5)(0.8)^2\left[\frac{1}{1 - \sin 50°} - 1\right] \approx \mathbf{6300\ N} \leftarrow \quad \textit{Ans.}$$

---

**P3.58** Consider incompressible flow in the entrance of a circular tube, as in Fig. P3.58. The inlet flow is uniform, $u_1 = U_o$. The flow at section 2 is developed pipe flow. Find the wall drag force $F$ as a function of ($p_1$, $p_2$, $\rho$, $U_o$, $R$) if the flow at section 2 is

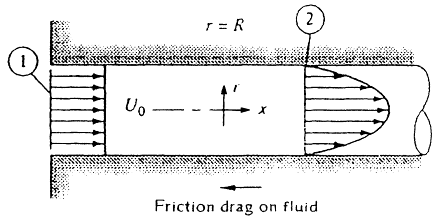


**Fig. P3.58**

(a) Laminar: $u_2 = u_{max}\left(1 - \dfrac{r^2}{R^2}\right)$

(b) Turbulent: $u_2 \approx u_{max}\left(1 - \dfrac{r}{R}\right)^{1/7}$

**Solution:** The CV encloses the inlet and outlet and is just inside the walls of the tube. We don't need to establish a relation between $u_{max}$ and $U_o$ by integration, because the results for these two profiles are given in the text. Note that $U_o = u_{av}$ at section (2). Now use these results as needed for the balance of forces:

$$\sum F_x = (p_1 - p_2)\pi R^2 - F_{drag} = \int_0^R u_2(\rho u_2\, 2\pi r\, dr) - U_o(\rho\pi R^2 U_o) = \rho\pi R^2 U_o^2(\beta_2 - 1)$$

We simply insert the appropriate momentum-flux factors $\beta$ from p. 136 of the text:

(a) Laminar: $\mathbf{F_{drag} = (p_1 - p_2)\pi R^2 - (1/3)\rho\pi R^2 U_o^2}$ *Ans.* (a)

(b) Turbulent, $\beta_2 \approx 1.020$: $\mathbf{F_{drag} = (p_1 - p_2)\pi R^2 - 0.02\rho\pi R^2 U_o^2}$ *Ans.* (b)

---

**P3.59** For the pipe-flow reducing section of Fig. P3.59, $D_1$ = 8 cm, $D_2$ = 5 cm, and $p_2$ = 1 atm. All fluids are at 20°C. If $V_1$ = 5 m/s and the manometer reading is $h$ = 58 cm, estimate the total horizontal force resisted by the flange bolts.

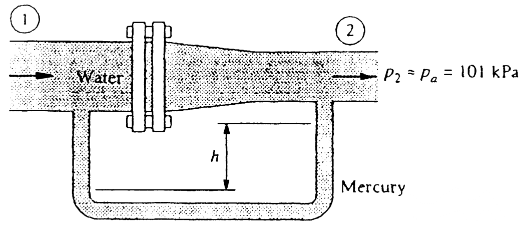


**Fig. P3.59**

**Solution:** Let the CV cut through the bolts and through section 2. For the given manometer reading, we may compute the upstream pressure:

$$p_1 - p_2 = (\gamma_{merc} - \gamma_{water})h = (132800 - 9790)(0.58 \text{ m}) \approx 71300 \text{ Pa (gage)}$$

Now apply conservation of mass to determine the exit velocity:

$$Q_1 = Q_2, \quad \text{or} \quad (5 \text{ m/s})(\pi/4)(0.08 \text{ m})^2 = V_2(\pi/4)(0.05)^2, \quad \text{solve for } V_2 \approx 12.8 \text{ m/s}$$

Finally, write the balance of horizontal forces:

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or:} \quad F_{bolts} = (71300)\frac{\pi}{4}(0.08)^2 - (998)\frac{\pi}{4}(0.08)^2(5.0)[12.8 - 5.0] \approx \mathbf{163\ N} \quad \textit{Ans.}$$

---

**P3.60** In Fig. P3.60 the jet strikes a vane which moves to the right at constant velocity $V_c$ on a frictionless cart. Compute (a) the force $F_x$ required to restrain the cart and (b) the power $P$ delivered to the cart. Also find the cart velocity for which (c) the force $F_x$ is a maximum and (d) the power $P$ is a maximum.

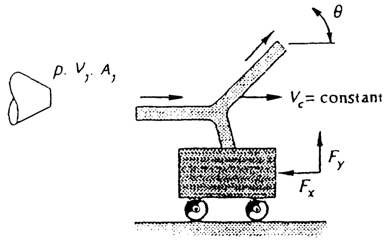



**Fig. P3.60**

**Solution:** Let the CV surround the vane and cart and move to the right at cart speed. The jet strikes the vane at *relative* speed Vj – Vc. The cart does not accelerate, so the horizontal force balance is

$$\sum F_x = -F_x = [\rho A_j(V_j - V_c)](V_j - V_c)\cos\theta - \rho A_j(V_j - V_c)^2$$

$$\text{or:}\quad F_x = \boldsymbol{\rho A_j(V_j - V_c)^2(1 - \cos\theta)} \quad \textit{Ans.}\ \text{(a)}$$

The power delivered is $P = V_cF_x = \boldsymbol{\rho A_j V_c(V_j - V_c)^2(1 - \cos\theta)}$ *Ans.* (b)

The maximum <u>force</u> occurs when the cart is fixed, or: $V_c = \mathbf{0}$ *Ans.* (c)

The maximum <u>power</u> occurs when $dP/dV_c = 0$, or: $V_c = \mathbf{V_j/3}$ *Ans.* (d)

---

**P3.61** Water at 20°C flows steadily through the box in Fig. P3.61, entering station (1) at 2 m/s. Calculate the (a) horizontal; and (b) vertical forces required to hold the box stationary against the flow momentum.

**Solution:** (a) Summing horizontal forces,

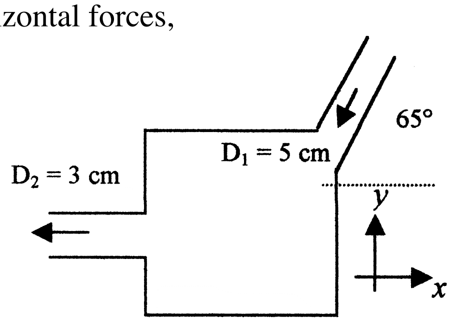



**Fig. P3.61**

$$\sum F_x = R_x = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in}$$

$$R_x = (998)\left[\left(\frac{\pi}{4}\right)(0.03^2)(5.56)\right](-5.56) - (998)\left[\left(\frac{\pi}{4}\right)(0.05^2)(2)\right](-2)(\cos 65°)$$

$$= \mathbf{-18.46}\ \boldsymbol{N} \quad \textit{Ans.}$$

$$Rx = \mathbf{18.5}\ \boldsymbol{N} \quad \textit{to the left}$$

$$\sum F_y = R_y = -\dot{m}_{in}u_{in} = -(998)\left(\frac{\pi}{4}\right)(0.05^2)(2)(-2\sin 65°) = \mathbf{7.1}\ \boldsymbol{N} \quad \textit{up}$$

---

**P3.62** Water flows through the duct in Fig. P3.62, which is 50 cm wide and 1 m deep into the paper. Gate BC completely closes the duct when $\beta = 90°$. Assuming one-dimensional flow, for what angle $\beta$ will the force of the exit jet on the plate be 3 kN?

**Solution:** The steady flow equation applied to the duct, $Q_1 = Q_2$, gives the jet velocity as $V_2 = V_1(1 - \sin\beta)$. Then for a force summation for a control volume around the jet's impingement area,

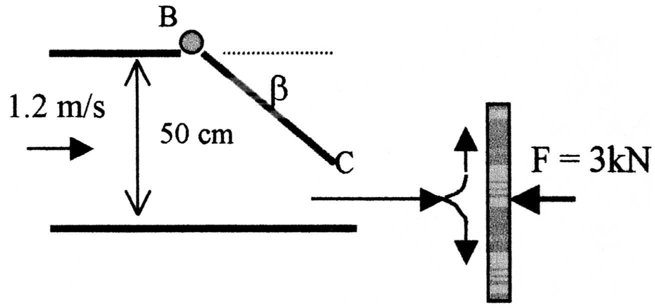



**Fig. P3.62**

$$\sum F_x = F = \dot{m}_j V_j = \rho(h_1 - h_1\sin\beta)(D)\left[\frac{1}{1-\sin\beta}\right]^2 (V_1^2)$$

$$\beta = \sin^{-1}\left[1 - \frac{\rho h_1 D V_1^2}{F}\right] = \sin^{-1}\left[1 - \frac{(998)(0.5)(1)(1.2)^2}{3000}\right] = \mathbf{49.5°} \quad \textit{Ans.}$$

---

**P3.63** The water tank in Fig. P3.63 stands on a frictionless cart and feeds a jet of diameter 4 cm and velocity 8 m/s, which is deflected 60° by a vane. Compute the tension in the supporting cable.

**Solution:** The CV should surround the tank and wheels and cut through the cable and the exit water jet. Then the horizontal force balance is

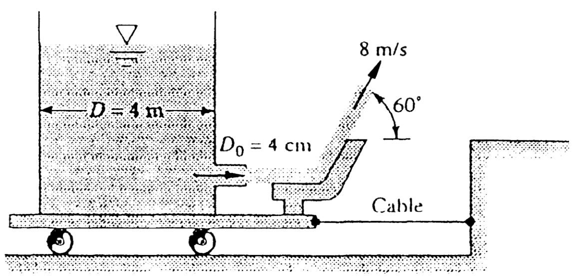



**Fig. P3.63**

$$\sum F_x = T_{cable} = \dot{m}_{out}u_{out} = (\rho AV_j)V_j\cos\theta = 998\left(\frac{\pi}{4}\right)(0.04)^2(8)^2\cos 60° = \mathbf{40\ N} \quad \textit{Ans.}$$

---

**P3.64** A pipe flow expands from (1) to (2), causing eddies as shown. Using the given CV and assuming p = $p_1$ on the corner annular ring, show that the downstream pressure is given by, neglecting wall friction,

$$p_2 = p_1 + \rho V_1^2\left(\frac{A_1}{A_2}\right)\left(1 - \frac{A_1}{A_2}\right)$$

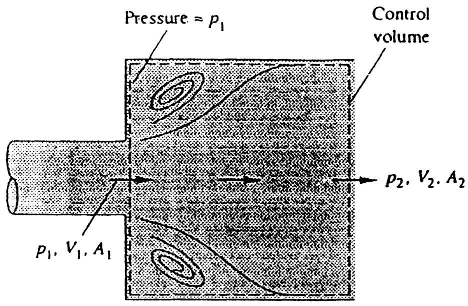



**Fig. P3.64**

**Solution:** From mass conservation, $V_1A_1 = V_2A_2$. The balance of x-forces gives

$$\sum F_x = p_1A_1 + p_{wall}(A_2 - A_1) - p_2A_2 = \dot{m}(V_2 - V_1), \quad \text{where } \dot{m} = \rho A_1V_1, \quad V_2 = V_1A_1/A_2$$

If $p_{wall} = p_1$ as given, this reduces to $\mathbf{p_2 = p_1 + \rho\frac{A_1}{A_2}V_1^2\left(1 - \frac{A_1}{A_2}\right)}$ *Ans.*

---

**P3.65** Water at 20°C flows through the elbow in Fig. P3.65 and exits to the atmosphere. The pipe diameter is $D_1$ = 10 cm, while $D_2$ = 3 cm. At a weight flow rate of 150 N/s, the pressure $p_1$ = 2.3 atm (gage). Neglecting the weight of water and elbow, estimate the force on the flange bolts at section 1.

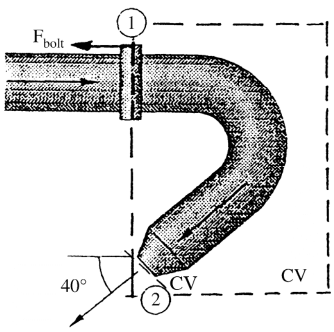



**Fig. P3.65**

**Solution:** First, from the weight flow, compute Q = (150 N/s)/(9790 N/m$^3$) = 0.0153 m$^3$/s. Then the velocities at (1) and (2) follow from the known areas:

$$V_1 = \frac{Q}{A_1} = \frac{0.0153}{(\pi/4)(0.1)^2} = 1.95\ \frac{m}{s}; \qquad V_2 = \frac{Q}{A_2} = \frac{0.0153}{(\pi/4)(0.03)^2} = 21.7\ \frac{m}{s}$$

The mass flow is $\rho A_1V_1 = (998)(\pi/4)(0.1)^2(1.95) \approx 15.25$ kg/s. Then the balance of forces in the *x*-direction is:

$$\sum F_x = -F_{bolts} + p_1A_1 = \dot{m}u_2 - \dot{m}u_1 = \dot{m}(-V_2\cos 40° - V_1)$$

solve for $F_{bolts} = (2.3 \times 101350)\frac{\pi}{4}(0.1)^2 + 15.25(21.7\cos 40° + 1.95) \approx \mathbf{2100\ N}$ *Ans.*

---

**P3.66** A 20°C water jet strikes a vane on a tank with frictionless wheels, as shown. The jet turns and falls into the tank without spilling. If $\theta = 30°$, estimate the horizontal force F needed to hold the tank stationary.

**Solution:** The CV surrounds the tank and wheels and cuts through the jet, as shown. We should *assume that the splashing into the tank does not increase the x-momentum of the water in the tank*. Then we can write the CV horizontal force relation:

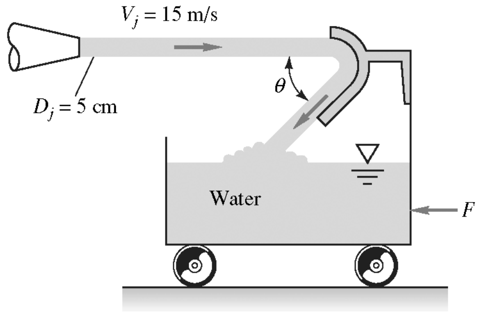



**Fig. P3.66**

$$\sum F_x = -F = \frac{d}{dt}\left(\int u\rho\, d\upsilon\right)_{tank} - \dot{m}_{in}u_{in} = 0 - \dot{m}V_{jet} \text{ independent of } \theta$$

$$\text{Thus} \quad F = \rho A_j V_j^2 = \left(998\ \frac{kg}{m^3}\right)\frac{\pi}{4}(0.05)^2\left(15\ \frac{m}{s}\right)^2 \approx \mathbf{440.9\ N} \quad \textit{Ans.}$$

---

**P3.67** Water at 20°C exits to the standard sea-level atmosphere through the split nozzle in Fig. P3.67. Duct areas are $A_1 = 0.02\ m^2$ and $A_2 = A_3 = 0.008\ m^2$. If $p_1 = 135$ kPa (absolute) and the flow rate is $Q_2 = Q_3 = 275\ m^3/h$, compute the force on the flange bolts at section 1.

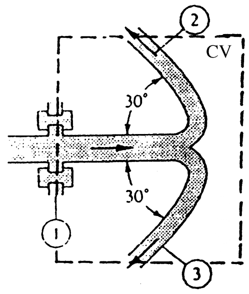



**Fig. P3.67**

**Solution:** With the known flow rates, we can compute the various velocities:

$$V_2 = V_3 = \frac{275/3600\ m^3/s}{0.008\ m^2} = 9.55\ \frac{m}{s}; \quad V_1 = \frac{550/3600}{0.02} = 7.64\ \frac{m}{s}$$

The CV encloses the split nozzle and cuts through the flange. The balance of forces is

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \rho Q_2(-V_2\cos 30°) + \rho Q_3(-V_3\cos 30°) - \rho Q_1(+V_1),$$

$$\text{or:} \quad F_{bolts} = 2(998)\left(\frac{275}{3600}\right)(9.55\cos 30°) + 998\left(\frac{550}{3600}\right)(7.64) + (135000 - 101350)(0.02)$$

$$= 1261 + 1165 + 673 \approx \mathbf{3100\ N} \quad \textit{Ans.}$$

---

**P3.68** A steady two-dimensional water jet, 4 cm thick with a weight flow rate of 1960 N/s, strikes an angled barrier as in Fig. P3.68. Pressure and water velocity are constant everywhere. Thirty percent of the jet passes through the slot. The rest splits symmetrically along the barrier. Calculate the horizontal force *F* needed, per unit thickness into the paper, to hold the barrier stationary.

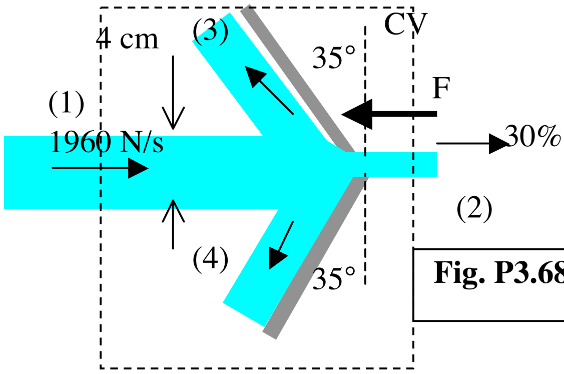



**Fig. P3.68**

*Solution*: For water take $\rho$ = 998 kg/m. The control volume (see figure) cuts through all four jets, which are numbered. The velocity of all jets follows from the weight flow at (1):

$$V_{1,2,3,4} = V_1 = \frac{\dot{w}_1}{\rho g A_1} = \frac{1960\,N/s}{(9.81 m/s^2)(998 kg/m^3)(0.04 m)(1 m)} = 5.0\frac{m}{s}$$

$$\dot{m}_1 = \frac{\dot{w}_1}{g} = \frac{1960 N/s-m}{9.81 N/s^2} = 200\frac{kg}{s-m}\,;\ \dot{m}_2 = 0.3\dot{m}_1 = 60\frac{kg}{s-m}\,;\ \dot{m}_3 = \dot{m}_4 = 70\frac{kg}{s-m}$$

Then the *x*-momentum relation for this control volume yields

$$\Sigma F_x = -F = \dot{m}_2 u_2 + \dot{m}_3 u_3 + \dot{m}_4 u_4 - \dot{m}_1 u_1 =$$

$$-F = (60)(5.0) + (70)(-5.0\cos 55^\circ) + (70)(-5.0\cos 55^\circ) - 200(5.0)\text{, or:}$$

$$F = 1000 + 201 + 201 - 300 \approx \mathbf{1100\ N}\ \text{per meter of width}\quad \textit{Ans.}$$

---

**P3.69** The 6-cm-diameter 20°C water jet in Fig. P3.69 strikes a plate containing a hole of 4-cm diameter. Part of the jet passes through the hole, and part is deflected. Determine the horizontal force required to hold the plate.

**Solution:** First determine the incoming flow and the flow through the hole:

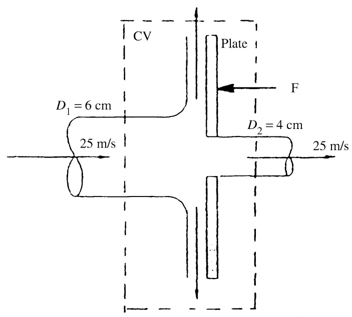



**Fig. P3.69**

$$Q_{in} = \frac{\pi}{4}(0.06)^2(25) = 0.0707\ \frac{m^3}{s},\quad Q_{hole} = \frac{\pi}{4}(0.04)^2(25) = 0.0314\ \frac{m^2}{s}$$

Then, for a CV enclosing the plate and the two jets, the horizontal force balance is

$$\sum F_x = -F_{plate} = \dot{m}_{hole}u_{hole} + \dot{m}_{upper}u_{upper} + \dot{m}_{lower}u_{lower} - \dot{m}_{in}u_{in}$$

$$= (998)(0.0314)(25) + 0 + 0 - (998)(0.0707)(25)$$

$$= 784 - 1764,\quad \text{solve for } \mathbf{F \approx 980\ N\ (to\ left)}\quad \textit{Ans.}$$

---

**P3.70** The box in Fig. P3.70 has three 12.5-mm holes on the right side. The volume flows of 20°C water shown are steady, but the details of the interior are not known. Compute the force, if any, which this water flow causes on the box.

**Solution:** First we need to compute the velocities through the various holes:

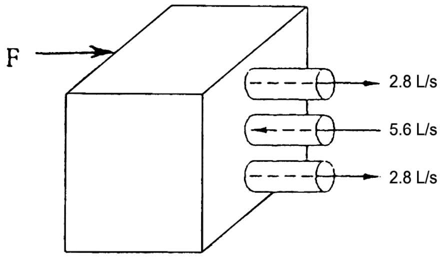



**Fig. P3.70**

$$V_{top} = V_{bottom} = \frac{2.8 \times 10^{-3}}{(\pi/4)(0.0125)^2} = 22.82 \text{ m/s}; \quad V_{middle} = 2V_{top} = 45.64 \text{ m/s}$$

Pretty fast, but do-able, I guess. Then make a force balance for a CV enclosing the box:

$$\sum F_x = F_{box} = -\dot{m}_{in}u_{in} + 2\dot{m}_{top}u_{top}, \quad \text{where} \quad u_{in} = -V_{middle} \quad \text{and} \quad u_{top} = V_{top}$$

$$\text{Solve for } F_{box} = (999.8)(5.6 \times 10^{-3})(45.64) + 2(999.8)(2.8 \times 10^{-3})(22.82) \approx \mathbf{383.3\ N} \quad \textit{Ans.}$$

---

**P3.71** The tank in Fig. P3.71 weighs 500 N empty and contains 600 L of water at 20°C. Pipes 1 and 2 have D = 6 cm and Q = 300 m$^3$/hr. What should the scale reading W be, in newtons?

**Solution:** Let the CV surround the tank, cut through the two jets, and slip just under the tank bottom, as shown. The relevant jet velocities are

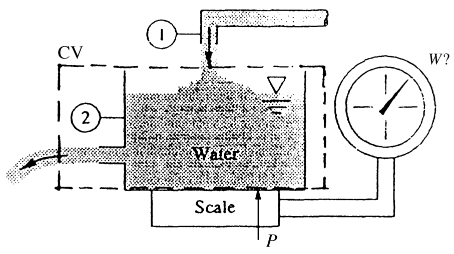



**Fig. P3.71**

$$V_1 = V_2 = \frac{Q}{A} = \frac{(300/3600) \text{ m}^3/\text{s}}{(\pi/4)(0.06 \text{ m})^2} \approx 29.5 \text{ m/s}$$

The scale reads force "P" on the tank bottom. Then the vertical force balance is

$$\sum F_z = P - W_{tank} - W_{water} = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \dot{m}[0 - (-V_1)]$$

$$\text{Solve for} \quad \mathbf{P} = 500 + 9790(0.6 \text{ m}^3) + 998\left(\frac{300}{3600}\right)(29.5) \approx \mathbf{8800\ N} \quad \textit{Ans.}$$

---

**P3.72** Gravel is dumped from a hopper, at a rate of 650 N/s, onto a moving belt, as in Fig. P3.72. The gravel then passes off the end of the belt. The drive wheels are 80 cm in diameter and rotate clockwise at 150 r/min. Neglecting system friction and air drag, estimate the power required to drive this belt.

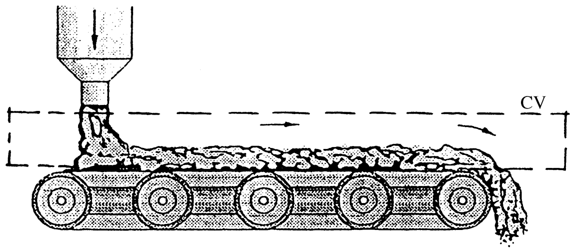



**Fig. P3.72**

**Solution:** The CV goes under the gravel on the belt and cuts through the inlet and outlet gravel streams, as shown. The no-slip belt velocity must be

$$V_{belt} = V_{outlet} = \Omega R_{wheel} = \left[150\frac{rev}{min}2\pi\frac{rad}{rev}\frac{1}{60}\frac{min}{s}\right](0.4\text{ m}) \approx 6.28\ \frac{m}{s}$$

Then the belt applies tangential force $F$ to the gravel, and the force balance is

$$\sum F_x = F_{\text{on belt}} = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in}, \quad \text{but } u_{in} = 0.$$

$$\text{Then}\quad F_{belt} = \dot{m}V_{out} = \left(\frac{650}{9.81}\ \frac{kg}{s}\right)\left(6.28\ \frac{m}{s}\right) = 416\text{ N}$$

The power required to drive the belt is $\boldsymbol{P} = FV_{belt} = (416)(6.28) \approx \mathbf{2600\ W}$ *Ans.*

---

**P3.73** The rocket in Fig. P3.73 has a supersonic exhaust, and the exit pressure $p_e$ is not necessarily equal to $p_a$. Show that the force $F$ required to hold this rocket on the test stand is $F = \rho_e A_e V_e^2 + A_e(p_e - p_a)$. Is this force $F$ what we term the *thrust* of the rocket?

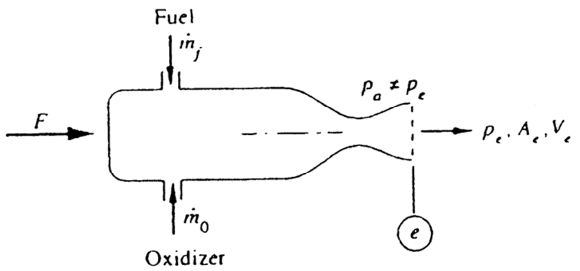



**Fig. P3.73**

**Solution:** The appropriate CV surrounds the entire rocket and cuts through the exit jet. Subtract $p_a$ everywhere so only exit pressure $\neq 0$. The horizontal force balance is

$$\sum F_x = F - (p_e - p_a)A_e = \dot{m}_e u_e - \dot{m}_f u_f - \dot{m}_o u_o, \quad \text{but } u_f = u_o = 0, \quad \dot{m}_e = \rho_e A_e V_e$$

$$\text{Thus}\quad \mathbf{F = \rho_e A_e V_e^2 + (p_e - p_a)A_e}\quad \text{(yes, the } \underline{\textbf{thrust}}\text{)}\quad \textit{Ans.}$$

---

**P3.74** A uniform rectangular plate, 40 cm long and 30 cm deep into the paper, hangs in air from a hinge at its top, 30-cm side. It is struck in its center by a horizontal 3-cm-diameter jet of water moving at 8 m/s. If the gate has a mass of 16 kg, estimate the angle at which the plate will hang from the vertical.

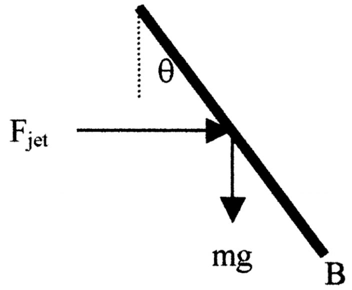



**Fig. P3.74**

**Solution:** The plate orientation can be found through force and moment balances. Find the force normal to the plate:

$$\sum F_n = F_n = \dot{m}_{jet} u_n = \rho A V u_n = (998)\left(\frac{\pi}{4}\right)(0.03^2)(8)(8\cos\theta) = 45.1\cos\theta \quad \textit{Newtons}$$

$$\sum M_{hinge} = 0 = -(45.1\cos\theta)(0.2\text{m}) + [(16)(9.81)\text{N}](0.2m)(\sin\theta);\ \tan\theta = 0.287, \quad \boldsymbol{\theta = 16°}$$

If the force and weight are centered in the plate, and the weight and jet flow are constant, the answer is independent of the length (40 cm) of the plate.

---

**P3.75** The dredger in Fig. P3.75 is loading sand (SG = 2.6) onto a barge. The sand leaves the dredger pipe at 1.2 m/s with a weight flux of 3780 N/s. Estimate the tension on the mooring line caused by this loading process.

**Solution:** The CV encloses the boat and cuts through the cable and the sand flow jet. Then,

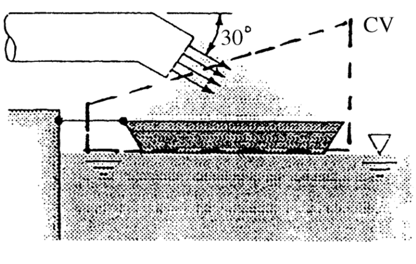



**Fig. P3.75**

$$\sum F_x = -T_{cable} = -\dot{m}_{sand} u_{sand} = -\dot{m} V_{sand} \cos\theta,$$

$$\text{or:} \quad T_{cable} = \left(\frac{3780}{9.81}\ \frac{\text{kg}}{\text{s}}\right)\left(1.2\ \frac{\text{m}}{\text{s}}\right)\cos 30° \approx \mathbf{400\ N} \quad \textit{Ans.}$$

---

**P3.76** Suppose that a deflector is deployed at the exit of the jet engine of Prob. 3.55, as shown in Fig. P3.76. What will the reaction $R_x$ on the test stand be now? Is this reaction sufficient to serve as a braking force during airplane landing?

**Solution:** From Prob. 3.55, recall that the essential data was

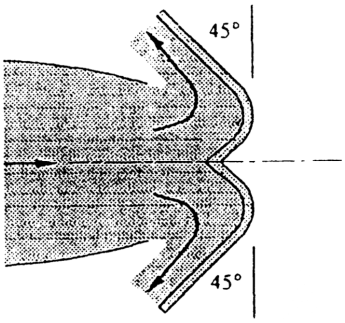



**Fig. P3.76**

$$V_1 = 250 \text{ m/s}, \quad V_2 = 900 \text{ m/s}, \quad \dot{m}_1 = 151 \text{ kg/s}, \quad \dot{m}_2 = 156 \text{ kg/s}$$

The CV should enclose the entire engine and also the deflector, cutting through the support and the 45° exit jets. Assume (unrealistically) that the exit velocity is *still* 900 m/s. Then,

$$\sum F_x = R_x = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in}, \quad \text{where } u_{out} = -V_{out}\cos 45° \quad \text{and} \quad u_{in} = V_1$$

$$\text{Then} \quad R_x = -156(900\cos 45°) - 151(250) = -137{,}000 \text{ N}$$

**The support reaction is to the left and equals 137 kN** *Ans.*

---

**P3.77** A thick elliptical cylinder immersed in a water stream creates the idealized wake shown. Upstream and downstream pressures are equal, and $U_o = 4$ m/s, L = 80 cm. Find the drag force on the cylinder per unit width into the paper. Also compute the dimensionless *drag coefficient* $C_D = 2F/(\rho U_o^2 bL)$.

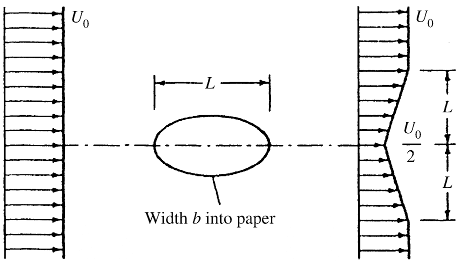



**Fig. P3.77**

**Solution:** This is a 'numerical' version of the "analytical" body-drag Prob. 3.49. The student still must make a CV analysis similar to Prob. 3.49 of this Manual. The wake is exactly the same shape, so the result from Prob. 3.49 holds here also:

$$F_{drag} = \frac{1}{3}\rho U_o^2 Lb = \frac{1}{3}(998)(4)^2(0.8)(1.0) \approx \mathbf{4260\ N} \quad \textit{Ans.}$$

The drag coefficient is easily calculated from the above result: **$C_D$ = 2/3**. *Ans.*

---

**P3.78** A pump in a tank of water directs a jet at 14 m/s and $12.6 \times 10^{-3} m^3/s$ (12.6 L/s) against a vane, as shown in the figure. Compute the force F to hold the cart stationary if the jet follows (a) path A; or (b) path B. The tank holds 2 $m^3$ of water at this instant.

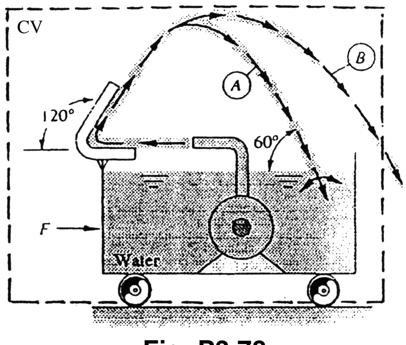


Fig. P3.78

**Solution:** The CV encloses the tank and passes through jet B.
(a) For jet path **A**, *no momentum flux crosses the CV*, therefore **F = 0** *Ans.* (a)

(b) For jet path B, there *is* momentum flux, so the *x*-momentum relation yields:

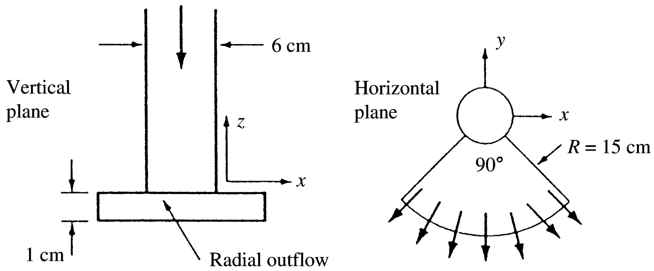
$$\sum F_x = F = \dot{m}_{out}u_{out} = \dot{m}_{jet}u_B$$

Now we don't really *know* $u_B$ exactly, but we make the reasonable assumption that the jet trajectory is *frictionless* and maintains its horizontal velocity component, that is, $u_B \approx V_{jet} \cos 60°$. Thus we can estimate

$$F = \dot{m}u_B = \left(999.8 \text{ kg/m}^3\right)\left(12.6 \times 10^{-3}\right)(14\cos 60°) \approx \mathbf{88.2\ N} \quad \textit{Ans. (b)}$$

---

**P3.79** Water at 20°C flows down a vertical 6-cm-diameter tube at 18.93 L/s, as in the figure. The flow then turns horizontally and exits through a 90° radial duct segment 1 cm thick, as shown. If the radial outflow is uniform and steady, estimate the forces ($F_x$, $F_y$, $F_z$) required to support this system against fluid momentum changes.




**Solution:** First convert 1893 L/s = 0.01893 $m^3$/s, hence the mass flow is $\rho$Q = 18.9 kg/s. The vertical-tube velocity (down) is $V_{tube} = 0.01893/[(\pi/4)(0.06)^2] = -6.69\ \mathbf{k}$ m/s. The exit tube area is $(\pi/2)R\Delta h = (\pi/2)(0.15)(0.01) = 0.002356\ m^2$, hence $V_{exit} = Q/A_{exit} = 0.01893/0.002356 = \mathit{8.03}$ m/s. Now estimate the force components:

$$\sum F_x = \mathbf{F_x} = \int u_{out}\, d\dot{m}_{out} = \int_{-45°}^{+45°} -V_{exit} \sin\theta\ \rho\Delta h R\, d\theta \equiv \mathbf{0} \quad \textit{Ans. (a)}$$

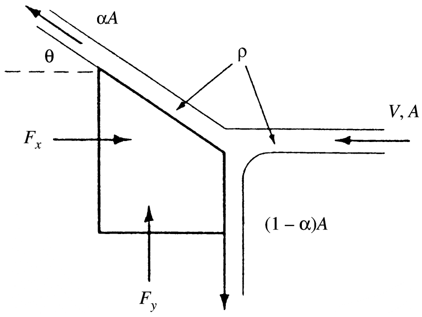
$$\sum F_y = \mathbf{F_y} = \int v_{out}\, d\dot{m}_{out} - \dot{m}v_{in} = \int_{-45°}^{+45°} -V_{exit} \cos\theta\, \rho\Delta h R\, d\theta - 0 = -V_{exit}\rho\Delta h R\sqrt{2}$$

*or:* $$\mathbf{F_y} = -(8.03)(998)(0.01)(0.15)\sqrt{2} \approx \mathbf{-17\ N} \quad \textit{Ans. (b)}$$

$$\sum F_z = \mathbf{F_z} = \dot{m}(w_{out} - w_{in}) = (18.9\ \mathit{kg/s})[0 - (-6.69\ \mathit{m/s})] \approx \mathbf{+126\ N} \quad \textit{Ans. (c)}$$

---

**P3.80** A liquid jet of density r and area A strikes a block and splits into two jets, as shown in the figure. All three jets have the same velocity V. The upper jet exits at angle $\theta$ and area $\alpha$A, the lower jet turns down at 90° and area $(1 - \alpha)$A. (a) Derive a formula for the forces ($F_x$,$F_y$) required to support the block against momentum changes.(b) Show that $F_y = 0$ only if $\alpha \geq 0.5$.(c) Find the values of $\alpha$ and $\theta$ for which both $F_x$ and $F_y$ are zero.




**Solution:** (a) Set up the *x*- and *y*-momentum relations:

$$\sum F_x = F_x = \alpha\dot{m}(-V\cos\theta) - \dot{m}(-V) \quad \textit{where } \dot{m} = \rho AV \textit{ of the inlet jet}$$

$$\sum F_y = F_y = \alpha\dot{m}V\sin\theta + (1-\alpha)\dot{m}(-V)$$

Clean this up for the final result:

$$F_x = \mathbf{\dot{m}V(1-\alpha\cos\theta)}$$

$$F_y = \mathbf{\dot{m}V(\alpha\sin\theta + \alpha - 1)} \quad \textit{Ans.} \text{ (a)}$$

(b) Examining $F_y$ above, we see that it can be zero only when,

$$\sin\theta = \frac{1-\alpha}{\alpha}$$

But this makes no sense if $\alpha < 0.5$, hence **$F_y = 0$ only if $\alpha \geq 0.5$**. *Ans.* (b)

(c) Examining $F_x$, we see that it can be zero only if $\cos\theta = 1/\alpha$, which makes no sense unless $\alpha = 1$, $\theta = 0°$. This situation also makes $F_x = 0$ above ($\sin\theta = 0$). Therefore the only scenario for which both forces are zero is the trivial case for which all the flow goes horizontally across a flat block:

$$F_x = F_y = 0 \quad \text{only if: } \alpha = \mathbf{1},\ \theta = \mathbf{0°} \quad \textit{Ans.} \text{ (c)}$$

---

**P3.81** A two-dimensional sheet of water, 10 cm thick and moving at 7 m/s, strikes a fixed wall inclined at 20° with respect to the jet direction. Assuming frictionless flow, find (a) the normal force on the wall per meter of depth, and the widths of the sheet deflected (b) upstream, and (c) downstream along the wall.

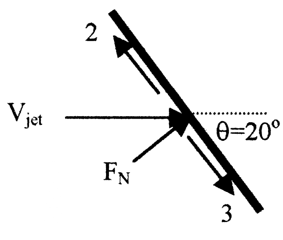



**Fig. P3.81**

**Solution:** (a) The force normal to the wall is due to the jet's momentum,

$$\sum F_N = -\dot{m}_{in}u_{in} = -(998)(0.1)(7^2)(\cos 70°) = \mathbf{1670}\ \boldsymbol{N/m} \quad \textit{Ans.}$$

(b) Assuming $V_1 = V_2 = V_3 = V_{jet}$, $V_jA_1 = V_jA_2 + V_jA_3$ where,

$$A_2 = A_1\sin\theta = (0.1)(1)(\sin 20°) = 0.034 \text{ m} \approx \mathbf{3\ cm} \quad \textit{Ans.}$$

(c) Similarly, $A_3 = A_1\cos\theta = (0.1)(1)(\cos 20°) = 0.094 \text{ m} \approx \mathbf{9.4\ cm}$ *Ans.*

---

**P3.82** Water at 20°C flows steadily through a reducing pipe bend, as in Fig. P3.82. Known conditions are $p_1$ = 350 kPa, $D_1$ = 25 cm, $V_1$ = 2.2 m/s, $p_2$ = 120 kPa, and $D_2$ = 8 cm. Neglecting bend and water weight, estimate the total force which must be resisted by the flange bolts.

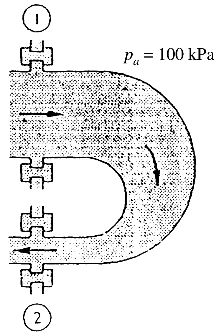


Fig. P3.82

**Solution:** First establish the mass flow and exit velocity:

$$\dot{m} = \rho_1 A_1 V_1 = 998\left(\frac{\pi}{4}\right)(0.25)^2(2.2) = 108\ \frac{kg}{s} = 998\left(\frac{\pi}{4}\right)(0.08)^2 V_2, \quad \text{or} \quad V_2 = 21.5\ \frac{m}{s}$$

The CV surrounds the bend and cuts through the flanges. The force balance is

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 + p_{2,gage}A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1, \quad \text{where } u_2 = -V_2 \quad \text{and} \quad u_1 = V_1$$

$$\text{or} \quad F_{bolts} = (350000 - 100000)\frac{\pi}{4}(0.25)^2 + (120000 - 100000)\frac{\pi}{4}(0.08)^2 + 108(21.5 + 2.2)$$

$$= 12271 + 101 + 2553 \approx \mathbf{14900\ N} \quad \textit{Ans.}$$

---

**P3.83** A fluid jet of diameter $D_1$ enters a cascade of moving blades at absolute velocity $V_1$ and angle $\beta_1$, and it leaves at absolute velocity $V_2$ and angle $\beta_2$, as in Fig. P3.83. The blades move at velocity $u$. Derive a formula for the power $P$ delivered to the blades as a function of these parameters.

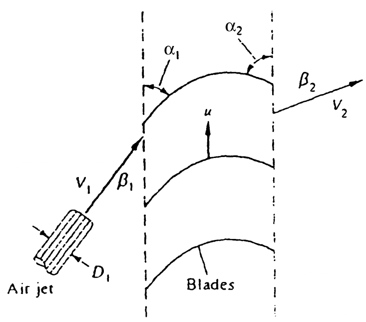
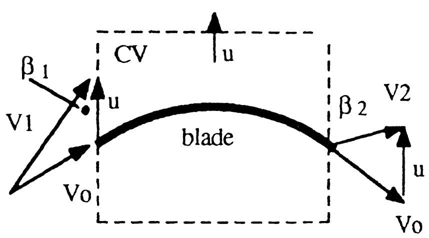


Fig. P3.83

**Solution:** Let the CV enclose the blades and move upward at speed $u$, so that the flow appears steady in that frame, as shown at right. The relative velocity $V_o$ may be eliminated by the law of cosines:




$$V_o^2 = V_1^2 + u^2 - 2V_1 u\cos\beta_1$$
$$= V_2^2 + u^2 - 2V_2 u\cos\beta_2$$

$$\text{solve for} \quad u = \frac{(1/2)\left(V_1^2 - V_2^2\right)}{V_1\cos\beta_1 - V_2\cos\beta_2}$$

Then apply momentum in the direction of blade motion:

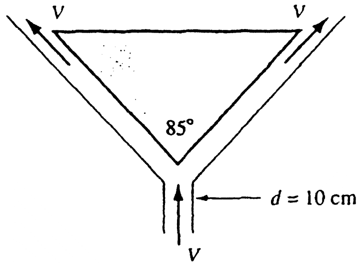
$$\sum F_y = F_{vanes} = \dot{m}_{jet}(V_{o1y} - V_{o2y}) = \dot{m}(V_1\cos\beta_1 - V_2\cos\beta_2), \quad \dot{m} = \rho A_1 V_1$$

The power delivered is P = Fu, which causes the parenthesis "$\cos\beta$" terms to cancel:

$$\mathbf{P = Fu = \frac{1}{2}\dot{m}_{jet}\left(V_1^2 - V_2^2\right)} \quad \textit{Ans.}$$

---

**P3.84** Air at 20°C and 1 atm enters the bottom of an 85° conical flowmeter duct at a mass flow rate of 0.3 kg/s, as shown in the figure. It supports a centered conical body by steady annular flow around the cone and exits at the same velocity as it enters. Estimate the weight of the body in newtons.




**Solution:** First estimate the velocity from the known inlet duct size:

$$\rho_{air} = \frac{p}{RT} = \frac{101350}{287(293)} = 1.205\ \frac{\text{kg}}{\text{m}^3},$$

$$thus \quad \dot{m} = 0.3 = \rho AV = (1.205)\frac{\pi}{4}(0.1)^2 V, \quad solve\ \ \mathbf{V = 31.7}\ \frac{\text{m}}{\text{s}}$$

Then set up the vertical momentum equation, the unknown is the body weight:

$$\sum F_z = -W = \dot{m}V\cos 42.5° - \dot{m}V = \dot{m}V(\cos 42.5° - 1)$$

$$Thus \quad \mathbf{W_{cone}} = (0.3)(31.7)(1 - \cos 42.5°) = \mathbf{2.5\ N} \quad Ans.$$

---

**P3.85** A river (1) passes over a "drowned" weir as shown, leaving at a new condition (2). Neglect atmospheric pressure and assume hydrostatic pressure at (1) and (2). Derive an expression for the force F exerted by the river on the obstacle. Neglect bottom friction.

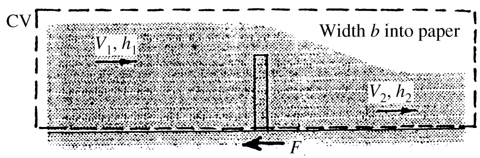



**Fig. P3.85**

**Solution:** The CV encloses (1) and (2) and cuts through the gate along the bottom, as shown. The volume flow and horizontal force relations give

$$V_1bh_1 = V_2bh_2$$

$$\sum F_x = -F_{weir} + \frac{1}{2}\rho g h_1(h_1 b) - \frac{1}{2}\rho g h_2(h_2 b) = (\rho h_1 b V_1)(V_2 - V_1)$$

Note that, except for the different geometry, the analysis is exactly the same as for the sluice gate in Ex. 3.10. The force result is the same, also:

$$\mathbf{F_{weir}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\rho}\mathbf{gb}\left(\mathbf{h_1^2 - h_2^2}\right) - \boldsymbol{\rho}\mathbf{h_1 b V_1^2}\left(\frac{\mathbf{h_1}}{\mathbf{h_2}} - \mathbf{1}\right) \quad Ans.$$

---

**P3.86** Torricelli's idealization of efflux from a hole in the side of a tank is $V \approx \sqrt{2gh}$, as shown in Fig. P3.86. The tank weighs 150 N when empty and contains water at 20°C. The tank bottom is on very smooth ice (static friction coefficient $\zeta \approx 0.01$). For what water depth $h$ will the tank just begin to move to the right?

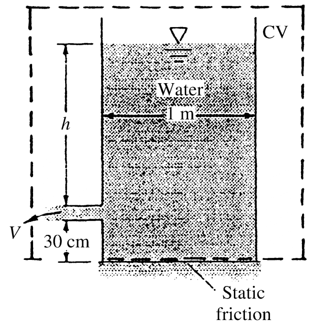



**Fig. P3.86**

**Solution:** The hole diameter is 9 cm. The CV encloses the tank as shown. The coefficient of static friction is $\zeta = 0.01$. The *x*-momentum equation becomes

$$\sum F_x = -\zeta W_{tank} = \dot{m}u_{out} = -\dot{m}\,V_{hole} = -\rho A V^2 = -\rho A(2gh)$$

$$\text{or:}\quad 0.01\left[(9790)\frac{\pi}{4}(1\text{ m})^2(h+0.3+0.09)+150\right] = 998\left(\frac{\pi}{4}\right)(0.09)^2(2)(9.81)h$$

Solve for $\mathbf{h \approx 0.66\ m}$ *Ans.*

---

**P3.87** The model car in Fig. P3.87 weighs 17 N and is to be accelerated from rest by a 1-cm-diameter water jet moving at 75 m/s. Neglecting air drag and wheel friction, estimate the velocity of the car after it has moved forward 1 m.

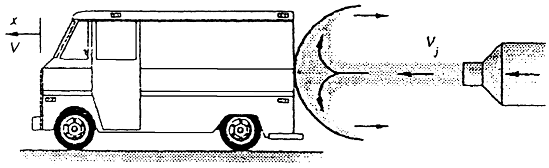



**Fig. P3.87**

**Solution:** The CV encloses the car, moves to the left at *accelerating* car speed V(t), and cuts through the inlet and outlet jets, which leave the CS at *relative* velocity $V_j - V$. The force relation is Eq. (3.50):

$$\sum F_x - \int a_{rel}\,dm = 0 - m_{car}a_{car} = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = -2\dot{m}_{jet}(V_j - V),$$

$$\text{or:}\quad m_{car}\frac{dV}{dt} = 2\rho A_j(V_j - V)^2$$

Except for the factor of "2," this is identical to the "cart" analysis of Example 3.12 on pages 172–173 of the text. The solution, for V = 0 at t = 0, is given there:

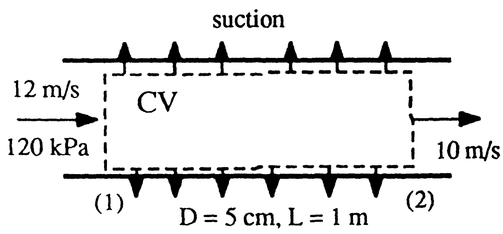
$$V = \frac{V_j^2 Kt}{1+V_jKt},\quad \text{where } K = \frac{2\rho A_j}{m_{car}} = \frac{2(998)(\pi/4)(0.01)^2}{(17/9.81)} = 0.0905\ \text{m}^{-1}$$

$$\text{Thus}\quad V\ (\text{in m/s}) = \frac{509t}{1+6.785t}\quad \text{and then compute distance } S = \int_0^t V\,dt$$

The initial acceleration is 509 m/s$^2$, quite large. Assuming the jet can follow the car without dipping, the car reaches <u>S = 1 m</u> at t ≈ 0.072 s, where **V ≈ 24.6 m/s**. *Ans.*

---

**P3.88** Gasoline at 20°C is flowing at $V_1$ = 12 m/s in a 5-cm-diameter pipe when it encounters a 1-m length of uniform radial wall suction. After the suction, the velocity has dropped to 10 m/s. If $p_1$ = 120 kPa, estimate $p_2$ if wall friction is neglected.




**Solution:** The CV cuts through sections 1 and 2 and the inside of the walls. We compute the mass flow at each section, taking $\rho \approx 680$ kg/m$^3$ for gasoline:

$$\dot{m}_1 = 680\left(\frac{\pi}{4}\right)(0.05)^2(12) = 16.02\ \frac{kg}{s};\quad \dot{m}_2 = 680\left(\frac{\pi}{4}\right)(0.05)^2(10) = 13.35\ \frac{kg}{s}$$

The difference, 16.02 – 13.35 = 2.67 kg/s, is sucked through the walls. If wall friction is neglected, the force balance (taking the momentum correction factors $\beta \approx 1.0$) is:

$$\sum F_x = p_1A_1 - p_2A_2 = \dot{m}_2V_2 - \dot{m}_1V_1 = (120000 - p_2)\frac{\pi}{4}(0.05)^2$$
$$= (13.35)(10) - (16.02)(12),\quad \text{solve for } p_2 \approx \mathbf{150\ kPa}\quad \textit{Ans.}$$

---

**P3.89** Air at 20°C and 1 atm flows in a 25-cm-diameter duct at 15 m/s, as in Fig. P3.89. The exit is choked by a 90° cone, as shown. Estimate the force of the airflow on the cone.

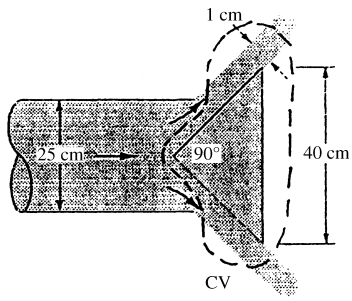


**Fig. P3.89**

**Solution:** The CV encloses the cone, as shown. We need to know exit velocity. The exit area is approximated as a ring of diameter 40.7 cm and thickness 1 cm:

$$Q = A_1V_1 = \frac{\pi}{4}(0.25)^2(15) = 0.736\ \frac{m^3}{s} = A_2V_2 \approx \pi(0.407)(0.01)V_2,\quad \text{or}\quad V_2 \approx 57.6\ \frac{m}{s}$$

The air density is $\rho$ = p/RT = (101350)/[287(293)] ≈ 1.205 kg/m$^3$. We are not given any pressures on the cone so we consider momentum only. The force balance is

$$\sum F_x = F_{cone} = \dot{m}(u_{out} - u_{in}) = (1.205)(0.736)(57.6\cos 45° - 15) \approx \mathbf{22.8\ N}\quad \textit{Ans.}$$

The force on the cone is *to the right* because we neglected pressure forces.

---

**P3.90** The thin-plate orifice in Fig. P3.90 causes a large pressure drop. For 20°C water flow at 1.893 $m^3$/min, with pipe $D$ = 10 cm and orifice $d$ = 6 cm, $p_1 - p_2 \approx$ 145 kPa. If the wall friction is negligible, estimate the force of the water on the orific plate.

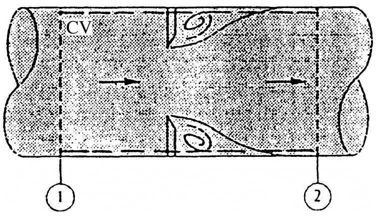



**Fig. P3.90**

**Solution:** The CV is inside the pipe walls, cutting through the orifice plate, as shown. At least to one-dimensional approximation, $V_1 = V_2$, so there is no momentum change. The force balance yields the force of the plate <u>on the fluid</u>:

$$\sum F_x = -F_{\text{plate on fluid}} + p_1A_1 - p_2A_2 - \tau_{\text{wall}}A_{\text{wall}} = \dot{m}(V_2 - V_1) \approx 0$$

$$\text{Since } \tau_{\text{wall}} \approx 0, \quad \text{we obtain } F_{\text{plate}} = (145000)\frac{\pi}{4}(0.1)^2 \approx \mathbf{1140\ N} \quad \textit{Ans.}$$

The force of the fluid on the plate is opposite to the sketch, or <u>to the right</u>.

---

**P3.91** For the water-jet pump of Prob. 3.38, add the following data: $p_1 = p_2$ = 172.4 kPa, and the distance between sections 1 and 3 is 2.032 m. If the average wall shear stress between sections 1 and 3 is 335 Pa, estimate the pressure $p_3$. Why is it higher than $p_1$?

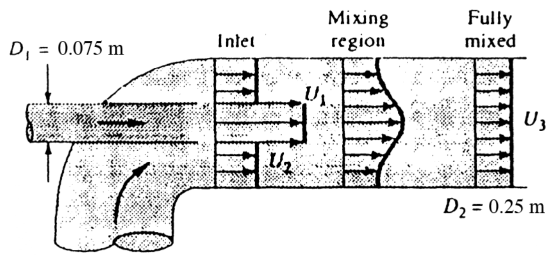



**Fig. P3.38**

**Solution:** The CV cuts through sections 1, 2, 3 and along the inside pipe walls. Recall from Prob. 3.36 that mass conservation led to the calculation **$V_3 \approx$ 6.33 m/s**. We need mass flows for each of the three sections:

$$\dot{m}_1 = 998\left(\frac{\pi}{4}\right)(0.0762)^2(40) \approx 182\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_2 = 998\left(\frac{\pi}{4}\right)[(0.254)^2 - (0.0762)^2](3) \approx 138\ \frac{\text{kg}}{\text{s}} \quad \text{and} \quad \dot{m}_3 \approx 182 + 138 \approx 320\ \frac{\text{kg}}{\text{s}}$$

Then the horizontal force balance will yield the (high) downstream pressure:

$$\sum F_x = p_1(A_1 + A_2) - p_3A_3 - \tau_{\text{wall}}\pi D_2 L = \dot{m}_3V_3 - \dot{m}_2V_2 - \dot{m}_1V_1$$

$$= (172400 - p_3)\frac{\pi}{4}(0.254)^2 - 335\pi(0.254)(2.032) = 320(6.33) - 138(3) - 182(40)$$

$$\text{Solve for} \quad p_3 \approx 274000\ \text{Pa} \qquad \textit{Ans.}$$

The pressure is high because the primary inlet kinetic energy at section (1) is converted by viscous mixing to pressure-type energy at the exit.

---

**P3.92** A vane turns a water jet through an angle $\alpha$, as shown in Fig. P3.92. Neglect friction on the vane walls. (*a*) What is the angle $\alpha$ for the support force to be in pure compression? (*b*) Calculate this compression force if the water velocity is 6.5 m/s and the jet cross-section is 25 cm$^2$.

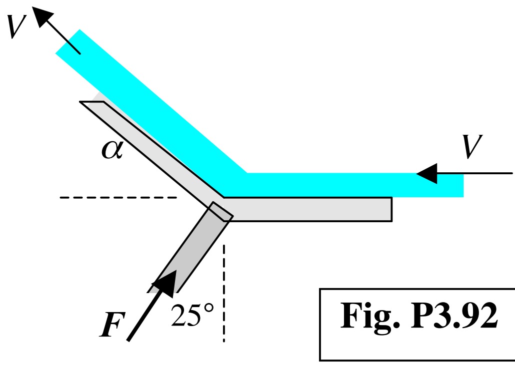



**Fig. P3.92**

**Solution**: (*a*) From the solution to Example 3.8, the support will be in pure compression (aligned with ***F***) if the vane angle is twice the support angle.

$$\text{Therefore} \quad \alpha = 2(25°) = \mathbf{50°} \quad \text{Ans.}(a)$$

(*b*) The mass flow of the jet is

$$\dot{m} = \rho A_{jet} V_{jet} = (999.8)(25\times 10^{-4}\,\text{m}^2)^2(6.5) = 16.25\ \text{kg/s}$$

Then, also from Example 3.8, the magnitude of the support force is

$$F = 2\dot{m}V\sin\frac{\alpha}{2} = 2(16.25\ \text{kg/s})(6.5\ \text{m/s})\sin\left(\frac{50^o}{2}\right) = \mathbf{89.3\ N} \quad \textit{Ans.}(b)$$

---

**P3.93** The boat in Fig. P3.93 is jet-propelled by a pump which develops a volume flow rate $Q$ and ejects water out the stern at velocity $V_j$. If the boat drag force is $F = kV^2$, where $k$ is a constant, develop a formula for the steady forward speed $V$ of the boat.

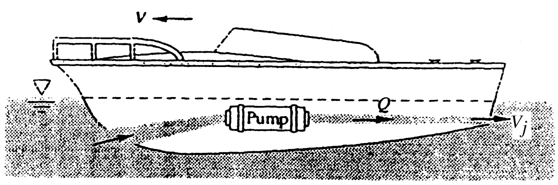



**Fig. P3.93**

**Solution:** Let the CV move to the left at boat speed V and enclose the boat and the pump's inlet and exit. Then the momentum relation is

$$\sum F_x = kV^2 = \dot{m}_{pump}(V_j + V - V_{inlet}) \approx \rho Q(V_j + V) \quad \text{if we assume } V_{inlet} << V_j$$

If, further, $V << V_j$, then the approximate solution is: $\mathbf{V \approx (\rho QV_j/k)^{1/2}}$ *Ans.*
If V and $V_j$ are comparable, then we solve a quadratic equation:

$$V \approx \boldsymbol{\zeta + [\zeta^2 + 2\zeta V_j]^{1/2}}, \quad \text{where } \boldsymbol{\zeta = \frac{\rho Q}{2k}} \quad \textit{Ans.}$$

---

**P3.94** Consider Fig. P3.38 as a general problem for analysis of a mixing ejector pump. If all conditions ($p$, $\rho$, $V$) are known at sections 1 and 2 and if the wall friction is negligible, derive formulas for estimating (a) $V_3$ and (b) $p_3$.

**Solution:** Use the CV in Prob. 3.91 but use symbols throughout. For volume flow,

$$V_1\frac{\pi}{4}D_1^2 + V_2\frac{\pi}{4}\left(D_2^2 - D_1^2\right) = V_3\frac{\pi}{4}D_2^2, \quad \text{or:} \quad V_3 = V_1\alpha + V_2(1-\alpha), \quad \alpha = (D_1/D_2)^2 \qquad \text{(A)}$$

Now apply $x$-momentum, assuming (quite reasonably) that $p_1 = p_2$:

$$(p_1 - p_3)\frac{\pi}{4}D_2^2 - \tau_w \pi D_2 L = \rho\frac{\pi}{4}D_2^2V_3^2 - \rho\frac{\pi}{4}\left(D_2^2 - D_1^2\right)V_2^2 - \rho\frac{\pi}{4}D_1^2V_1^2$$

Clean up: $$\mathbf{p_3 = p_1 - \frac{4L\tau_w}{D_2} + \rho\left[\alpha V_1^2 + (1-\alpha)V_2^2 - V_3^2\right]} \quad \text{where } \boldsymbol{\alpha} = \left(\frac{\mathbf{D_1}}{\mathbf{D_2}}\right)^2 \qquad \textit{Ans.}$$

You have to insert $V_3$ into this answer from Eq. (A) above, but the algebra is messy.

---

**P3.95** As shown in Fig. P3.95, a liquid column of height $h$ is confined in a vertical tube of cross-sectional area $A$ by a stopper. At $t = 0$ the stopper is suddenly removed, exposing the bottom of the liquid to atmospheric pressure. Using a control-volume analysis of mass and vertical momentum, derive the differential equation for the downward motion $V(t)$ of the liquid. Assume one-dimensional, incompressible, frictionless flow.

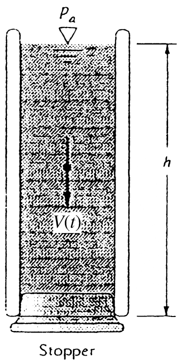



**Fig. P3.95**

**Solution:** Let the CV enclose the cylindrical blob of liquid. With density, area, and the blob volume constant, mass conservation requires that V = V(t) only. The CV accelerates downward at blob speed V(t). Vertical (downward) force balance gives

$$\sum F_{down} - \int a_{rel}\,dm = \frac{d}{dt}\left(\int V_{down}\rho\,d\upsilon\right) + \dot{m}_{out}V_{out} - \dot{m}_{in}V_{in} = 0$$

$$\text{or:} \quad m_{blob}g + \Delta p\,A - \tau_w A_w - a m_{blob} = 0$$

$$\text{Since } \Delta p = 0 \text{ and } \tau = 0, \text{ we are left with } \mathbf{a_{blob} = \frac{dV}{dt} = g} \qquad \textit{Ans.}$$

---

**P3.96** Extend Prob. 3.95 to include a linear (laminar) average wall shear stress of the form $\tau \approx cV$, where c is a constant. Find V(t), assuming that the wall area remains constant.

**Solution:** The downward momentum relation from Prob. 3.95 above now becomes

$$0 = m_{blob}g - \tau_w \pi DL - m_{blob}\frac{dV}{dt}, \quad \text{or} \quad \frac{dV}{dt} + \zeta V = g, \quad \text{where } \zeta = \frac{c\pi DL}{m_{blob}}$$

where we have inserted the laminar shear $\tau = cV$. The blob mass equals $\rho(\pi/4)D^2L$. For V = 0 at t = 0, the solution to this equation is

$$\mathbf{V = \frac{g}{\zeta}(1 - e^{-\zeta t}), \quad \text{where } \zeta = \frac{c\pi DL}{m_{blob}} = \frac{4c}{\rho D}} \quad \textit{Ans.}$$

---

**P3.97** A more involved version of Prob. 3.95 is the elbow-shaped tube in Fig. P3.97, with constant cross-sectional area *A* and diameter $D << h, L$. Assume incompressible flow, neglect friction, and derive a differential equation for *dV/dt* when the stopper is opened. *Hint:* Combine two control volumes, one for each leg of the tube.

**Solution:** Use two CV's, one for the vertical blob and one for the horizontal blob, connected as shown by pressure.

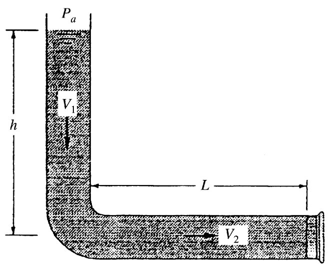
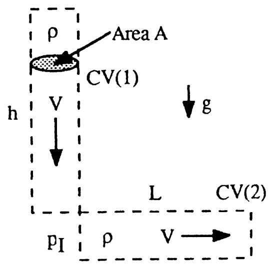



**Fig. P3.97**




From mass conservation, $V_1 = V_2 = V(t)$. For CV's #1 and #2,

$$\sum F_{down} - \int a_{rel}\, dm = \Delta(\dot{m}v) = 0 = (p_{atm} - p_I)A + \rho gAh - m_1\frac{dV}{dt} \qquad \text{(No. 1)}$$

$$\sum F_x - \int a_{rel}\, dm = \Delta(\dot{m}u) = 0 = (p_I - p_{atm})A + 0 - m_2\frac{dV}{dt} \qquad \text{(No. 2)}$$

Add these two together. The pressure terms cancel, and we insert the two blob masses:

$$\rho gAh - (\rho Ah + \rho AL)\frac{dV}{dt} = 0, \quad \text{or:} \quad \mathbf{\frac{dV}{dt} = g\frac{h}{L+h}} \quad \textit{Ans.}$$

---

**P3.98** According to Torricelli's theorem, the velocity of a fluid draining from a hole in a tank is $V \approx (2gh)^{1/2}$, where $h$ is the depth of water above the hole, as in Fig. P3.98. Let the hole have area $A_o$ and the cylindrical tank have cross-section area $A_b$. Derive a formula for the time to drain the tank from an initial depth $h_o$.

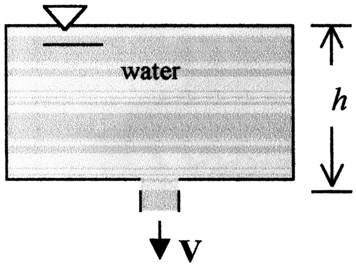



**Fig. P3.98**

**Solution:** For a control volume around the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} = 0$$

$$\rho A_b \frac{dh}{dt} = -\dot{m}_{out} \approx -\rho A_o \sqrt{2gh}$$

$$\int_{h_o}^{0} \frac{dh}{\sqrt{h}} = \int_0^t \frac{A_o\sqrt{2g}}{A_b}\, dt; \qquad \boldsymbol{t = \frac{A_b}{A_o}\sqrt{\frac{h_o}{2g}}} \quad \textit{Ans.}$$

---

**P3.99** A water jet 7.5 cm in diameter strikes a concrete (SG = 2.3) slab which rests freely on a level floor. If the slab is 30 cm wide into the paper, calculate the jet velocity which will just begin to tip the slab over.

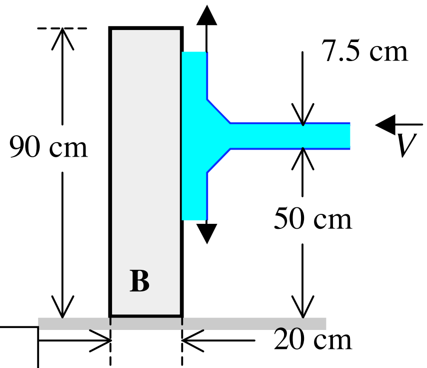



Fig. P3.99

**Solution**: For water let $\rho = 999.8$ kg/m$^3$. Find the water force and then take moments about the lower left corner of the slab, point B. A control volume around the water flow yields

$$\sum F_x = F_{on\,jet} = \Sigma \dot{m}_{out} u_{out} - \Sigma \dot{m}_{in} u_{in} = \dot{m}_{out}(0) - \rho A V(-V), \quad F = \rho A V^2$$

$$\sum M_B = (\rho A V^2)(0.5375\text{ m}) - W_{slab}(0.1\text{ m}), \; W_{slab} = (2.3\times 1000)(0.2\text{ m})(0.9\text{ m})(0.3\text{ m})\times 9.81\text{ m/s}^2$$

$$= 1218.4\text{ N}$$

Thus $(999.8)\frac{\pi}{4}(0.075\text{ m})^2 V^2 (0.5375\text{ m}) = (1218.4\text{ N})(0.1\text{ m})$, solve for $V_{jet} = \mathbf{7.16\ m/s}$ *Ans.*

---

**3.100** A cylindrical water tank discharges through a well-rounded orifice to hit a plate, as in Fig. P3.100. Use the Torricelli formula of Prob. P3.86 to estimate the exit velocity. (*a*) If, at this instant, the force *F* required to hold the plate is 40 N, what is the depth *h* ? (*b*) If the tank surface is dropping at the rate of 5 cm every 2 seconds, what is the tank diameter *D*?

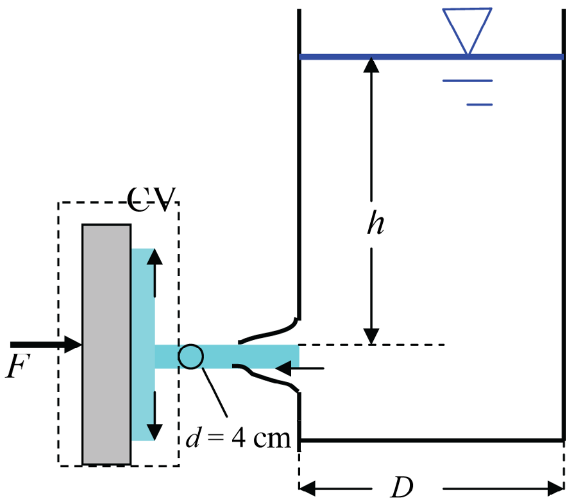



Fig. P3.100

**Solution**: For water take $\rho = 998$ kg/m$^3$. The control volume surrounds the plate and yields

$$\Sigma F_x = F = -\dot{m}_{in}u_{in} = -\dot{m}_{jet}(-V_{jet}) = \rho A_{jet}V_{jet}(V_{jet}) = \rho\frac{\pi}{4}d^2V_{jet}^2$$

$$\text{But Torricelli says} \quad V_{jet}^2 = 2gh \;; \quad \text{Thus} \quad h = \frac{F}{\rho(\pi/4)d^2(2g)}$$

$$\textit{Given data}: \; h = \frac{40\,N}{(998\,kg/m^3)(\pi/4)(0.04\,m)^2(2)(9.81\,m/s^2)} = \mathbf{1.63\,m} \quad \textit{Ans.(a)}$$

(b) In 2 seconds, *h* drops from 1.63m to 1.58m, not much change. So, instead of a laborious calculus solution, find $Q_{\text{jet,av}}$ for an average depth $h_{\text{av}} = (1.63+1.58)/2 = 1.605$ m:

$$Q_{av} = A_{jet}\sqrt{2gh_{av}} = \frac{\pi}{4}(0.04\,m)^2\sqrt{2(9.81\,m/s^2)(1.605\,m)} \approx 0.00705\,m^3/s$$

$$\text{Equate } Q\Delta t = A_{\text{tank}}\Delta h \text{, or: } D = \sqrt{\frac{Q\Delta t}{(\pi/4)\Delta h}} = \sqrt{\frac{(0.00705)(2\,s)}{(\pi/4)(0.05\,m)}} \approx \mathbf{0.60\,m} \quad \textit{Ans.(b)}$$

**P3.101** Extend Prob. 3.95 to the case of the liquid motion in a frictionless U-tube whose liquid column is displaced a distance Z upward and then released, as in Fig. P3.101. Neglect the short horizontal leg and combine control-volume analyses for the left and right legs to derive a single differential equation for *V*(*t*) of the liquid column.

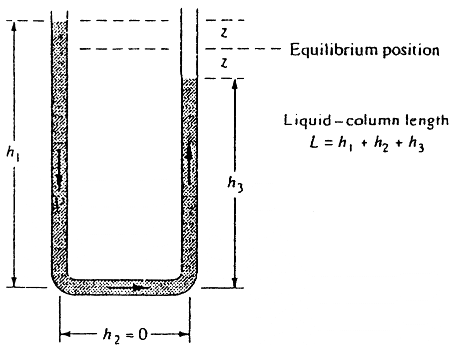
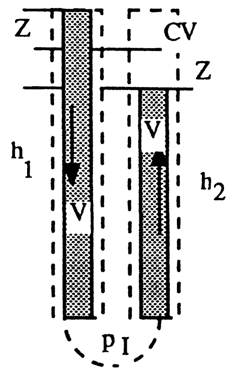



**Fig. P3.101**




**Solution:** As in Prob. 3.97, break it up into <u>two moving CV's</u>, one for each leg, as shown. By mass conservation, the velocity V(t) is the same in each leg. Let $p_I$ be the bottom pressure in the (very short) cross-over leg. Neglect wall shear stress. Now apply vertical momentum to each leg:

$$\text{Leg\#1:}\quad \sum F_{down} - \int a_{rel}\, dm$$

$$= (p_a - p_I)A + \rho g A h_1 - m_1 \frac{dV}{dt} = 0$$

$$\text{Leg\#2:}\quad \sum F_{up} - \int a_{rel}\, dm = (p_I - p_a)A - \rho g A h_2 - m_2 \frac{dV}{dt} = 0$$

Add these together. The pressure terms will cancel. Substitute for the h's as follows:

$$\rho g A(h_1 - h_2) = \rho g A(2Z) = (m_1 + m_2)\frac{dV}{dt} = \rho A(h_1 + h_2)\frac{dV}{dt} = \rho A L \frac{dV}{dt}$$

Since $V = -\dfrac{dZ}{dt}$, we arrive at, finally, $\mathbf{\dfrac{d^2Z}{dt^2} + \dfrac{2g}{L}Z = 0}$ *Ans.*

The solution is a simple harmonic oscillation: $Z = C\cos\left[t\sqrt{(2g/L)}\right] + D\sin\left[t\sqrt{(2g/L)}\right]$.

---

**P3.102** Extend Prob. 3.101 to include a linear (laminar) average wall shear stress resistance of the form $\tau \approx 8\mu V/D$, where $\mu$ is the fluid viscosity. Find the differential equation for $dV/dt$ and then solve for $V(t)$, assuming an initial displacement $z = z_o$, $V = 0$ at $t = 0$. The result should be a damped oscillation tending toward $z = 0$.

**Solution:** The derivation now includes wall shear stress on each leg (see Prob. 3.101):

$$\text{Leg\#1:}\quad \sum F_{down} - \int a_{rel}\, dm = \Delta p\, A + \rho g A h_1 - \tau_w \pi D h_1 - m_1 \frac{dV}{dt} = 0$$

$$\text{Leg\#2:}\quad \sum F_{up} - \int a_{rel}\, dm = -\Delta p\, A - \rho g A h_2 - \tau_w \pi D h_2 - m_2 \frac{dV}{dt} = 0$$

Again add these two together: the pressure terms cancel, and we obtain, if $A = \pi D^2/4$,

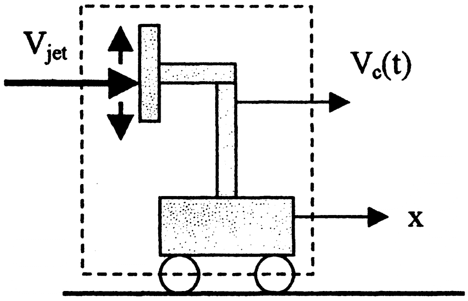
$$\mathbf{\frac{d^2Z}{dt^2} + \frac{4\tau_w}{\rho D} + \frac{2g}{L}Z = 0,} \quad \text{where } \mathbf{\tau_w = \frac{8\mu V}{D}} \quad \textit{Ans.}$$

The shear term is equal to the linear damping term $(\mathbf{32\mu/\rho D^2})(\mathbf{dZ/dt})$. If we assume an initial static displacement Z = $Z_o$, V = 0, at t = 0, we obtain the damped oscillation

$$\mathbf{Z = Z_o e^{-t/t^*}\cos(\omega t)}, \quad \text{where } \mathbf{t^* = \frac{\rho D^2}{16\mu}} \quad \text{and} \quad \omega = \sqrt{\mathbf{2g/L}} \quad \textit{Ans.}$$

---

**P3.103** As an extension of Ex. 3.10, let the plate and cart be unrestrained, with frictionless wheels. Derive (a) the equation of motion for cart velocity $V_c(t)$; and (b) the time required for the cart to accelerate to 90% of jet velocity. (c) Compute numerical values for (b) using the data from Ex. 3.10 and a cart mass of 2 kg.




**Solution:** (a) Use Eq. (3.49) with $\mathbf{a}_{rel}$ equal to the cart acceleration and $\sum F_x = 0$:

$$\sum F_x - a_{x,rel} m = \int u\rho \mathbf{V}\cdot\mathbf{n}\,dA = -m_c\frac{dV_c}{dt} = \boldsymbol{-\rho_j A_j (V_j - V_c)^2} \quad \textit{Ans.}\text{ (a)}$$

The above 1st-order differential equation can be solved by separating the variables:

$$\int_0^{V_c}\frac{dV_c}{(V_j - V_c)^2} = K\int_0^t dt, \quad \textit{where } K = \frac{\rho A_j}{m_c}$$

$$\textit{Solve for: } \frac{V_c}{V_j} = \frac{V_j K t}{1 + V_j K t} = 0.90 \quad \textit{if} \quad \mathbf{t_{90\%}} = \frac{9}{KV_j} = \frac{\mathbf{9m_c}}{\boldsymbol{\rho}\mathbf{A_j V_j}} \quad \textit{Ans.}\text{ (b)}$$

$$\textit{For the Example } 3.10 \textit{ data, } \mathbf{t_{90\%}} = \frac{9(2\ kg)}{(1000\ \text{kg/m}^3)(0.0003\ \text{m}^2)(20\ \text{m/s})} \approx \mathbf{3.0\ s} \quad \textit{Ans.}\text{ (c)}$$

---

**P3.104** Let the rocket of Fig. E3.12 start at z = 0, with constant exit velocity and exit mass flow, and rise vertically with zero drag. (a) Show that, as long as fuel burning continues, the vertical height $S(t)$ reached is given by

$$S = \frac{V_e M_o}{\dot m}[\zeta\ln\zeta - \zeta + 1], \quad \textit{where } \zeta = 1 - \frac{\dot m t}{M_o}$$

(b) Apply this to the case $V_e$ = 1500 m/s and $M_o$ = 1000 kg to find the height reached after a burn of 30 seconds, when the final rocket mass is 400 kg.

**Solution:** (a) Ignoring gravity effects, integrate the equation of the projectile's velocity (from E3.12):

$$S(t) = \int V(t)dt = \int_0^t \left[-V_e \ln\left(1 - \frac{\dot{m}t}{M_o}\right)\right]dt$$

$$Let\ \zeta = 1 - \frac{\dot{m}t}{M_o},\quad then\ d\zeta = -\frac{\dot{m}}{M_o}dt\quad and\ the\ integral\ becomes,$$

$$S(t) = (-V_e)\left[\frac{-M_o}{\dot{m}}\right]\int_1^{\zeta}(\zeta\ln\zeta)d\zeta = \left(\frac{V_eM_o}{\dot{m}}\right)[\zeta\ln\zeta - \zeta]_1^{\zeta} = \left(\frac{V_eM_o}{\dot{m}}\right)[\zeta\ln\zeta - \zeta + 1]$$

(b) Substituting the numerical values given,

$$\dot{m} = \frac{\Delta M}{\Delta t} = \frac{M_f - M_o}{\Delta t} = \frac{1000\text{ kg} - 400\text{ kg}}{30\text{ s}} = 20\text{ kg/s}\quad and\quad \zeta = 1 - \frac{(20\text{ kg/s})(30\text{ s})}{1000\text{ kg}} = 0.40$$

$$S(t = 30\text{ s}) = \frac{(1500\text{ m/s})(1000\text{ kg})}{(20\text{ kg/s})}[0.4\ \ln(0.4) - (0.4) + 1] = \mathbf{17,500}\text{ m}\quad Ans.$$

---

**P3.105** Suppose that the solid-propellant rocket of Prob. 3.37 is built into a missile of diameter 70 cm and length 4 m. The system weighs 1800 N, which includes 700 N of propellant. Neglect air drag. If the missile is fired vertically from rest at sea level, estimate (a) its velocity and height at fuel burnout and (b) the maximum height it will attain.

**Solution:** The theory of Example 3.12 holds until burnout. Now $M_o$ = 1800/9.81 = 183.5 kg, and recall from Prob. 3.37 that $V_e$ = 1150 m/s and the exit mass flow is 11.8 kg/s. The fuel mass is 700/9.81 = 71.4 kg, so burnout will occur at $t_{burnout}$ = 71.4/11.8 = 6.05 s. Then Example 3.12 predicts the velocity at burnout:

$$V_b = -1150\ \ln\left(1 - \frac{11.8(6.05)}{183.5}\right) - 9.81(6.05) \approx \mathbf{507\ \frac{m}{s}}\quad Ans.\ (a)$$

Meanwhile, Prob. 3.104 gives the formula for altitude reached at burnout:

$$S_b = \frac{183.5(1150)}{11.8}[1 + (0.611)\{\ln(0.611) - 1\}] - \frac{1}{2}(9.81)(0.605)^2 \approx \mathbf{1393\ m}\quad Ans.\ (a)$$

where "0.611" = 1 – 11.8(6.05)/183.5, that is, the mass ratio at burnout. After burnout, with drag neglected, the missile moves as a falling body. Maximum height occurs at

$$\Delta t = \frac{V_o}{g} = \frac{507}{9.81} = 51.7\text{ s, whence}$$

$$S = S_o + \frac{1}{2}g\Delta t^2 = 1393 + (1/2)(9.81)(51.7)^2 \approx \mathbf{14500\ m}\quad Ans.\ (b)$$

---

**P3.106** Water at 20°C flows steadily through the tank in Fig. P3.106. Known conditions are $D_1$ = 8 cm, $V_1$ = 6 m/s, and $D_2$ = 4 cm. A rightward force $F$ = 70 N is required to keep the tank fixed. (*a*) What is the velocity leaving section 2? (*b*) If the tank cross-section is 1.2 m$^2$, how fast is the water surface $h(t)$ rising or falling?

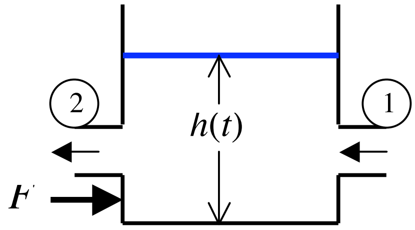



Fig. P3.106

**Solution**: First, for water at 20°C, $\rho$ = 998 kg/m$^3$. (*a*) For a control volume around the tank,

$$\sum F_x = F = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}_2(-V_2) - \dot{m}_1(-V_1)$$

$$\text{or: } 70\,N = [(998)\frac{\pi}{4}(0.04m)^2 V_2](-V_2) + [(998)\frac{\pi}{4}(0.08m)^2(6\frac{m}{s})](6\frac{m}{s})$$

$$= -1.254\,V_2^2 + 180.6\,N\,,\ solve\ V_2 = \sqrt{\frac{70-180.6}{-1.254}} = \mathbf{9.39}\,\frac{m}{s} \quad Ans.(a)$$

(*b*) The mass flows at 1 and 2 are *not* equal. The difference in volume flow moves the surface:

$$A_{\text{tank}}\frac{dh}{dt} = Q_1 - Q_2 = \frac{\pi}{4}(0.08m)^2(6\frac{m}{s}) - \frac{\pi}{4}(0.04m)^2(9.39\frac{m}{s})$$

$$\text{or: } (1.2m^2)\frac{dh}{dt} = 0.0302 - 0.0118 = 0.0184\,\frac{m^3}{s}\,,\ \text{solve}\ \frac{dh}{dt} \approx +\mathbf{0.0153}\,\frac{m}{s}\uparrow \quad Ans.(b)$$

---

**P3.107** As can often be seen in a kitchen sink when the faucet is running, a high-speed channel flow ($V_1$, $h_1$) may "jump" to a low-speed, low-energy condition ($V_2$, $h_2$) as in Fig. P3.107. The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Use the continuity and momentum relations to find $h_2$ and $V_2$ in terms of ($h_1$, $V_1$).

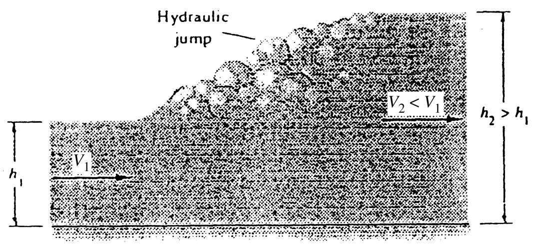



**Fig. P3.107**

**Solution:** The CV cuts through sections 1 and 2 and surrounds the jump, as shown. Wall shear is neglected. There are no obstacles. The only forces are due to hydrostatic pressure:

$$\sum F_x = 0 = \frac{1}{2}\rho g h_1(h_1 b) - \frac{1}{2}\rho g h_2(h_2 b) = \dot{m}(V_2 - V_1),$$

$$\text{where } \dot{m} = \rho V_1 h_1 b = \rho V_2 h_2 b$$

$$\text{Solve for} \quad V_2 = \mathbf{V_1 h_1 / h_2} \quad \text{and} \quad h_2/h_1 = \mathbf{-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 8V_1^2/(gh_1)}} \quad Ans.$$

---

**P3.108** Suppose that the solid-propellant rocket of Prob. 3.37 is mounted on a 1000-kg car to propel it up a long slope of 15°. The rocket motor weighs 900 N, which includes 500 N of propellant. If the car starts from rest when the rocket is fired, and if air drag and wheel friction are neglected, estimate the maximum distance that the car will travel up the hill.

**Solution:** This is a variation of Prob. 3.105, except that "*g*" is now replaced by "*g* sin$\theta$." Recall from Prob. 3.37 that the rocket mass flow is 11.8 kg/s and its exit velocity is 1150 m/s. The rocket fires for $t_b$ = (500/9.81)/11.8 = 4.32 sec, and the initial mass is $M_o$ = (1000 + 900/9.81) = 1092 kg. Then the differential equation for uphill powered motion is

$$m\frac{dV}{dt} = \dot{m}V_e - mg\sin\theta, \quad m = M_o - \dot{m}t$$

This integrates to: $V(t) = -V_e \ln(1 - \dot{m}t/M_o) - gt\sin\theta \quad \text{for } t \le 4.32 \text{ s.}$

After burnout, the rocket coasts uphill with the usual falling-body formulas with "*g* sin$\theta$." The distance travelled during rocket power is modified from Prob. 3.104:

$$S = (M_oV_e/\dot{m})[1 + (1 - \dot{m}t/M_o)\{\ln(1 - \dot{m}t/M_o) - 1\}] - \frac{1}{2}gt^2\sin\theta$$

Apply these to the given data at burnout to obtain

$$V_{burnout} = -1150\ \ln(0.9533) - \frac{1}{2}(9.81)\sin 15°(4.32) \approx 44.0 \text{ m/s}$$

$$S_{burnout} = \frac{1092(1150)}{11.8}[1 + 0.9533\{\ln(0.9533) - 1\}] - \frac{1}{2}(9.81)\sin 15°(4.32)^2 \approx 94 \text{ m}$$

The rocket then coasts uphill a distance $\Delta S$ such that $V_b^2$ = $2g\Delta S \sin\theta$, or $\Delta S$ = $(44.0)^2/[2(9.81)\sin 15°] \approx 381$ m. The total distance travelled is 381 + 94 ≈ **475 m** *Ans.*

---

**P3.109** A rocket is attached to a rigid horizontal rod hinged at the origin as in Fig. P3.109. Its initial mass is $M_o$, and its exit properties are $\dot{m}$ and $V_e$ relative to the rocket. Set up the differential equation for rocket motion, and solve for the angular velocity $\omega(t)$ of the rod. Neglect gravity, air drag, and the rod mass.

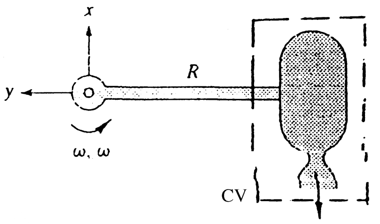


**Fig. P3.109**

**Solution:** The CV encloses the rocket and moves at (accelerating) rocket speed $\Omega$(t). The rocket arm is free to rotate, there is no force parallel to the rocket motion. Then we have

$$\sum F_{tangent} = 0 - \int a_{rel}\, dm = \dot{m}(-V_e), \quad \text{or} \quad mR\frac{d\Omega}{dt} = \dot{m}V_e, \quad \text{where } m = M_o - \dot{m}t$$

Integrate, with $\Omega = 0$ at t = 0, to obtain $\mathbf{\Omega = -\frac{V_e}{R}\ln\left(1 - \frac{\dot{m}t}{M_o}\right)}$ *Ans.*

---

**P3.110** Extend Prob. 3.109 to the case where the rocket has a linear air drag force $F = cV$, where $c$ is a constant. Assuming no burnout, solve for $\omega(t)$ and find the *terminal* angular velocity, i.e., the final motion when the angular acceleration is zero. Apply to the case $M_o = 6$ kg, $R = 3$ m, $m = 0.05$ kg/s, $V_e = 1100$ m/s, and $c = 0.075$ N·s/m to find the angular velocity after 12 s of burning.

**Solution:** If linear resistive drag is added to Prob. 3.109, the equation of motion becomes

$$m\frac{d\Omega}{dt} = \frac{\dot{m}V_e}{R} - C\Omega, \quad \text{where } m = M_o - \dot{m}t, \text{ with } \Omega = 0 \text{ at } t = 0$$

The solution is found by separation of variables:

$$\text{If } B = \dot{m}V_e/R, \quad \text{then } \int_0^{\Omega}\frac{d\Omega}{B - C\Omega} = \int_0^{t}\frac{dt}{M_o - \dot{m}t}, \quad \text{or: } \Omega = \frac{\mathbf{B}}{\mathbf{C}}\left[\mathbf{1} - \left(\mathbf{1} - \frac{\dot{\mathbf{m}}\mathbf{t}}{\mathbf{M_o}}\right)^{\mathbf{C}/\dot{\mathbf{m}}}\right] \quad \textit{Ans.}\text{(a)}$$

Strictly speaking, <u>there is no terminal velocity</u>, but if we set the acceleration equal to zero in the basic differential equation, we obtain an estimate $\mathbf{\Omega_{term} = mV_e/(RC)}$. *Ans.* (b)

For the given data, at t = 12 s, we obtain the angular velocity

$$\text{At } t = 12 \text{ s:} \quad \Omega = \frac{(0.05)(1100)}{(3.0)(0.075)}\left[1 - \left(1 - \frac{0.05(12)}{6.0}\right)^{\frac{0.075}{0.05}}\right] \approx \mathbf{36\ \frac{rad}{sec}} \quad \textit{Ans.}\text{ (c)}$$

---

**P3.111** Actual air flow past a parachute creates a variable distribution of velocities and directions. Let us model this as a circular air jet, of diameter half the parachute diameter, which is turned completely around by the parachute, as in Fig. P3.111.

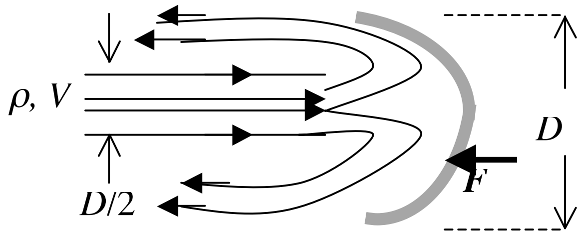



**Fig. P3.111**

(*a*) Find the force $F$ required to support the chute.

(*b*) Express this force as a dimensionless *drag coefficient*, $C_D = F/[(1/2)\rho V^2(\pi/4)D^2]$ and compare with Table 7.3.

**Solution**: This model is crude, compared to velocity-field theory, but gives the right order of magnitude. (*a*) Let the control volume surround the parachute and cut through the oncoming and leaving air streams:

$$\sum F_x = -F = \dot{m}u_{out} - \dot{m}u_{in} = \dot{m}(-V) - \dot{m}(+V),$$

$$\text{or: } F = 2\dot{m}V = 2(\rho A_{jet}V)V = 2\rho\left[\frac{\pi}{4}\left(\frac{D}{2}\right)^2\right]V^2 = \mathbf{\frac{\pi}{8}\rho D^2 V^2} \quad \textit{Ans.}(a)$$

(*b*) Express this approximate result as a dimensionless drag coefficient:

$$C_D = \frac{F}{(1/2)\rho V^2(\pi/4)D^2} = \frac{(\pi/8)\rho V^2 D}{(1/2)\rho V^2(\pi/4)D^2} \approx \mathbf{1.0} \qquad Ans.(b)$$

From Table 7.3, actual measurements show a parachute drag coefficient of about **1.2**. Not bad!

---

**P3.112** The cart in Fig. P3.112 moves at constant velocity $V_o$ = 12 m/s and takes on water with a scoop 80 cm wide which dips $h$ = 2.5 cm into a pond. Neglect air drag and wheel friction. Estimate the force required to keep the cart moving.

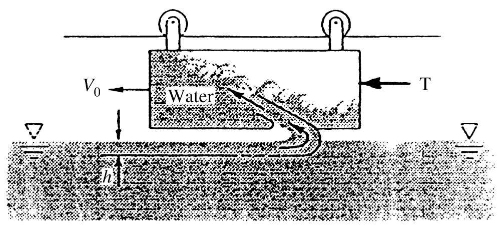



**Fig. P3.112**

**Solution:** The CV surrounds the cart and scoop and moves to the left at cart speed $V_o$. Momentum *within* the cart fluid is neglected. The horizontal force balance is

$$\sum F_x = -\text{Thrust} = -\dot{m}_{scoop}V_{inlet}, \quad \text{but} \quad V_{inlet} = V_o \ \text{(water motion relative to scoop)}$$

$$\text{Therefore} \quad \text{Thrust} = \dot{m}V_o = [998(0.025)(0.8)(12)](12) \approx \mathbf{2900\ N} \quad Ans.$$

---

**P3.113** A rocket sled of mass $M$ is to be decelerated by a scoop, as in Fig. P3.113, which has width $b$ into the paper and dips into the water a depth $h$, creating an upward jet at 60°. The rocket thrust is $T$ to the left. Let the initial velocity be $V_o$, and neglect air drag and wheel friction. Find an expression for $V(t)$ of the sled for (a) $T = 0$ and (b) finite $T \neq 0$.

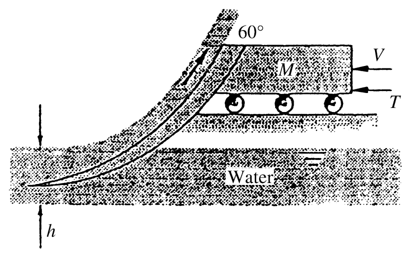



**Fig. P3.113**

**Solution:** The CV surrounds the sled and scoop and moves to the *left* at sled speed V(t). Let $x$ be positive <u>to the left</u>. The horizontal force balance is

$$\sum F_x = T - M\frac{dV}{dt} = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}(-V\cos\theta) - \dot{m}(-V), \quad \dot{m} = \rho bhV$$

$$\text{or:} \quad M_{sled}\frac{dV}{dt} = T - CV^2, \quad C = \rho bh(1-\cos\theta)$$

Whether or not thrust T = 0, the variables can be separated and integrated:

$$\text{(a) } T = 0: \quad \int_{V_o}^{V}\frac{dV}{V^2} = -\frac{C}{M}\int_0^t dt, \quad \text{or:} \quad \mathbf{V = \frac{V_o}{1 + CV_o t/M}} \quad Ans.\text{(a)}$$

$$\text{(b) } T > 0: \quad \int_{V_o}^{V}\frac{M\,dV}{T - CV^2} = \int_0^t dt, \quad \text{or:} \quad \mathbf{V = V_{final}\tanh[\alpha t + \phi]} \quad Ans.\text{(b)}$$

$$\text{where} \quad V_{final} = [T/\rho bg(1-\cos\theta)]^{1/2}, \quad \alpha = [T\rho bh(1-\cos\theta)]^{1/2}/M, \quad \phi = \tanh^{-1}(V_o/V_f)$$

<u>This solution only applies</u> when $V_o < V_{final}$, which may not be the case for a speedy sled.

---

**P3.114** For the boundary layer flow in Fig. 3.10, let the exit velocity profile, at x = $L$, simulate turbulent flow, $u \approx U_o(y/\delta)^{1/7}$. (*a*) Find a relation between $h$ and $\delta$. (*b*) Find an expression for the drag force $F$ on the plate between 0 and $L$.

**Solution**: (*a*) Since the upper and lower boundaries of the control volume are streamlines, the mass flow in, at $x$=0, must equal the mass flow out, at $x$=$L$.

$$\int d\dot{m}_{in} = \int_0^h \rho U_o b\,dy = \rho U_o b h = \int d\dot{m}_{out} = \int_0^\delta \rho U_o (\frac{y}{\delta})^{1/7} b\,dy = \frac{7}{8}\rho U_o b\delta$$

$$\text{after cancellation,} \quad h = \frac{7}{8}\delta \qquad \textit{Ans.(a)}$$

(*b*) Instead of blindly using Karman's formula from Example 3.10, derive the drag force in straightforward control volume momentum-integral fashion:

$$\sum F_x = -F = \int u_{out}\, d\dot{m}_{out} - \int u_{in}\, d\dot{m}_{in} = \int_0^\delta U_o(\frac{y}{\delta})^{1/7}[\rho U_o(\frac{y}{\delta})^{1/7}]b\,dy - \int_0^h U_o \rho U_o\, b\,dy$$

$$\text{or:} \quad F = \rho U_o^2 b\delta(\frac{7}{8} - \frac{7}{9}) = \frac{7}{72}\rho U_o^2 b\delta \qquad \textit{Ans.(b)}$$

---

**P3.115** Consider the same conditions as given in Prob. P3.64, but the pressure on the corner annular ring equals to $p(r) = p_1 + (r - R_1)(P_1 - P_2)$ for $R_2 \ge r \ge R_1$, where $R_2$ and $R_1$ are the radius of the pipe at ② and ① respectively. Derive the downstream pressure.

**Solution**: Linear momentum equation in $x$-direction is given as

$$\sum F_x = P_1 A_1 + \int_{R_1}^{R_2} p\,dA - P_2 A_2 = \dot{m}(V_2 - V_1) \text{where } \dot{m} = \rho A_1 V_1$$

$$V_2 = \frac{V_1 A_1}{A_2}$$

$$\therefore \quad P_1 A_1 + \int_{R_1}^{R_2} [P_1 + (r - R_1)(P_1 - P_2)]dA = \rho A_1 V_1^2 \left(\frac{A_1}{A_2} - 1\right), \; dA = 2\pi r dr$$

with some manipulations

$$\pi\left(R_2^2 + \frac{2}{3}(R_2^3 - R_1^3) - R_1(R_2^2 - R_1^2)\right)P_1$$

$$-\pi\left(\frac{2}{3}(R_2^3 - R_1^3) - R_1(R_2^2 - R_1^2)\right)P_2 = \rho A_1 V_1^2\left(\frac{A_1}{A_2} - 1\right)$$

$$\therefore \quad P_2 = \frac{\pi\left(R_2^2 + \frac{2}{3}(R_2^3 - R_1^3) - R_1(R_2^2 - R_1^2)\right)P_1 + \rho A_1 V_1^2\left(1 - \frac{A_1}{A_2}\right)}{\pi\left(\frac{2}{3}(R_2^3 - R_1^3) - R_1(R_2^2 - R_1^2)\right)}$$

---

**P3.116** In the firefighting nozzle model shown in Fig. 3.116, water steadily flows in the *z* direction at the base diameter *D* with flow rate *Q* under pressure *P*. Water exits through the nozzle diameter *d*. Calculate the forces to hold the nozzle at its base.

**Solution**: From conservation of mass,

$$\vec{V}_1 = \frac{Q\vec{k}}{A_1} = \frac{4Q}{\pi D^2}\vec{k}$$
$$|V_2| = \frac{Q}{A_2} = \frac{4Q}{\pi d^2}$$
$$\vec{V}_2 = |V_2|(\sin\phi\cos\theta\,\vec{\mathrm{i}} + \sin\phi\sin\theta\,\vec{\mathrm{j}} + \cos\phi\,\vec{\mathrm{k}})$$

Consider linear momentum equation,

$$\vec{F} = \frac{\partial}{\partial t}\int_{CV}\vec{V}\rho dV + \int_{CS}\vec{V}\rho\vec{V}\cdot d\vec{A}$$

Since we consider our calculation at steady flow condition, we then have

$$\vec{F} = \vec{F}_S + \vec{F}_B = \int_{CS}\vec{V}\rho\vec{V}\cdot d\vec{A}$$

Assume that the nozzle with water weighs WN, we would have

$$\vec{F}_S + \vec{F}_B = pA_1\vec{k} - W\vec{k} + \vec{F}_{base} = \dot{m}_{\text{out}}\vec{V}_2 - \dot{m}_{\text{in}}\vec{V}_1$$
$$\therefore\quad \vec{F}_{base} = \rho Q\left(\frac{4Q}{\pi d^2}(\sin\phi\cos\theta\,\vec{\mathrm{i}} + \sin\phi\sin\theta\,\vec{\mathrm{j}} + \cos\phi\,\vec{\mathrm{k}})\frac{-4Q\vec{k}}{\pi D^2}\right) + (W - pA_1)\vec{k}$$
$$\vec{F}_{base} = \frac{4\rho Q^2}{\pi d^2}(\sin\phi\cos\theta\,\vec{\mathrm{i}} + \sin\phi\sin\theta\,\vec{\mathrm{j}}) + \left[\frac{4\rho Q^2}{\pi}\left(\frac{1}{d^2}\cos\phi - \frac{1}{D^2}\right) + w - \frac{\pi\rho D^2}{4}\right]\vec{k}$$

---

**P3.117** Repeat Prob. 3.54 by assuming that $p_1$ is unknown and using Bernoulli's equation with no losses. Compute the new bolt force for this assumption. What is the head loss between 1 and 2 for the data of Prob. 3.54?

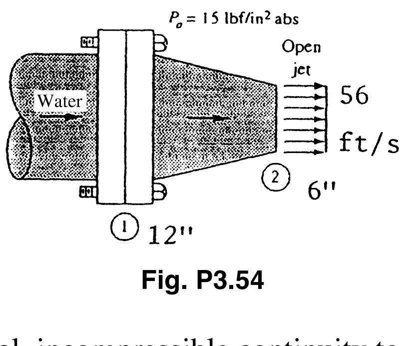



**Fig. P3.54**

**Solution:** Use one-dimensional, incompressible continuity to find $V_1$:

$$V_1 A_1 = V_1(\pi/4)(0.3)^2 = V_2 A_2 = (17\text{ m/s})\frac{\pi}{4}(0.15)^2\ ,\quad \text{or}\quad V_1 = 4.25\text{ m/s}$$

Bernoulli's equation with no losses to estimate p1 with $\Delta z = 0$:

$$\frac{p_1}{\gamma} + \frac{(4.25)^2}{2(9.81)} \approx \frac{103.42\times 10^3}{(998)(9.81)} + \frac{(17)^2}{2(9.81)}, \quad \text{solve for } p_{1,\text{ideal}} \approx \mathbf{238.62\ kPa(abs)}$$

From the *x*-momentum CV analysis of Prob. 3.54, the bolt force is given by

$$F_{\text{bolts}} = p_{2,\text{gage}}A_2 - \dot{m}(V_2 - V_1)$$
$$= \left(238.62\times 10^3 - 103.42\times 10^3\right)\frac{\pi}{4}(0.3)^2 - 998\left(\frac{\pi}{4}\right)(0.3)^2(4.25)(17-4.25) \approx \mathbf{57340\ N} \quad \textit{Ans.}$$

We can estimate the friction head loss in Prob. 3.54 from the steady flow energy equation, with p1 taken to be the value of 262 kPa given in that problem:

$$\frac{262\times 10^3}{(998)(9.81)} + \frac{(4.25)^2}{2(9.81)} = \frac{103.42\times 10^3}{(998)(9.81)} + \frac{(17)^2}{2(9.81)} + h_f, \quad \text{solve for } h_f \approx \mathbf{2.4\ m} \quad \textit{Ans.}$$

---

**P3.118** Extend the siphon analysis of Ex. 3.22 as follows. Let $p_1$ = 1 atm and let the fluid be hot water at 60°C. Let $z_{1,2,4}$ be the same, with $z_3$ unknown. Find the value of $z_3$ for which the water might begin to vaporize.

**Solution**: Given $p_1$ = 101350 Pa and recall that $z_1$ = 60 cm, $z_2$ = -25 cm, and $z_4$ was not needed. Then note that, because of steady-flow one-dimensional continuity, from Ex. 3.22,

$$V_3 = V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2(9.81)[0.6-(-0.25)]} = 4.08\, m/s$$

For cavitation, $p_3$ should drop down to the vapor pressure of water at 60°C, which from Table A.5 is 19.92 kPa. And, from Table A.3, the density of water at 60°C is 983 kg/m³. Now write Bernoulli from point 1 to point 3 at the top of the siphon:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_3}{\rho} + \frac{V_3^2}{2} + gz_3$$

$$\frac{101350}{983} + \frac{0^2}{2} + (9.81)(0.6m) = \frac{19920\, Pa}{983\, kg/m^3} + \frac{(4.08 m/s)^2}{2} + 9.81 z_3$$

$$103.1 + 0 + 5.9 = 20.3 + 8.3 + 9.81 z_3, \quad \textit{Solve} \quad z_3 \approx \frac{80.4}{9.81} = \mathbf{8.2\ m} \quad \textit{Ans.}$$

That's pretty high, so the writer does not think cavitation is a problem with this siphon.

---

**P3.119** A jet of alcohol strikes the vertical plate in Fig. P3.119. A force $F \approx 425$ N is required to hold the plate stationary. Assuming there are no losses in the nozzle, estimate (a) the mass flow rate of alcohol and (b) the absolute pressure at section 1.

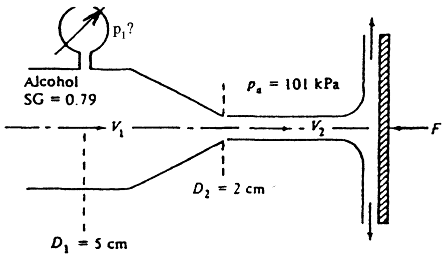



Fig. 3.119

**Solution:** A momentum analysis of the plate (e.g. Prob. 3.45) will give

$$F = \dot{m}V_2 = \rho A_2 V_2^2 = 0.79(998)\frac{\pi}{4}(0.02)^2 V_2^2 = 425 \text{ N,}$$

$$\text{solve for } V_2 \approx 41.4 \text{ m/s}$$

$$\text{whence} \quad \dot{m} = 0.79(998)(\pi/4)(0.02)^2(41.4) \approx \mathbf{10.3\ kg/s} \quad \textit{Ans.}\text{ (a)}$$

We find $V_1$ from mass conservation and then find $p_1$ from Bernoulli with no losses:

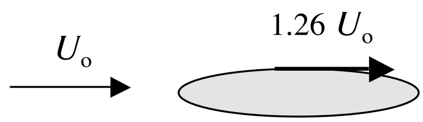
$$\text{Incompressible mass conservation:} \quad V_1 = V_2(D_2/D_1)^2 = (41.4)\left(\frac{2}{5}\right)^2 \approx 6.63 \text{ m/s}$$

$$\text{Bernoulli, } z_1 = z_2\text{:} \quad p_1 = p_2 + \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = 101000 + \frac{0.79(998)}{2}[(41.4)^2 - (6.63)^2]$$

$$\approx \mathbf{760{,}000\ Pa} \quad \textit{Ans.}\text{ (b)}$$

---

**P3.120** An airplane is flying at 134 m/s at 4000 m standard altitude. As is typical, the air velocity relative to the upper surface of the wing, near its maximum thickness, is 26 percent higher than the plane's velocity. Using Bernoulli's equation, calculate the absolute pressure at this point on the wing. Neglect elevation changes and compressibility.




**Solution**: Fix the frame of steady flow relative to the wing. Let point 1 be the oncoming stream and point 2 be the maximum thickness point. From Table A.5, at 4000 m, $p$ = 61,633 Pa, and $\rho$ = 0.8191 kg/m$^3$. Then the velocity $U_2$ at the max thickness point is 1.26(134) = 169 m/s. Then, from the figure,

$$p_1 + \frac{1}{2}\rho U_1^2 \approx (61633) + \frac{1}{2}(0.8191)(134)^2 = p_2 + \frac{1}{2}\rho U_2^2 = p_2 + \frac{1}{2}(0.8191)(169)^2$$

$$\text{Solve for } p_2 = \mathbf{57{,}300}\ \textit{Pa} \text{ on the upper surface} \quad \textit{Ans.}$$

If the elevation change were accounted for, the answer would differ by *less than one* Pascal.

---

**P3.121** Water flows through a circular nozzle, exits into the air as a jet, and strikes a plate, as in Fig. P3.121. The force required to hold the plate steady is 70 N. Assuming frictionless one-dimensional flow, estimate (a) the velocities at sections (1) and (2); (b) the mercury manometer reading *h*.

**Solution:** (a) First examine the momentum of the jet striking the plate,

$$\sum F = F = -\dot{m}_{in}u_{in} = -\rho A_2 V_2^2$$

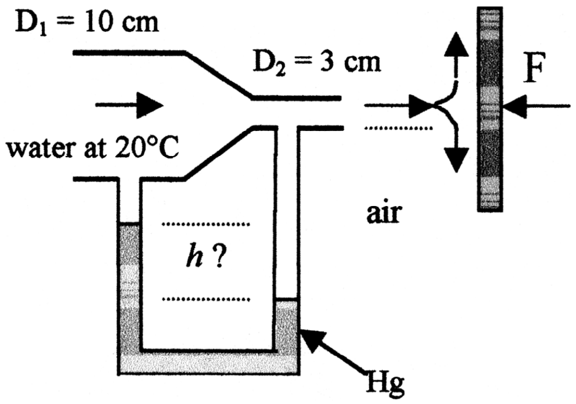



**Fig. P3.121**

$$70\ N = -(998)\left(\frac{\pi}{4}\right)(0.03^2)(V_2^2) \quad \mathbf{V_2 = 9.96\ m/s} \quad \textit{Ans.}\ (a)$$

$$\textit{Then} \quad V_1 = \frac{V_2 A_2}{A_1} = \frac{(9.96)\left(\frac{\pi}{4}\right)(0.03^2)}{\frac{\pi}{4}(0.1^2)} \quad \textit{or} \quad \mathbf{V_1 = 0.9\ m/s} \quad \textit{Ans.}\ (a)$$

(b) Applying Bernoulli,

$$p_2 - p_1 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \frac{1}{2}(998)(9.96^2 - 0.9^2) = 49{,}100\ Pa$$

And from our manometry principles,

$$h = \frac{\Delta p}{\Delta(\rho g)} = \frac{49{,}100}{(133{,}100 - 9790)} \approx \mathbf{0.40\ m} \quad \textit{Ans.}\ (b)$$

**P3.122** A free liquid jet, as in Fig. P3.122, has constant ambient pressure and small losses; hence from Bernoulli's equation $z + V^2/(2g)$ is constant along the jet. For the fire nozzle in the figure, what are (a) the minimum and (b) the maximum values of $\theta$ for which the water jet will clear the corner of the building? For which case will the jet velocity be higher when it strikes the roof of the building?

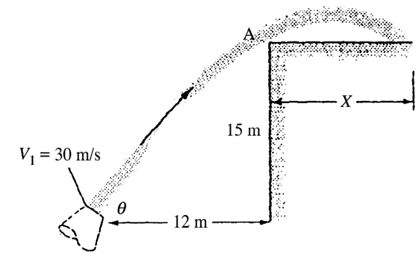



**Fig. P3.122**

**Solution:** The two extreme cases are when the jet just touches the corner A of the building. For these two cases, Bernoulli's equation requires that

$$V_1^2 + 2gz_1 = (30)^2 + 2g(0) = V_A^2 + 2gz_A = V_A^2 + 2(9.81)(15), \quad \text{or:} \quad V_A = 24.6\,\frac{m}{s}$$

The jet moves like a frictionless particle as in elementary particle dynamics:

$$\text{Vertical motion:} \quad z = (V_1 \sin\theta)\,t - \frac{1}{2}gt^2; \quad \text{Horizontal motion:} \quad x = (V_1 \cos\theta)\,t$$

Eliminate "t" between these two and apply the result to point A:

$$z_A = 15 = x_A \tan\theta - \frac{g x_A^2}{2V_1^2 \cos^2\theta} = 12\,\tan\theta - \frac{(9.81)(12)^2}{2(30)^2 \cos^2\theta}; \quad \text{clean up and rearrange:}$$

$$\tan\theta = 1.25 + 0.0654\,\sec^2\theta, \quad \text{solve for } \theta = \approx\mathbf{85.865°} \quad \textit{Ans.}\text{ (a)} \quad \text{and} \quad \mathbf{55.46°} \quad \textit{Ans.}\text{ (b)}$$

Path (b) is shown in the figure, where the jet just grazes the corner A and goes over the top of the roof. Path (a) goes nearly straight up, to z = 45.63 m, then falls down to pt. A. In both cases, the velocity when the jet strikes point A is the same, 24.6 m/s.

---

**P3.123** For the container of Fig. P3.123 use Bernoulli's equation to derive a formula for the distance *X* where the free jet leaving horizontally will strike the floor, as a function of *h* and *H*. For what ratio *h*/*H* will *X* be maximum? Sketch the three trajectories for *h*/*H* = 0.25, 0.5, and 0.75.

**Solution:** The velocity out the hole and the time to fall from hole to ground are given by

$$V_o = \sqrt{2g(H-h)} \qquad t_{fall} = \sqrt{2h/g}$$

Then the distance travelled horizontally is

$$X = V_o t_{fall} = \mathbf{2\sqrt{h(H-h)}} \quad \textit{Ans.}$$

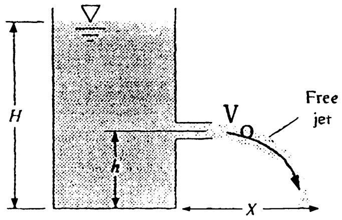
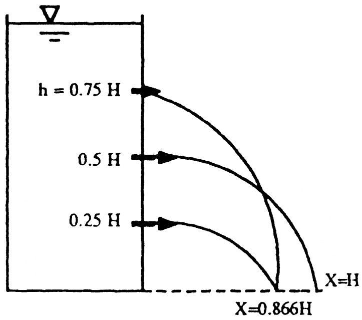



**Fig. P3.123**




Maximum X occurs at **h = H/2**, or **$X_{max}$ = H**. When h = 0.25H or 0.75H, the jet travels out to X = 0.866H. These three trajectories are shown in the sketch on the previous page.

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**P3.124** Water at 20°C, in the pressurized tank of Fig. P3.124, flows out and creates a vertical jet as shown. Assuming steady frictionless flow, determine the height *H* to which the jet rises.

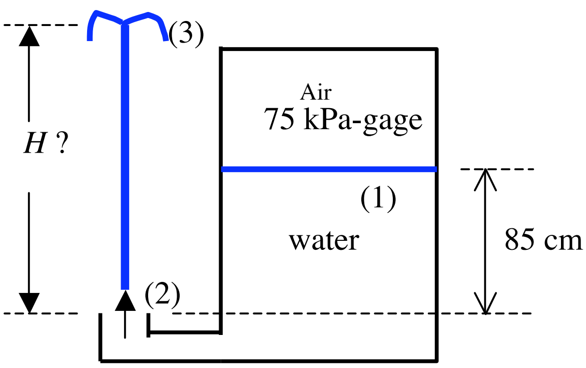


Fig. P3.124

**Solution**: This is a straightforward Bernoulli problem. Let the water surface be (1), the exit plane be (2), and the top of the vertical jet be (3). Let $z_2 = 0$ for convenience. If we are clever, we can bypass (2) and write Bernoulli directly from (1) to (3):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + z_3 \ , \quad or:$$

$$\frac{75000}{(9.81)(998)} + 0 - 0.85m = 0 + 0 + H$$

$$Solve \quad H = 7.66\,m + 0.85\,m = \mathbf{8.51\,m} \quad \quad Ans.$$

If we took an intermediate step from (1) to (2), we would find $V_2^2/2g = 8.51$ m, and then going from (2) to (3) would convert the velocity head into pure elevation, because $V_3 = 0$.

---

**P3.125** Bernoulli's 1738 treatise *Hydrodynamica* contains many excellent sketches of flow patterns. One, however, redrawn here as Fig. P3.125, seems physically misleading. What is wrong with the drawing?

**Solution:** If friction is neglected and the exit pipe is fully open, then pressure in the closed "piezometer" tube would be atmospheric and the fluid would not rise at all in the tube. The open jet coming from the hole in the tube would have $V \approx \sqrt{(2gh)}$ and would rise up to nearly the same height as the water in the tank.

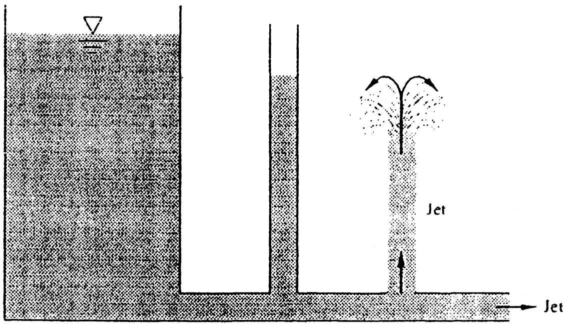


**Fig. P3.125**

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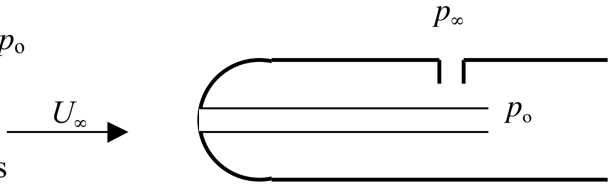
**P3.126** A long fixed tube with a rounded nose, aligned with an oncoming flow, can be used to measure velocity. Measurements are made of the pressure at (1) the front nose and (2) a hole in the side of the tube further along, where the pressure nearly equals stream pressure. (*a*) Make a sketch of this device and show how the velocity is calculated. (*b*) For a particular sea-level air flow, the difference between nose pressure and side pressure is 10.3 kPa. What is the air velocity, in km/h?

**Solution**: (*a*) The front nose measures $p_o$ and the side hole measures the stream pressure $p_\infty$. With no elevation changes, the Bernoulli equation predicts




$$U_\infty = \sqrt{\frac{2(p_o - p_\infty)}{\rho}} \qquad Ans.(a)$$

The device is, of course, called a *Pitot-static tube* and is in wide use in fluids engineering. (*b*) For sea-level conditions, take $\rho_{air} = 1.2255\ \text{kg/m}^3$.

$$U_\infty = \sqrt{\frac{2(p_o - p_\infty)}{\rho}} = \sqrt{\frac{2(10342\ \text{Pa})}{1.2255\ \text{kg/m}^3}} = 130\ \text{m/s} = \mathbf{468}\ \text{km/h} \quad Ans.(b)$$

---

**P3.127** The manometer fluid in Fig. P3.127 is mercury. Estimate the volume flow in the tube if the flowing fluid is (a) gasoline and (b) nitrogen, at 20°C and 1 atm.

**Solution:** For gasoline (a) take $\rho = 680.3$ kg/m$^3$. For nitrogen (b), R ≈ 297 J/kg · °C and $\rho = p/RT =$ (101350)/[(297)(293)] ≈ 1.165 kg/m$^3$. For mercury, take $\rho \approx$ 13,575.1 kg/ m$^3$. The pitot tube (2) reads stagnation pressure, and the wall hole (1) reads static pressure. Thus Bernoulli's relation becomes, with $\Delta z = 0$,

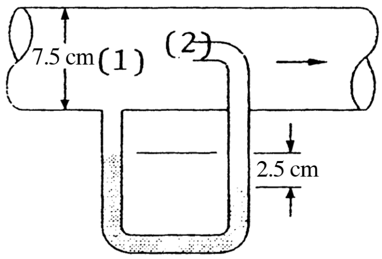



**Fig. P3.127**

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2, \quad \text{or} \quad V_1 = \sqrt{2(p_2 - p_1)/\rho}$$

The pressure difference is found from the manometer reading, for each fluid in turn:

$$\text{(a) Gasoline:} \quad \Delta p = (\rho_{Hg} - \rho)gh = (13575.1 - 680.3)(9.81)(2.5 \times 10^{-2}) \approx 3162.5 \text{ N/m}^2$$

$$V_1 = [2(3162.5)/680.3]^{1/2} = 3.05 \text{ m/s}, \; Q = V_1A_1 = (3.05)\left(\frac{\pi}{4}\right)(7.5 \times 10^{-2})^2 = \mathbf{13.5\ L/s} \; \textit{Ans.} \text{ (a)}$$

$$\text{(b) } N_2\text{:} \quad \Delta p = (\rho_{Hg} - \rho)gh = (13575.1 - 1.165)(9.81)(2.5 \times 10^{-2}) \approx 3329 \text{ N/m}^2$$

$$V_1 = [2(3329)/1.165]]^{1/2} = 75.6 \text{ m/s}, \; Q = V_1A_1 = (75.6)\left(\frac{\pi}{4}\right)(7.5 \times 10^{-2})^2 \approx \mathbf{334\ L/s} \; \textit{Ans.} \text{ (b)}$$

---

**P3.128** In Fig. P3.128 the flowing fluid is $CO_2$ at 20°C. Neglect losses. If $p_1$ = 170 kPa and the manometer fluid is Meriam red oil (SG = 0.827), estimate (a) $p_2$ and (b) the gas flow rate in m$^3$/h.

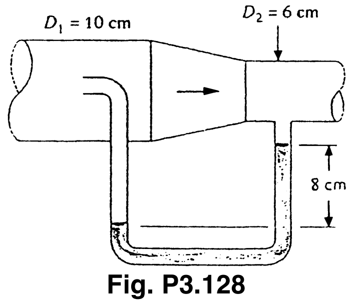



**Fig. P3.128**

**Solution:** Estimate the $CO_2$ density as $\rho = p/RT$ = (170000)/[189(293)] ≈ 3.07 kg/m$^3$. The manometer reading gives the down-stream pressure:

$$p_1 - p_2 = (\rho_{oil} - \rho_{CO_2})gh = [0.827(998) - 3.07](9.81)(0.08) \approx 645 \text{ Pa}$$

$$\text{Hence} \quad p_2 = 170{,}000 - 645 \approx \mathbf{169400\,Pa} \quad \textit{Ans.} \text{ (a)}$$

Now use Bernoulli to find $V_2$, assuming $p_1 \approx$ stagnation pressure ($V_1 = 0$):

$$p_1 + \frac{1}{2}\rho(0)^2 \approx p_2 + \frac{1}{2}\rho V_2^2,$$

$$\text{or:}\quad V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} = \sqrt{\frac{2(645)}{3.07}} \approx 20.5\ \frac{m}{s}$$

$$\text{Then}\quad Q = V_2A_2 = (20.5)(\pi/4)(0.06)^2 = 0.058\ m^3/s \approx \mathbf{209\ \frac{m^3}{hr}}\quad \textit{Ans.}\ (b)$$

---

**P3.129** The cylindrical water tank in Fig. P3.129 is being filled at a volume flow $Q_1$ = 3.785 L/min, while the water also drains from a bottom hole of diameter $d$ = 6 mm. At time $t = 0$, $h = 0$. Find ($a$) an expression for $h(t)$ and ($b$) the eventual maximum water depth $h_{max}$. Assume that Bernoulli's steady-flow equation is valid.

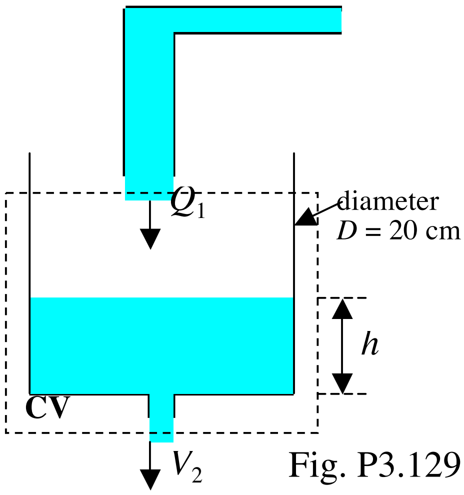


Fig. P3.129

**Solution**: Bernoulli predicts that $V_2 \approx \sqrt{(2gh)}$.

Convert $Q_1$ = 6.309E-5 m$^3$/s. A control volume around the tank gives the mass balance:

$$\frac{dm}{dt}\Big|_{system} = 0 = \frac{d}{dt}(Ah) - Q_1 + A_2\sqrt{2gh}\ ,\ \text{where}\ A = \frac{\pi}{4}D^2\ \text{and}\ A_2 = \frac{\pi}{4}d^2$$

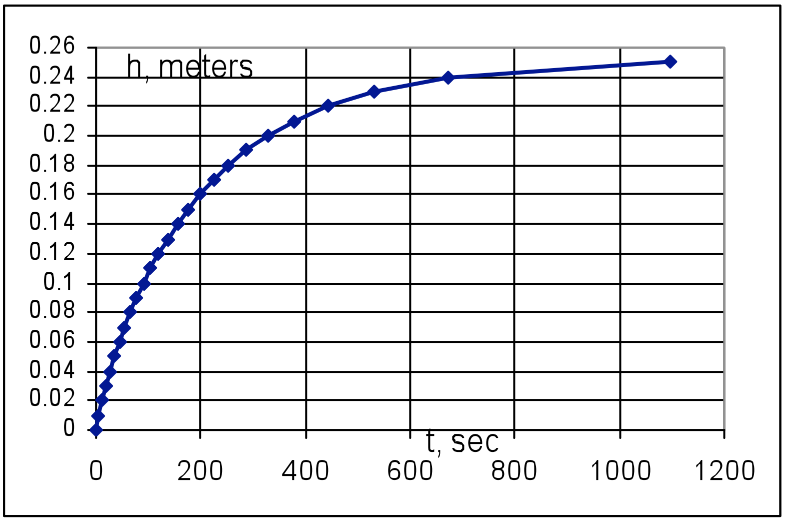
Rearrange, separate the variables, and integrate:

$$\int_0^{h(t)} \frac{dh}{Q_1 - A_2\sqrt{2gh}} = \frac{1}{A}\int_0^t dt$$

($a$) The integration is a bit tricky and laborious. Here is the writer's result:

$$t = \frac{2Q_1 A}{\alpha^2}\ln\left(\frac{Q_1}{Q_1 - \alpha\sqrt{h}}\right) - \frac{2A\sqrt{h}}{\alpha}\ ,\quad \text{where}\ \alpha = A_2\sqrt{2g}\qquad \textit{Ans.}(a)$$

($a$) A graph of $h$ versus $t$ for the particular given data is as follows:




($b$) The water level rises fast and then slower and is asymptotic to the value $h_{max} = 0.254$ m. This is when the outflow through the hole exactly equals the inflow from the pipe:

$$Q_1 = A_2\sqrt{2gh_{max}}\ ,\quad or:\quad 6.309E-5\frac{m^3}{s} = \frac{\pi}{4}(0.006m)^2\sqrt{2(9.81)h_{max}}$$

$$Solve\ for\quad h_{max} = \mathbf{0.254\,m}\qquad Ans.(b)$$

---

**P3.130** The air-cushion vehicle in Fig. P3.130 brings in sea-level standard air through a fan and discharges it at high velocity through an annular skirt of 3-cm clearance. If the vehicle weighs 50 kN, estimate (a) the required airflow rate and (b) the fan power in kW.

**Solution:** The air inside at section 1 is nearly stagnant (V ≈ 0) and supports the weight and also drives the flow out of the interior into the atmosphere:

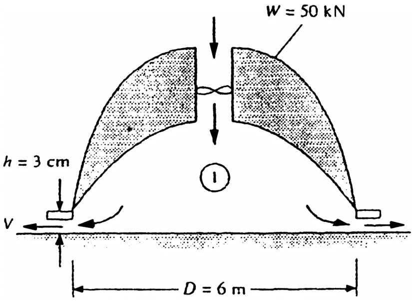



**Fig. P3.130**

$$p_1 \approx p_{o1}:\quad p_{o1} - p_{atm} = \frac{\text{weight}}{\text{area}} = \frac{50{,}000\text{ N}}{\pi(3\text{ m})^2} = \frac{1}{2}\rho V_{exit}^2 = \frac{1}{2}(1.205)V_{exit}^2 \approx 1768\text{ Pa}$$

$$\text{Solve for}\quad V_{exit} \approx 54.2\text{ m/s, whence}\quad Q_e = A_e V_e = \pi(6)(0.03)(54.2) = 30.6\ \frac{m^3}{s}$$

Then the power required by the fan is $P = Q_e\Delta p = (30.6)(1768) \approx \mathbf{54000\ W}$ *Ans.*

---

**P3.131** A necked-down section in a pipe flow, called a *venturi*, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in Fig. P3.131. Using Bernoulli's equation with no losses, derive an expression for the velocity $V_1$ which is just sufficient to bring reservoir fluid into the throat.

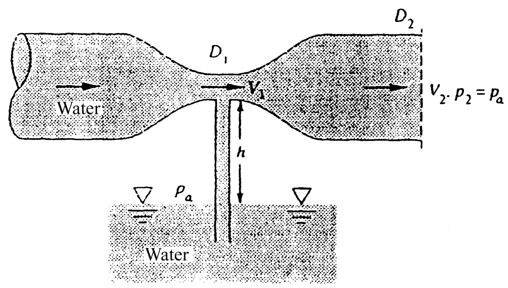



**Fig. P3.131**

**Solution:** Water will begin to aspirate into the throat when $p_a - p_1 = \rho gh$. Hence:

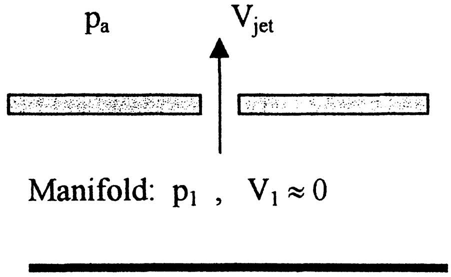
$$\text{Volume flow: } V_1 = V_2(D_2/D_1)^2; \quad \text{Bernoulli } (\Delta z = 0)\text{: } p_1 + \frac{1}{2}\rho V_1^2 \approx p_{atm} + \frac{1}{2}\rho V_2^2$$

$$\text{Solve for } p_a - p_1 = \frac{\rho}{2}(\alpha^4 - 1)V_2^2 \geq \rho gh, \quad \alpha = \frac{D_2}{D_1}, \quad \text{or: } \mathbf{V_2 \geq \sqrt{\frac{2gh}{\alpha^4 - 1}}} \quad \textit{Ans.}$$

$$\textit{Similarly, } \mathbf{V_{1,\,min} = \alpha^2 V_{2,\,min} = \sqrt{\frac{2gh}{1 - (D_1/D_2)^4}}} \quad \textit{Ans.}$$

---

**P3.132** Suppose you are designing a $0.9 \times 1.8$-m air-hockey table, with 1.6-mm-diameter holes spaced every mm in a rectangular pattern (2592 holes total), the required jet speed from each hole is 15 m/s. You must select an appropriate blower. Estimate the volumetric flow rate (in $m^3$/min) and pressure rise (in Pa) required. *Hint*: Assume the air is stagnant in the large manifold under the table surface, and neglect frictional losses.




**Solution:** Assume an air density of about sea-level, **1.21 kg/m³**. Apply Bernoulli's equation through any single hole, as in the figure:

$$p_1 + \cancel{\frac{\rho}{2}V_1^2} = p_a + \frac{\rho}{2}V_{jet}^2, \quad or\text{:}$$

$$\Delta p_{required} = p_1 - p_a = \frac{\rho}{2}V_{jet}^2 = \frac{1.21}{2}(15)^2 = \mathbf{136.125\ Pa} \quad \textit{Ans.}$$

The total volume flow required is

$$Q = VA_{1-hole}(\#\ of\ holes) = \left(15\ \frac{\text{m}}{\text{s}}\right)\frac{\pi}{4}\left(1.6\times10^{-3}\text{m}\right)^2(2592\ holes)$$

$$= \mathbf{4.7\ \frac{m^3}{min}} \quad \textit{Ans.}$$

It wasn't asked, but the power required would be

$$P = Q\ \Delta\text{p} = (0.078\ \text{m}^3/\text{s})(136.125\ \text{Pa}) = 10.64\ \text{Watts}.$$

---

**P3.133** The liquid in Fig. P3.133 is kerosine at 20°C. Estimate the flow rate from the tank for (a) no losses and (b) pipe losses $h_f \approx 4.5V^2/(2g)$.

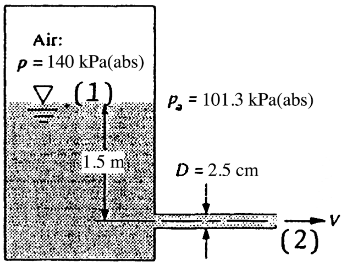



**Fig. P3.133**

**Solution:** For kerosine let $\gamma$ = 8044.2 N. Let (1) be the surface and (2) the exit jet:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 = \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_f,\quad \text{with } z_2 = 0 \text{ and } V_1 \approx 0,\ h_f = K\frac{V_2^2}{2g}$$

$$\text{Solve for}\quad \frac{V_2^2}{2g}(1+K) = z_1+\frac{p_1-p_2}{\gamma} = 1.5+\frac{\left(140\times10^3-101.3\times10^3\right)}{8044.2} \approx 6.31\ \text{m}$$

We are asked to compute two cases (a) no losses; and (b) substantial losses, K ≈ 4.5:

$$\text{(a) K = 0:}\quad V_2 = \left[\frac{2(9.81)(6.31)}{1+0}\right]^{1/2} = 11.13\ \frac{\text{m}}{\text{s}},\quad Q = 11.13\frac{\pi}{4}\left(2.5\times10^{-2}\right)^2 \approx \mathbf{5.46\ \frac{L}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) K = 4.5:}\quad V_2 = \sqrt{\frac{2(9.81)(6.31)}{1+4.5}} = 4.74\ \frac{\text{m}}{\text{s}},\quad Q = 4.74\frac{\pi}{4}\left(2.5\times10^{-2}\right)^2 \approx \mathbf{2.33\ \frac{L}{s}} \quad \textit{Ans.}\text{ (b)}$$

---

**P3.134** An open water jet exits from a nozzle into sea-level air, as shown, and strikes a stagnation tube. If the centerline pressure at section (1) is 110 kPa and losses are neglected, estimate (a) the mass flow in kg/s; and (b) the height *H* of the fluid in the tube.

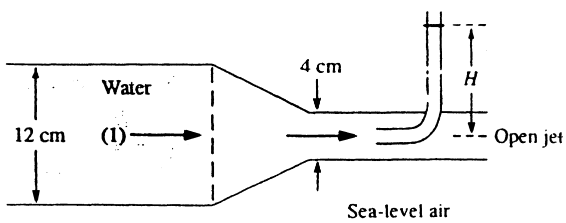



**Fig. P3.134**

**Solution:** Writing Bernoulli and continuity between pipe and jet yields jet velocity:

$$p_1 - p_a = \frac{\rho}{2}V_{jet}^2\left[1-\left(\frac{D_{jet}}{D_1}\right)^4\right] = 110000 - 101350 = \frac{998}{2}V_{jet}^2\left[1-\left(\frac{4}{12}\right)^4\right],$$

$$solve \quad V_{jet} = \mathbf{4.19}\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\textit{Then the mass flow is} \quad \dot{m} = \rho A_{jet}V_{jet} = 998\frac{\pi}{4}(0.04)^2(4.19) = \mathbf{5.25}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans. (a)}$$

(b) The water in the stagnation tube will rise above the jet surface by an amount equal to the stagnation pressure head of the jet:

$$\mathbf{H} = R_{jet} + \frac{V_{jet}^2}{2g} = 0.02\ \mathrm{m} + \frac{(4.19)^2}{2(9.81)} = 0.02 + 0.89 = \mathbf{0.91\ m} \quad \textit{Ans. (b)}$$

---

**P3.135** A *venturi meter*, shown in Fig. P3.135, is a carefully designed constriction whose pressure difference is a measure of the flow rate in a pipe. Using Bernoulli's equation for steady incompressible flow with no losses, show that the flow rate $Q$ is related to the manometer reading $h$ by

$$Q = \frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{\frac{2gh(\rho_M-\rho)}{\rho}}$$

where $\rho_M$ is the density of the manometer fluid.

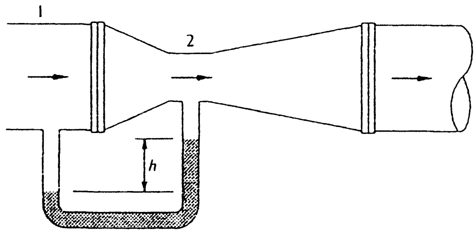



**Fig. P3.135**

**Solution:** First establish that the manometer reads the pressure difference between 1 and 2:

$$p_1 - p_2 = (\rho_M - \rho)gh \qquad (1)$$

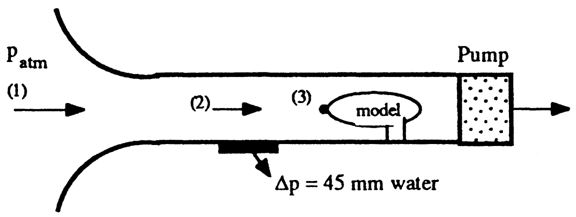
Then write incompressible Bernoulli's equation and continuity between (1) and (2):

$$(\Delta z = 0): \quad \frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_2}{\rho} + \frac{V_2^2}{2} \quad \text{and} \quad V_2 = V_1(D_1/D_2)^2, \quad Q = A_1V_1 = A_2V_2$$

$$\text{Eliminate } V_2 \text{ and } (p_1 - p_2) \text{ from (1) above:} \quad \mathbf{Q} = \frac{\mathbf{A_2}\sqrt{\mathbf{2gh}(\rho_M-\rho)/\rho}}{\sqrt{\mathbf{1-(D_2/D_1)^4}}} \quad \textit{Ans.}$$

---

**P3.136** A wind tunnel draws in sea-level standard air from the room and accelerates it into a 1-m by 1-m test section. A pressure transducer in the test section wall measures $\Delta p$ = 45 mm water between inside and outside. Estimate (a) the test section velocity in mi/hr; and (b) the absolute pressure at the nose of the model.




**Solution:** (a) First apply Bernoulli from the atmosphere (1) to (2), using the known $\Delta p$:

$$p_a - p_2 = 45 \text{ mm } H_2O = 441 \text{ Pa}; \quad \rho_a = 1.225 \text{ kg/m}^3; \quad p_1 + \frac{\rho}{2}V_1^2 \approx p_2 + \frac{\rho}{2}V_2^2$$

$$\text{Since } V_1 \approx 0 \text{ and } p_1 = p_a, \text{ we obtain } V_2 = \sqrt{\frac{2\,\Delta p}{\rho}} = \sqrt{\frac{2(441)}{1.225}} = \frac{26.8 \times 3600}{1000} = \mathbf{96.48\ \frac{km}{hr}} \quad \textit{Ans.}\text{ (a)}$$

(b) Bernoulli from 1 to 3: both velocities = 0, so $p_{nose} = p_a \approx$ **101350 Pa**. *Ans.* (b)

---

**P3.137** In Fig. P3.137 the fluid is gasoline at 20°C at a weight flux of 120 N/s. Assuming no losses, estimate the gage pressure at section 1.

**Solution:** For gasoline, $\rho$ = 680 kg/m$^3$. Compute the velocities from the given flow rate:

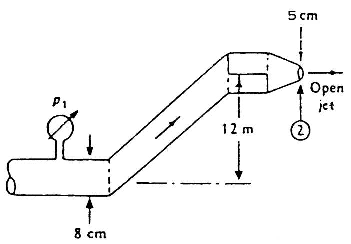



**Fig. P3.137**

$$Q = \frac{\dot{W}}{\rho g} = \frac{120 \text{ N/s}}{680(9.81)} = 0.018\ \frac{m^3}{s},$$

$$V_1 = \frac{0.018}{\pi(0.04)^2} = 3.58\ \frac{m}{s}; \quad V_2 = \frac{0.018}{\pi(0.025)^2} = 9.16\ \frac{m}{s}$$

Now apply Bernoulli between 1 and 2:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \approx \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2, \quad \text{or:} \quad \frac{p_1}{\rho} + \frac{(3.58)^2}{2} + 0 \approx \frac{0(\text{gage})}{680} + \frac{(9.16)^2}{2} + 9.81(12)$$

$$\text{Solve for} \quad p_1 \approx \mathbf{104{,}000\ Pa\,(gage)} \quad \textit{Ans.}$$

---

**P3.138** In Fig. P3.138 both fluids are at 20°C. If $V_1$ = 0.5 m/s and losses are neglected, what should the manometer reading *h* m be?

**Solution:** By continuity, establish $V_2$:

$$V_2 = V_1(D_1/D_2)^2 = 0.5(3/1)^2 = 4.5\ \frac{m}{s}$$

Now apply Bernoulli between 1 and 2 to establish the pressure at section 2:

$$p_1 + \frac{\rho}{2}V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}V_2^2 + \rho g z_2,$$

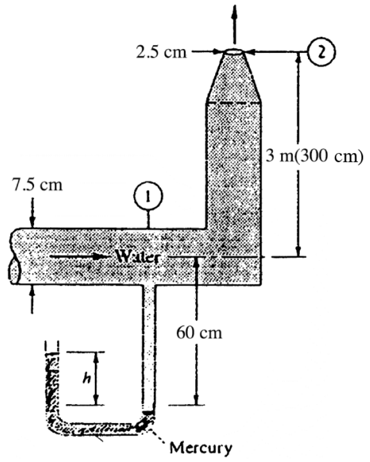



**Fig. P3.138**

$$\text{or:}\quad p_1 + (998/2)(0.5)^2 + 0 \approx 0 + (998/2)(4.5)^2 + (998)(9.81)(10),\quad p_1 = 107.9\text{ kPa}$$

This is *gage* pressure. Now the manometer *reads* gage pressure, so

$p_1 - p_a = 107.9\text{ kPa} = (\rho_{merc} - \rho_{water})gh = (13552 - 998)(9.81)h$, solve for $h \approx$ **87.61 cm** *Ans.*

---

**P3.139** Extend the siphon analysis of Ex. 3.14 to account for friction in the tube, as follows. Let the friction head loss in the tube be correlated as $5.4(V_{tube})^2/(2g)$, which approximates turbulent flow in a 2-m-long tube. Calculate the exit velocity in m/s and the volume flow rate in $cm^3/s$. We repeat the sketch of Ex. 3.14 for convenience.

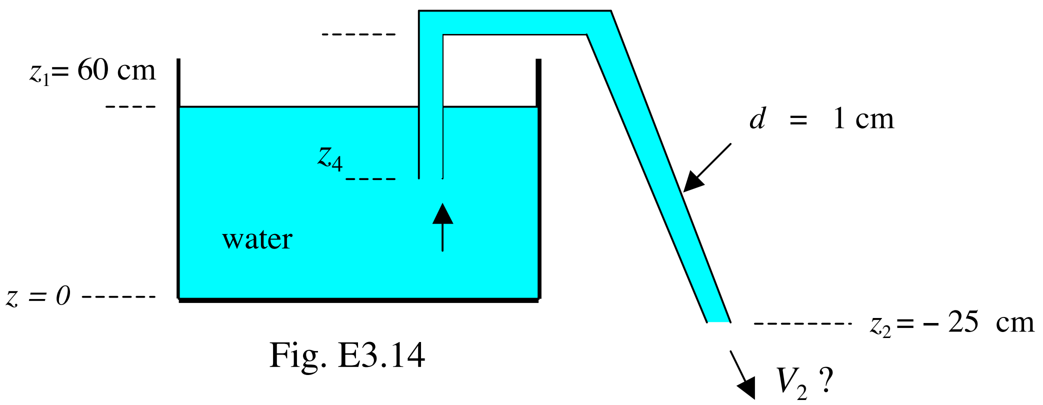



Fig. E3.14

**Solution**: Write the steady flow energy equation from the water surface (1) to the exit (2):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \ , \quad \text{where } h_f = 5.4\frac{V_{tube}^2}{2g}$$

The tube area is constant, hence $V_{\text{tube}} = V_2$. Also, $p_1 = p_2$ and $V_1 \approx 0$. Thus we obtain

$$z_1 - z_2 = 0.6m - (-0.25m) = 0.85m = \frac{V_2^2}{2g}(1+5.4)$$

$$Solve\ \ V_2 = \sqrt{\frac{2(9.81m/s^2)(0.85m)}{1+5.4}} = \mathbf{1.61\frac{m}{s}} \qquad Ans.$$

$$and\ \ Q = V_2A_2 = (1.61\frac{m}{s})\frac{\pi}{4}(0.01m)^2 = 0.000167\frac{m^3}{s} = \mathbf{127\frac{cm^3}{s}} \ \ Ans.$$

Tube friction has reduced the flow rate by more than 60%.

---

**P3.140** If losses are neglected in Fig. P3.140, for what water level $h$ will the flow begin to form vapor cavities at the throat of the nozzle?

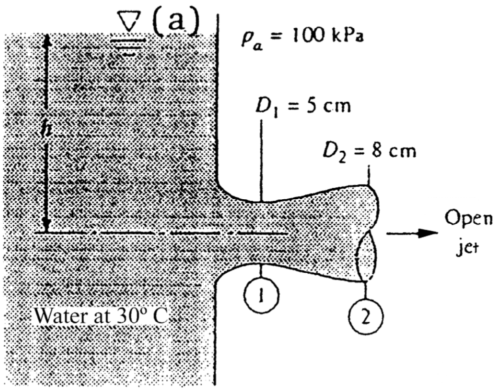



**Fig. P3.140**

**Solution:** Applying Bernoulli from (a) to (2) gives Torricelli's relation: $V_2 = \sqrt{2gh}$. Also,

$$V_1 = V_2(D_2/D_1)^2 = V_2(8/5)^2 = 2.56V_2$$

Vapor bubbles form when $p_1$ reaches the <u>vapor pressure</u> at 30°C, $p_{vap} \approx 4242$ Pa (from Table A.5), while $\rho \approx 996$ kg/m$^3$ at 30°C (Table A.1). Apply Bernoulli between 1 and 2:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \approx \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2, \quad \text{or:} \quad \frac{4242}{996} + \frac{(2.56V_2)^2}{2} + 0 \approx \frac{100000}{996} + \frac{V_2^2}{2} + 0$$

$$\text{Solve for} \quad V_2^2 = 34.62 = 2gh, \quad \text{or} \quad h = 34.62/[2(9.81)] \approx \mathbf{1.76\ m} \quad Ans.$$

---

**P3.141** For the 40°C water flow in Fig. P3.141, estimate the volume flow through the pipe, assuming no losses; then explain what is wrong with this seemingly innocent question. If the actual flow rate is $Q = 40\ \text{m}^3/\text{h}$, compute (a) the head loss in m and (b) the constriction diameter $D$ which causes cavitation, assuming that the throat divides the head loss equally and that changing the constriction causes no additional losses.

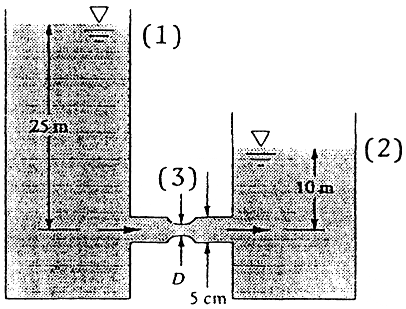



**Fig. P3.141**

**Solution:** Apply Bernoulli between 1 and 2:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 \approx \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2, \quad \text{or:} \quad 0+0+25 \approx 0+0+10, \quad \text{or:} \quad \mathbf{25 = 10\ ??}$$

This is madness, what happened? The answer is that this problem <u>cannot be free of losses</u>. There is a **<u>15-m loss</u>** as the pipe-exit jet dissipates into the downstream reservoir. *Ans.* (a)

(b) Examining analysis (a) shows that the head loss is 15 meters. For water at 40°C, the vapor pressure is 7375 Pa (Table A.5), and the density is 992 kg/m$^3$ (Table A.1). Now write Bernoulli between (1) and (3), assuming a head loss of 15/2 = 7.5 m:

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1 = \frac{p_3}{\rho}+\frac{V_3^2}{2}+gz_3+\frac{g}{2}h_{f,total}, \quad \text{where } V_3 = \frac{Q}{A_3} = \frac{40/3600}{(\pi/4)D^2} = \frac{0.0141}{D^2}$$

$$\text{Thus} \quad \frac{101350}{992}+0+9.81(25) \approx \frac{7375}{992}+\frac{(0.0141/D^2)^2}{2}+0+(9.81)(7.5)$$

$$\text{Solve for} \quad D^4 \approx 3.75\text{E–}7\ \text{m}^4, \quad \text{or} \quad D \approx 0.0248\ \text{m} \approx 25\ \text{mm} \quad \textit{Ans.(b)}$$

This corresponds to $V_3 \approx 23$ m/s.

---

**P3.142** The 35°C water flow of Fig. P3.142 discharges to sea-level standard atmosphere. Neglecting losses, for what nozzle diameter $D$ will cavitation begin to occur? To avoid cavitation, should you increase or decrease $D$ from this critical value?

**Solution:** At 35°C the vapor pressure of water is approximately 5600 Pa (Table A.5). Bernoulli from the surface to point 3 gives the

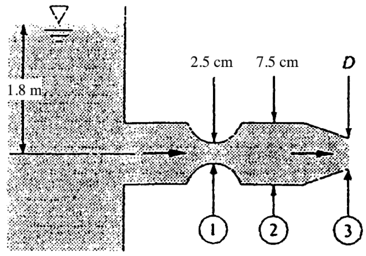



**Fig. P3.142**

Torricelli result $V_3 = \sqrt{(2gh)} = \sqrt{[2(9.81)(1.8)]} \approx 5.94$ m/s. We can ignore section 2 and write Bernoulli from (1) to (3), with $p_1 = p_{vap}$ and $\Delta z = 0$:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_2}{\rho} + \frac{V_2^2}{2}, \quad \text{or:} \quad \frac{5600}{994} + \frac{V_1^2}{2} \approx \frac{101,325}{994} + \frac{V_3^2}{2},$$

$$\text{but also } V_1 = V_3\left(\frac{D}{2.5 \times 10^{-2}}\right)^2$$

Eliminate $V_1$ and introduce $V_3 = 5.94 \dfrac{m}{s}$ to obtain $D^4 = 1.26 \times 10^{-6}$, $\mathbf{D \approx 3.4\ cm}$ *Ans.*

To avoid cavitation, we would keep **D < 3.4 cm**, which will keep $p_1 > p_{vapor}$.

---

**P3.143** Air, assumed frictionless, flows through a tube, exiting to sea-level atmosphere. Diameters at 1 and 3 are 5 cm, while $D_2 = 3$ cm. What mass flow of air is required to suck water up 10 cm into section 2 of Fig. P3.143?

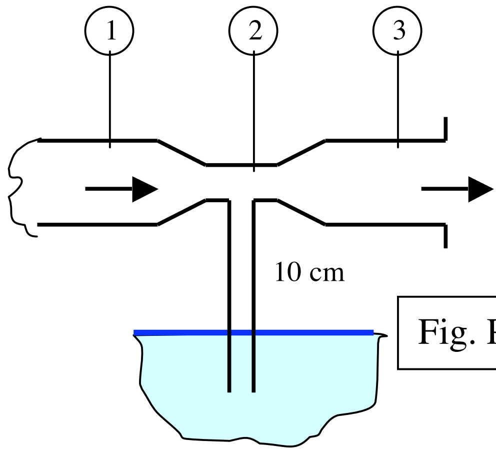



Fig. P3.143

**Solution**: For sea-level, take $\rho_{air} = 1.2255$ kg/m$^3$. Section 2 must be less than atmospheric. How much less? Determine the pressure change for 10 cm of water:

$$\Delta p = p_3 - p_2 = \rho_{water}\, g\, h = (998\, kg/m^3)(9.81\, m/s^2)(0.1\, m) = 979\ Pa$$

This must be the pressure difference between sections 2 and 3. From Bernoulli's equation,

$$p_3 - p_2 = \frac{\rho}{2}(V_2^2 - V_3^2) \quad \text{plus continuity: } V_3 = \frac{A_2}{A_3}V_2 = \frac{D_2^2}{D_3^2}V_2 = \frac{9}{25}V_2$$

$$\text{or: } 979\,Pa = \frac{1.2255}{2}(V_2^2)[1 - (\frac{9}{25})^2]; \ \textit{Solve for } V_2 = 42.8\frac{m}{s}, V_3 = 15.4\frac{m}{s}$$

$$\text{Finally, } \dot{m}_{air} = \rho A_3 V_3 = (1.2255\frac{kg}{m^3})\frac{\pi}{4}(0.05m)^2(15.4\frac{m}{s}) = \mathbf{0.037}\frac{kg}{s} \quad \textit{Ans.}$$

---

**P3.144** In Fig. P3.144 the piston drives water at 20°C. Neglecting losses, estimate the exit velocity $V_2$ m/s. If $D_2$ is further constricted, what is the maximum possible value of $V_2$?

**Solution:** Find $p_1$ from a freebody of the piston:

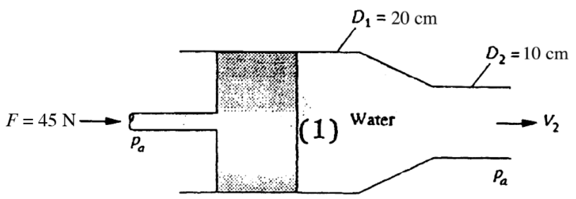



**Fig. P3.144**

$$\sum F_x = F + p_a A_1 - p_1 A_1, \quad \text{or:} \quad p_1 - p_a = \frac{45\text{ N}}{(\pi/4)(0.2)^2} \approx 1432.4\text{ N/m}^2$$

Now apply continuity and Bernoulli from 1 to 2:

$$V_1 A_1 = V_2 A_2, \quad \text{or} \quad V_1 = \frac{1}{4}V_2; \quad \frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_a}{\rho} + \frac{V_2^2}{2}$$

$$\text{Introduce } p_1 - p_a \text{ and substitute for } V_1 \text{ to obtain } \quad V_2^2 = \frac{2(1432.4)}{998(1 - 1/16)},$$

$$V_2 = \mathbf{1.75}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}$$

If we reduce section 2 to a pinhole, $V_2$ will drop off slowly until $V_1$ vanishes:

$$\text{Severely constricted section 2:} \quad V_2 = \sqrt{\frac{2(1432.4)}{998(1-0)}} \approx \mathbf{1.69}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}$$

---

**P3.145** For the sluice gate flow of Example 3.10, use Bernoulli's equation, along the surface, to estimate the flow rate $Q$ as a function of the two water depths. Assume constant width $b$.

**Solution**: Along surface 1, down the inside of the gate, and along surface 2 is a streamline of the flow. Therefore Bernoulli applies if we neglect friction, and we can use continuity also:

$$\text{Continuity:}\quad Q_1 = V_1 h_1 b = Q_2 = V_2 h_2 b\ ,\ \ or: V_1 = V_2 (h_2 / h_1)$$

$$\text{Bernoulli:}\ \ \frac{p_1}{\rho} + \frac{V_1^2}{2g} + h_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + h_2$$

$$\text{But } p_1 = p_2 = p_{atmosphere},\ \ \text{thus } V_2^2 - V_1^2 = 2g(h_1 - h_2)$$

Eliminate $V_1$ from continuity and solve for $V_2$, hence solve for <u>$Q$:</u>

$$V_2 = \sqrt{\frac{2g(h_1 - h_2)}{1-(h_2/h_1)^2}}\ ;\ \ Q = b h_2 \sqrt{\frac{2g(h_1 - h_2)}{1-(h_2/h_1)^2}} = b h_2 h_1 \sqrt{\frac{2g}{h_1 + h_2}} \quad Ans$$

---

**P3.146** In the spillway flow of Fig. P3.146, the flow is assumed uniform and hydrostatic at sections 1 and 2. If losses are neglected, compute (a) $V_2$ and (b) the force per unit width of the water on the spillway.

**Solution:** For mass conservation,

$$V_2 = V_1 h_1 / h_2 = \frac{5.0}{0.7} V_1 = 7.14 V_1$$

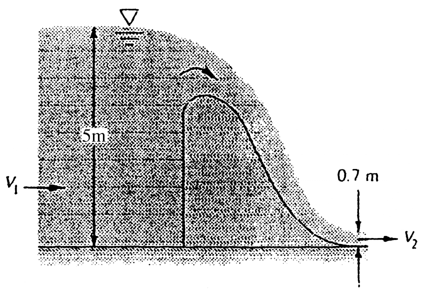



**Fig. P3.146**

(a) Now apply Bernoulli from 1 to 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_1 \approx \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_2;\quad \text{or:}\quad 0 + \frac{V_1^2}{2g} + 5.0 \approx 0 + \frac{(7.14V_1)^2}{2g} + 0.7$$

$$\text{Solve for } V_1^2 = \frac{2(9.81)(5.0 - 0.7)}{[(7.14)^2 - 1]},\ \text{ or } V_1 = \mathbf{1.30}\ \frac{\mathbf{m}}{\mathbf{s}},\quad V_2 = 7.14V_1 = \mathbf{9.28}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\text{ (a)}$$

(b) To find the force on the spillway (F ←), put a CV around sections 1 and 2 to obtain

$$\sum F_x = -F + \frac{\gamma}{2} h_1^2 - \frac{\gamma}{2} h_2^2 = \dot{m}(V_2 - V_1),\quad \text{or, using the given data,}$$

$$F = \frac{1}{2}(9790)[(5.0)^2 - (0.7)^2] - 998[(1.30)(5.0)](9.28 - 1.30) \approx \mathbf{68300}\ \frac{\mathbf{N}}{\mathbf{m}} \quad \textit{Ans.}\text{ (b)}$$

---

**P3.147** For the water-channel flow of.Fig. P3.147, $h_1$ = 1.5 m, $H$ = 4 m, and $V_1$ = 3 m/s. Neglecting losses and assuming uniform flow at sections 1 and 2, find the downstream depth $h_2$, and show that *two* realistic solutions are possible.

**Solution:** Combine continuity and Bernoulli between 1 and 2:

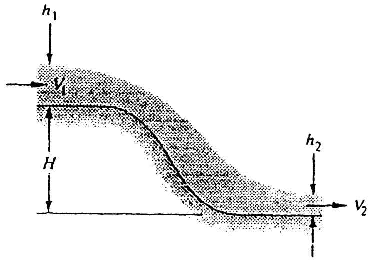



**Fig. P3.147**

$$V_2 = V_1\frac{h_1}{h_2} = \frac{3(1.5)}{h_2}; \quad \frac{V_1^2}{2g} + h_1 + H \approx \frac{V_2^2}{2g} + h_2 = \frac{V_1^2}{2(9.81)} + 1.5 + 4 \approx \frac{(4.5/h_2)^2}{2(9.81)} + h_2$$

Combine into a cubic equation: $h_2^3 - 5.959\, h_2^2 + 1.032 = 0$. The three roots are:

$h_2 = -0.403$ m (impossible); $h_2 = +$**5.93 m** (subcritical);

$h_2 = +$**0.432 m** (supercritical) *Ans.*

---

**P3.148** For the water channel flow of Fig. P3.148, $h_1$ = 15 cm, H = 70 cm, and $V_1$ = 5 m/s. Neglecting losses and assuming uniform flow at sections 1 and 2, find the downstream depth $h_2$. Show that *two* realistic solutions are possible.

**Solution:** The analysis is quite similar to Prob. 3.185 - continuity + Bernoulli:

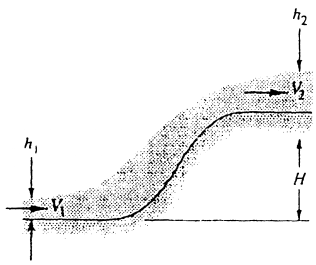



**Fig. P3.148**

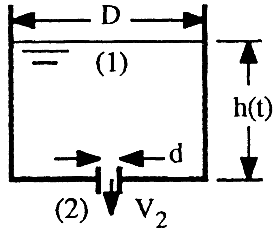
$$V_2 = V_1\frac{h_1}{h_2} = \frac{5(0.15)}{h_2}; \quad \frac{V_1^2}{2g} + h_1 = \frac{V_2^2}{2g} + h_2 + H = \frac{V_1^2}{2(9.81)} + 0.15 = \frac{(0.75/h_2)^2}{2(9.81)} + h_2 + 0.7$$

Combine into a cubic equation: $h_2^3 - 0.724\, h_2^2 + 0.0287 = 0$. The three roots are:

$h_2 = -0.178$ m (impossible); $h_2 = +$**0.658 m** (subcritical);

$h_2 = +$**0.245 m** (supercritical) *Ans.*

---




**P3.149** A cylindrical tank of diameter $D$ contains liquid to an initial height $h_o$. At time $t = 0$ a small stopper of diameter $d$ is removed from the bottom. Using Bernoulli's equation with no losses, derive (a) a differential equation for the free-surface height $h(t)$ during draining and (b) an expression for the time $t_o$ to drain the entire tank.

**Solution:** Write continuity and the unsteady Bernoulli relation from 1 to 2:

$$\int_1^2 \frac{\partial V}{\partial t}ds + \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1; \quad \text{Continuity: } V_2 = V_1\frac{A_1}{A_2} = V_1\left(\frac{D}{d}\right)^2$$

The integral term $\int \frac{\partial V}{\partial t}ds \approx \frac{dV_1}{dt}h$ is very small and will be neglected, and $p_1 = p_2$.. Then

$$V_1 \approx \left[\frac{2gh}{\alpha - 1}\right]^{1/2}, \text{ where } \alpha = (D/d)^4\text{; but also } V_1 = -\frac{dh}{dt}, \text{ separate and integrate:}$$

$$\int_{h_o}^{h} \frac{dh}{h^{1/2}} = -\left[\frac{2g}{\alpha - 1}\right]^{1/2}\int_o^t dt, \quad \text{or: } \mathbf{h} = \left[\mathbf{h_o^{1/2}} - \left\{\frac{\mathbf{g}}{\mathbf{2(\alpha - 1)}}\right\}^{1/2}\mathbf{t}\right]^2, \quad \alpha = \left(\frac{D}{d}\right)^4 \quad \textit{Ans.}\text{(a)}$$

(b) the tank is empty when [] = 0 in (a) above, or $\mathbf{t_o = [2(\alpha - 1)g/h_o]^{1/2}}$. *Ans.* (b)

---

**P3.150** The large tank of incompressible liquid in Fig. P3.150 is at rest when, at $t = 0$, the valve is opened to the atmosphere. Assuming $h \approx$ constant (negligible velocities and accelerations in the tank), use the unsteady frictionless Bernoulli equation to derive and solve a differential equation for $V(t)$ in the pipe.

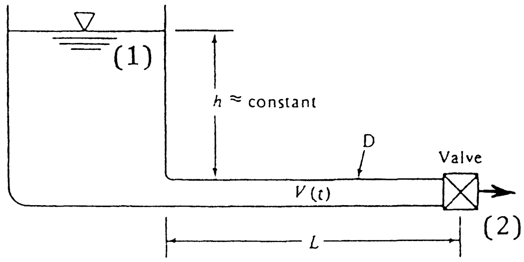



**Fig. P3.150**

**Solution:** Write unsteady Bernoulli from 1 to 2:

$$\int_1^2 \frac{\partial V}{\partial t}ds + \frac{V_2^2}{2} + gz_2 \approx \frac{V_1^2}{2} + gz_1, \quad \text{where } p_1 = p_2,\ V_1 \approx 0,\ z_2 \approx 0, \text{ and } z_1 = h = \text{const}$$

The integral approximately equals $\frac{dV}{dt}L$, so the diff. eqn. is $2L\frac{dV}{dt} + V^2 = 2gh$

This first-order ordinary differential equation has an exact solution for V = 0 at t = 0:

$$\mathbf{V = V_{final}\tanh\left(\frac{V_{final}t}{2L}\right)}, \quad \text{where } \mathbf{V_{final}} = \sqrt{\mathbf{2gh}} \quad \textit{Ans.}$$

---

**P3.151** A 6-cm-diameter free water jet, in sea-level air at 101,350 Pa, strikes perpendicular to a flat wall. If the water stagnation pressure at the wall is 213,600 Pa, estimate the force required to support the wall against jet momentum.

**Solution**: For standard conditions, take $\rho_{water} = 998\ kg/m^3$. The moving free jet must be at sea-level pressure; hence the stagnation pressure value allows us to compute the jet velocity:

$$p_o = 213{,}600\,Pa = p_{jet} + \frac{\rho}{2}V_{jet}^2 = 101{,}350 + \frac{998}{2}V_{jet}^2$$

$$Solve\ for\ V_{jet} = \sqrt{225\ m^2/s^2} = 15\ \frac{m}{s}$$

From many other momentum problems of this jet-striking-wall type, we found that

$$F_{support} = \rho A_{jet} V_{jet}^2 = (998)[\frac{\pi}{4}(0.06\,m)^2](15\frac{m}{s})^2 \approx \mathbf{635\ N} \quad Ans.$$

---

**P3.152** The incompressible-flow form of Bernoulli's relation, Eq. (3.54), is accurate only for Mach numbers less than about 0.3. At higher speeds, variable density must be accounted for. The most common assumption for compressible fluids is *isentropic flow of an ideal gas*, or $p = C\rho^k$, where $k = c_p/c_v$. Substitute this relation into Eq. (3.52), integrate, and eliminate the constant $C$. Compare your compressible result with Eq. (3.54) and comment.

**Solution:** We are to integrate the differential Bernoulli relation with variable density:

$$p = C\rho^k, \quad \text{so} \quad dp = kC\rho^{k-1}d\rho, \quad k = c_p/c_v$$

Substitute this into the Bernoulli relation:

$$\frac{dp}{\rho} + V\,dV + g\,dz = \frac{kC\rho^{k-1}d\rho}{\rho} + V\,dV + g\,dz = 0$$

$$\text{Integrate:} \quad \int kC\rho^{k-2}d\rho + \int V\,dV + \int g\,dz = \int 0 = \text{constant}$$

The first integral equals $kC\rho^{k-1}/(k-1) = kp/[\rho(k-1)]$ from the isentropic relation. Thus the compressible isentropic Bernoulli relation can be written in the form

$$\mathbf{\frac{kp}{(k-1)\rho} + \frac{V^2}{2} + gz = constant} \quad Ans.$$

It looks quite different from the incompressible relation, which only has "$p/\rho$." It becomes more clear when we make the ideal-gas substitution $p/\rho = RT$ and $c_p = kR/(k-1)$. Then we obtain the equivalent of the adiabatic, no-shaft-work energy equation:

$$\mathbf{c_pT + \frac{V^2}{2} + gz = constant} \quad Ans.$$

---

**P3.153** The pump in Fig. P3.153 draws gasoline at 20°C from a reservoir. Pumps are in big trouble if the liquid vaporizes (cavitates) before it enters the pump.
(a) Neglecting losses and assuming a flow rate of 250 L/min, find the limitations on ($x$, $y$, $z$) for avoiding cavitation. (b) If pipe-friction losses are included, what additional limitations might be important?

**Solution:** (a) From Table A.3, $\rho = 680$ kg/ m$^3$ and $p_v = 5.51E+4$.

$$z_2 - z_1 = y + z = \frac{p_1 - p_2}{\rho g} = \frac{(p_a + \rho g y) - p_v}{\rho g}$$

$$y + z = \frac{(100{,}000 - 55{,}100)}{(680)(9.81)} + y \quad z = 6.73\ m$$

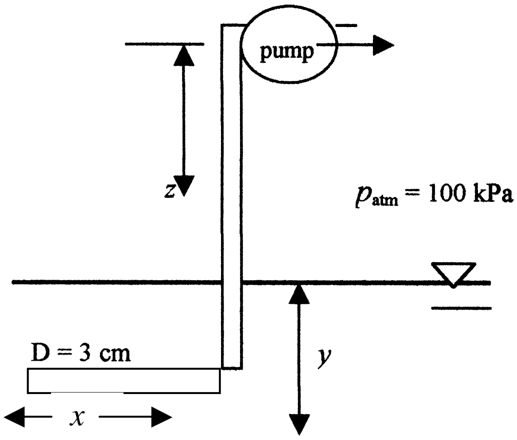



**Fig. P3.153**

Thus make length z appreciably less than 6.73m (25% less), or **z < 5 m**. *Ans.* (a)

(b) **Total pipe length ($x + y + z$) restricted by friction losses**. *Ans.* (b)

---

**P3.154** For the system of Prob. 3.153, let the pump exhaust gasoline at 240 L/min to the atmosphere through a 3-cm-diameter opening, with no cavitation, when $x$ = 3 m, $y$ = 2.5 m, and $z$ = 2 m. If the friction head loss is $h_{loss} \approx 3.7(V^2/2g)$, where $V$ is the average velocity in the pipe, estimate the horsepower required to be delivered by the pump.

**Solution:** Since power is a function of $h_{pump}$, Bernoulli is required. Thus calculate the velocity,

$$V = \frac{Q}{A} = \frac{\left(240\frac{\text{L}}{\text{min}}\cdot\frac{1}{60}\cdot\frac{\text{min}}{\text{s}}\cdot 10^{-3}\cdot\frac{\text{m}^3}{\text{L}}\right)}{\frac{\pi}{4}(0.03^2)} = 5.8 \text{ m/s}$$

The pump head may then be found,

$$\frac{p_1}{\gamma}+z_1=\frac{p_2}{\gamma}+z_2+h_f-h_p+\frac{V_j^2}{2g}$$

$$\frac{100{,}000+(680)(9.81)(2.5)}{(680)(9.81)}-2.5=\frac{100{,}000}{(680)(9.81)}+2+\frac{3.7(5.8^2)}{2(9.81)}-h_p+\frac{(5.8^2)}{2(9.81)}$$

$$h_p=10.05\ m$$

$$P=\gamma Q h_p=(680)(9.81)(0.0041)(10.05)\quad \mathbf{P=275\ W=0.37\ hp}\quad \textit{Ans.}$$

---

**P3.155** By neglecting friction, (*a*) use the Bernoulli equation between surfaces 1 and 2 to estimate the volume flow through the orifice, whose diameter is 3 cm. (*b*) Why is the result to part (*a*) absurd? (*c*) Suggest a way to resolve this paradox and find the true flow rate.

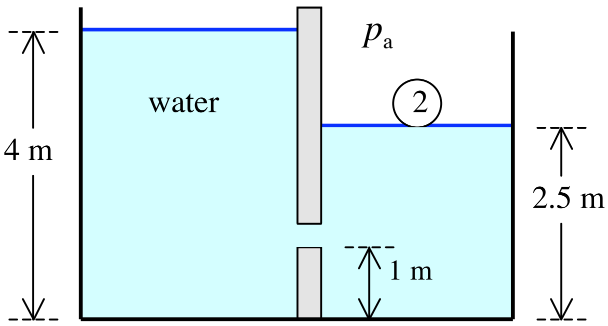



Fig. P3.155

**Solution**: (*a*) The incompressible Bernoulli equation between surfaces 1 and 2 yields

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_a}{9790}+\frac{0^2}{2(9..81)}+4=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_a}{9790}+\frac{0^2}{2(9..81)}+2.5$$

This gives the absurd result **4 m = 2.5 m ?** *Ans.*(*a*)

(*b*) The absurd result arises because the flow is *not frictionless*. The jet of water passing through the orifice loses all of its kinetic energy by viscous dissipation in the right-side tank. (*c*) As we shall see in Chap. 6, we add an orifice-exit *head loss* equal to the jet kinetic energy:

$$z_1-z_2=h_{f,jet}=\frac{V_{jet}^2}{2g},\quad\text{or}\quad V_{jet}=\sqrt{2(9.81\frac{m}{s^2})(4m-2.5m)}$$

$$\text{Solve for}\quad V_{jet}=5.42\frac{m}{s},\quad Q=AV=\frac{\pi}{4}(0.03m)^2(5.42\frac{m}{s})\approx \mathbf{0.0038\ \frac{m^3}{2}}\quad \textit{Ans.}(c)$$

---

**P3.156** The horizontal lawn sprinkler in Fig. P3.156 has a water flow rate of 15 L/min introduced vertically through the center. Estimate (a) the retarding torque required to keep the arms from rotating and (b) the rotation rate (r/min) if there is no retarding torque.

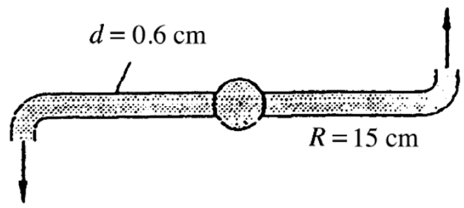



**Fig. P3.156**

**Solution:** The flow rate is 15 L/min = $2.5 \times 10^{-4} \text{m}^3/\text{s}$, and $\rho = 998$ kg/m$^3$. The velocity issuing from each arm is $\text{Vo} = \left(2.5 \times 10^{-4}/2\right)/\left[(\pi/4)\left(6 \times 10^{-3}\right)^2\right] \approx 4.42$ m/s. Then:

(a) From Example 3.15, $\omega = \dfrac{V_o}{R} - \dfrac{T_o}{\rho Q R^2}$ and, if there is no motion ($\omega = 0$),

$$T_o = \rho Q R V_o = (998)(2.5 \times 10^{-4})(0.15)(4.42) \approx \mathbf{0.165\ N\text{-}m} \quad \textit{Ans.}\ \text{(a)}$$

(b) If $T_o = 0$, then $\omega_{\text{no friction}} = V_o/R = \dfrac{4.42 \text{ m/s}}{0.15 \text{ m}} = 29.5 \dfrac{\text{rad}}{\text{s}} \approx \mathbf{281.7} \dfrac{\mathbf{rev}}{\mathbf{min}}$ *Ans.* (b)

---

**P3.157** In Prob. 3.65 find the torque caused around flange 1 if the center point of exit 2 is 1.2 m directly below the flange center.

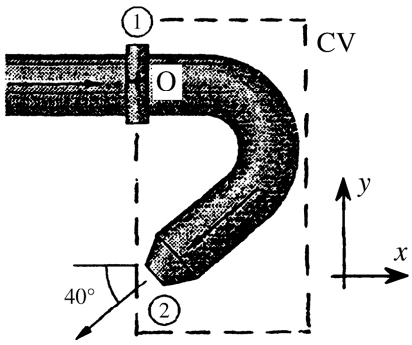



**Fig. P3.65**

**Solution:** The CV encloses the elbow and cuts through flange (1). Recall from Prob. 3.65 that $D_1 = 10$ cm, $D_2 = 3$ cm, weight flow = 150 N/s, whence $V_1 = 1.95$ m/s and $V_2 = 21.7$ m/s. Let "O" be in the center of flange (1). Then $\mathbf{r}_{O2} = -1.2\mathbf{j}$ and $\mathbf{r}_{O1} = 0$.

The pressure at (1) passes through O, thus causes no torque. The moment relation is

$$\sum M_O = \mathbf{T}_O = \dot{m}[(\mathbf{r}_{O2} \times \mathbf{V}_2) - (\mathbf{r}_{O1} \times \mathbf{V}_1)] = \left(\frac{150}{9.81}\ \frac{\text{kg}}{\text{s}}\right)[(-1.2\mathbf{j}) \times (-16.6\mathbf{i} - 13.9\mathbf{j})]$$

$$\text{or:} \quad \mathbf{T}_O = -305\mathbf{k}\ \text{N}\cdot\text{m} \quad \textit{Ans.}$$

---

**P3.158** The wye joint in Fig. P3.158 splits the pipe flow into equal amounts $Q/2$, which exit, as shown, a distance $R_o$ from the axis. Neglect gravity and friction. Find an expression for the torque $T$ about the $x$ axis required to keep the system rotating at angular velocity $\Omega$.

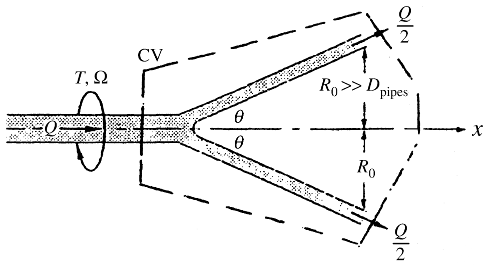



**Fig. P3.158**

**Solution:** Let the CV enclose the junction, cutting through the inlet pipe and thus exposing the required torque $T$. If $y$ is "up" in the figure, the absolute exit velocities are

$$\mathbf{V}_{upper} = V_o\cos\theta\,\mathbf{i} + V_o\sin\theta\,\mathbf{j} + R_o\Omega\,\mathbf{k};\quad \mathbf{V}_{lower} = V_o\cos\theta\,\mathbf{i} - V_o\sin\theta\,\mathbf{j} - R_o\Omega\,\mathbf{k}$$

where $V_o = Q/(2A)$ is the exit velocity relative to the pipe walls. Then the moments about the $x$ axis are related to angular momentum fluxes by

$$\sum\mathbf{M}_{axis} = T\mathbf{i} = (\rho Q/2)(R_o\mathbf{j})\times\mathbf{V}_{upper} + (\rho Q/2)(-R_o\mathbf{j})\times\mathbf{V}_{lower} - \rho Q(\mathbf{r}_{inlet}\mathbf{V}_{inlet})$$

$$= \frac{\rho Q}{2}\left(R_o^2\Omega\,\mathbf{i} - R_oV_o\Omega\,\mathbf{k}\right) + \frac{\rho Q}{2}\left(R_o^2\Omega\,\mathbf{i} + R_oV_o\Omega\,\mathbf{k}\right) - \rho Q(0)$$

Each arm contributes to the torque via relative velocity ($\Omega R_o$). Other terms with $V_o$ cancel.

$$\text{Final torque result:}\quad T = \rho Q R_o^2\Omega = \dot{\mathbf{m}}\mathbf{R}_o^2\,\boldsymbol{\Omega}\quad \textit{Ans.}$$

---

**P3.159** Modify Ex. 3.19 so that the arm starts up from rest and spins up to its final rotation speed. The moment of inertia of the arm about O is $I_o$. Neglect air drag. Find $d\omega/dt$ and integrate to determine $\omega$ (t), assuming $\omega = 0$ at t = 0.

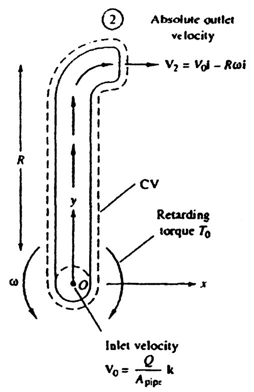



**Fig. 3.16** View from above of a single arm of a rotating lawn sprinkler.

**Solution:** The CV is shown. Apply clockwise moments:

$$\sum\mathbf{M}_o - \int(\mathbf{r}\times\mathbf{a}_{rel})\,dm = \int_{CS}(\mathbf{r}\times\mathbf{V})\,d\dot{m},$$

$$\text{or:}\quad -T_o - I_o\frac{d\omega}{dt} = \rho Q(R^2\omega - RV_o),$$

$$\text{or:}\quad \frac{d\omega}{dt} + \frac{\rho Q R^2}{I_o}\omega = \frac{\rho Q R V_o - T_o}{I_o}$$

Integrate this first-order linear differential equation, with $\omega = 0$ at t = 0. The result is:

$$\omega = \left(\frac{\mathbf{V_o}}{\mathbf{R}} - \frac{\mathbf{T_o}}{\rho\mathbf{QR}^2}\right)\left[\mathbf{1} - \mathbf{e}^{-\rho\mathbf{QR}^2\mathbf{t}/\mathbf{I_o}}\right]\quad \textit{Ans.}$$

---

**P3.160** The 3-arm lawn sprinkler of Fig. P3.160 receives 20°C water through the center at 2.7 $m^3$/hr. If collar friction is neglected, what is the steady rotation rate in rev/min for (a) $\theta = 0°$; (b) $\theta = 40°$?

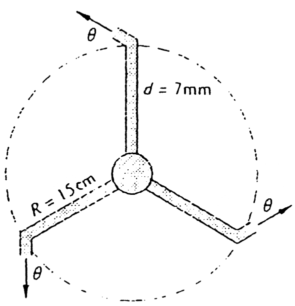



Fig. P3.160

**Solution:** The velocity exiting each arm is

$$V_o = \frac{Q/3}{(\pi/4)d^2} = \frac{2.7/[(3600)(3)]}{(\pi/4)(0.007)^2} = 6.50\ \frac{m}{s}$$

With negligible air drag and bearing friction, the steady rotation rate (Example 3.15) is

$$\omega_{final} = \frac{V_o\cos\theta}{R} \quad \text{(a) } \theta = 0°: \quad \omega = \frac{(6.50)\cos 0°}{0.15\ m} = 43.3\ \frac{rad}{s} = \mathbf{414}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) } \theta = 40°: \quad \omega = \omega_o\cos\theta = (414)\cos 40° = \mathbf{317}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans.}\text{ (b)}$$

---

**P3.161** Water at 20°C flows at 114 L/min through the 2-cm-diameter double pipe bend of Fig. P3.161. The pressures are $p_1$ = 207 kPa and $p_2$ = 165 kPa. Compute the torque $T$ at point $B$ necessary to keep the pipe from rotating.

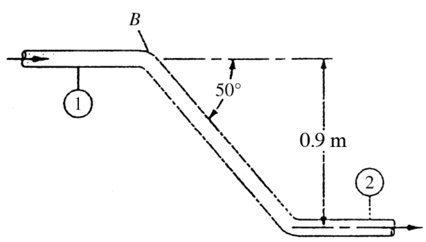



Fig. P3.161

**Solution:** This is similar to Example 3.13, of the text. The volume flow Q = 114 L/min = 1.9 $\times 10^{-3}$ $m^3$/s, and $\rho$ = 998 kg/$m^3$. Thus the mass flow $\rho$Q = 1.9 kg/s. The velocity in the pipe is

$$V_1 = V_2 = Q/A = \frac{1.9\times 10^{-3}}{(\pi/4)(2\times 10^{-2})^2} = 6.05\ \frac{m}{s}$$

If we take torques about point B, then the distance "$h_1$," from p. 143, = 0, and $h_2$ = 0.9 m. The final torque at point B, from "Ans. (a)" on p. 143 of the text, is

$$T_B = h_2(p_2A_2 + \dot{m}V_2) = (0.9\ m)\left[(165000)\frac{\pi}{4}\left(2\times 10^{-2}\right)^2 + (1.9)(6.05)\right] \approx \mathbf{57\ N\cdot m} \quad \textit{Ans.}$$

---

**P3.162** The centrifugal pump of Fig. P3.162 has a flow rate $Q$ and exits the impeller at an angle $\theta_2$ relative to the blades, as shown. The fluid enters axially at section 1. Assuming incompressible flow at shaft angular velocity $\omega$, derive a formula for the power $P$ required to drive the impeller.

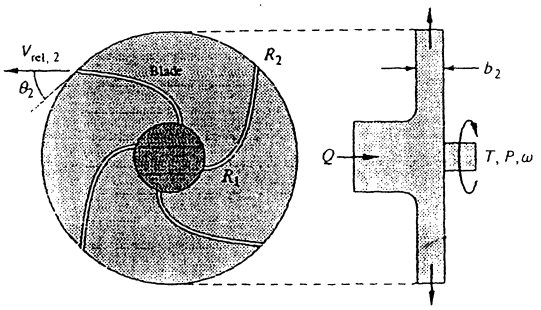
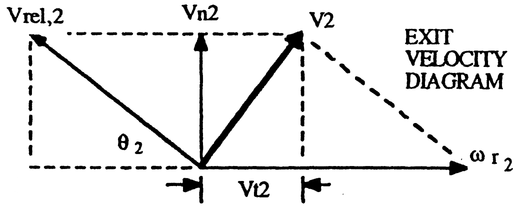



**Fig. P3.162**

**Solution:** Relative to the blade, the fluid exits at velocity $V_{rel,2}$ tangent to the blade, as shown in Fig. P3.123. But the Euler turbine formula, Ans. (a) from Example 3.14 of the text,




$$\text{Torque } T = \rho Q(r_2V_{t2} - r_1V_{t1})$$
$$\approx \rho Q r_2 V_{t2} \text{ (assuming } V_{t1} \approx 0)$$

involves the *absolute fluid velocity tangential to the blade circle* (see Fig. 3.13). To derive this velocity we need the "velocity diagram" shown above, where absolute exit velocity $V_2$ is found by adding blade tip rotation speed $\omega r_2$ to $V_{rel,2}$. With trigonometry,

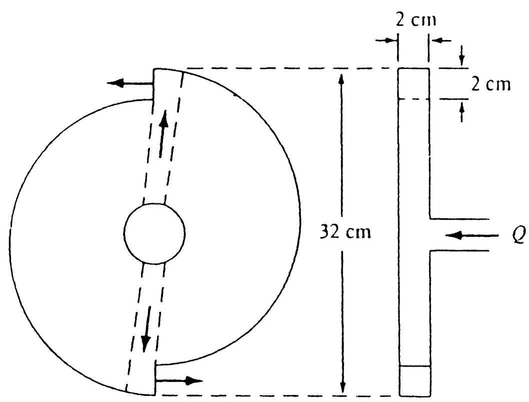
$$V_{t2} = r_2\omega - V_{n2}\cot\theta_2, \quad \text{where } V_{n2} = Q/A_{exit} = \frac{Q}{2\pi r_2 b_2} \text{ is the normal velocity}$$

With torque T known, the power required is $P = T\omega$. The final formula is:

$$\mathbf{P = \rho Q r_2 \omega \left[ r_2\omega - \left(\frac{Q}{2\pi r_2 b_2}\right)\cot\theta_2 \right]} \quad \textit{Ans.}$$

---

**P3.163** A simple turbomachine is con-structed from a disk with two internal ducts which exit tangentially through square holes, as in the figure. Water at 20°C enters the disk at the center, as shown. The disk must drive, at 250 rev/min, a small device whose retarding torque is 1.5 N·m. What is the proper mass flow of water, in kg/s?



**Solution:** This problem is a disguised version of the lawn-sprinkler arm in Example 3.15. For that problem, the steady rotating speed, with retarding torque $T_o$, was

$$\omega = \frac{V_o}{R} - \frac{T_o}{\rho Q R^2}, \quad \textit{where } V_o \textit{ is the exit velocity and } R \textit{ is the arm radius.}$$

Enter the given data, noting that $Q = 2V_o(\Delta L_{exit})^2$ is the total volume flow from the two arms:

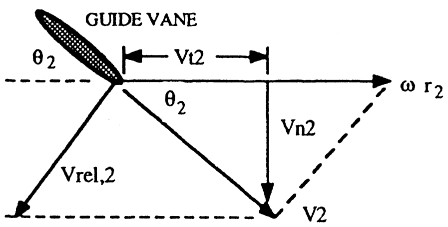
$$\omega = 250\left(\frac{2\pi}{60}\right)\frac{rad}{\text{s}} = \frac{V_o}{0.16\text{ m}} - \frac{1.5\ N\cdot\text{m}}{998(2V_o)(0.02\text{ m})^2(0.16\text{ m})^2}, \quad \textit{solve } V_o = \mathbf{6.11}\ \frac{\text{m}}{\text{s}}$$

The required mass flow is thus,

$$\dot{\mathbf{m}} = \rho Q = \left(998\ \frac{\text{kg}}{\text{m}^3}\right)\left(2\left(6.11\ \frac{\text{m}}{\text{s}}\right)\right)(0.02\text{ m})^2 = \mathbf{2.44}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}$$

---

**P3.164** Reverse the flow in Fig. P3.162, so that the system operates as a radial-inflow *turbine*. Assuming that the outflow into section 1 has no tangential velocity, derive an expression for the power *P* extracted by the turbine.




**Solution:** The Euler turbine formula, "Ans. (a)" from Example 3.14 of the text, is valid in reverse, that is, for a turbine with inflow at section 2 and outflow at section 1. The torque developed is

$$T_o = \rho Q(r_2V_{t2} - r_1V_{t1}) \approx \rho Q r_2 V_{t2} \quad \text{if} \quad V_{t1} \approx 0$$

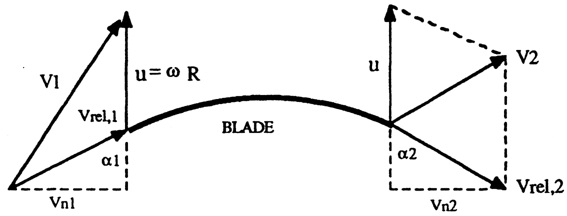
The velocity diagram is reversed, as shown in the figure. The fluid enters the turbine at angle $\theta_2$, which can only be ensured by a guide vane set at that angle. The absolute tangential velocity component is directly related to inlet normal velocity, giving the final result

$$V_{t2} = V_{n2}\cot\theta_2, \quad V_{n2} = \frac{Q}{2\pi r_2 b_2},$$

$$\text{thus} \quad P = \omega T_o = \boldsymbol{\rho Q \omega r_2\left(\frac{Q}{2\pi r_2 b_2}\right)\cot\theta_2} \quad \textit{Ans.}$$

---

**P3.165** Revisit the turbine cascade system of Prob. 3.83, and derive a formula for the power $P$ delivered, using the *angular-momentum theorem* of Eq. (3.59).




**Solution:** To use the angular momentum theorem, we need the inlet and outlet velocity diagrams, as in the figure. The Euler turbine formula becomes

$$T_o = \rho Q(r_1V_{t1} - r_2V_{t2}) \approx \rho QR(V_{t1} - V_{t2})$$

since the blades are at nearly constant radius R. From the velocity diagrams, we find

$$V_{t1} = u + V_{n1}\cot\alpha_1; \quad V_{t2} = u - V_{n2}\cot\alpha_2, \quad \text{where } V_{n1} = V_{n2} = V_1\cos\beta_1$$

The normal velocities are equal by virtue of mass conservation across the blades. Finally,

$$P = \rho Q\omega R(V_{t1} - V_{t2}) = \boldsymbol{\rho QuV_n(\cot\alpha_1 + \cot\alpha_2)} \quad \textit{Ans.}$$

---

**P3.166** A centrifugal pump delivers 15 $m^3$/min of water at 20°C with a shaft rotating at 1750 rpm. Neglect losses. If $r_1$ =15 cm, $r_2$ = 36 cm, $b_1 = b_2$ = 4.5 cm, $V_{t1}$ = 3 m/s, and $V_{t2}$ = 33 m/s, compute the absolute velocities (a) $V_1$ and (b) $V_2$, and (c) the ideal horsepower required.

**Solution:** First convert 15 $m^3$/min = 0.25 $m^3$/s and 1750 rpm = 183 rad/s. For water, take $\rho$ = 998 kg/$m^3$. The normal velocities are determined from mass conservation:

$$V_{n1} = \frac{Q}{2\pi r_1 b_1} = \frac{0.25}{2\pi(0.15)(0.045)} = 5.89\ \frac{m}{s}; \quad V_{n2} = \frac{Q}{2\pi r_2 b_2} = 2.46\ \frac{m}{s}$$

Then the desired absolute velocities are simply the resultants of $V_t$ and $V_n$:

$$V_1 = \left[(3)^2 + (5.89)^2\right]^{1/2} = \mathbf{6.61\ \frac{m}{s}} \qquad V_2 = \left[(33)^2 + (2.46)^2\right]^{1/2} = \mathbf{33.1\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a, b)}$$

The ideal power required is given by Euler's formula:

$$P = \rho Q\omega(r_2V_{t2} - r_1V_{t1}) = (998)(0.25)(183)[(0.36)(33.1) - (0.15)(3)]$$
$$= 523{,}520.4\ W \approx \mathbf{702\ hp} \quad \textit{Ans.}\text{ (c)}$$

---

**P3.167** The pipe bend of Fig. P3.167 has $D_1$ = 27 cm and $D_2$ = 13 cm. When water at 20°C flows through the pipe at 15.12 m$^3$/min, $p_1$ = 194 kPa (gage). Compute the torque required at point *B* to hold the bend stationary.

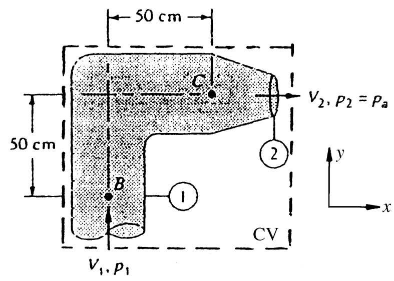


**Fig. P3.167**

**Solution:** First convert Q = 15.12 m$^3$/ min = 0.252 m$^3$/s. We need the exit velocity:

$$V_2 = Q/A_2 = \frac{0.252}{(\pi/4)(0.13)^2} = 19.0\ \frac{m}{s} \quad \text{Meanwhile, } V_1 = Q/A_1 = 4.4\ \frac{m}{s}$$

We don't really need V1, because it passes through B and has no angular momentum. The angular momentum theorem is then applied to point B:

$$\sum \mathbf{M}_B = \mathbf{T}_B + \mathbf{r_1} \times p_1A_1\mathbf{j} + \mathbf{r_2} \times p_2A_2(-\mathbf{i}) = \dot{m}(\mathbf{r_2} \times \mathbf{V_2} - \mathbf{r_1} \times \mathbf{V_1})$$

But r1 and p2 are zero,

$$\text{hence} \quad \mathbf{T}_B = \dot{m}(\mathbf{r_2} \times \mathbf{V_2}) = \rho Q[(0.5\mathbf{i} + 0.5\mathbf{j}) \times (19.0\mathbf{i})]$$

Thus, finally, $\mathbf{T}_B$ = (998)(0.252)(0.5)(19.0)(−**k**) ≈ **−2400 k** N·m (clockwise) *Ans.*

---

**P3.168** Extend Prob. 3.51 to the problem of computing the center of pressure *L* of the normal face $F_n$, as in Fig. P3.168. (At the center of pressure, no moments are required to hold the plate at rest.) Neglect friction. Express your result in terms of the sheet thickness $h_1$ and the angle $\theta$ between the plate and the oncoming jet 1.

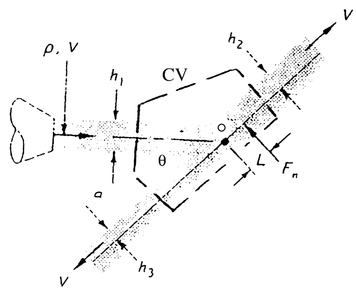


**Fig. P3.168**

**Solution:** Recall that in Prob. 3.51 of this Manual, we found h2 = (h1/2)(1 + cos$\theta$) and that h3 = (h1/2)(1 − cos$\theta$), where $\theta$ is the angle between the plate and the horizontal. . The force on the plate was Fn = $\rho$QVsin$\theta$. Take clockwise moments about O, where the jet strikes the plate, and use the angular momentum theorem:

$$\sum M_o = -F_nL = \dot{m}_2|\mathbf{r_{2O}} \times \mathbf{V_2}|_z + \dot{m}_3|\mathbf{r_{3O}} \times \mathbf{V_3}|_z - \dot{m}_1|\mathbf{r_{1O}} \times \mathbf{V_1}|_z$$
$$= \rho Vh_2(h_2V/2) + \rho Vh_3(-h_3V/2) - 0 = (1/2)\rho V^2\left(h_2^2 - h_3^2\right)$$

$$\text{Thus} \quad L = -\frac{(1/2)\rho V^2\left(h_2^2 - h_3^2\right)}{\rho V^2 h_1 \sin\theta} = -\frac{\left(h_2^2 - h_3^2\right)}{2h_1\sin\theta} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{h_1}\ \mathbf{cot}\boldsymbol{\theta} \quad \textit{Ans.}$$

The latter result follows from the (h1, h2, h3) relations in 3.51. The C.P. is <u>below</u> point O.

---

**P3.169** The waterwheel in Fig. P3.169 is being driven at 200 r/min by a 46-m/s jet of water at 20°C. The jet diameter is 6.5 cm. Assuming no losses, what is the horse-power developed by the wheel? For what speed Ω r/min will the horsepower developed be a maximum? Assume that there are many buckets on the waterwheel.

**Solution:** First convert Ω = 200 rpm = 20.9 rad/s. The bucket velocity = $V_b$ = ΩR = (20.9)(1.2) = 25.1 m/s. From Prob. 3.56 of this Manual, if there are many buckets, the entire (absolute) jet mass flow does the work:

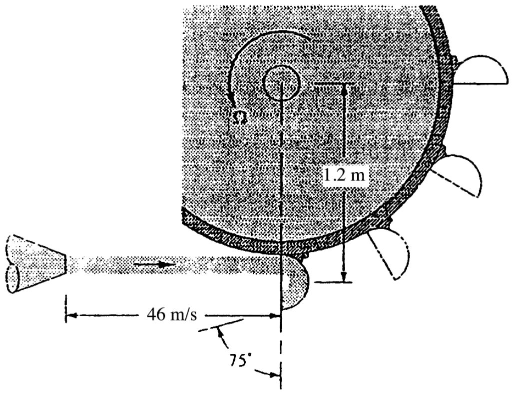



**Fig. P3.169**

$$P = \dot{m}_{jet} V_b (V_{jet} - V_b)(1 - \cos 165°) = \rho A_{jet} V_{jet} V_b (V_{jet} - V_b)(1.966)$$

$$= (998)\frac{\pi}{4}(0.065)^2(46)(25.1)(46 - 25.1)(1.966)$$

$$= 157{,}111.7 \text{ W} \approx \mathbf{210\ hp} \quad \textit{Ans.}$$

<u>Prob. 3.56</u>: Max. power is for $V_b = V_{jet}/2$ = 23 m/s, or Ω = 18.75 rad/s = **179 rpm** *Ans.*

---

**P3.170** A rotating dishwasher arm delivers at 60°C to six nozzles, as in Fig. P3.170. The total flow rate is 12 L/min. Each nozzle has a diameter of 0.5 cm. If the nozzle flows are equal and friction is neglected, estimate the steady rotation rate of the arm, in r/min.

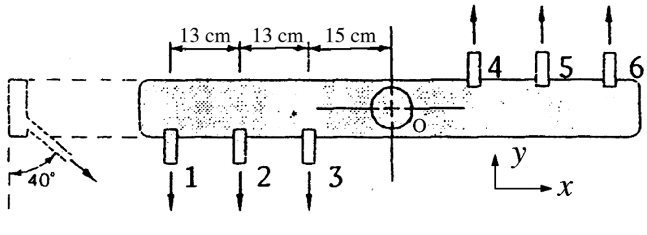



**Fig. P3.170**

**Solution:** First we need the mass flow and velocity from each hole "i," $i$ = 1 to 6:

$$V_i = \frac{Q_i}{A_i} = \frac{12 \times 10^{-3}/60}{\frac{\pi}{4}(0.005)^2} \approx 1.7 \ \frac{\text{m}}{\text{s}} \qquad \dot{m}_i = \frac{\rho Q}{6} = 998\left(\frac{12 \times 10^{-3}/60}{6}\right) = 0.0333 \ \frac{\text{kg}}{\text{s}}$$

Recall Example 3.15 from the text. For each hole, we need the absolute velocity, $V_i - \Omega r_i$. The angular momentum theorem is then applied to moments about point O:

$$\sum M_O = T_O = \sum \dot{m}_i(\mathbf{r}_{iO} \times \mathbf{V}_{i,abs}) - \dot{m}_{in}V_{in} = \sum \dot{m}_i r_i (V_i \cos 40° - \Omega r_i)$$

All the velocities and mass flows from each hole are equal. Then, if $T_O = 0$ (no friction),

$$\Omega = \frac{\sum \dot{m}_i r_i V_i \cos 40°}{\sum \dot{m}_i r_i^2} = V_i \cos 40° \frac{\sum r_i}{\sum r_i^2} = (1.7)(0.766)\frac{1.68}{0.538} = 4.07 \ \frac{\text{rad}}{\text{s}} = \mathbf{38.8\ rpm} \quad \textit{Ans.}$$

---

**P3.171** A liquid of density $\rho$ flows through a 90° bend as in Fig. P3.171 and issues vertically from a uniformly porous section of length L. Neglecting weight, find a result for the support torque M required at point O.

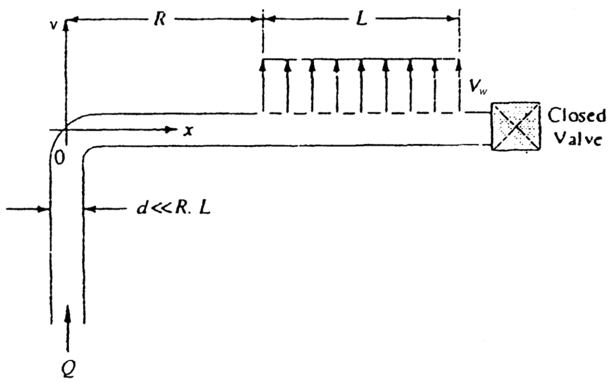



**Fig. P3.171**

**Solution:** Mass conservation requires

$$Q = \int_0^L V_w(\pi d)\,dx = V_w \pi d L, \quad \text{or:} \quad \frac{dQ}{dx} = \pi d V_w$$

Then the angular momentum theorem applied to moments about point O yields

$$\sum \mathbf{M}_O = \mathbf{T}_O = \int_{CS} (\mathbf{r}_O \times \mathbf{V})\,d\dot{m}_{out} = \mathbf{k}\int_0^L (R + x)V_w \rho\pi d V_w\,dx$$

$$= \frac{\mathbf{k}}{2}\rho\pi d V_w^2[(R+x)^2 - R^2]\Big|_0^L$$

Substitute $V_w \pi d L = Q$ and clean up to obtain $\mathbf{T}_O = \rho Q V_w\left(R + \frac{L}{2}\right)\mathbf{k}$ *Ans.*

---

**P3.172** Using the information given in P3.116, calculate the moment at the nozzle's base.

**Solution:** Consider angular momentum equation

$$\sum M_0 = \frac{\partial}{\partial t}\left[\int_{CV}(\vec{r}\times\vec{V})\rho d\mathrm{V}\right] + \int_{CS}(\vec{r}\times\vec{V})\rho(\vec{V}\cdot\vec{n})d\mathrm{A}$$

Under steady flow condition, we have

$$\sum M_0 = \int_{CS}(\vec{r}\times\vec{V})\rho(\vec{V}\cdot\vec{n})d\mathrm{A}$$

Since water flows through the nozzle's base, there is no moment from flow in the control volume (nozzle).

$$\sum M_0 = (\vec{r}\times\vec{V})\rho Q$$

$$\sum M_0 = -\rho Q \mathrm{V}_{\mathrm{nozzle}}(\ell_1 \sin\phi \sin\theta\ \vec{i} + \ell_1 \sin\phi \cos\theta\ \vec{j})$$

$$Q = \frac{\pi d^2}{d}\mathrm{V}_{\mathrm{nozzle}}$$

$$\therefore\quad \sum M_0 = -\frac{\pi\rho d^2}{4}\mathrm{V}^2_{\mathrm{nozzle}}(\ell_1 \sin\phi \sin\theta\ \vec{i} + \ell_1 \sin\phi \cos\theta\ \vec{j})$$

---

**P3.173** Given a steady isothermal flow of water at 20°C through the device in Fig. P3.173. Heat-transfer, gravity, and temperature effects are negligible. Known data are $D_1$ = 9 cm, $Q_1$ = 220 m$^3$/h, $p_1$ = 150 kPa, $D_2$ = 7 cm, $Q_2$ = 100 m$^3$/h, $p_2$ = 225 kPa, $D_3$ = 4 cm, and $p_3$ = 265 kPa. Compute the rate of shaft work done for this device and its direction.

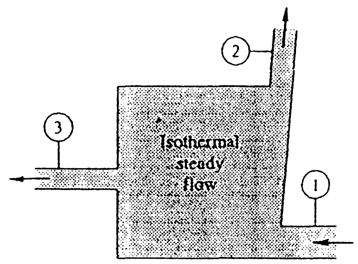



**Fig. P3.173**

**Solution:** For continuity, $Q_3 = Q_1 - Q_2$ = 120 m$^3$/hr. Establish the velocities at each port:

$$\mathrm{V}_1 = \frac{\mathrm{Q}_1}{\mathrm{A}_1} = \frac{220/3600}{\pi(0.045)^2} = 9.61\ \frac{\mathrm{m}}{\mathrm{s}};\quad \mathrm{V}_2 = \frac{100/3600}{\pi(0.035)^2} = 7.22\ \frac{\mathrm{m}}{\mathrm{s}};\quad \mathrm{V}_3 = \frac{120/3600}{\pi(0.02)^2} = 26.5\ \frac{\mathrm{m}}{\mathrm{s}}$$

With gravity and heat transfer and internal energy neglected, the energy equation becomes

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_3\left(\frac{p_3}{\rho_3} + \frac{V_3^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho_2} + \frac{V_2^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho_1} + \frac{V_1^2}{2}\right),$$

$$\text{or:}\quad -\dot{W}_s/\rho = \frac{100}{3600}\left[\frac{225000}{998}+\frac{(7.22)^2}{2}\right]+\frac{120}{3600}\left[\frac{265000}{998}+\frac{(26.5)^2}{2}\right]$$
$$+\frac{220}{3600}\left[\frac{150000}{998}+\frac{(9.61)^2}{2}\right]$$

Solve for the shaft work: $\dot{W}_s = 998(-6.99-20.56+12.00) \approx \mathbf{-15500\ W}$ *Ans.*
(negative denotes work done <u>on</u> the fluid)

---

**P3.174** A power plant on a river, as in Fig. P3.174, must eliminate 55 MW of waste heat to the river. The river conditions upstream are $Q_1$ = 2.5 m$^3$/s and $T_1$ = 18°C. The river is 45 m wide and 2.7 m deep. If heat losses to the atmosphere and ground are negligible, estimate the downstream river conditions ($Q_0$, $T_0$).

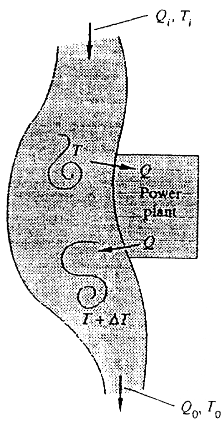



**Fig. P3.174**

**Solution:** For water, take $c_p \approx$ 4280 J/kg · °C. For an overall CV enclosing the entire sketch,

$$\dot{Q} = \dot{m}_{out}(c_p T_{out}) - \dot{m}_{in}(c_p T_{in}),$$

or: $55{,}000{,}000\ \text{W} \approx (998\times 2.5)[4280T_{out} - 4280(18)]$, solve for $T_{out} \approx \mathbf{23.15°C}$ *Ans.*

The power plant flow is "internal" to the CV, hence $Q_{out} = Q_{in} = \mathbf{2.5\ m^3/s}$. *Ans.*

---

**P3.175** For the conditions of Prob. 3.174, if the power plant is to heat the nearby river water by no more than 12°C, what should be the minimum flow rate $Q$, in m$^3$/s, through the plant heat exchanger? How will the value of $Q$ affect the downstream conditions ($Q_0$, $T_0$)?

**Solution:** Now let the CV <u>only enclose the power plant</u>, so that the flow going through the plant shows as an inlet and an outlet. The CV energy equation, with no work, gives

$$\dot{Q}_{plant} = \dot{m}_{out}c_pT_{out} - \dot{m}_{in}c_pT_{in} = (998)Q_{plant}(4280)(12°C)\quad \text{since } Q_{in} = Q_{out}$$

$$\text{Solve for}\quad Q_{plant} = \frac{55{,}000{,}000}{(998)(4280)(12)} \approx \mathbf{1.07\ m^3/s}\quad \textit{Ans.}$$

It's a lot of flow, but if the river water mixes well, the <u>downstream flow is still the same</u>.

---

**P3.176** Multnomah Falls in the Columbia River Gorge has a sheer drop of 165 m. Use the steady flow energy equation to estimate the water temperature rise, in °C, resulting.

**Solution:** For water, convert $c_p$ = 4200 J/(kg·K). Use the steady flow energy equation in the form of Eq. (3.70), with "1" upstream at the top of the falls:

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - q$$

Assume adiabatic flow, $q = 0$ (although evaporation might be important), and neglect the kinetic energies, which are much smaller than the potential energy change. Solve for

$$\Delta h = c_p\Delta T \approx g(z_1 - z_2), \quad \text{or:} \quad \Delta T = \frac{9.81(165)}{4200} \approx \mathbf{0.39°C} \quad \textit{Ans.}$$

---

**P3.177** When the pump in Fig. P3.177 draws 220 $m^3$/h of water at 20°C from the reservoir, the total friction head loss is 5 m. The flow discharges through a nozzle to the atmosphere Estimate the pump power in kW delivered to the water.

**Solution:** Let "1" be at the reservoir surface and "2" be at the nozzle exit, as shown. We need to know the exit velocity:

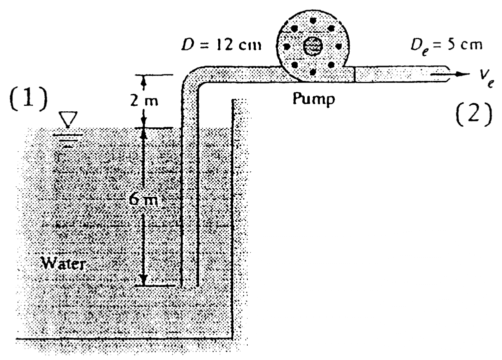



**Fig. P3.177**

$$V_2 = Q/A_2 = \frac{220/3600}{\pi(0.025)^2} = 31.12\ \frac{m}{s}, \quad \text{while } V_1 \approx 0 \text{ (reservoir surface)}$$

Now apply the steady flow energy equation from (1) to (2):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 0 = 0 + (31.12)^2/[2(9.81)] + 2 + 5 - h_p, \quad \text{solve for } h_p \approx 56.4 \text{ m.}$$

The pump power $P = \rho g Q h_p$ = (998)(9.81)(220/3600)(56.4)

= 33700 W = **33.7 kW** *Ans.*

---

**P3.178** A steam turbine operates steadily under the following conditions. At the inlet, $p = 2.5$ MPa, $T = 450°C$, and $V = 40$ m/s. At the outlet, $p = 22$ kPa, $T = 70°C$, and $V = 225$ m/s. (*a*) If we neglect elevation changes and heat transfer, how much work is delivered to the turbine blades, in kJ/kg? (*b*) If the mass flow is 10 kg/s, how much total power is delivered? (*c*) Is the steam wet as it leaves the exit?

**Solution**: This problem is made to order for the general steady-flow energy equation (3.70).

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - q + w_s + w_v$$

The viscous work $w_v$ is zero because the control volume has all no-slip surfaces. Look up the two enthalpies of steam, in the Steam Tables or with EES:

At 2.5 MPa and 450°C, $h_1 = 3351$ kJ/kg (or 3,351,000 J/kg or $m^2/s^2$)
At 22 kPa and 70°C, $h_2 = 2628$ kJ/kg (or 2,628,000 J/kg or $m^2/s^2$)

The energy equation thus becomes

$$3{,}351{,}000 + (1/2)(40m/s)^2 + 0 = 2{,}628{,}000 + (1/2)(225m/s)^2 - 0 + w_s + 0$$
$$\Delta h + \Delta(KE) = 723{,}100 - 24{,}500 = 698{,}600\ J/kg = +\mathbf{698.6}\ kJ/kg \quad Ans.(a)$$

(*b*) For the given mass flow of 10 kg/s of steam, the overall power extracted is

$$\text{Power} = (10\text{ kg/s})(698.6\text{ kJ/kg}) = 6986\text{ kJ/s} \approx \mathbf{7.0\ \ MW} \qquad Ans.(b)$$

(*c*) For the exit pressure of 22 kPa, EES states that the saturation temperature of steam is 62°C, less than the exit temperature of 70°C. The ***exit is just barely into the superheat region***. *Ans.*(*c*)

**P3.179** Consider a turbine extracting energy from a penstock in a dam, as in the figure. For turbulent flow (Chap. 6) the friction head loss is $h_f = CQ^2$, where the constant C depends upon penstock dimensions and water physical properties. Show that, for a given penstock and river flow Q, the maximum turbine power possible is $P_{max} = 2\rho gHQ/3$ and occurs when $Q = (H/3C)^{1/2}$.

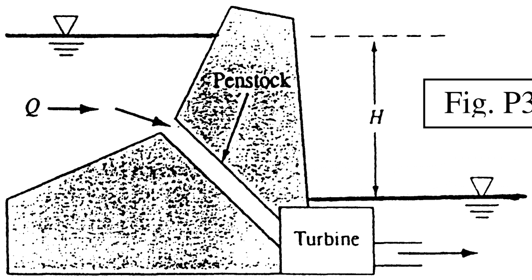



Fig. P3.179

**Solution:** Write the steady flow energy equation from point 1 on the upper surface to point 2 on the lower surface:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + H = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f + h_{turbine}$$

But $p_1 = p_2 = p_{atm}$ and $V_1 \approx V_2 \approx 0$. Thus the turbine head is given by

$$h_t = H - h_f = H - CQ^2,$$

$$or: \quad Power = P = \rho g Q h_t = \rho g Q H - \rho g C Q^3$$

Differentiate and set equal to zero for max power and appropriate flow rate:

$$\frac{dP}{dQ} = \rho g H - 3\rho g C Q^2 = 0 \quad if \quad \mathbf{Q = \sqrt{H/3C}} \quad Ans.$$

$$Insert\ Q\ in\ P\ to\ obtain\ \mathbf{P_{max}} = \boldsymbol{\rho g Q}\left(\frac{\mathbf{2H}}{\mathbf{3}}\right) \quad Ans.$$

---

**P3.180** The long pipe in Fig. 3.180 is filled with water at 20°C. When valve *A* is closed, $p_1 - p_2 = 75$ kPa. When the valve is open and water flows at 500 m$^3$/h, $p_1 - p_2 = 160$ kPa. What is the friction head loss between 1 and 2, in m, for the flowing condition?

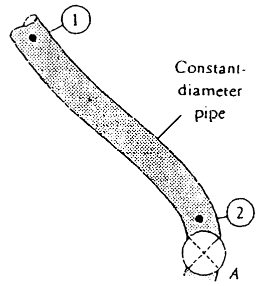



**Fig. P3.180**

**Solution:** With the valve closed, there is no velocity or friction loss:

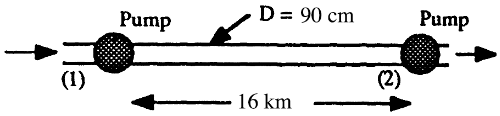
$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2, \quad \text{or:} \quad z_2 - z_1 = \frac{p_1 - p_2}{\rho g} = \frac{75000}{998(9.81)} \approx 7.66 \text{ m}$$

When the valve is open, the velocity is the same at (1) and (2), thus "d" is not needed:

$$\text{With flow:} \quad h_f = \frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2) = \frac{160000}{998(9.81)} + 0 - 7.66 \approx \mathbf{8.7\ m} \quad Ans.$$

---

**P3.181** A 90-cm-diameter pipeline carries oil (SG = 0.89) at 1 million barrels per day (bbl/day) (1 bbl = 159 L). The friction head loss is 13 m/1000 m of pipe. It is planned to place pumping stations every 16 km along the pipe. Estimate the horsepower which must be delivered to the oil by each pump.




**Solution:** Since ΔV and Δz are zero, the energy equation reduces to

$$h_f = \frac{\Delta p}{\rho g}, \quad \text{and} \quad h_f = 0.013\,\frac{\text{m-loss}}{\text{m-pipe}}(16\text{ km}) \approx 208\text{ m}$$

Convert the flow rate from 1E6 bbl/day to 159000 $m^3$/day to **1.84** $m^3$/s. Then the power is

$$P = Q\Delta p = \gamma Q h_f = (890)(9.81)(1.84)(208) = 3.34\times 10^6\,\text{W} \approx \mathbf{4479.2\ hp} \quad \textit{Ans.}$$

---

**P3.182** The *pump-turbine* system in Fig. P3.182 draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower to upper reservoirs to restore the situation. For a design flow rate of 57 $m^3$/ min in either direction, the friction head loss is 5.2 m. Estimate the power in kW (a) extracted by the turbine and (b) delivered by the pump.

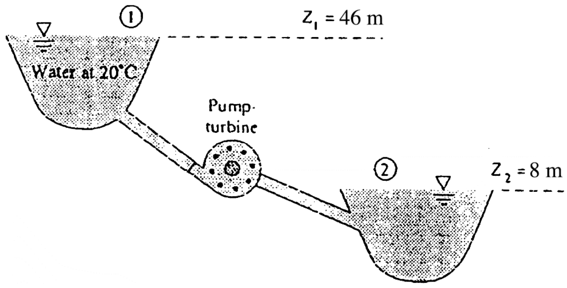



**Fig. P3.182**

**Solution:** (a) With the turbine, "1" is upstream:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f + h_t,$$

$$\text{or:}\quad 0 + 0 + 46 = 0 + 0 + 8 + 5.2 + h_t$$

Solve for $h_t$ = 32.8 m. Convert Q = 57 $m^3$/min = **0.95** $m^3$/s. Then the turbine power is

$$P = \gamma Q h_{turb} = (998)(9.81)(0.95)(32.8) = 3.05\times 10^5\,\text{W} \approx \mathbf{409\ hp} \quad \textit{Ans.}\text{ (a)}$$

(b) For pump operation, point "2" is upstream:

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_f - h_p,$$

$$\text{or:}\quad 0 + 0 + 8 = 0 + 0 + 46 + 5.2 - h_p$$

$$\text{Solve for}\quad h_p \approx 43.2\text{ m}$$

The pump power is Ppump = $\gamma Q h_p$ = (998)(9.81)(0.95)(43.2)

$$= 4.02\times 10^5\,\text{W} \approx \mathbf{539\ hp} \qquad \textit{Ans.}\text{ (b)}$$

---

**P3.183** Water at 20°C is delivered from one reservoir to another through a long 8-cm-diameter pipe. The lower reservoir has a surface elevation $z_2$ = 80 m. The friction loss in the pipe is correlated by the formula $h_{loss} \approx 17.5(V^2/2g)$, where $V$ is the average velocity in the pipe. If the steady flow rate through the pipe is 1.89 m³/min, estimate the surface elevation of the higher reservoir.

**Solution:** We may apply Bernoulli here. Convert 1.89 m³/min = 0.0315 m³/s.

$$h_f = \frac{17.5V^2}{2g} = z_1 - z_2$$

$$\frac{17.5}{2(9.81\text{ m/s}^2)}\left[\frac{0.0315m^3/s}{(\pi/4)(0.08m)^2}\right]^2 = z_1 - 80\text{ m}$$

$$z_1 \approx \mathbf{115\ m} \quad \textit{Ans.}$$

---

**P3.184** A fireboat draws seawater (SG = 1.025) from a submerged pipe and discharges it through a nozzle, as in Fig. P3.184. The total head loss is 2 m. If the pump efficiency is 75 percent, what horsepower motor is required to drive it?

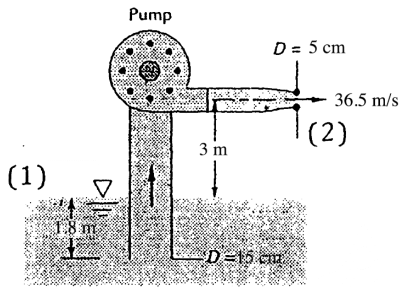



**Fig. P3.184**

**Solution:** For seawater, $\gamma$ = 1.025(1000)(9.81) = 10055.25 N. The energy equation becomes

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0 + 0 + 0 = 0 + \frac{(36.5)^2}{2(9.81)} + 3 + 2 - h_p$$

Solve for $h_p$ = 72.9 m. The flow rate is $Q = V_2A_2 = (36.5)(\pi/4)(0.05)^2 = 0.0717$ m³/s. Then

$$P_{pump} = \frac{\gamma Q h_p}{\text{efficiency}} = \frac{(10055.25)(0.0717)(72.9)}{0.75} = 70045.8\text{ W} \approx \mathbf{94\ hp} \quad \textit{Ans.}$$

---

**P3.185** A device for measuring liquid viscosity is shown in Fig. P3.185. With the parameters ($\rho$, $L$, $H$, $d$) known, the flow rate $Q$ is measured and the viscosity calculated, assuming a laminar-flow pipe loss from Chap. 6, $h_f = (32\mu LV)/(\rho g d^2)$. Heat transfer and all other losses are negligible. (*a*) Derive a formula for the viscosity $\mu$ of the fluid. (*b*) Calculate $\mu$ for the case $d = 2$ mm, $\rho = 800$ kg/m$^3$, $L = 95$ cm, $H = 30$ cm, and $Q = 760$ cm$^3$/h. (*c*) What is your guess of the fluid in part (*b*)? (*d*) Verify that the Reynolds number $Re_d$ is less than 2000 (laminar pipe flow).

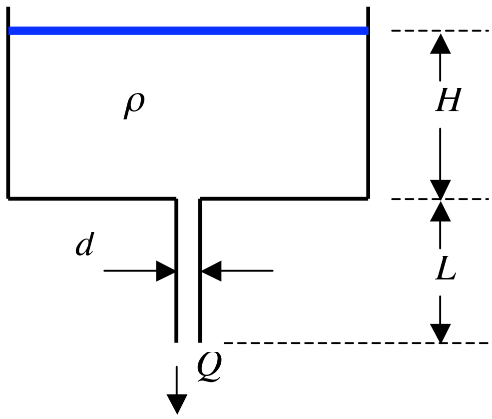



Fig. P3.185

**Solution**: Use energy Eq. (3.75) so we don't forget the *laminar* kinetic energy correction factor $\alpha$:

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_{turbine} - h_{pump} + h_f$$

$$0 + 0 + H + L = 0 + \frac{\alpha_2 V_2^2}{2g} + 0 + 0 - 0 + \frac{32\mu L V_2}{\rho g d^2}$$

Introduce $Q = (\pi/4)(d^2)(V_2)$, rewrite, and solve for $\mu$:

$$\mu = \frac{\pi \rho g d^4}{128 L Q}(H + L) - \frac{\alpha_2 \rho Q}{16\pi L} \qquad Ans.(a)$$

(*b*) Introduce the given data and compute the viscosity of the liquid. Convert $Q$ to (760cm$^3$/h)/(3600s/h)/(1E6cm$^3$/m$^3$) = 2.11E-7 m$^3$/s. Recall that $\alpha_2 = 2.0$. Then

$$\mu = \frac{\pi(800)(9.81)(0.002)^4}{128(0.95)(2.11E-7)}(0.30 + 0.95) - \frac{(2.0)(800)(2.11E-7)}{16\pi(0.95)}$$

$$= 0.0192 - 0.000007 = \mathbf{0.0192}\ \frac{kg}{m-s} \qquad Ans.(b)$$

(*c*) From Table A.4, both $\mu$ and $\rho$ seem to fit **kerosene** very well. *Ans.*(*c*)

(*d*) Check the diameter Reynolds number of this flow:

$$Re_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi \mu d} = \frac{4(800)(2.11E-7)}{\pi(0.0192)(0.002)} = \mathbf{5.6} \ll 2000 \text{ OK, laminar}$$

The flow is so slow (0.067 m/s) that the kinetic energy term is negligible.

**P3.186** The horizontal pump in Fig. P3.186 discharges 20°C water at 57 m$^3$/h. Neglecting losses, what power in kW is delivered to the water by the pump?

**Solution:** First we need to compute the velocities at sections (1) and (2):

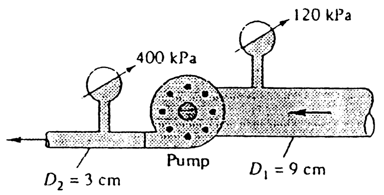



**Fig. P3.186**

$$V_1 = \frac{Q}{A_1} = \frac{57/3600}{\pi(0.045)^2} = 2.49\ \frac{m}{s};\quad V_2 = \frac{Q}{A_2} = \frac{57/3600}{\pi(0.015)^2} = 22.4\ \frac{m}{s}$$

Then apply the steady flow energy equation across the pump, neglecting losses:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or: } \frac{120000}{9790} + \frac{(2.49)^2}{2(9.81)} + 0 = \frac{400000}{9790} + \frac{(22.4)^2}{2(9.81)} + 0 + 0 - h_p,\quad \text{solve for } h_p \approx 53.85 \text{ m}$$

Then the pump power is $P_p = \gamma Q h_p = 9790\left(\frac{57}{3600}\right)(53.85) = 8350 \text{ W} = \mathbf{8.4\ kW}$ *Ans.*

---

**P3.187** Steam enters a horizontal turbine at 2.413 MPa absolute, 580°C, and 3.7 m/s and is discharged at 33.5 m/s and 25°C saturated conditions. The mass flow is 1.13 kg/s, and the heat losses are 16.28 kJ/kg of steam. If head losses are negligible, how much horsepower does the turbine develop?

**Solution:** We have to use the Steam Tables to find the enthalpies. State (2) is *saturated* vapor at 25°C, for which we find $h_2 \approx 2547.3$ kJ/kg. At state (1), 2.413 MPa and 580°C, we find $h_1 \approx 3641.1$ kJ/kg. The heat loss is 16.28 kJ/kg. The steady flow energy equation is best written on a per-mass basis:

$$q - w_s = h_2 + \frac{1}{2}V_2^2 - h_1 - \frac{1}{2}V_1^2,\quad \text{or:}$$

$$-16.28\times10^3 - w_s = 2547.3\times10^3 + (33.5)^2/2 - 3641.1\times10^3 - (3.7)^2/2,\quad \text{solve for } w_s \approx 1077 \text{ kJ/kg}$$

The result is positive because work is done by the fluid. The turbine power at 100% is

$$P_{turb} = \dot{m}w_s = (1.13)(1077\times10^3) = 1{,}216{,}971.26 \approx \mathbf{1630\ hp}\quad \textit{Ans.}$$

---

**P3.188** Water at 20°C is pumped at 5.7 $m^3$/ min from the lower to the upper reservoir, as in Fig. P3.188. Pipe friction losses are approximated by $h_f \approx 27V^2/(2g)$, where $V$ is the average velocity in the pipe. If the pump is 75 percent efficient, what horse-power is needed to drive it?

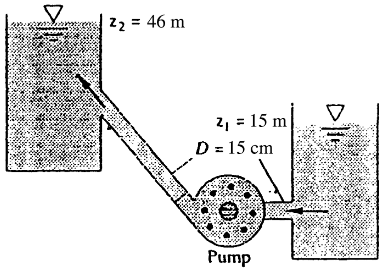



**Fig. P3.188**

**Solution:** First evaluate the average velocity in the pipe and the friction head loss:

$$Q = \frac{5.7}{60} = 0.095\frac{m^3}{s}, \quad \text{so } V = \frac{Q}{A} = \frac{0.095}{\pi(0.075)^2} = 5.38\ \frac{m}{s} \quad \text{and} \quad h_f = 27\frac{(5.38)^2}{2(9.81)} \approx \mathbf{39.77\ m}$$

Then apply the steady flow energy equation:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0 + 0 + 15 = 0 + 0 + 46 + 39.77 - h_p$$

$$\text{Thus } h_p = 70.77\text{ m}, \quad \text{so } P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(998)(9.81)(0.095)(70.77)}{0.75}$$

$$= 87{,}762.9\text{ W} \approx \mathbf{118\ hp} \quad \textit{Ans.}$$

---

**P3.189** A typical pump has a head which, for a given shaft rotation rate, varies with the flow rate, resulting in a *pump performance curve* as in Fig. P3.189. Suppose that this pump is 75 percent efficient and is used for the system in Prob. 3.188. Estimate (a) the flow rate, in $m^3$/min, and (b) the horsepower needed to drive the pump.

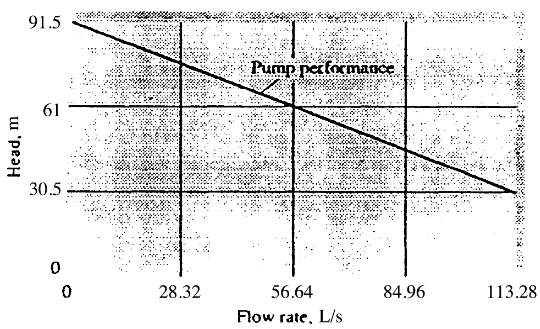



**Fig. P3.189**

**Solution:** This time <u>we do not know the flow rate</u>, but the pump head is $h_p \approx 91.5 - 0.538Q$, with Q in cubic meter per second. The energy equation directly above becomes,

$$0+0+15 = 0+0+46+(27)\frac{V^2}{2(9.81)} - (91.5-0.538Q), \quad \text{where } Q = V\frac{\pi}{4}(0.15\text{m})^2$$

This becomes the quadratic $V^2 + 6.93\times10^{-3}V - 43.96 = 0,$

solve for $V \approx 6.63$ m/s or $Q = 0.117\text{m}^3/\text{s}\,(7.02\text{m}^3/\text{min})$

$$\text{Then the power is } P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(9790.38)(0.117)[91.5-0.538(0.117)]}{0.75}$$

$$= 139{,}652\text{ W} \approx \mathbf{187\ hp} \quad \textit{Ans.}$$

---

**P3.190** The insulated tank in Fig. P3.190 is to be filled from a high-pressure air supply. Initial conditions in the tank are $T$ = 20°C and $p$ = 200 kPa. When the valve is opened, the initial mass flow rate into the tank is 0.013 kg/s. Assuming an ideal gas, estimate the initial rate of temperature rise of the air in the tank.

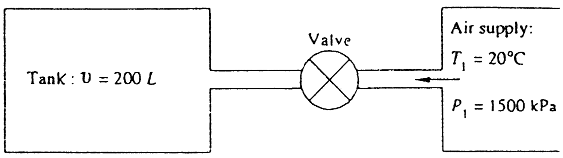



**Fig. P3.190**

**Solution:** For a CV surrounding the tank, with *unsteady* flow, the energy equation is

$$\frac{d}{dt}\left(\int e\rho d\upsilon\right) - \dot{m}_{in}\left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right) = \dot{Q} - \dot{W}_{shaft} = 0, \quad \text{neglect } V^2/2 \text{ and gz}$$

$$\text{Rewrite as} \quad \frac{d}{dt}(\rho\upsilon c_v T) \approx \dot{m}_{in}c_p T_{in} = \rho\upsilon c_v\frac{dT}{dt} + c_v T\upsilon\frac{d\rho}{dt}$$

where $\rho$ and T are the instantaneous conditions inside the tank. The CV mass flow gives

$$\frac{d}{dt}\left(\int \rho d\upsilon\right) - \dot{m}_{in} = 0, \quad \text{or:} \quad \upsilon\frac{d\rho}{dt} = \dot{m}_{in}$$

Combine these two to eliminate $\upsilon(d\rho/dt)$ and use the given data for air:

$$\left.\frac{dT}{dt}\right|_{tank} = \frac{\dot{m}(c_p - c_v)T}{\rho\upsilon c_v} = \frac{(0.013)(1005-718)(293)}{\left[\frac{200000}{287(293)}\right](0.2\text{ m}^3)(718)} \approx \mathbf{3.2}\ \frac{\mathbf{°C}}{\mathbf{s}} \quad \textit{Ans.}$$

---

**P3.191** The pump in Fig. P3.191 creates a 20°C water jet oriented to travel a maximum horizontal distance. System friction head losses are 6.5 m. The jet may be approximated by the trajectory of frictionless particles. What power must be delivered by the pump?

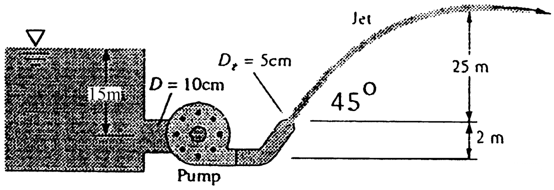



**Fig. P3.191**

**Solution:** For maximum travel, the jet must exit at $\theta = 45°$, and the exit velocity must be

$$V_2 \sin\theta = \sqrt{2g\Delta z_{max}} \quad \text{or:} \quad V_2 = \frac{[2(9.81)(25)]^{1/2}}{\sin 45°} \approx 31.32\ \frac{m}{s}$$

The steady flow energy equation for the piping system may then be evaluated:

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 15 = 0 + (31.32)^2/[2(9.81)] + 2 + 6.5 - h_p, \quad \text{solve for } h_p \approx 43.5 \text{ m}$$

$$\text{Then} \quad P_{pump} = \gamma Q h_p = (9790)\left[\frac{\pi}{4}(0.05)^2(31.32)\right](43.5) \approx \mathbf{26200\ W} \quad \textit{Ans.}$$

---

**P3.192** The large turbine in Fig. P3.192 diverts the river flow under a dam as shown. System friction losses are $h_f = 3.5V^2/(2g)$, where $V$ is the average velocity in the supply pipe. For what river flow rate in $m^3/s$ will the power extracted be 25 MW? Which of the *two* possible solutions has a better "conversion efficiency"?

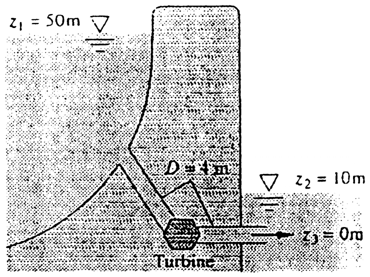



**Fig. P3.192**

**Solution:** The flow rate is the unknown, with the turbine power known:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f + h_{turb}, \quad \text{or:} \quad 0 + 0 + 50 = 0 + 0 + 10 + h_f + h_{turb}$$

$$\text{where} \quad h_f = 3.5V_{pipe}^2/(2g) \quad \text{and} \quad h_p = P_p/(\gamma Q) \quad \text{and} \quad V_{pipe} = \frac{Q}{(\pi/4)D_{pipe}^2}$$

Introduce the given numerical data (e.g. $D_{pipe}$ = 4 m, $P_{pump}$ = 25E6 W) and solve:

$$Q^3 - 35410Q + 2.261E6 = 0, \quad \text{with roots } Q = \mathbf{+76.5}, \mathbf{+137.9}, \text{ and } -214.4\ m^3/s$$

The *negative* Q is nonsense. The large Q (137.9$m^3$/s) gives large friction loss, $h_f \approx 21.5$ m. The smaller Q (76.5 $m^3$/s) gives $h_f \approx 6.6$ m, about right. Select $Q_{river} \approx$ **76.5 $m^3$/s**. *Ans.*

---

**P3.193** Kerosene at 20°C flows through the pump in Fig. P3.193 at 65 L/s. Head losses between 1 and 2 are 2.4 m, and the pump delivers 6 kW to the flow. What should the mercury-manometer reading $h$ m be?

**Solution:** First establish the two velocities:

$$V_1 = \frac{Q}{A_1} = \frac{65\times 10^{-3}\,\mathrm{m}^3/\mathrm{s}}{(\pi/4)(0.075)^2}$$

$$= 14.71\ \frac{\mathrm{m}}{\mathrm{s}};\quad V_2 = \frac{1}{4}V_1 = 3.68\ \frac{\mathrm{m}}{\mathrm{s}}$$

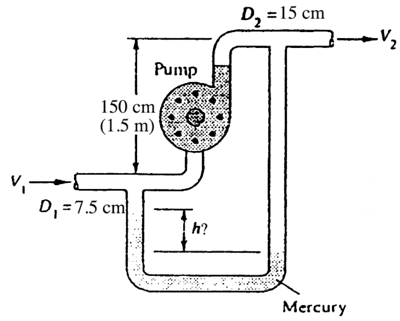



**Fig. P3.193**

For kerosene take $\rho$ = 804 kg/m$^3$ or $\gamma_k$ = (804)(9.81) = 7887.24 N. For mercury take $\gamma_m$ = (13600)(9.81) = 133,416 N. Then apply a manometer analysis to determine the pressure difference between points 1 and 2:

$$p_2 - p_1 = (\gamma_m - \gamma_k)h - \gamma_k\Delta z = (133416 - 7887.24)h - (7887.24)(1.5)$$

$$= 125{,}528.76h - 11{,}830.86\ \frac{\mathrm{N}}{\mathrm{m}^2}$$

Now apply the steady flow energy equation between points 1 and 2:

$$\frac{p_1}{\gamma_k} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_k} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,\quad \text{where } h_p = \frac{P}{\gamma_k Q} = \frac{6000}{(7887.24)(65\times 10^{-3})} = 11.70\ \mathrm{m}$$

$$\text{Thus:}\quad \frac{p_1}{7887.24} + \frac{(14.71)^2}{2(9.81)} + 0 = \frac{p_2}{7887.24} + \frac{(3.68)^2}{2(9.81)} + 1.5 + 2.4 - 11.7$$

Solve $p_2 - p_1 = 143.1$ kPa

Now, with the pressure difference known, apply the manometer result to find $h$:

$$p_2 - p_1 = 143{,}062.8 = 125{,}528.76\ h - 11{,}830.86,$$

$$\text{or:}\quad h = \frac{143{,}062.8 + 11{,}830.86}{125{,}528.76} = \mathbf{1.23\ m}\quad \textit{Ans.}$$

## FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE3.1 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the flow rate is 160 gal/min, what is the average velocity at section 1?

**(a) 2.6 m/s** (b) 0.81 m/s (c) 93 m/s (d) 23 m/s (e) 1.62 m/s

FE3.2 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the flow rate is 160 gal/min and friction is neglected, what is the gage pressure at section 1?

(a) 1.4 kPa (b) 32 kPa (c) 43 kPa **(d) 22 kPa** (e) 123 kPa

FE3.3 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the exit velocity is $V_2$ = 8 m/s and friction is neglected, what is the axial flange force required to keep the nozzle attached to pipe 1?

(a) 11 N **(b) 36 N** (c) 83 N (d) 123 N (e) 110 N

FE3.4 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the manometer fluid has a specific gravity of 1.6 and $h$ = 66 cm, with friction neglected, what is the average velocity at section 2?

(a) 4.55 m/s (b) 2.4 m/s **(c) 2.8 m/s** (d) 5.55 m/s (e) 3.4 m/s

FE3.5 A jet of water 3 cm in diameter strikes normal to a plate as in Fig. FE3.5. If the force required to hold the plate is 23 N, what is the jet velocity?

(a) 2.85 m/s **(b) 5.7 m/s** (c) 8.1 m/s (d) 4.0 m/s (e) 23 m/s

FE3.6 A fireboat pump delivers water to a vertical nozzle with a 3:1 diameter ratio, as in Fig. FE3.6. If friction is neglected and the flow rate is 1.895 $m^3$/min, how high will the outlet water jet rise?

(a) 2.0 m (b) 9.8 m **(c) 32 m** (d) 64 m (e) 98 m

FE3.7 A fireboat pump delivers water to a vertical nozzle with a 3:1 diameter ratio, as in Fig. FE3.6. If friction is neglected and the pump increases the pressure at section 1 to 51 kPa (gage), what will be the resulting flow rate?

**(a) 708 L/min** (b) 753 L/min (c) 810 L/min (d) 1359 L/min (e) 534 L/min

FE3.8 A fireboat pump delivers water to a vertical nozzle with a 3:1 diameter ratio, as in Fig. FE3.6. If duct and nozzle friction are neglected and the pump provides 3.75 m of head to the flow, what will be the outlet flow rate?

(a) 322 L/min (b) 454 L/min **(c) 583 L/min** (d) 821 L/min (e) 1079 L/min

FE3.9 Water flowing in a smooth 6-cm-diameter pipe enters a venturi contraction with a throat diameter of 3 cm. Upstream pressure is 120 kPa. If cavitation occurs in the throat at a flow rate of 155 gal/min, what is the estimated fluid vapor pressure, assuming ideal frictionless flow?

(a) 6 kPa (b) 12 kPa (c) 24 kPa **(d) 31 kPa** (e) 52 kPa

FE3.10 Water flowing in a smooth 6-cm-diameter pipe enters a venturi contraction with a throat diameter of 4 cm. Upstream pressure is 120 kPa. If the pressure in the throat is 50 kPa, what is the flow rate, assuming ideal frictionless flow?

(a) 28 L/min (b) 893 L/min **(c) 996 L/min** (d) 2820 L/min (e) 3986/min

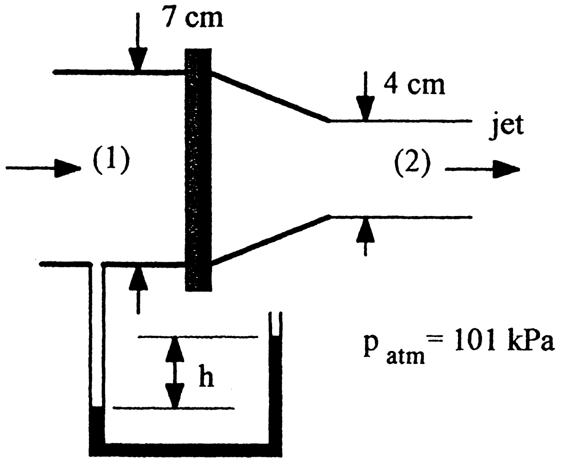



Fig. FE3.1

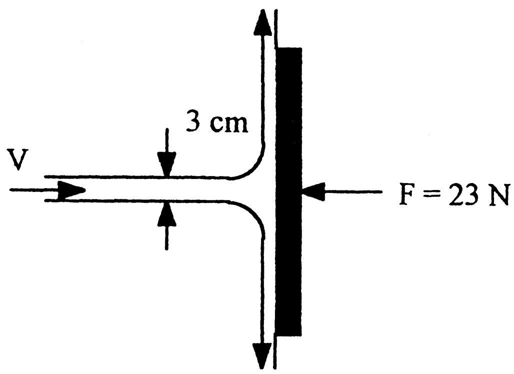



Fig. FE3.5

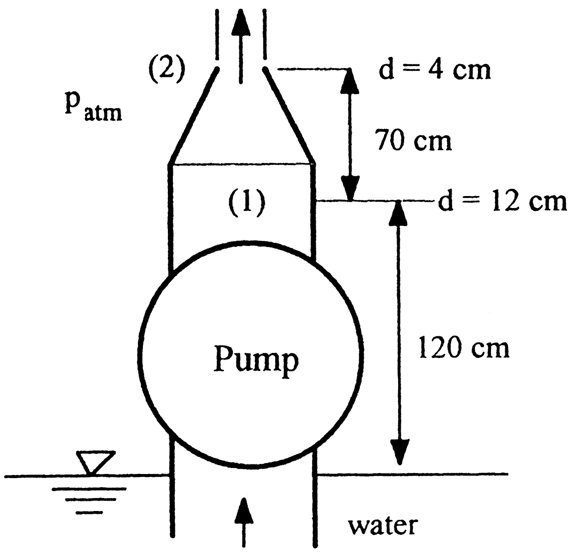
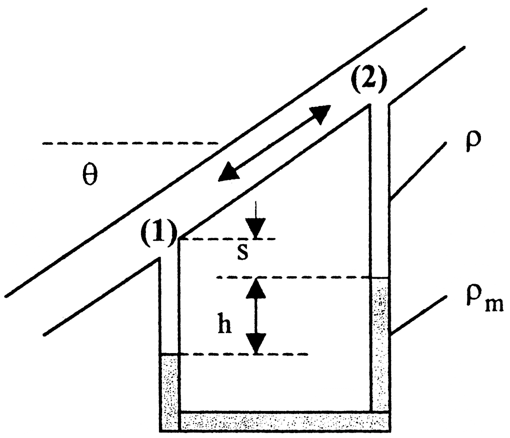



Fig. FE3.6

## COMPREHENSIVE PROBLEMS

**C3.1** In a certain industrial process, oil of density $\rho$ flows through the inclined pipe in the figure. A U-tube manometer with fluid density $\rho_m$, measures the pressure difference between points 1 and 2, as shown. The flow is steady, so that fluids in the U-tube are stationary. (a) Find an analytic expression for $p_1 - p_2$ in terms of system parameters. (b) Discuss the conditions on *h* necessary for there to be no flow in the pipe. (c) What about flow *up*, from 1 to 2? (d) What about flow *down*, from 2 to 1?




**Solution:** (a) Start at 1 and work your way around the U-tube to point 2:

$$p_1 + \rho g s + \rho g h - \rho_m g h - \rho g s - \rho g \Delta z = p_2,$$

$$or: \quad \boldsymbol{p_1 - p_2 = \rho g \Delta z + (\rho_m - \rho) g h} \quad \text{where } \Delta z = z_2 - z_1 \quad Ans.\ (a)$$

(b) If there is no flow, the pressure is entirely hydrostatic, therefore $\Delta p = \rho g$ and, since $\rho_m \neq \rho$, it follows from Ans. (a) above that **h = 0** *Ans.* (b)

(c) If *h* is positive (as in the figure above), $p_1$ is greater than it would be for no flow, because of head losses in the pipe. **Thus, if h > 0, flow is *up* from 1 to 2**. *Ans.* (c)

(d) If *h* is negative, $p_1$ is less than it would be for no flow, because the head losses act against hydrostatics. **Thus, if h < 0, flow is *down* from 2 to 1**. *Ans.* (d)

Note that *h* is a direct measure of flow, regardless of the angle $\theta$ of the pipe.

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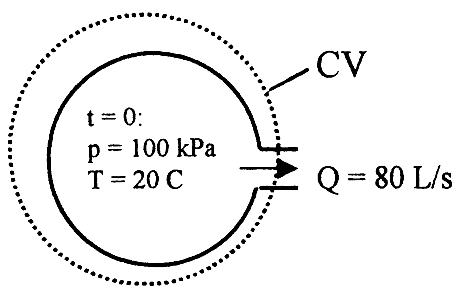
**C3.2** A rigid tank of volume $\upsilon$ = 1.0 m$^3$ is initially filled with air at 20°C and $p_o$ = 100 kPa. At time t = 0, a vacuum pump is turned on and evacuates air at a constant volume flow rate Q = 80 L/min (regardless of the pressure). Assume an ideal gas and an isothermal process. (a) Set up a differential equation for this flow. (b) Solve this equation for t as a function of ($\upsilon$, Q, p, $p_o$). (c) Compute the time in minutes to pump the tank down to p = 20 kPa. [Hint: Your answer should lie between 15 and 25 minutes.]




**Solution:** The control volume encloses the tank, as shown. The CV mass flow relation becomes

$$\frac{d}{dt}\left(\int \rho d\upsilon\right) + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

Assuming that $\rho$ is constant throughout the tank, the integral equals $\rho\upsilon$, and we obtain

$$\upsilon \frac{d\rho}{dt} + \rho Q = 0, \quad or: \quad \int \frac{d\rho}{\rho} = -\frac{Q}{\upsilon}\int dt, \quad yielding\ \ln\left(\frac{\rho}{\rho_o}\right) = -\frac{Qt}{\upsilon}$$

Where $\rho_o$ is the initial density. But, for an isothermal ideal gas, $\rho/\rho_o$ = p/po. Thus the time required to pump the tank down to pressure p is given by

$$\boldsymbol{t = -\frac{\upsilon}{Q}\ln\left(\frac{p}{p_o}\right)} \quad \textit{Ans.}\text{ (a, b)}$$

(c) For our particular numbers, noting Q = 80 L/min = 0.080 m$^3$/min, the time to pump a 1 m$^3$ tank down from 100 to 20 kPa is

$$t = -\frac{1.0\text{ m}^3}{0.08\text{ m}^3/\text{min}}\ln\left(\frac{20}{100}\right) = \textbf{20.1 min} \quad \textit{Ans.}\text{ (c)}$$

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**C3.3** Suppose the same steady water jet as in Prob. 3.45 (jet velocity 8 m/s and jet diameter 10 cm) impinges instead on a cup cavity as shown in the figure. The water is turned 180° and exits, due to friction, at lower velocity, $V_e$ = 4 m/s. (Looking from the left, the exit jet is a circular annulus of outer radius R and thickness *h*, flowing toward the viewer.) The cup has a radius of curvature of 25 cm. Find (a) the thickness *h* of the exit jet, and (b) the force *F* required to hold the cupped object in place. (c) Compare part (b) to Prob. 3.45, where F = 500 N, and give a physical explanation as to why F has changed.

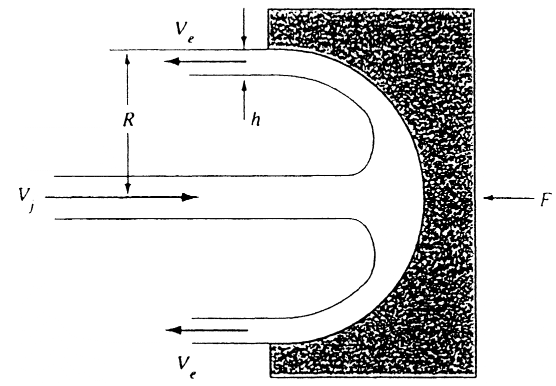

**Fig. C3.3**

**Solution:** For a steady-flow control volume enclosing the block and cutting through the jets, we obtain $\Sigma Q_{in} = \Sigma Q_{out}$, or:

$$V_j\frac{\pi}{4}D_j^2 = V_e\pi[R^2 - (R-h)^2], \quad or: \quad \boldsymbol{h = R - \sqrt{R^2 - \frac{V_j}{V_e}\frac{D_j^2}{4}}} \quad \textit{Ans.}\text{ (a)}$$

For our particular numbers,

$$h = 0.25 - \sqrt{(0.25)^2 - \frac{8}{4}\frac{(0.1)^2}{4}} = 0.25 - 0.2398 = 0.0102\text{ m} = \textbf{1.02 cm} \quad \textit{Ans.}\text{ (a)}$$

(b) Use the momentum relation, assuming no net pressure force except for F:

$$\sum F_x = -F = \dot{m}_{jet}(-V_e) - \dot{m}_{jet}(V_j), \quad or: \quad \boldsymbol{F = \rho V_j\frac{\pi}{4}D_j^2(V_j + V_e)} \quad \textit{Ans.}\text{ (b)}$$

For our particular numbers:

$$F = 998(8)\frac{\pi}{4}(0.1)^2(8+4) = \textbf{752 N} \quad \textbf{to the left} \quad \textit{Ans.}\text{ (b)}$$

(c) The answer to Prob. 3.45 was 502 N. We get **50% more** because we turned through 180°, not 90°. *Ans.* (c)

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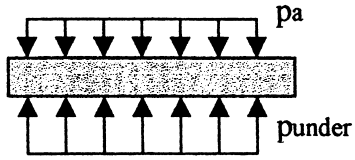
**C3.4** The air flow beneath an air hockey puck is very complex, especially since the air jets from the table impinge on the puck at various points asymmetrically. A reasonable approximation is that, at any given time, the gage pressure on the bottom of the puck is halfway between zero (atmospheric) and the stagnation pressure of the impinging jets, $p_o = 1/2\ \rho V_{jet}^2$. (a) Find the velocity $V_{jet}$ required to support a puck of weight $W$ and diameter $d$, with air density $\rho$ as a parameter. (b) For $W = 0.22$ N and $d = 6.35$ cm, estimate the required jet velocity in m/s.




**Solution:** (a) The puck has atmospheric pressure on the top and slightly higher on the bottom:

$$(p_{under} - p_a)A_{puck} = W = \frac{1}{2}\left(0 + \frac{\rho}{2}V_{jet}^2\right)\frac{\pi}{4}d^2, \quad \textit{Solve for}\ \ \mathbf{V_{jet} = \frac{4}{d}\sqrt{\frac{W}{\pi\rho}}} \quad \textit{Ans.}\ \text{(a)}$$

For our particular numbers, W = 0.22 N and d = 6.35 cm, we assume sea-level air, $\rho = 1.22\ \text{kg/m}^3$, and obtain

$$V_{jet} = \frac{4}{(0.0635)}\sqrt{\frac{0.22\ \text{N}}{\pi(1.22\text{kg/m}^3)}} = \mathbf{15.1\ m/s} \quad \textit{Ans.}\ \text{(b)}$$

**C3.5** Neglecting friction sometimes leads to odd results. You are asked to analyze and discuss the following example in Fig. C3.5. A fan blows air vertically through a duct from section 1 to section 2, as shown. Assume constant air density $\rho$. Neglecting frictional losses, find a relation between the required fan head $h_p$ and the flow rate and the elevation change. Then explain what may be an unexpected result.

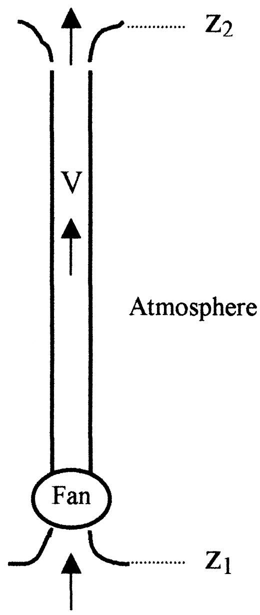



**Fig. C3.5**

**Solution:** Neglecting frictional losses, $h_f = 0$, and Bernoulli becomes,

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+z_2+\frac{V_2^2}{2g}-h_p$$

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2+\rho g(z_1-z_2)}{\rho g}+\frac{V_2^2}{2g}+z_2-h_p$$

Since the fan draws from and exhausts to atmosphere, $V_1 = V_2 \approx 0$. Solving for $h_p$,

$$\boldsymbol{h_p=\rho g(z_1-z_2)+\rho g z_2-\rho g z_1=0} \quad \textit{Ans.}$$

Without friction, and with $V_1 = V_2$, the energy equation predicts that $\boldsymbol{h_p = 0}$**!** Because the air has insignificant weight, as compared to a heavier fluid such as water, the power input required to lift the air is also negligible.