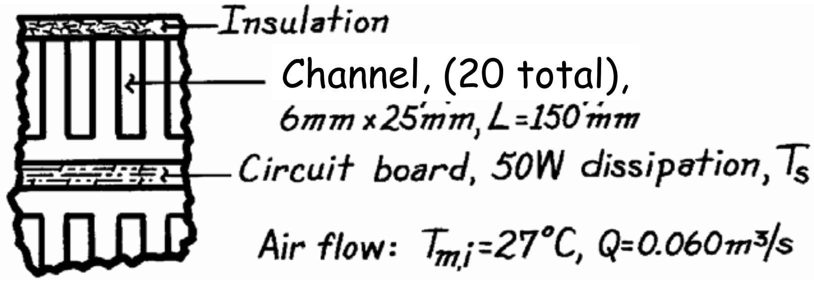
PROBLEM 8.90

KNOWN: Heat sink with 20 passages for air flow removes power dissipation from circuit board.

FIND: Operating temperature of the board and pressure drop across the sink.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation, (3) Negligible thermal resistance between the circuit boards and air passages, (4) Sink surface and board are isothermal at T_s.

PROPERTIES: *Table A-4*, Air ($\overline{T} \approx 310$ K,1 atm): $\rho = 1.1281$ kg/m^3, $c_p = 1008$ J/kg·K, $\nu = 16.89 \times 10^{-6}$ m^2/s, k = 0.0270 W/m·K, Pr = 0.706.

ANALYSIS: The air outlet temperature follows from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m}\,c_p}\right).$$

The mass flow rate for both heat sinks is

$$\dot{m} = \rho\dot{\forall} = 1.1281 \text{ kg/m}^3 \times 0.060 \text{ m}^3/\text{s} = 6.77 \times 10^{-2} \text{ kg/s}$$

and the Reynolds number for a rectangular passage is

$$Re_D = \frac{u_m D_h}{\nu}$$

where $D_h = 4A_c / P = 4(6 \text{ mm} \times 25 \text{ mm})/2(6+25)\text{mm} = 9.68 \text{ mm}$

$$u_m = \frac{\dot{m}/20}{\rho A_c} = \frac{6.77 \times 10^{-2} \text{ kg/s}/20}{1.1281 \text{ kg/m}^3 (6 \times 25) \times 10^{-6} \text{m}^2} = 20.0 \text{ m/s}$$

giving $Re_D = \dfrac{20.0 \text{ m/s} \times 9.68 \times 10^{-3} \text{ m}}{16.89 \times 10^{-6} \text{ m}^2/\text{s}} = 11,460.$

Assume the flow is turbulent and fully developed and using the Dittus-Boelter correlation find

$$Nu_D = 0.023 Re^{4/5} Pr^{0.4} = 0.023(11,460)^{4/5}(0.706)^{0.4} = 35.4$$

$$h = \frac{Nu \cdot k}{D_h} = \frac{35.4 \times 0.027 \text{ W/m} \cdot \text{K}}{0.00968 \text{ m}} = 98.6 \text{ W/m}^2 \cdot \text{K}.$$

Continued …

PROBLEM 8.90 (Cont.)

From an overall energy balance on the sink,

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) \qquad T_{m,o} = T_{m,i} + q/\dot{m}\, c_p$$

$$T_{m,o} = 27^\circ\text{C} + 50\ \text{W}/6.77\times10^{-2}\ \text{kg/s}\times1008\ \text{J/kg}\cdot\text{K} = 27.73^\circ\text{C}$$

Hence, the operating temperature of the circuit board for these conditions is

$$\frac{T_s - 27.73}{T_s - 27} = \exp\left[-\frac{2\left(6+25\right)\times10^{-3}\ \text{m}\times0.150\text{m}\times98.6\ \text{W/m}^2\cdot\text{K}}{\left(6.77\times10^{-2}\ \text{kg/s/20}\right)\times1008\ \text{J/kg}\cdot\text{K}}\right]$$

$$T_s = 30.1^\circ\text{C}.$$ <

The pressure drop in the rectangular passage for the smooth surface condition follows from Eqs. 8.22 and 8.21

$$\Delta p = f\,\frac{\rho\, u_m^2}{D_h}\,L$$

where

$$f = (0.790\ln(Re_D) - 1.64)^{-2} = 0.032.$$

$$\Delta p = 0.0320\,\frac{1.1281\ \text{kg/m}^3\left(20.0\ \text{m/s}\right)^2}{0.00968\ \text{m}}\times0.150\ \text{m} = 224\ \text{N/m}^2.$$ <

COMMENTS: (1) The analysis has been simplified by assuming the channel is rectangular and all four sides experience heat transfer. Since the insulated surface is a small portion of the total passage surface area, the effect can't be very large. (2) The power required to move the air through the heat sink is $P_{elec} = \dot{\forall}\,\Delta p = 0.060\ \text{m}^3/\text{s} \times 224\ \text{N/m}^2 = 13.4$ W. (3) The assumption $\overline{T} \approx 310$ K for evaluating properties is an overestimate. The calculation could be repeated for $\overline{T} = 300$ K for greater accuracy.