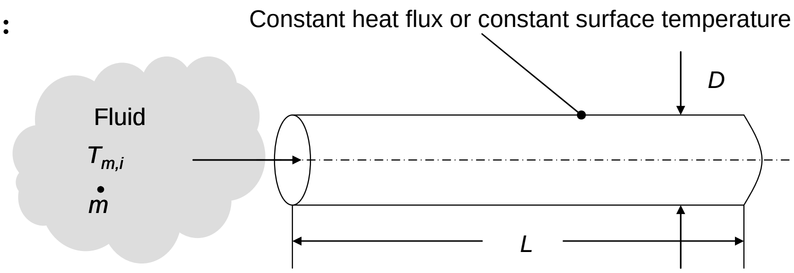
PROBLEM 8.11

KNOWN: Mass flow rate in a circular tube, tube length and diameter, thermal conditions.

FIND: (a) Expression for $(T_s(x = L) - T_{m,i})/q$ for constant heat flux conditions, (b) $(T_s - T_{m,i})/q$ for constant surface temperature conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties.

ANALYSIS (a) From Newton's law of cooling,

$$T_s(x=L) - T_m(x=L) = q''/h \tag{1}$$

and from an energy balance on the entire tube,

$$T_m(x=L) = T_{m,i} + q/\dot{m}c_p \tag{2}$$

Combining Eqs. (1) and (2) and noting that $q = q''\pi DL$ yields

$$\frac{T_s(x=L) - T_{m,i}}{q} = \frac{1}{\pi DLh} + \frac{1}{\dot{m}c_p}$$

Substituting the expression for the local Nusselt number, $Nu_D = hD/k$ gives

$$\frac{T_s(x=L) - T_{m,i}}{q} = \frac{1}{Nu_D \pi Lk} + \frac{1}{\dot{m}c_p} \qquad <$$

(b) From Eq. (8.41b)

$$\frac{T_s - T_m(x=L)}{T_s - T_{m,i}} = \exp\left(-\frac{\pi D\bar{h}L}{\dot{m}c_p}\right) \tag{3}$$

Combining Eqs. (2) and (3) yields

$$1 - \frac{q/\dot{m}c_p}{(T_s - T_{m,i})} = \exp\left(-\frac{\pi D\bar{h}L}{\dot{m}c_p}\right)$$

Continued…

which may be rearranged to yield

$$\frac{q}{\left(T_s - T_{m,i}\right)} = \dot{m}c_p\left[1 - \exp\left(-\frac{\pi D\bar{h}L}{\dot{m}c_p}\right)\right]$$

or

$$\frac{(T_s - T_{m,i})}{q} = \frac{1}{\dot{m}c_p\left[1 - \exp\left(-\frac{\pi \overline{Nu}_D kL}{\dot{m}c_p}\right)\right]} \qquad <$$

COMMENTS: (1) The ratio on the LHS is a *figure of merit* that, in many applications, is sought to be minimized. (2)The two terms on the RHS of the final expression for the constant heat flux case may be thought of as thermal resistances. The first term on the RHS is a thermal resistance associated with heat transfer between the fluid and the tube wall at the tube exit, and the second term is associated with the increase in temperature between the tube inlet and the tube exit. (3) As the argument of the exponential term increases in magnitude for the constant surface temperature expression, the mean outlet temperature approaches the surface temperature value (see Eq. 3), and the figure of merit expression reduces to Eq. (2).