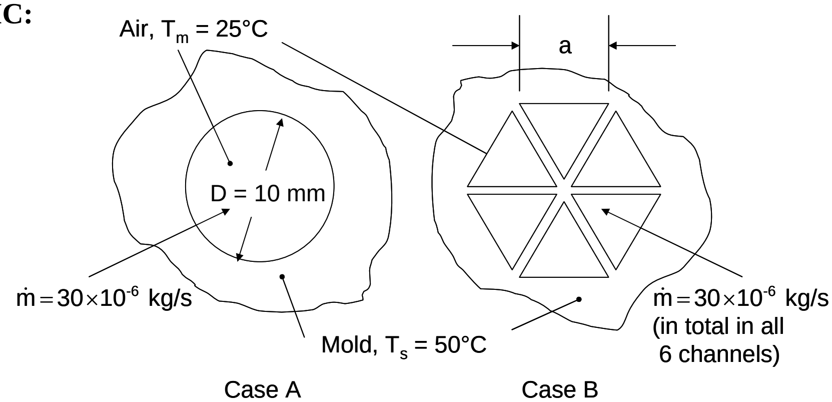
PROBLEM 8.81

KNOWN: Inlet temperature and mass flow rate of air flow. Geometry and dimensions of channels through a mold. Mold temperature.

FIND: (a) Heat transferred to the air for case A, (b) Heat transferred to the air for case B, and (c) pressure drop for both cases.

SCHEMATIC:



ASSUMPTIONS: (1) Flow is hydrodynamically and thermally fully developed, (2) Mold temperature is uniform. (3) Narrow fins between channels in case B are at the mold temperature.

PROPERTIES: Table A-4, Air (T ≈ 310 K assumed, 1atm): $\rho = 1.128$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 189.3 \times 10^{-7}$ N·s/m^2, k = 0.027 W/m·K.

ANALYSIS:

(a) The Reynolds number is

$$Re_D = \frac{4\,\dot{m}}{\pi\,D\,\mu} = \frac{4 \times 30 \times 10^{-6}\ \text{kg/s}}{\pi \times 0.01\ \text{m} \times 189.3 \times 10^{-7}\ \text{N}\cdot\text{s/m}^2} = 202$$

Thus, the flow is laminar. Since it has also been assumed that the flow is fully developed and the mold temperature is uniform, the Nusselt number is

$$Nu_D = 3.66$$

Thus $\quad h = Nu_D k/D = 3.66 \times 0.027$ W/m·K/0.01 m = 9.88 W/m^2·K.

The outlet temperature can be found from Equation 8.41b,

$$T_{m,o} = T_s + (T_{m,i} - T_s)\exp\left(-\frac{P\,L}{\dot{m}\,c_p}\bar{h}\right)$$

$$= 50°\text{C} + (25°\text{C} - 50°\text{C})\exp\left(-\frac{\pi \times 0.01\ \text{m} \times 0.1\ \text{m} \times 9.88\ \text{W/m}^2\cdot\text{K}}{30 \times 10^{-6}\ \text{kg/s} \times 1007\ \text{J/kg}\cdot\text{K}}\right)$$

$$= 41.0°\text{C}$$

Thus

$$q = \dot{m}c_p(T_{m,o} - T_{m,i}) = 30 \times 10^{-6}\ \text{kg/s} \times 1007\ \text{J/kg}\cdot\text{K} \times (41.0°\text{C} - 25°\text{C}) = 0.485\ \text{W} \quad <$$

(b) We first determine the dimensions of the triangular channels from the requirement that the total area is the same as case A.

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$$\pi D^2/4 = 6a^2/2$$

$$a = \left(\frac{\pi}{12}\right)^{1/2} D = \left(\frac{\pi}{12}\right)^{1/2} \times 10\text{ mm} = 5.1\text{ mm}$$

and the flowrate in one channel is 5×10^{-6} kg/s.
The hydraulic diameter is $D_h = 4A_c/P = 4(a^2/2)/3a = 2a/3 = 3.4$mm.
The Reynolds number is

$$Re_D = \frac{4\,\dot{m}}{\pi\, D_h\, \mu} = \frac{4 \times 5 \times 10^{-6}\text{ kg/s}}{\pi \times 0.0034\text{ m} \times 189.3 \times 10^{-7}\text{ N}\cdot\text{s/m}^2} = 98.6$$

so the flow is laminar. From Table 8.1, the Nusselt number is $Nu_D = 2.47$, so

$$h = Nu_D k/D_h = 2.47 \times 0.027\text{ W/m}\cdot\text{K}/0.0034\text{ m} = 19.6\text{ W/m}^2\cdot\text{K}.$$

The outlet temperature is

$$T_{m,o} = T_s + (T_{m,i} - T_s)\exp\left(-\frac{P\,L}{\dot{m}\,c_p}\bar{h}\right)$$

$$= 50°\text{C} + (25°\text{C} - 50°\text{C})\exp\left(-\frac{3 \times 0.0051\text{ m} \times 0.1\text{ m} \times 19.0\text{ W/m}^2\cdot\text{K}}{5 \times 10^{-6}\text{ kg/s} \times 1007\text{ J/kg}\cdot\text{K}}\right)$$

$$= 49.9°\text{C}$$

Then using the total flowrate to account for all six channels,

$$q = \dot{m}c_p(T_{m,o} - T_{m,i}) = 30 \times 10^{-6}\text{ kg/s} \times 1007\text{ J/kg}\cdot\text{K} \times (49.9°\text{C} - 25°\text{C}) = 0.753\text{ W} \qquad <$$

(c) The friction factor for case A is $f = 64/Re_D = 64/202 = 0.317$. The pressure drop is, from Equation 8.22a,

$$\Delta p = f\frac{\rho u_m^2}{2D}L$$

with $u_m = \dot{m}/\rho A_c = 30 \times 10^{-6}\text{ kg/s}/(1.128\text{ kg/m}^3 \times \pi\,(0.01\text{ m})^2/4) = 0.339\text{ m/s}$. Thus

$$\Delta p = 0.317 \times \frac{1.128\text{ kg/m}^3 \times (0.339\text{ m/s})^2}{2 \times 0.01\text{ m}} \times 0.1\text{ m} = 0.205\text{ Pa} \qquad <$$

For Case B, from Table 8.1, $f = 53/Re_D = 53/98.6 = 0.538$, and $u_m = 0.339$ m/s as in Case A. Thus

$$\Delta p = f\frac{\rho u_m^2}{2D_h}L = 0.538 \times \frac{1.128\text{ kg/m}^3 \times (0.339\text{ m/s})^2}{2 \times 0.0034\text{ m}} \times 0.1\text{ m} = 1.02\text{ Pa} \qquad <$$

COMMENTS: (1) Segmenting the channel into six smaller sections increases the heat transfer by 55%, but at the expense of almost a five-fold increase in the pressure drop. (2) For the circular duct, the hydrodynamic entry length, is $x_{fd,h} = 0.05\, Re_D\, D = 0.1$ m, so it is not fully developed as assumed. For the triangular duct, $x_{fd,h} = 0.05\, Re_D\, D_h = 0.02$ m, so the assumption is more appropriate. The thermal development length is shorter, since $Pr = 0.7$.