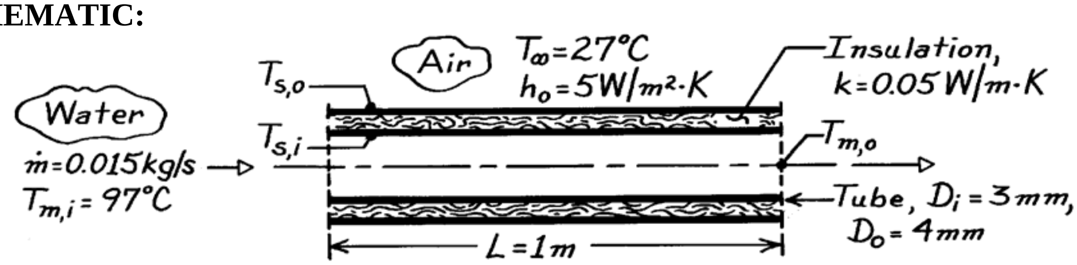
PROBLEM 8.65

KNOWN: Water flow rate and inlet temperature for a thin-walled tube of prescribed length and diameter.

FIND: Water outlet temperature for each of the following conditions: (a) Tube surface maintained at 27°C, (b) Insulation applied and outer surface maintained at 27°C, (c) Insulation applied and outer surface exposed to ambient air at 27°C.

SCHEMATIC:



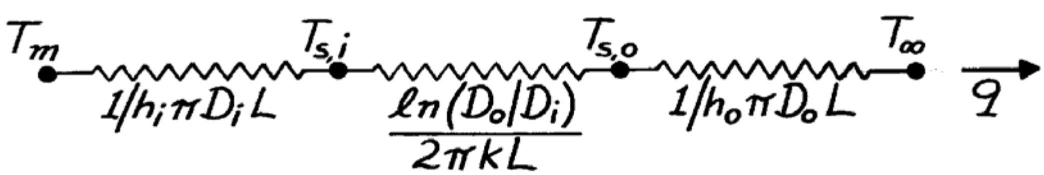
ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed flow throughout the tube, (3) Negligible tube wall conduction resistance, (4) Negligible contact resistance between tube wall and insulation, (5) Uniform outside convection coefficient.

PROPERTIES: Assume water cools to $T_{m,o}$ = 27°C with no insulation but that cooling is negligible ($T_{m,o}$ = 97°C) with insulation. *Table A-4*, Water $\left(\bar{T}_m = 335K\right)$: c_p = 4186 J/kg·K, $\mu = 453 \times 10^{-6}$ N·s/m^2, k =0.656 W/m·K, Pr = 2.88; *Table A-4*, Water ($T_{m,i}$ = 370K): c_p = 4214 J/kg·K, $\mu = 289 \times 10^{-6}$ N·s/m^2, k = 0.679 W/m·K, Pr = 1.80.

ANALYSIS: For each of the three cases, heat is transferred from the warm water to a surface (or the air) which is at a fixed temperature (27°C). Accordingly, an expression of the form given by Eq. 8.41b may be used to determine the outlet temperature of the water, so long as the appropriate heat transfer coefficient is used. In particular, each of the cases can be described by Eq. 8.45a.

$$\frac{\Delta T_o}{\Delta T_i} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}\, c_p}\right)$$

Referring to the thermal circuit associated with heat transfer from the water,



and the UA product may be evaluated as

$$UA = \left(\Sigma R_t\right)^{-1}.$$

(a) For the first case: $T_{s,i} = 27^\circ C \quad \Delta T_i = T_{m,i} - T_{s,i} = 70^\circ C \quad UA = h_i \pi\, D_i L.$

$$Re_D = \frac{4\dot{m}}{\pi\, D_i \mu} = \frac{4 \times 0.015 \text{ kg/s}}{\pi\left(0.003\text{m}\right)453 \times 10^{-6}\, \text{N} \cdot \text{s/m}^2} = 14{,}053.$$

Continued …

From Eq. 8.60,

$$h_i = \frac{k}{D_i} 0.023\, Re_D^{4/5}\, Pr^{0.30} = \frac{0.656\ W/m\cdot K}{0.003m}(0.023)\ (14{,}053)^{4/5}(2.88)^{0.3} = 14{,}373\ W/m^2\cdot K.$$

$$\Delta T_o = \Delta T_i \exp\left(-\frac{h_i \pi D_i L}{\dot{m}\, c_p}\right) = 70^\circ C \exp\left(-\frac{14{,}373 \frac{W}{m^2\cdot K}\pi\times 0.003m\times 1m}{0.015\ kg/s\times 4186\ J/kg\cdot K}\right) = 8.1^\circ C$$

$$T_{m,o} = \Delta T_o + T_{s,i} = 8.1^\circ C + 27^\circ C = 35.1^\circ C. \qquad <$$

(b) For the second case: $T_{s,o}$ = 27°C with

$$\Delta T_i = T_{m,i} - T_{s,o} = 70^\circ C \qquad UA = \left[\left(1/h_i \pi D_i L\right) + \ell n\left(D_o / D_i\right)/2\pi kL\right]^{-1}.$$

With $Re_D = \dfrac{4\,\dot{m}}{\pi D_i \mu} = \dfrac{4\times 0.015\ kg/s}{\pi(0.003m)289\times 10^{-6} N\cdot s/m^2} = 22{,}028$

$$h_i = \frac{k}{D_i} 0.023\, Re_D^{4/5}\, Pr^{0.3} = \frac{0.679\ W/m\cdot K}{0.003m}(0.023)\ (22{,}028)^{4/5}(1.80)^{0.3} = 18{,}511\ W/m^2\cdot K.$$

It follows that

$$UA = \left[\frac{1}{18{,}511\ \pi\times 0.003} + \frac{\ell n(0.004/0.003)}{2\pi(0.05)}\right]^{-1} = \left[5.73\times 10^{-3} + 0.916\right]^{-1} = 1.085\ W/K$$

and the outlet temperature is

$$\Delta T_o = 70^\circ C \exp\left(-\frac{1.085\ W/K}{0.015\ kg/s\times 4214\ J/kg\cdot K}\right) = 68.8^\circ C$$

$$T_{m,o} = \Delta T_o + T_{s,o} = 68.8^\circ C + 27^\circ C = 95.8^\circ C. \qquad <$$

(c) For the third case: T_∞ = 27°C, $\Delta T_i = T_{m,i} - T_\infty$ = 70°C and

$$UA = \left[\left(1/h_i \pi D_i L\right) + \ell n\left(D_o / D_i\right)/2\pi kL + \left(1/h_o \pi D_o L\right)\right]^{-1}$$

$$UA = \left[5.73\times 10^{-3} + 0.916 + \frac{1}{5\pi(0.004)}\right]^{-1} = \left[5.73\times 10^{-3} + 0.916 + 15.92\right]^{-1} = 0.0594\ W/K$$

$$\Delta T_o = 70^\circ C \exp\left(-\frac{0.0594\ W/K}{0.015\ kg/s\times 4214\ J/kg\cdot K}\right) = 69.9^\circ C$$

$$T_{m,o} = \Delta T_o + T_\infty = 69.9^\circ C + 27^\circ C = 96.9^\circ C. \qquad <$$

COMMENTS: Note that $R_{conv,o} >> R_{cond,insul} >> R_{conv,i}$,