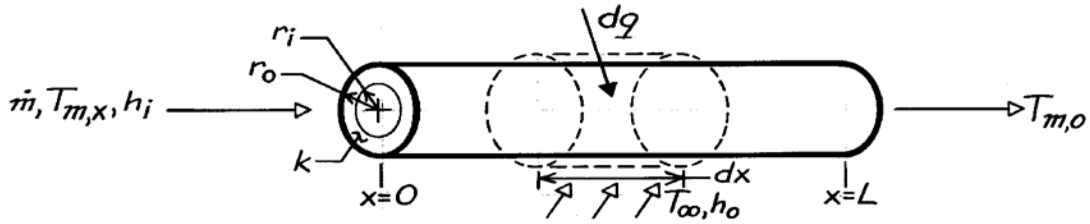
PROBLEM 8.22

KNOWN: Heat transfer between fluid flow over a tube and flow through the tube.

FIND: Axial variation of mean temperature for inner flow.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of Eq. 8.34, (2) Negligible axial conduction, (3) Constant c_p, (4) Uniform T_∞.

ANALYSIS: From Eq. 8.36,

$$dq = \dot{m}\, c_p d\, T_m$$

with

$$dq = U dA \left(T_\infty - T_m\right) = UP\left(T_\infty - T_m\right) dx.$$

The overall heat transfer coefficient may be defined in terms of the inner or outer surface area, with

$$U_i P_i = U_o P_o.$$

For the inner surface, from Eq. 3.36,

$$U_i = \left[\frac{1}{h_i} + \frac{r_i}{k} \ln \frac{r_o}{r_i} + \frac{r_i}{r_o}\frac{1}{h_o}\right]^{-1}.$$

Hence,

$$\frac{d\, T_m}{T_\infty - T_m} = +\frac{UP}{\dot{m}\, c_p} dx$$

or, with $\Delta T \equiv T_\infty - T_m$,

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d\left(\Delta T\right)}{\Delta T} = -\frac{P}{\dot{m}\, c_p} \int_0^L U dx.$$

Hence,

$$\ln \frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}\, c_p}\left(\frac{1}{L}\int_0^L U dx\right)$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p}\overline{U}\right). \quad <$$

COMMENTS: The development and results parallel those for a constant surface temperature, with $\overline{U}$ and T_∞ replacing $\overline{h}$ and T_s.