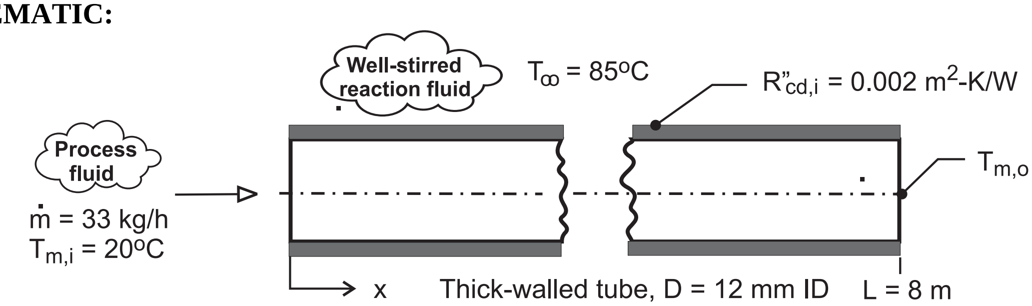
PROBLEM 8.41

KNOWN: Water flow through a thick-walled tube immersed in a well stirred, hot reaction tank maintained at 85°C; conduction thermal resistance of the tube wall based upon the inner surface area is $R''_{cd} = 0.002\ m^2 \cdot K/W$.

FIND: (a) The outlet temperature of the process fluid, $T_{m,o}$; assume, and then justify, fully developed flow and thermal conditions within the tube; and (b) Do you expect $T_{m,o}$ to increase or decrease if the combined entry condition exists within the tube? Estimate the outlet temperature of the process fluid for this condition.

SCHEMATIC:



ASSUMPTIONS: (1) Flow is fully developed, part (a), (2) Constant properties, (3) Incompressible liquid with negligible viscous dissipation, and (4) Constant wall temperature heating, with $T_s \approx T_\infty$ because of small thermal resistance associated with well-stirred reaction fluid.

PROPERTIES: *Table A-6*, *Water* ($T_m = (T_{m,o} + T_{m,i})/2 = 315$ K): $c_p = 4179$ J/kg·K, $\mu = 6.31 \times 10^{-4}$ N·s/m², k = 0.634 W/m·K, Pr = 4.16.

ANALYSIS: (a) The outlet temperature is determined from the rate equation, Eq. 8.45a, written as

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\overline{U} A_s}{\dot{m} c_p}\right) \tag{1}$$

where the overall coefficient, based upon the inner surface area of the tube is expressed in terms of the convection and conduction thermal resistances,

$$\frac{1}{\overline{U}} = \frac{1}{\overline{h}} + R''_{cd,i} \tag{2}$$

To estimate $\overline{h}$, begin by characterizing the flow

$$Re_D = 4\dot{m}/\pi D \mu \tag{3}$$

$$Re_D = 4(33/3600\ kg/s)/\pi \times 0.012\ m \times 6.31 \times 10^{-4}\ N \cdot s/m^2 = 1540$$

Consider the flow as laminar, and assuming fully developed conditions, estimate $\overline{h}$ with the correlation of Eq. 8.55,

$$\overline{Nu}_D = \overline{h} D/k = 3.66 \tag{4}$$

$$\overline{h} = 3.66 \times 0.634\ W/m \cdot K / 0.012\ m = 193\ W/m^2 \cdot K$$

From Eq. (2),

$$\overline{U} = \left[1/193\ W/m^2 \cdot K + 0.002\ m^2 \cdot K/W\right]^{-1} = 139\ W/m^2 \cdot K$$

and from Eq. (1), with $A_s = \pi DL$, calculate $T_{m,o}$.

Continued …

PROBLEM 8.41 (Cont.)

$$\frac{85-T_{m,o}}{85-20}=\exp\left(-\frac{139\ \mathrm{W/m^2\cdot K}\times\pi\times 0.012\ \mathrm{m}\times 8\ \mathrm{m}}{33/3600\ \mathrm{kg/s}\times 4179\ \mathrm{J/kg\cdot K}}\right)$$

$$T_{m,o}=63°\mathrm{C}$$ <

Fully developed flow and thermal conditions are justified if the tube length is much greater than the fully developed lengths $x_{fd,h}$ and $x_{fd,t}$. From Eqs. 8.3 and 8.23,

$$x_{fd,h}=0.05\ Re_D\ D=0.05\times 1540\times 0.012\ \mathrm{m}=0.92\ \mathrm{m}$$
$$x_{fd,t}=x_{fd,h}\ Pr=0.92\ \mathrm{m}\times 4.16=3.8\ \mathrm{m}$$

That is, while fully developed velocity conditions may be justifiable, the length is only twice that required to reach *thermally* fully developed conditions.

(b) We expect the calculated outlet temperature to be larger if the combined entrance effect exists, since the average heat transfer coefficient would be larger relative to that associated with fully-developed conditions. <

Considering combined entry length conditions, estimate the convection coefficient using the Baehr and Stephan correlation, Eq. 8.58, where from Eq. 8.56, $Gz_D = (D/L)Re_D Pr = (0.012\ \mathrm{m}/8\ \mathrm{m}) \times 1540 \times 4.16 = 9.61$:

$$\overline{Nu}_D=\frac{\bar{h}D}{k}=\frac{\dfrac{3.66}{\tanh[2.264Gz_D^{-1/3}+1.7Gz_D^{-2/3}]}+0.0499Gz_D\tanh(Gz_D^{-1})}{\tanh(2.432\,Pr^{1/6}\,Gz_D^{-1/6})} \qquad (5)$$

$$\bar{h}=\frac{0.634\ \mathrm{W/m\cdot K}}{0.012\ \mathrm{m}}\left(\frac{\dfrac{3.66}{\tanh[2.264\times 9.61^{-1/3}+1.7\times 9.61^{-2/3}]}+0.0499\times 9.61\tanh(9.61^{-1})}{\tanh(2.432\times 4.16^{1/6}\times 9.61^{-1/6})}\right)=225\ \mathrm{W/m^2\cdot K}$$

which is a 17% increase over the fully developed analysis result. Using the foregoing relations, find

$$U=155\ \mathrm{W/m^2\cdot K} \qquad T_{m,o}=65.9°\mathrm{C}$$ <

COMMENTS: (1) The thermophysical properties for the fully developed correlation should be evaluated at the mean fluid temperature $T_m = (T_{m,o} + T_{m,i})/2 = 316$ K. This is very close to the assumed value of 315 K. (2) For the Baehr and Stephan correlation, the properties are also evaluated at T_m. (3) For this case where the tube length is about twice $x_{fd,t}$, the average heat transfer coefficient is larger than the fully developed value, as we would expect. (4) The thermal entry length correlation due to Hausen yields $\bar{h}$ = 222 W/m^2·K, close to the combined entry length value. This is not surprising, considering that the Prandtl number of 4.16 is close to meeting the Pr > 5 condition for the Hausen correlation to be valid.