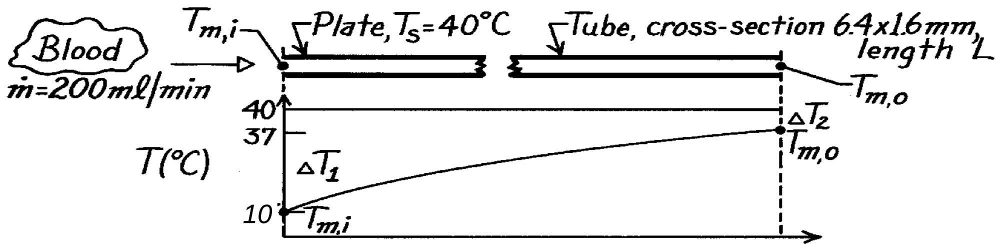
PROBLEM 8.88

KNOWN: Heat exchanger to warm blood from a storage temperature 10°C to 37° at 200 ml/min. Tubing has rectangular cross-section 6.4 mm × 1.6 mm sandwiched between plates maintained at 40°C.

FIND: (a) Length of tubing and (b) Assessment of assumptions to indicate whether analysis under- or over-estimates length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible liquid with negligible viscous dissipation, (3) Blood flow is fully developed, (4) Blood has properties of water, and (5) Negligible tube wall and contact resistance.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m \approx 300$ K): $c_{p,f} = 4179$ J/kg·K, $\rho_f = 1/v_f = 997$ kg/m^3, $\nu_f = \mu_f v_f = 8.58 \times 10^{-7}$ m^2/s, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) From an overall energy balance and the rate equation,

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = \overline{h} A_s \Delta T_{LMTD} \tag{1}$$

where

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} = \frac{(40-15)-(40-37)}{\ln(25/3)} = 11.7^\circ \text{C}.$$

To estimate $\overline{h}$, find the Reynolds number for the rectangular tube,

$$Re_D = \frac{u_m D_h}{\nu} = \frac{0.326 \text{ m/s} \times 0.00256 \text{ m}}{8.58 \times 10^{-7} \text{m}^2/\text{s}} = 973$$

where

$$D_h = 4\, A_c / P = 4(6.4 \text{ mm} \times 1.6 \text{ mm}) / 2(6.4 + 1.6) \text{mm} = 2.56 \text{ mm}$$

$$A_c = (6.4 \text{ mm} \times 1.6 \text{ mm}) = 1.024 \times 10^{-5} \text{m}^2$$

$$u_m = \dot{m}/\rho A_c = \dot{\forall}/A_c = 200 \text{ m}\ell/60 \text{ s}\left(10^{-6} \text{m}^3/\text{m}\ell\right)/1.024 \times 10^{-5} \text{m}^2 = 0.326 \text{ m/s}.$$

Hence the flow is laminar, but assuming fully developed flow with an isothermal surface from Table 8.1 with b/a = 6.4/1.6 = 4,

$$Nu_D = \frac{h D_h}{k} = 4.44 \qquad h = \frac{4.44 \times 0.613 \text{ W/m} \cdot \text{K}}{0.00256 \text{ m}} = 1063 \text{ W/m}^2 \cdot \text{K}.$$

Continued …

From Eq. (1) with

$$A_s = PL = 2(6.4 + 1.6) \times 10^{-3}\,\text{m} \times L = 1.6 \times 10^{-2} L$$

$$\dot{m} = \rho A_c u_m = 997\ \text{kg/m}^3 \times 1.024 \times 10^{-5}\,\text{m}^2 \times 0.326\ \text{m/s} = 3.328 \times 10^{-3}\ \text{kg/s}$$

the length of the rectangular tubing can be found from Eq. (1) as

$$3.328 \times 10^{-3}\ \text{kg/s} \times 4179\ \text{J/kg}\cdot\text{K}(37 - 10)\text{K} = 1063\ \text{W/m}^2\cdot\text{K} \times 1.6 \times 10^{-2}\,\text{L}\text{m}^2 \times 11.7\ \text{K}$$

$$L = 1.9\ \text{m}. \qquad <$$

(b) Consider these comments with regard to whether the analysis under- or over-estimates the length,

- ⇒ With $x_{fd,h} \approx 0.05 D_h Re_D = 0.12$ m and $x_{fd,t} = x_{fd,h} Pr = 0.73$ m, the thermal development may not be negligible and would contribute to increasing heat transfer; the present analysis over predicts the length,
- ⇒ negligible tube wall resistance - depends upon materials of construction; if plastic, analysis under predicts length,
- ⇒ negligible thermal contact resistance between tube and heating plate - if present, analysis under predicts length.