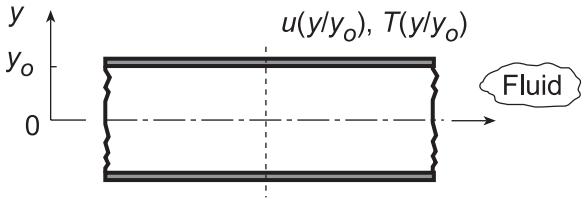
PROBLEM 8.8

KNOWN: Velocity and temperature profiles for laminar flow in a parallel plate channel.

FIND: Mean velocity, u_m, and mean (or bulk) temperature, T_m, at this axial position. Plot the velocity and temperature distributions. Comment on whether values of u_m and T_m appear reasonable.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar incompressible flow, (2) Constant properties.

ANALYSIS: The prescribed velocity and temperature profiles (m/s and °C, respectively) are

$$u(y) = 0.75\left[1-(y/y_o)^2\right] \qquad T(y) = 5.0 + 95.66(y/y_o)^2 - 47.83(y/y_o)^4 \quad (1,2)$$

The mean velocity, u_m, follows from its definition, Eq. 8.7,

$$\dot{m} = \rho A_c u_m = \rho \int_{A_c} u(y)\cdot dA_c$$

where the flow cross-sectional area is $dA_c = 1\cdot dy$, and $A_c = 2y_o$,

$$u_m = \frac{1}{A_c}\int_{A_c} u(y)\cdot dy = \frac{1}{2y_o}\int_{-y_o}^{+y} u(y)dy \tag{3}$$

$$u_m = \frac{1}{2y_o}\cdot y_o \int_{-1}^{+1} 0.75\left[1-(y/y_o)^2\right] d(y/y_o)$$

$$u_m = 1/2\left\{0.75\left[(y/y_o) - 1/3(y/y_o)^3\right]\right\}_{-1}^{+1}$$

$$u_m = 1/2\times 0.75\left\{[1-1/3]-[-1+1/3]\right\} = 1/2\times 0.75\times 4/3 = 2/3\times 0.75 = 0.50\,\text{m/s} \qquad <$$

The mean temperature, T_m, follows from its definition, Eq. 8.25,

$$\dot{E}_t = \dot{m}c_v T_m \qquad \text{where} \qquad \dot{m} = \rho A_c u_m$$

$$\rho A_c u_m c_p T_m = \rho c_p \int_{A_c} u(y)\cdot T(y)\, dA_c$$

Hence, substituting velocity and temperature profiles,

$$T_m = \frac{1}{u_m A_c}\int_{-y_o}^{+y_o} u(y)\cdot T(y)\, dy \tag{4}$$

$$T_m = \frac{1}{(0.5\,\text{m/s})2y_o}\, y_o \int_{-1}^{+1}\left\{0.75\left[1-(y/y_o)^2\right]\right\}\left\{5.0+95.66(y/y_o)^2 - 47.83(y/y_o)^4\right\} d(y/y_o)$$

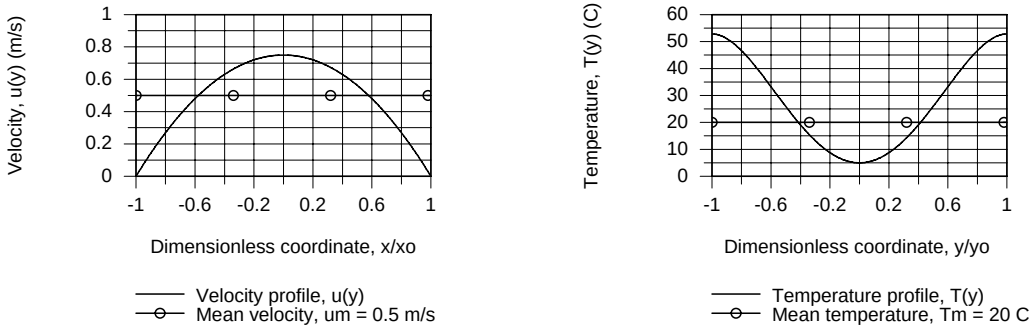
$$T_m = \frac{0.75}{0.5\times 2}\left\{\left[5(y/y_o) + 31.89(y/y_o)^3 - 9.57(y/y_o)^5\right] - \left[1.67(y/y_o)^3 + 19.13(y/y_o)^5 - 6.83(y/y_o)^7\right]\right\}_{-1}^{+1}$$

$$T_m = \frac{0.75}{0.5\times 2}\left\{[27.32-13.97]-[-27.32-(-13.97)]\right\} = 20.0^\circ\text{C} \qquad <$$

Continued...

PROBLEM 8.8 (Cont.)

The velocity and temperature profiles along with the u_m and T_m values are plotted below.



For the velocity profile, the mean velocity is 2/3 that of the centerline velocity, $u_m = 2u(0)/3$. Note that the areas above and below the u_m line appear to be equal. Considering the temperature profile, we'd expect the mean temperature to be closer to the centerline temperature since the velocity profile weights the integral toward the centerline.

COMMENTS: The integrations required to obtain u_m and T_m, Eqs. (3) and (4), could also be performed using the intrinsic function *INTEGRAL (y,x)* in the *IHT Workspace*.