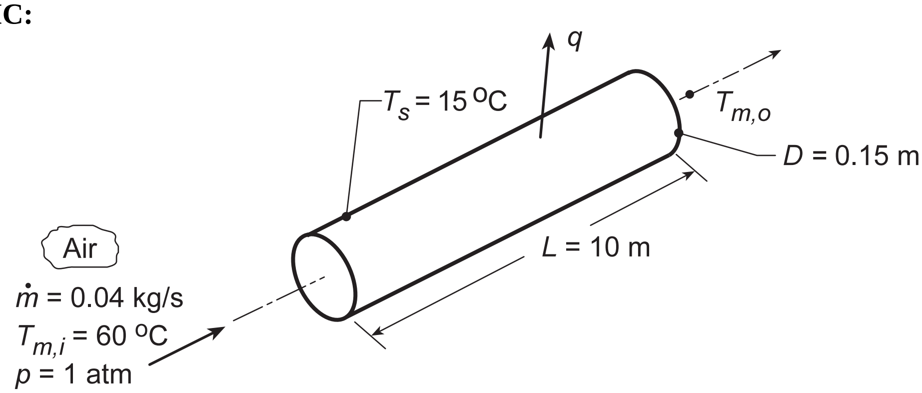
PROBLEM 8.42

KNOWN: Flow rate and temperature of atmospheric air entering a duct of prescribed diameter, length and surface temperature.

FIND: (a) Air outlet temperature and duct heat loss for the prescribed conditions and (b) Calculate and plot q and Δp for the range of diameters, $0.1 \le D \le 0.2$ m, maintaining the total surface area, $A_s = \pi DL$, at the same value as part (a). Explain the trade off between the heat transfer rate and pressure drop.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3)Ideal gas with negligible viscous dissipation and pressure variation, (4) Uniform surface temperature, (5) Fully developed flow conditions.

PROPERTIES: *Table A.4*, Air ($\bar{T}_m \approx 310$ K, 1 atm): $\rho = 1.128$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 189 \times 10^{-7}$ N·s/m^2, k = 0.027 W/m·K, Pr = 0.706.

ANALYSIS: (a) With

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.04\,\text{kg/s}}{\pi(0.15\,\text{m})189\times 10^{-7}\,\text{N}\cdot\text{s/m}^2} = 17{,}965$$

the flow is turbulent. Assuming fully developed conditions throughout the tube, it follows from the Dittus-Boelter correlation, Eq. 8.60, that

$$\bar{h} = \frac{k}{D}0.023\,Re_D^{4/5}\,Pr^{0.3} = \frac{0.027\,\text{W/m}\cdot\text{K}}{0.15\,\text{m}}0.023(17{,}965)^{4/5}(0.706)^{0.3} = 9.44\,\text{W/m}^2\cdot\text{K}\,.$$

Hence, from the energy balance relation, Eq. 8.41b,

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{\pi DL}{\dot{m}c_p}\bar{h}\right)$$

$$T_{m,o} = 15^\circ\text{C} + 45^\circ\text{C}\exp\left(-\frac{\pi(0.15\,\text{m})10\,\text{m}(9.44\,\text{W/m}^2\cdot\text{K})}{0.04\,\text{kg/s}(1007\,\text{J/kg}\cdot\text{K})}\right) = 29.9^\circ\text{C} \qquad <$$

From the overall energy balance, Eq. 8.34, it follows that

$$q = \dot{m}c_p(T_{m,o} - T_{m,i}) = 0.04\,\text{kg/s}\times 1007\,\text{J/kg}\cdot\text{K}(29.9 - 60)^\circ\text{C} = -1212\,\text{W}\,. \qquad <$$

From Eq. 8.22a, the pressure drop is

$$\Delta p = f\,\frac{\rho u_m^2}{2D}L$$

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and for the smooth surface conditions, Eq. 8.21 can be used to evaluate the friction factor,

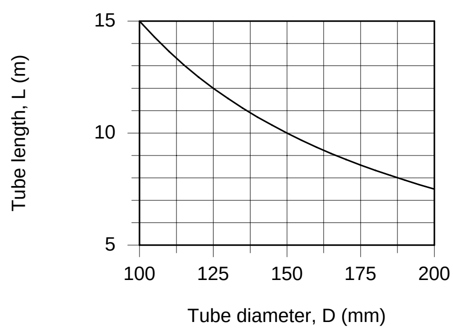
$$f = \left(0.790\ln\left(Re_D\right) - 1.64\right)^{-2} = \left(0.790\ln\left(17,965\right) - 1.64\right)^{-2} = 0.0269$$

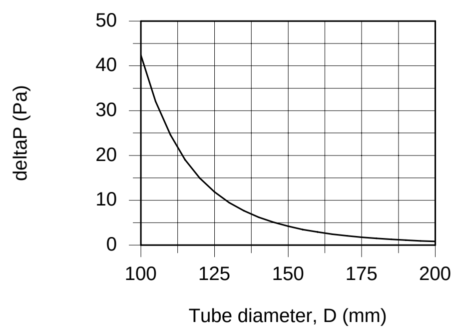
Hence, the pressure drop is

$$\Delta p = 0.0269\frac{1.128\,kg/m^3\left(2.0\,m/s\right)^2}{2\times 0.15\,m}\times 10\,m = 4.03\,N/m^2 \qquad <$$

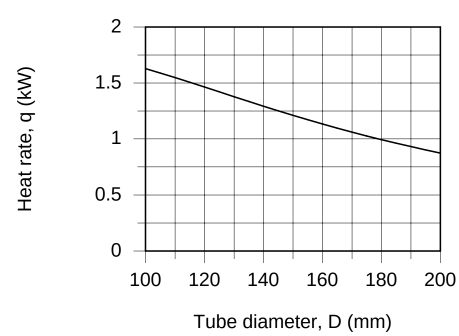
where $u_m = \dot{m}/\rho A_c = 0.04\,kg/s\Big/1.128\,kg/m^3 \times \left(\pi 0.15^2\,m^2/4\right) = 2.0\,m/s$.

(b) For the prescribed conditions of part (a), $A_s = \pi DL = \pi(0.15\text{ m}) \times 10\text{ m} = 4.712\text{ m}^2$, using the *IHT Correlations Tool*, *Internal Flow* for fully developed *Turbulent Flow* along with the energy balance equation, rate equation and pressure drop equations used above, the heat rate q and Δp are calculated and plotted below.





From above, as D increases, L decreases so that A_s remains unchanged. The decrease in heat rate with increasing diameter is nearly linear, while the pressure drop decreases markedly. This is the trade off: increased heat rate requires a more significant increase in pressure drop, and hence fan blower power requirements.



COMMENTS: (1) To check the calculations, compute q from Eq. 8.43, where $\Delta T_{\ell m}$ is given by Eq. 8.44. It follows that $\Delta T_{\ell m}$ = -27.1°C and q = -1206 W. The small difference in results may be attributed to round-off error.

(2) For part (a), a slight improvement in accuracy may be obtained by evaluating the properties at $\overline{T}_m$ = 318 K: $\overline{h}$ = 9.42 $W/m^2 \cdot K$, $T_{m,o}$ = 303 K = 30°C, q = -1211 W, f = 0.0271 and $\Delta p = 4.20\ N/m^2$.