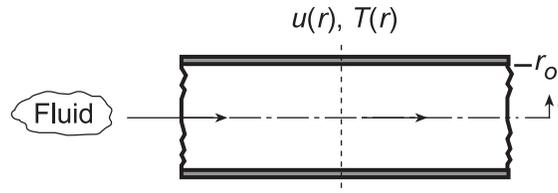


PROBLEM 8.7

KNOWN: Velocity and temperature profiles for laminar flow in a tube of radius $r_o = 10$ mm.

FIND: Mean (or bulk) temperature, T_m , at this axial position.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar incompressible flow, (2) Constant properties.

ANALYSIS: The prescribed velocity and temperature profiles, (m/s and K, respectively) are

$$u(r) = 0.1 [1 - (r/r_o)^2] \quad T(r) = 344.8 + 75.0 (r/r_o)^2 - 18.8 (r/r_o)^4 \quad (1,2)$$

For incompressible flow with constant c_v in a circular tube, from Eq. 8.26, the mean temperature and u_m , the mean velocity, from Eq. 8.8 are, respectively,

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} u(r) \cdot T(r) \cdot r \cdot dr \quad u_m = \frac{2}{r_o^2} \int_0^{r_o} u(r) \cdot r \cdot dr \quad (3,4)$$

Substituting the velocity profile, Eq. (1), into Eq. (4) and integrating, find

$$u_m = \frac{2}{r_o^2} \int_0^{r_o} 0.1 [1 - (r/r_o)^2] (r/r_o) d(r/r_o) = 2 \left\{ 0.1 \left[\frac{1}{2} (r/r_o)^2 - \frac{1}{4} (r/r_o)^4 \right] \right\}_0^1 = 0.05 \text{ m/s}$$

Substituting the profiles and u_m into Eq. (3), find

$$T_m = \frac{2}{(0.05 \text{ m/s}) r_o^2} \int_0^{r_o} \left\{ 0.1 [1 - (r/r_o)^2] \right\} \left\{ 344.8 + 75.0 (r/r_o)^2 - 18.8 (r/r_o)^4 \right\} \cdot (r/r_o) \cdot d(r/r_o)$$

$$T_m = 4 \int_0^1 \left\{ [344.8 (r/r_o) + 75.0 (r/r_o)^3 - 18.8 (r/r_o)^5] - [344.8 (r/r_o)^3 + 75.0 (r/r_o)^5 - 18.8 (r/r_o)^7] \right\} d(r/r_o)$$

$$T_m = 4 \{ [172.40 + 18.75 - 3.13] - [86.20 + 12.50 - 2.35] \} = 367 \text{ K} \quad <$$

The velocity and temperature profiles appear as shown below. Do the values of u_m and T_m found above compare with their respective profiles as you thought? Is the fluid being heated or cooled?

