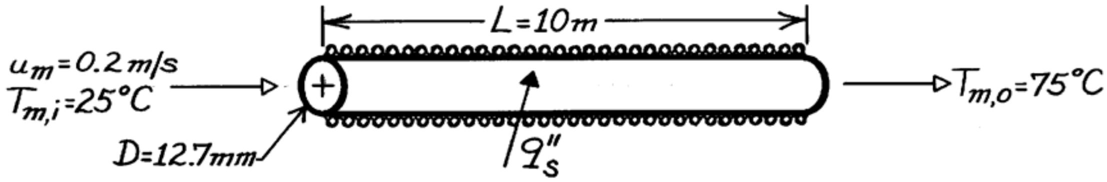
PROBLEM 8.27

KNOWN: Inlet and outlet temperatures and velocity of fluid flow in tube. Tube diameter and length.

FIND: Surface heat flux and temperatures at x = 0.5 and 10 m.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to surroundings, (4) Incompressible liquid with negligible viscous dissipation, (5) Negligible axial conduction.

PROPERTIES: Pharmaceutical (given): $\rho = 1000\ \text{kg/m}^3$, $c_p = 4000$ J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, k = 0.80 W/m·K, Pr = 10.

ANALYSIS: With

$$\dot{m} = \rho V A = 1000\ \text{kg/m}^3 (0.2\ \text{m/s}) \pi (0.0127\ \text{m})^2 / 4 = 0.0253\ \text{kg/s}$$

Eq. 8.34 yields

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.0253\ \text{kg/s}(4000\ \text{J/kg}\cdot\text{K})50\ \text{K} = 5060\ \text{W}.$$

The required heat flux is then

$$q_s'' = q/A_s = 5060\ \text{W}/\pi(0.0127\ \text{m})10\ \text{m} = 12{,}682\ \text{W/m}^2. \qquad <$$

With

$$Re_D = \rho V D/\mu = 1000\ \text{kg/m}^3 (0.2\ \text{m/s}) 0.0127\ \text{m}/2\times10^{-3}\ \text{kg/s}\cdot\text{m} = 1270$$

the flow is laminar and Eq. 8.23 yields

$$x_{fd,t} = 0.05 Re_D\, Pr\, D = 0.05(1270)10(0.0127\ \text{m}) = 8.06\ \text{m}.$$

Hence, with fully developed hydrodynamic and thermal conditions at x = 10 m, Eq. 8.53 yields

$$h(10\ \text{m}) = Nu_{D,fd}(k/D) = 4.36(0.80\ \text{W/m}\cdot\text{K}/0.0127\ \text{m}) = 274.6\ \text{W/m}^2\cdot\text{K}.$$

Hence, from Newton's law of cooling,

$$T_{s,o} = T_{m,o} + \left(q_s''/h\right) = 75^\circ\text{C} + \left(12{,}682\ \text{W/m}^2/274.6\ \text{W/m}^2\cdot\text{K}\right) = 121^\circ\text{C}. \qquad <$$

At x = 0.5 m, $(x/D)/(Re_D Pr) = 0.0031$ and Figure 8.10 yields $Nu_D \approx 8$ for a thermal entry region with uniform surface heat flux. Hence, h(0.5 m) = 503.9 W/m²·K and, since T_m increases linearly with x, $T_m(x = 0.5\ \text{m}) = T_{m,i} + (T_{m,o} - T_{m,i})(x/L) = 27.5^\circ\text{C}$. It follows that

$$T_s(x = 0.5\ \text{m}) \approx 27.5^\circ\text{C} + \left(12{,}682\ \text{W/m}^2/503.9\ \text{W/m}^2\cdot\text{K}\right) = 52.7^\circ\text{C}. \qquad <$$