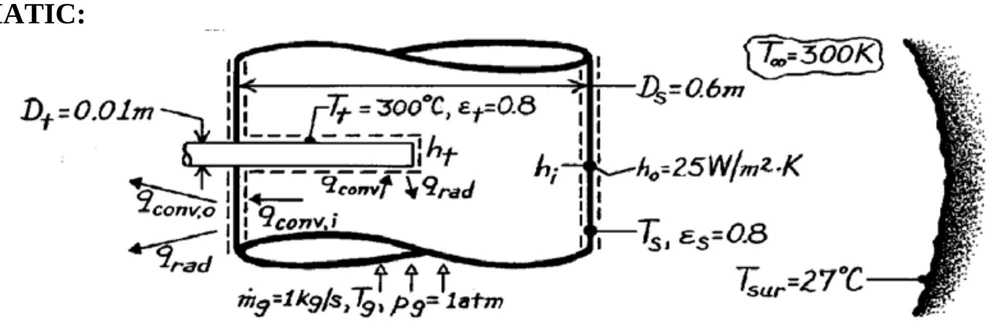
PROBLEM 8.75

KNOWN: Temperature recorded by a thermocouple inserted in a stack containing flue gases with a prescribed flow rate. Diameters and emissivities of thermocouple tube and gas stack. Conditions associated with stack surroundings.

FIND: Equations for predicting thermocouple error and error associated with prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Flue gas has properties of air at $T_g \approx 327°C$, (3) Stack forms a large enclosure about the thermocouple tube and surroundings form a large enclosure around the stack, (4) Stack surface energy balance is unaffected by heat loss to tube, (5) Gas flow is fully developed, (6) Negligible conduction along thermocouple tube, (7) Stack wall is thin.

PROPERTIES: *Table A-4*, Air ($T_g \approx 600K$, $p_g = 1$ atm): $\rho = 0.58\ kg/m^3$, $\mu = 305.8 \times 10^{-7}\ N\cdot s/m^2$, $\nu = 52.7\times 10^{-6}\ m^2/s$, $k = 0.0469\ W/m\cdot K$, $Pr = 0.685$.

ANALYSIS: Determination of the thermocouple error necessitates determining the gas temperature T_g and relating it to the thermocouple temperature T_t. From an energy balance applied to a control surface about the thermocouple,

$$q_{conv} = q_{rad} \qquad \text{or} \qquad h_t A_t \left(T_g - T_t\right) = \varepsilon_t \sigma A_t \left(T_t^4 - T_s^4\right).$$

Hence

$$T_g = T_t + \frac{\varepsilon_t \sigma}{h_t}\left(T_t^4 - T_s^4\right). \qquad (1) \quad <$$

However, T_s is unknown and must be determined from an energy balance on the stack wall.

$$q_{conv,i} = q_{conv,o} + q_{rad}$$

$$h_i A_s \left(T_g - T_s\right) = h_o A_s \left(T_s - T_\infty\right) + \varepsilon_s \sigma A_s \left(T_s^4 - T_{sur}^4\right)$$

or

$$T_g = T_s + \frac{h_o}{h_i}\left(T_s - T_\infty\right) + \frac{\varepsilon_s \sigma}{h_i}\left(T_s^4 - T_{sur}^4\right). \qquad (2) \quad <$$

T_g and T_s may be determined by simultaneously solving Eqs. (1) and (2). For the prescribed conditions

$$Re_{Dt} = \frac{\rho V D_t}{\mu} = \frac{\rho\left(\dot{m}_g / \rho\pi D_s^2/4\right) D_t}{\mu} = \frac{4\ \dot{m}_g D_t}{\pi\mu D_s^2} = \frac{4\times 1\ kg/s \times 0.01\ m}{\pi \times 305.8\times 10^{-7}\ N\cdot s/m^2\ \left(0.6\ m\right)^2} = 1157.$$

Continued …

PROBLEM 8.75 (Cont.)

Assuming $(Pr/Pr_s) = 1$, it follows from the Zukauskus correlation

$$\overline{Nu}_D = 0.26 Re_{Dt}^{0.6} Pr^{0.37}$$

where C = 0.26 and m = 0.6 from Table 7.4. Hence

$$h_t = \frac{0.0469\ W/m\cdot K}{0.01\ m}(1157)^{0.6}(0.685)^{0.37} \times 0.26 = 73\ W/m^2\cdot K.$$

Hence, from Eq. (1) $\quad T_g = 573\ K + \dfrac{0.8\times 5.67\times 10^{-8}\ W/m^2\cdot K^4}{73\ W/m^2\cdot K}\left(573^4 - T_s^4\right)K^4$

$$T_g = 573\ K + 67\ K - 6.214\times 10^{-10} T_s^4 = 640 - 6.214\times 10^{-10} T_s^4. \qquad (1a)$$

Also, $\quad Re_{Ds} = \dfrac{4\,\dot{m}_g}{\pi\, D_s \mu} = \dfrac{4\times 1\ kg/s}{\pi(0.6\ m)305.8\times 10^{-7}\ N\cdot s/m^2} = 6.94\times 10^4$

and the gas flow is turbulent. Hence from the Dittus-Boelter correlation,

$$h_i = \frac{k}{D_s} 0.023 Re_{Ds}^{4/5} Pr^{0.3} = \frac{0.0469\ W/m\cdot K}{0.6\ m}\times 0.023\left(6.94\times 10^4\right)^{4/5}\times(0.685)^{0.3} = 12\ W/m^2\cdot K.$$

Hence from Eq. (2)

$$T_g = T_s + \frac{25}{12}(T_s - 300\ K) + \frac{0.8\times 5.67\times 10^{-8}\ W/m^2\cdot K^4}{12\ W/m^2\cdot K}\left[T_s^4 - 300^4\right]K^4$$

$$T_g = T_s + 2.083 T_s - 625\ K + 3.78\times 10^{-9} T_s^4 - 30.6\ K = -655.6\ K + 3.083 T_s + 3.78\times 10^{-9} T_s^4. \qquad (2a)$$

Solve Eqs. (1a) and (2a) by trial-and-error. Assume values for T_s and determine T_g from (1a) and (2a). Continue until values of T_g agree.

T_s (K)	T_g (K) $\rightarrow$ (1a)	T_g (K) $\rightarrow$ (2a)
400	624	674
375	628	575
387	626	622
388	626	626

Hence $\quad T_s = 388\ K,\ T_g = 626\ K$

and the thermocouple error is $\quad T_g - T_t = 626\ K - 573\ K = 53°C.$ $\quad <$

COMMENTS: The thermocouple error results from radiation exchange between the thermocouple tube and the cooler stack wall. Anything done to $\uparrow T_s$ would $\downarrow$ this error (e.g., $\downarrow h_o$ or $\uparrow T_\infty$ and T_{sur}). The error also $\downarrow$ with $\uparrow h_t$. The error could be reduced by installing a radiation shield around the tube.