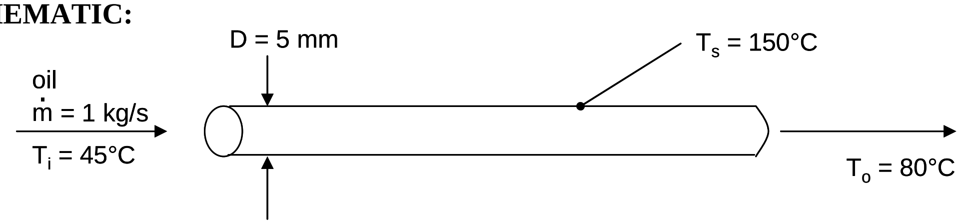
PROBLEM 8.29

KNOWN: Oil flow rate. Pipe diameter. Inlet, outlet, and pipe surface temperatures.

FIND: Length of tube required to achieve desired outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Incompressible flow, (3) Negligible viscous dissipation.

PROPERTIES: Table A-5, Engine oil (T_i = 45°C = 318 K): $\mu_i = 16.3 \times 10^{-2}$ N·s/m²; (T_o = 80°C = 353 K): $\mu_o = 3.25 \times 10^{-2}$ N·s/m².

ANALYSIS: We begin by calculating the Reynolds numbers at the inlet and outlet, from Equation 8.6,

$$Re_{Di} = \frac{4\,\dot{m}}{\pi\, D\, \mu_i} = \frac{4 \times 1\ \text{kg/s}}{\pi \times 0.005\ \text{m} \times 16.3 \times 10^{-2}\text{N}\cdot\text{s/m}^2} = 1560$$

$$Re_{Do} = \frac{4 \times 1\ \text{kg/s}}{\pi \times 0.005\ \text{m} \times 3.25 \times 10^{-2}\text{N}\cdot\text{s/m}^2} = 7840$$

Therefore the flow is laminar at the inlet and turbulent at the outlet. The transition occurs when Re_D = 2300, that is, where

$$\mu = \frac{4\,\dot{m}}{\pi\, D\, 2300} = \frac{4 \times 1\ \text{kg/s}}{\pi \times 0.005\ \text{m} \times 2300} = 11.1 \times 10^{-2}\text{N}\cdot\text{s/m}^2$$

From Table A-5, this occurs at a transition temperature of $T_{m,t}$ = 325 K = 52°C. Now we proceed to analyze separately the heat transfer in the laminar and turbulent regions.

Laminar Region. The mean temperature in the laminar region is $\overline{T}_{m1}$ = (45°C + 52°C)/2 = 48.5°C = 321.5 K. The properties are c_{p1} = 1999 J/kg·K, $\mu_1 = 13.2 \times 10^{-2}$ N·s/m², k_1 = 0.143 W/m·K, Pr_1 = 1851. We recalculate the Reynolds number,

$$Re_{D1} = \frac{4\,\dot{m}}{\pi\, D\, \mu_1} = \frac{4 \times 1\ \text{kg/s}}{\pi \times 0.005\ \text{m} \times 13.2 \times 10^{-2}\text{N}\cdot\text{s/m}^2} = 1930$$

The hydrodynamic and thermal entry lengths are given by

$$x_{fd,h} = 0.05\, Re_{Di}\, D = 0.05 \times 1930 \times 0.005\ \text{m} = 0.48\ \text{m}$$

$$x_{fd,t} = x_{fd,h} \cdot Pr_i = 0.48\ \text{m} \times 1851 = 890\ \text{m}$$

Based on this information, we assume the flow is hydrodynamically developed but thermally developing, and use Equations 8.56 and 8.57 for the Nusselt number (with Pr > 5),

$$\overline{Nu}_{D1} = \overline{h}_1 D/k_1 = 3.66 + \frac{0.0668\,(D/L_1)\,Re_{D1}\,Pr_1}{1 + 0.04\left[(D/L_1)\,Re_{D1}\,Pr_1\right]^{2/3}} \qquad (1)$$

where L_1 is the length of the laminar region, which is as yet unknown. We can also use Equation 8.42 for the mean temperature variation:

$$\frac{T_s - T_{m,t}}{T_s - T_i} = \exp\left(-\frac{\pi\, D\, L_1}{\dot{m}\, c_{p1}}\overline{h}_1\right)$$

Continued…

Solving for $\overline{h}_1 L_1$, we have

$$\overline{h}_1 L_1 = -\frac{\dot{m} c_{p1}}{\pi D}\ln\left(\frac{T_s - T_{m,t}}{T_s - T_i}\right) = -\frac{1\text{ kg/s} \times 1999\text{ J/kg}\cdot\text{K}}{\pi \times 0.005\text{ m}}\ln\left(\frac{150°\text{C} - 52°\text{C}}{150°\text{C} - 45°\text{C}}\right)$$

$$= 8780\text{ W/m}\cdot\text{K} \qquad (2)$$

We can solve by iterating between Equations (1) and (2). Beginning with the estimate $\overline{Nu}_{D1} = 3.66$, we find $\overline{h}_1 = 3.66\, k_1/D = 105$ W/m^2·K. From Equation (2), $L_1 = 84$ m. Then from Equation (1), $\overline{Nu}_{D1} = 22.3$ and $\overline{h}_1 = 639$ W/m^2·K. Continuing the iterations, we find $\overline{Nu}_{D1} = 16.9$, $\overline{h}_1 = 484$ W/m^2·K, and $L_1 = 18.1$ m.

Turbulent Range. The mean temperature in the turbulent region is $\overline{T}_{m2} = (52°\text{C} + 80°\text{C})/2 = 66°\text{C} = 339$ K. The properties are $c_{p2} = 2072$ J/kg·K, $\mu_2 = 5.62 \times 10^{-2}$ N·s/m^2, $k_2 = 0.139$ W/m·K, $Pr_2 = 834$. Thus

$$Re_{D2} = \frac{4\dot{m}}{\pi D \mu_2} = 4530$$

We assume the flow is fully-developed hydrodynamically and thermally and use Equation 8.62,

$$Nu_{D2} = \frac{(f/8)(Re_{D2} - 1000)Pr_2}{1 + 12.7(f/8)^{1/2}(Pr_2^{2/3} - 1)}$$

where from Equation 8.21,

$$f = (0.790 \ln Re_{D2} - 1.64)^{-2} = (0.790 \ln(4530) - 1.64)^{-2} = 0.0398$$

Thus

$$Nu_{D2} = \frac{(0.0398/8)(4530 - 1000)\,834}{1 + 12.7(0.0398/8)^{1/2}(834^{2/3} - 1)} = 184$$

and $h_2 = Nu_{D2} k_2/D = 5120$ W/m^2·K. Then the required length L_2 can be found from Equation 8.42, expressed between the transition point and the outlet,

$$\frac{T_s - T_o}{T_s - T_{m,t}} = \exp\left(-\frac{\pi D L_2}{\dot{m} c_{p2}}\overline{h}_2\right)$$

$$L_2 = -\frac{\dot{m} c_{p2}}{\pi D \overline{h}_2}\ln\left(\frac{T_s - T_o}{T_s - T_{m,t}}\right) = -\frac{1\text{ kg/s} \times 2072\text{ J/kg}\cdot\text{K}}{\pi \times 0.005\text{ m} \times 5120\text{ W/m}^2\cdot\text{K}}\ln\left(\frac{150°\text{C} - 80°\text{C}}{150°\text{C} - 52°\text{C}}\right)$$

$$= 8.7\text{ m}$$

The total required length is $L = L_1 + L_2 = 26.8$ m. <

COMMENTS: (1) If we had simply calculated the properties based on the mean temperature of $\overline{T}_m = (45°\text{C} + 80°\text{C})/2 = 62.5°\text{C} = 335.5$ K, we would have found $Re_D = 3810$. Assuming the flow to be turbulent throughout would have resulted in a higher average Nusselt number, $\overline{Nu}_D = 159$, and correspondingly lower total length, $L = 11.9$ m. The variation of properties with temperature can be very important for some fluids such as oils. (2) If the oil were being cooled by exposure to a cooler wall, the Reynolds number could decrease from a turbulent to a laminar value. The flow would likely not completely "relaminarize," and the heat transfer in the section for which $Re_D < 2300$ would fall between the values calculated using laminar and turbulent Nusselt number correlations.