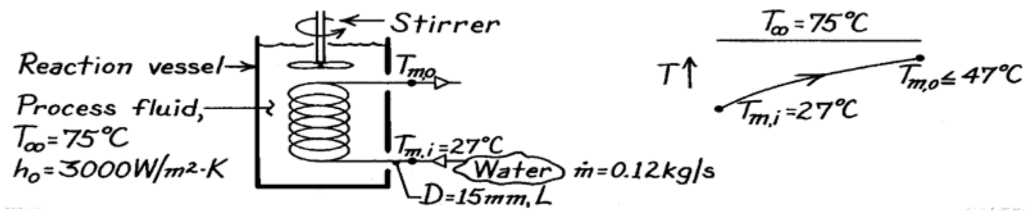
PROBLEM 8.68

KNOWN: Reaction vessel with process fluid at 75°C cooled by water at 27°C and 0.12 kg/s through 15 mm tube. High convection coefficient on outside of tube (3000 $W/m^2 \cdot K$) created by vigorous stirring.

FIND: (a) Maximum heat transfer rate if outlet temperature of water cannot exceed $T_{m,o}$ = 47°C, and (b) Required tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2)Incompressible liquid with negligible viscous dissipation, (3) Negligible thermal resistance of tube wall.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = (47+27)^\circ C/2 = 310K\right)$: $\rho = 1/v_f$ = 993.1 kg/m^3, c_p = 4178 J/kg·K, $\mu = 695 \times 10^{-6}$ $N \cdot s/m^2$, k = 0.628 W/m·K, Pr = 4.62.

ANALYSIS: (a) From an overall energy balance on the tube with $T_{m,o}$ = 47°C,

$$q_{max} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.12 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K} (47-27)^\circ \text{C} = 10{,}027 \text{ W.} \qquad <$$

(b) For the constant surface temperature heating condition, from Eq. 8.45a,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p}\overline{U}\right) \qquad \text{where} \qquad 1/U = 1/\overline{h}_o + 1/\overline{h}_i.$$

For internal flow in the tube, find

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.12 \text{ kg/s}}{\pi \times 0.015 \text{ m} \times 695 \times 10^{-6} \text{N} \cdot \text{s/m}^2} = 14{,}656$$

and the flow is turbulent. Assuming fully developed flow, use the Dittus-Boelter correlation with n = 0.4 (heating),

$$Nu_D = h_i D/k = 0.023 Re_D^{4/5} Pr^{0.4}$$

$$h_i = \left[0.628 \text{ W/m} \cdot \text{K}/0.015 \text{ m}\right] \times 0.023 (14{,}656)^{4/5} (4.62)^{0.4} = 3822 \text{ W/m}^2 \cdot \text{K}.$$

Hence, 1/U = [1/3000 + 1/3822] $m^2 \cdot K/W$ or U = 1680 $W/m^2 \cdot K$. From the energy balance relation with $P = \pi D$, find

$$\frac{(75-47)^\circ \text{C}}{(75-27)^\circ \text{C}} = \exp\left(-\frac{\pi (0.015 \text{ m}) L \times 1680 \text{ W/m}^2 \cdot \text{K}}{0.12 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K}}\right) \qquad L = 3.4 \text{ m.} \qquad <$$

COMMENTS: Note that L/D = 227 and the fully developed flow assumption is appropriate.