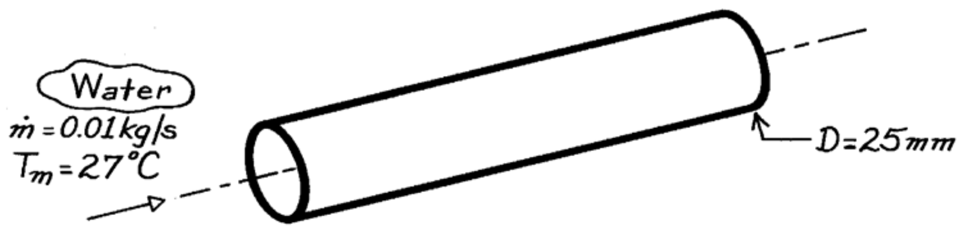
PROBLEM 8.1

KNOWN: Flowrate and temperature of water in fully developed flow through a tube of prescribed diameter.

FIND: Maximum velocity and pressure gradient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal flow, (3) Horizontal tube.

PROPERTIES: *Table A-6*, Water (300 K): $\rho = 998\ \text{kg/m}^3$, $\mu = 855 \times 10^{-6}\ \text{N·s/m}^2$.

ANALYSIS: From Eq. 8.6,

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01\ \text{kg/s}}{\pi(0.025\text{m})\ 855 \times 10^{-6}\ \text{kg/m} \cdot \text{s}} = 596.$$

Hence the flow is laminar and the velocity profile is given by Eq. 8.15,

$$\frac{u(r)}{u_m} = 2\left[1 - (r/r_o)^2\right].$$

The maximum velocity is therefore at r = 0, the centerline, where

$$u(0) = 2\ u_m.$$

From Eq. 8.5

$$u_m = \frac{\dot{m}}{\rho\pi\ D^2/4} = \frac{4 \times 0.01\ \text{kg/s}}{998\ \text{kg/m}^3 \times \pi(0.025\text{m})^2} = 0.020\ \text{m/s},$$

hence

$$u(0) = 0.041\ \text{m/s}.$$

Combining Eqs. 8.16 and 8.19, the pressure gradient is

$$\frac{dp}{dx} = -\frac{64}{\text{Re}_D}\ \frac{\rho u_m^2}{2D}$$

$$\frac{dp}{dx} = -\frac{64}{596} \times \frac{998\ \text{kg/m}^3\ (0.020\ \text{m/s})^2}{2 \times 0.025\ \text{m}} = -0.86\ \text{kg/m}^2 \cdot \text{s}^2$$

$$\frac{dp}{dx} = -0.86\text{N/m}^2 \cdot \text{m} = -0.86 \times 10^{-5}\ \text{bar/m}.$$ <