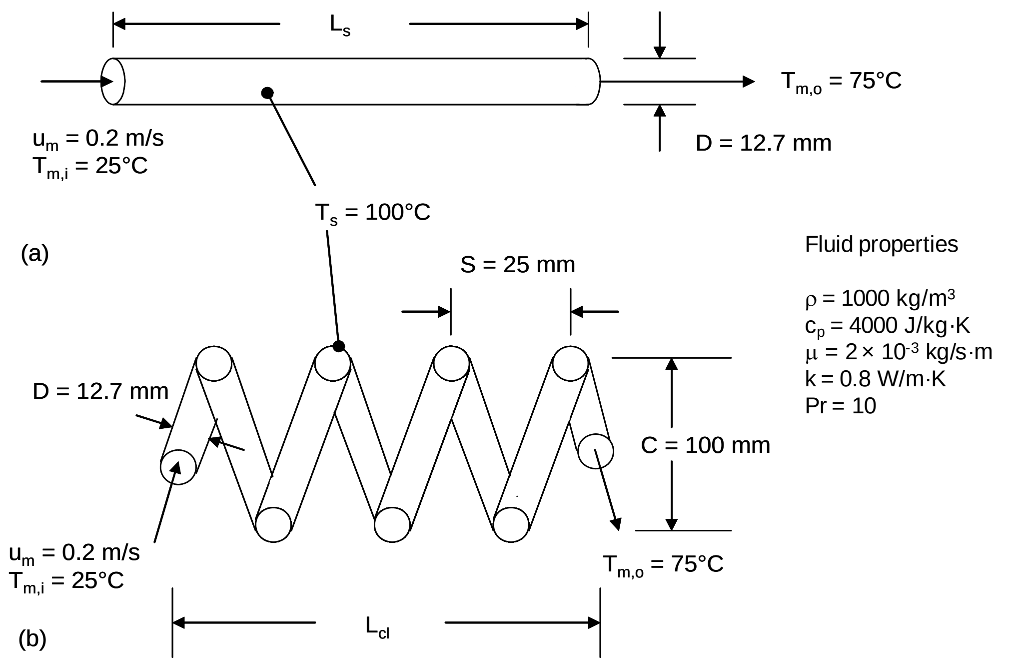
PROBLEM 8.101

KNOWN: Inlet and desired outlet temperature of a pharmaceutical fluid flowing in a straight tube or coiled tube of known diameter. Inlet velocity and tube surface temperature.

FIND: (a) Length of straight tube needed to achieve the desired outlet temperature, (b) Length of coiled tube to achieve the desired outlet temperature, (c) Pressure drops associated with the straight and coiled tubes, (d) Steam condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Incompressible liquid and negligible viscous dissipation, (3) Steady-state conditions, (4) fully developed hydrodynamic conditions at the entrance.

PROPERTIES: Steam (Table A.6): h_{fg} (T = 100°C) = 2257 kJ/kg. Pharmaceutical (given): ρ = 1000 kg/m^3, c_p = 4000 J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, k = 0.80 W/m·K, Pr = 10.

ANALYSIS:

(a) From Problem 8.27, $Re_D = \rho u_m D/\mu = 1270$ and the flow is laminar for both cases. Hence, augmentation is expected to occur in the coiled tube. For the straight tube case a, the Hausen correlation is written as

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = 3.66 + \frac{0.0668 \times (D/L_s)\, Re_D Pr}{1 + 0.04\left[(D/L_s)\, Re_D Pr\right]^{2/3}}$$

which may be rearranged to yield

Continued…

$$\overline{h} = \frac{k}{D}\left\{3.66 + \frac{0.0668\ (D/L_s)\ Re_D Pr}{1 + 0.04\left[(D/L_s)\ Re_D Pr\right]^{2/3}}\right\}$$

$$\overline{h} = \frac{0.80\ W/mK}{12.7 \times 10^{-3}\ m}\left\{3.66 + \frac{0.0668(12.7 \times 10^{-3}\ m/L_s) \times 1270 \times 10}{1 + 0.04\left[\left(12.7 \times 10^{-3}\ m/L_s\right) \times 1270 \times 10\right]^{2/3}}\right\} \qquad (1)$$

From Problem 8.27 $\dot{m} = 0.0253$ kg/s and the tube perimeter is
$P = \pi D = \pi \times 12.7 \times 10^{-3}\ m = 39.9 \times 10^{-3}\ m$

Equation 8.41b may be written

$$\frac{100°C - 75°C}{100°C - 25°C} = \exp\left(-\frac{39.9 \times 10^{-3} m \times L_s}{0.0253\ kg/s \times 4000\ J/kg \cdot K} \times \overline{h}\right) \qquad (2)$$

Equations (1) and (2) may be solved simultaneously to yield

$L_s = 9.77$ m, ($\overline{h}$ = 286 W/m^2·K) <

(b) For the coiled tube,

$$Re_D (D/C)^{1/2} = 1270 \times (12.7/100)^{1/2} = 452.6$$

Therefore, C/D = 100/12.7 = 7.87 > 3, Equation 8.77 yields

$$a = \left(1 + \frac{957(C/D)}{Re_D^2 Pr}\right) = \left(1 + \frac{957 \times (100/12.7)}{1270^2 \times 10}\right) = 1.0005$$

$$b = 1 + \frac{0.477}{Pr} = 1 + \frac{0.477}{10} = 1.0477$$

Therefore Equation 8.76 becomes

$$Nu_D = \left[\left(3.66 + \frac{4.343}{1.0005}\right)^3 + 1.158 \times \left(\frac{1270 \times (12.7/100)^{1/2}}{1.0477}\right)^{3/2}\right]^{1/3} = 22.18$$

Therefore,

$$h = Nu_D \frac{k}{D} = 22.18 \times \frac{0.80\ W/m \cdot K}{12.7 \times 10^{-3}\ m} = 1397\ W/m^2 \cdot K$$

Equation 8.41b may be written

Continued…

PROBLEM 8.101 (Cont.)

$$\frac{100°C - 75°C}{100°C - 25°C} = \exp\left(-\frac{3.99 \times 10^{-3} m \times L_c}{0.0253 \text{ kg/s} \times 4000 \text{ J/kg}\cdot K} \times 1397 \text{ W/m}^2 \cdot K\right)$$

or $L_c = 2.00$ m

The number of coil turns is $N = \frac{L_c}{\pi C} = \frac{2.00 \text{ m}}{\pi \times 100 \times 10^{-3} \text{ m}} = 6.4$

The coil length is $L_{cl} = NS = 6.4 \times 25 \times 10^{-3} \text{ m} = 159 \times 10^{-3} \text{ m} = 159 \text{ mm}$ <

(c) The flow is hydrodynamically fully-developed in the straight tube. From Equations 8.19 and 8.22a,

$$\Delta p_s = \frac{64}{Re_D}\frac{\rho u_m^2}{2D} L_s = \frac{64}{1270} \times \frac{1000 \text{ kg/m}^3 \times (0.2 \text{ m/s})^2}{2 \times 12.7 \times 10^{-3} \text{ m}} \times 9.77 \text{ m} = 775 \text{ N/m}^2$$ <

For the coiled tube, Equation 8.75b is

$$f = \frac{7.2}{Re_D^{0.5}} (D/C)^{0.25} = \frac{7.2}{1270^{0.5}} \times \left(\frac{12.7}{100}\right)^{0.25} = 0.121$$

$$\Delta p_c = f\frac{\rho u_m^2}{2D} L_c = 0.121 \times \frac{1000 \text{ kg/m}^3 \times (0.2 \text{ m/s})^2}{2 \times 12.7 \times 10^{-3} \text{ m}} \times 2.00 \text{ m} = 379 \text{ N/m}^2$$ <

(d) The steam condensation rate, $\dot{m}_{st}$, is

$$\dot{m}_{st} h_{fg} = \dot{m} c_p (T_{m,o} - T_{m,i}) = u_m \rho A c_p (T_{m,o} - T_{m,i})$$

or

$$\dot{m}_{st} = \frac{0.2 \text{ m/s} \times 1000 \text{ kg/m}^3 \times \pi \times (12.7 \times 10^{-3} \text{ m})^2 \times 4000 \text{ J/kg}\cdot K \times (75 - 25)°C}{4 \times 2257 \times 10^3 \text{ J/kg}}$$

$$\dot{m}_{st} = 2.25 \times 10^{-3} \text{ kg/s}$$ <

COMMENTS: (1) For the straight tube, $x_{fd,t} = 0.05 Re_D Pr D = 0.05 \times 1270 \times 10 \times 12.7 \times 10^{-3}$ m = 8m. The value of the entrance length for the coiled tube will be 20 to 50 percent shorter than for the straight tube or between approximately 4 and 6 m. The flow in the coiled tube is not fully developed, and actual heat transfer rates will exceed those predicted using Equation 8.76. (2) The coiled tube requires (2/9.77) × 100 = 20 percent of the tube length relative to the straight tube case. (3) The coil length is (0.159/9.77) × 100 = 1.6 percent that of the straight tube. (4) The pressure drop in the coiled tube is (379/775) × 100 = 48 percent that of the straight tube. (5) The coiled tube will induce secondary flow in the pharmaceutical, thereby reducing radial temperature gradients in the liquid.