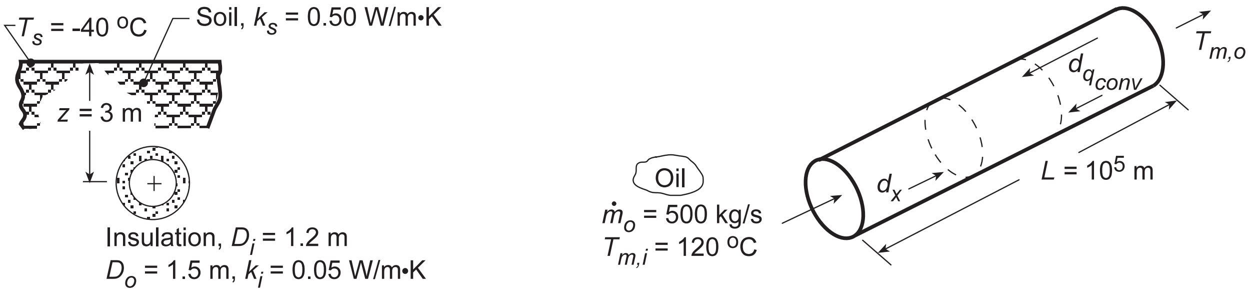
PROBLEM 8.63

KNOWN: Length, diameter, insulation characteristics and burial depth of a pipe. Ground surface temperature. Inlet temperature, flow rate and properties of oil flowing through pipe.

FIND: (a) An expression for the oil outlet temperature, (b) Oil outlet temperature and pipe heat transfer rate for prescribed conditions, and (c) Design information for trade off between burial depth of pipe (z) and pipe insulation thickness (t) on the heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Two-dimensional conduction in soil, (4) Negligible pipe wall thermal resistance, (5) Total resistance to heat loss is independent of x, (6) Oil is incompressible liquid with negligible viscous dissipation.

PROPERTIES: Oil (given): $\rho_o = 900\ \text{kg/m}^3$, $c_{p,o} = 2000$ J/kg·K, $\nu_o = 8.5 \times 10^{-4}\ \text{m}^2/\text{s}$, $k_o = 0.140$ W/m·K, $Pr_o = 10^4$; Soil (given): $k_s = 0.50$ W/m·K; Insulation (given): $k_i = 0.05$ W/m·K.

ANALYSIS: (a) From Eq. 8.36 for a differential control volume in the oil and the rate equation

$$dq_{conv} = \dot{m}_o c_{p,o} dT_m = dq = \left(T_s - T_m\right)/R_{tot} \tag{1}$$

where the total resistance is expressed as

$$R_{tot} = R_{conv} + R_{cond,i} + R_{cond,s} = \left(\bar{h}\pi D dx\right)^{-1} + \frac{\ln\left(D_o/D_i\right)}{2\pi k_i dx} + \frac{1}{k_s S}$$

$$R_{tot} = \left(\frac{1}{\bar{h}\pi D_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi k_i} + \frac{\cosh^{-1}\left(2z/D_o\right)}{2\pi k_s}\right)/\,dx = R'_{tot}/dx \tag{2}$$

where, from Table 4.1,

$$S = 2\pi dx \Big/ \cosh^{-1}\left(2z/D_o\right) \tag{3}$$

It follows that

$$\frac{\left(T_s - T_m\right)dx}{R'_{tot}} = \dot{m}_o c_{p,o} dT_m \qquad\qquad \frac{dT_m}{T_s - T_m} = \frac{dx}{\dot{m}_o c_{p,o} R'_{tot}}$$

Integrating between inlet and outlet conditions

$$\int_{T_{m,i}}^{T_{m,o}} \frac{dT_m}{T_m - T_s} = -\int_0^L \frac{dx}{\dot{m}_o c_{p,o} R'_{tot}}\,.$$

Assuming R'_{tot} to be independent of x and integrating,

$$\frac{T_{m,o} - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{L}{\dot{m}_o c_{p,o} R'_{tot}}\right). \tag{3}$$ <

Continued...

(b) To calculate $T_{m,o}$ for the prescribed conditions, begin by evaluating $\bar{h}$, where

$$Re_D = \frac{4\dot{m}_o}{\pi D_i \rho_o \nu_o} = \frac{4 \times 500\,\mathrm{kg/s}}{\pi(1.2\,\mathrm{m})900\,\mathrm{kg/m^3} \times 8.5 \times 10^{-4}\,\mathrm{m^2/s}} = 694 \quad (4)$$

Hence, the flow is laminar, and with $Pr_o > 5$, the Hausen correlation is appropriate,

$$\overline{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{2/3}} \quad (5)$$

$$Gz_D = (D_i/L) Re_D\, Pr = \left(\frac{1.2}{10^5}\right)(694)10^4 = 83.3 \qquad \overline{Nu}_D = 6.82$$

$$\bar{h} = \frac{k}{D_i} 6.82 = \frac{0.14\,\mathrm{W/m \cdot K}}{1.2\,\mathrm{m}} 6.82 = 0.80\,\mathrm{W/m^2 \cdot K}$$

From Eq. (2), the overall thermal resistance is

$$R'_{tot} = \frac{1}{0.8\,\mathrm{W/m^2 \cdot K}\pi(1.2\,\mathrm{m})} + \frac{\ln(1.5/1.2)}{2\pi(0.05\,\mathrm{W/m \cdot K})} + \frac{\cosh^{-1}(4)}{2\pi(0.5\,\mathrm{W/m \cdot K})}$$

$$R'_{tot} = (0.33 + 0.71 + 0.66)\,\mathrm{K \cdot m/W} = 1.70\,\mathrm{K \cdot m/W}$$

and the oil outlet temperature can be calculated as

$$\frac{T_{m,o} - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{10^5\,\mathrm{m}}{500\,\mathrm{kg/s} \times 2000\,\mathrm{J/kg \cdot K} \times 1.7\,\mathrm{K \cdot m/W}}\right) = 0.943$$

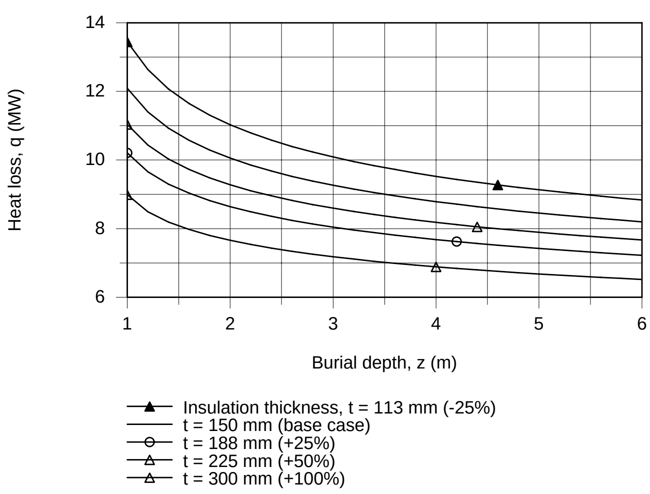
$T_{m,o} = 110.9°C$ $<$

The total rate of heat transfer *from* the pipeline is then

$$q = \dot{m}_o c_{p,o}\left(T_{m,i} - T_{m,o}\right) \quad (6)$$

$$q = 500\,\mathrm{kg/s} \times 2000\,\mathrm{J/kg \cdot K}(120 - 110.9)^\circ\mathrm{C} = 9.1 \times 10^6\,\mathrm{W}.$$ $<$

(c) Using the *IHT Workspace* with the foregoing equations, an analysis was performed to determine the heat loss, q, as a function of burial depth for the range, $1 \le z \le 6$ m, for thicknesses of insulation which are -25%, +25%, +50% and 100% that of the base case, $t = r_o - r_1 = 150$ mm.



Continued...

From this information, the operations manager can compare the costs associated with burial depth and insulation thickness with respect to acceptable heat loss.

COMMENTS: (1) Since the thermal entry region is very long, $x_{fd,t} \approx 0.05DRe_DPr = 4.16 \times 10^5$ m, h_x will be changing with x throughout the pipe. A more accurate solution would therefore be one in which Eq. (1) is integrated numerically, in a step-by-step fashion. For example, the integration could involve a step width of $\Delta x = 10^3$ m, with h and R'_t evaluated at each step.

(2) The three contributions to the total thermal resistance are comparable.

(3) In IHT 3.0, the inverse hyperbolic cosine function is "invcosh," so the shape factor can be found as:

```
// Shape factor:
S = 2 * pi / invcosh(arg)
arg = 2*z/Do
z = 3
Do = 1.5
```

Note that the argument of a function must be calculated separately in IHT. That is, we cannot use invcosh(2*z/Do).