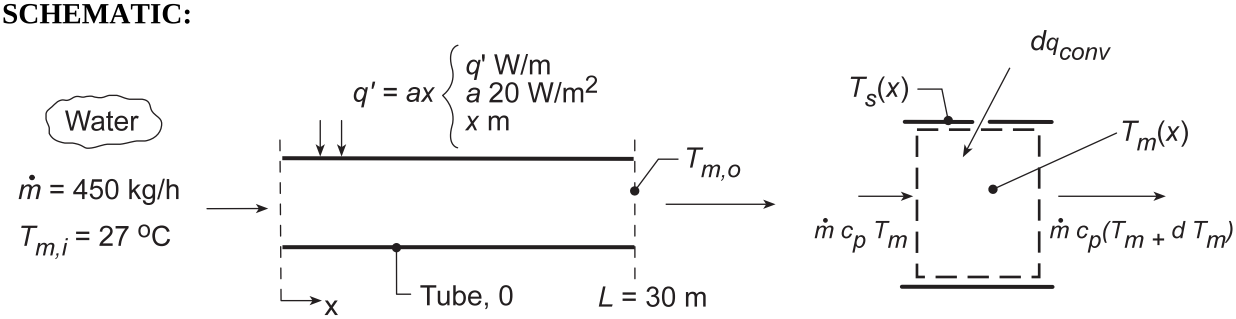
PROBLEM 8.12

KNOWN: Internal flow with prescribed wall heat flux as a function of distance.

FIND: (a) Beginning with a properly defined differential control volume, the temperature distribution, $T_m(x)$, (b) Outlet temperature, $T_{m,o}$, (c) Sketch $T_m(x)$, and $T_s(x)$ for fully developed *and* developing flow conditions, and (d) Value of uniform wall flux q_s'' (instead of $q_s' = ax$) providing same outlet temperature as found in part (a); sketch $T_m(x)$ and $T_s(x)$ for this heating condition.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A.6*, Water (300 K): $c_p = 4.179$ kJ/kg·K.

ANALYSIS: (a) Applying energy conservation to the control volume above,

$$dq_{conv} = \dot{m}c_p dT_m \qquad (1)$$

where $T_m(x)$ is the mean temperature at any cross-section and $dq_{conv} = q' \cdot dx$. Hence,

$$ax = \dot{m}c_p \frac{dT_m}{dx}. \qquad (2)$$

Separating and integrating with proper limits gives

$$a\int_{x=0}^{x} x\,dx = \dot{m}c_p \int_{T_{m,i}}^{T_m(x)} dT_m \qquad T_m(x) = T_{m,i} + \frac{ax^2}{2\dot{m}c_p} \qquad (3,4) <$$

(b) To find the outlet temperature, let x = L, then

$$T_m(L) = T_{m,o} = T_{m,i} + aL^2 / 2\dot{m}c_p. \qquad (5)$$

Solving for $T_{m,o}$, we find

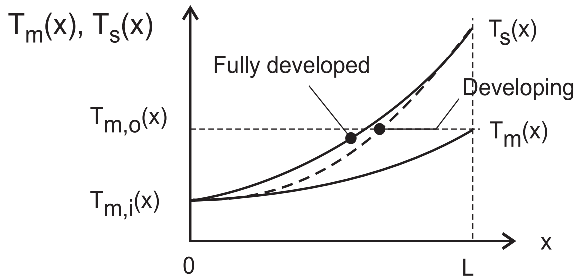
$$T_{m,o} = 27^\circ C + \frac{20\,W/m^2\left(30\,m^2\right)}{2\left(450\,kg/h/\left(3600\,s/h\right)\right)\times 4179\,J/kg\cdot K} = 27^\circ C + 17.2^\circ C = 44.2^\circ C. \quad <$$

(c) For *linear wall heating,* $q_s' = ax$, the fluid temperature distribution along the length of the tube is quadratic as prescribed by Eq. (4). From the convection rate equation,

$$q_s' = h(x)\cdot \pi D\left(T_s(x) - T_m(x)\right) \qquad (6)$$

For fully developed flow conditions, h(x) = h is a constant; hence, $T_s(x) - T_m(x)$ increases linearly with x. For developing conditions, h(x) will decrease with increasing distance along the tube eventually achieving the fully developed value.

Continued...



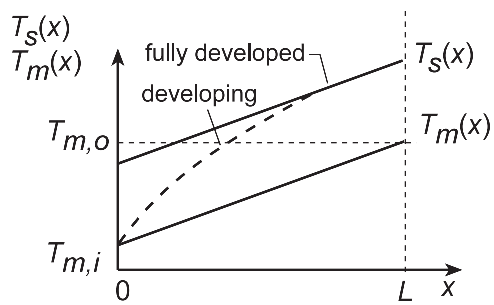
(d) For *uniform wall heat flux heating*, the overall energy balance on the tube yields

$$q = q_s'' \pi DL = \dot{m} c_p \left(T_{m,o} - T_{m,i} \right)$$

Requiring that $T_{m,o}$ = 44.2°C from part (a), find

$$q_s'' = \frac{(450/3600)\,\mathrm{kg/s} \times 4179\,\mathrm{J/kg\cdot K}\,(44.2 - 27)\,\mathrm{K}}{\pi D \times 30\,\mathrm{m}} = 95.3/D\ \mathrm{W/m^2} \qquad <$$

where D is the diameter (m) of the tube which, when specified, would permit determining the required heat flux, q_s''. For uniform heating, Section 8.3.2, we know that $T_m(x)$ will be linear with distance. $T_s(x)$ will also be linear for fully developed conditions and appear as shown below when the flow is developing.



COMMENTS: (1) Note that c_p should be evaluated at T_m = (27 + 44)°C/2 = 309 K.

(2) Why did we show $T_s(0) = T_m(0)$ for both types of history when the flow was developing?

(3) Why must $T_m(x)$ be linear with distance in the case of uniform wall flux heating?