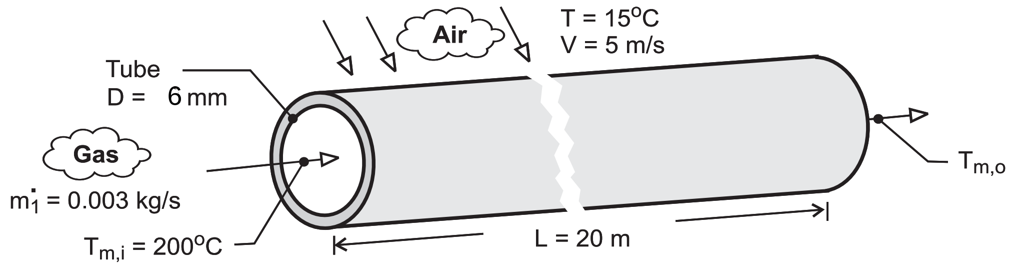
PROBLEM 8.70

KNOWN: Exhaust gases at 200°C and mass rate 0.03 kg/s enter tube of diameter 6 mm and length 20 m. Tube experiences cross-flow of autumn winds at 15°C and 5 m/s.

FIND: Average heat transfer coefficients for (a) exhaust gas inside tube and (b) air flowing across outside of tube, (c) Estimate overall coefficient and exhaust gas temperature at outlet of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation, (3) Negligible tube wall resistance, (4) Exhaust gas properties are those of air, (5) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (assume $T_{m,o} \approx 15°C$, hence $\overline{T}_m$ = 380 K, 1 atm): c_p = 1012 J/kg·K, k = 0.0323 W/m·K, $\mu = 221.6 \times 10^{-7}$ N·s/m^2, Pr = 0.694; Air (T_∞ = 15°C = 288 K, 1 atm): k = 0.0253 W/m·K, $\nu = 14.82 \times 10^{-6}$ m^2/s, Pr = 0.710; Air ($\overline{T}_s \approx 90°C$ = 363 K, 1 atm): Pr = 0.698.

ANALYSIS: (a) For the *internal flow* through the tube assuming a value for $T_{m,o}$ = 15°C, find

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 0.003 \text{ kg/s}}{\pi \times 0.006 \text{ m} \times 221.6 \times 10^{-7} \text{ N} \cdot \text{s/m}^2} = 2.873 \times 10^4.$$

Hence the flow is turbulent and, since L/D >> 10, fully developed. Using the Dittus-Doelter correlation with n = 0.3,

$$Nu_D = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023\left(2.873 \times 10^4\right)^{0.8} (0.694)^{0.3} = 76.0$$

$$h_i = Nu \cdot k/D = 76.0 \times 0.0323 \text{ W/m} \cdot \text{K}/0.006 \text{ m} = 409 \text{ W/m}^2 \cdot \text{K}.$$ <

(b) For *cross-flow* over the circular tube, find using thermophysical properties at T_∞,

$$Re_D = \frac{VD}{\nu} = \frac{5 \text{ m/s} \times 0.006 \text{ m}}{14.82 \times 10^{-6} \text{ m}^2/\text{s}} = 2024$$

and using the Zukauskus correlation with C = 0.26, m = 0.6, and n = 0.37,

$$Nu_D = C Re_D^m Pr^n \left(Pr/Pr_s\right)^{1/4} = 0.26(2024)^{0.6}\, 0.710^{0.37} (0.710/0.698)^{0.25} = 22.2$$

where Pr_s is evaluated at $\overline{T}_s$. Hence,

$$h_o = Nu_D \cdot k/D = 22.2 \times 0.0253 \text{ W/m} \cdot \text{K}/0.006 \text{ m} = 93.4 \text{ W/m}^2 \cdot \text{K}.$$ <

Continued …

PROBLEM 8.70 (Cont.)

(c) Assuming the thermal resistance of the tube wall is negligible,

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i} = \left(\frac{1}{93.4} + \frac{1}{409}\right) \mathrm{m^2 \cdot K/W} \qquad U = 76.1\ \mathrm{W/m^2 \cdot K}.$$ <

The gas outlet temperature can be determined from the expression where $P = \pi D$.

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PUL}{\dot{m}\, c_p}\right) = \exp\left(-\frac{\pi \times 0.006\ \mathrm{m} \times 76.1\ \mathrm{W/m^2 \cdot K} \times 20\ \mathrm{m}}{0.003\ \mathrm{kg/s} \times 1012\ \mathrm{J/kg \cdot K}}\right)$$

$$\frac{15 - T_{m,o}}{(15 - 200)^\circ \mathrm{C}} = 7.9 \times 10^{-5}$$

$$T_{m,o} = 15^\circ \mathrm{C}.$$ <

COMMENTS: (1) With $T_{m,o}$ = 15°C, find $\overline{T}_m$ = 380 K; hence thermophysical properties for the internal flow correlation were evaluated at a reasonable temperature. Note that the gas is cooled from 200°C to the ambient air temperature, $T_{m,o} = T_\infty$, over the 20-m length!

(2) The average wall surface temperature, $\overline{T}_s$, follows from an energy balance on the wall surface,

$$\frac{\overline{T}_m - \overline{T}_s}{\overline{T}_s - T_{inf}} = \frac{h_i}{h_o}$$

and substituting numerical values, find $\overline{T}_s = 90°\mathrm{C} = 363\ \mathrm{K}$, the value we assumed for evaluating Pr_s. Can you draw a thermal circuit to represent this energy balance relation?

(3) When using the Zukauskus correlation, it is reasonable to evaluate Pr_s at the $\overline{T}_m$ for the first trial. For gases the assumption is a safe one, but for liquids, especially oils, additional trials will be required since the Prandtl number may be strongly dependent upon temperature.