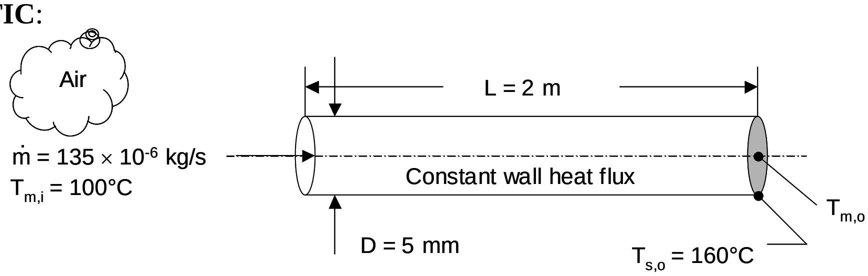
PROBLEM 8.30

KNOWN: Diameter and length of tube, air flow rate, air temperature and pressure at the tube inlet. Surface temperature at the tube exit.

FIND: (a) The heat transfer rate of the problem. (b) Conditions at the tube exit for reduced tube length. (c) Conditions at the tube exit for increased air flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible viscous dissipation.

PROPERTIES: *Table A.4*, Air ($\overline{T}_m \approx 400$ K, p = 1 atm): $\mu = 230.1\times10^{-7}$ N·s/m², Pr = 0.690, k = 0.0338 W/m·K, c_p = 1014 J/kg·K.

ANALYSIS: (a) We begin by calculating the Reynolds number

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times135\times10^{-6}\text{ kg/s}}{\pi\times0.005\text{m}\times230.1\times10^{-7}\,\text{N}\cdot\text{s/m}^2} = 1494$$

Therefore, the flow is laminar. The hydrodynamic and thermal entrance lengths are

$$x_{fd,h} = 0.05DRe_D = 0.05\times0.005\text{ m}\times1494 = 0.37\text{ m}$$
$$x_{fd,t} = x_{fd,h}Pr = 0.37\text{ m}\times0.690 = 0.26\text{ m}$$

Therefore, the flow is fully-developed at the tube exit. For fully-developed laminar flow with constant heat flux conditions, the Nusselt number is Nu_D = 4.36. Therefore, the local heat transfer coefficient at the tube exit is

$$h = 4.36k/D = 4.36\times0.0338\text{ W/m}\cdot\text{K}/0.005\text{m} = 29.47\text{ W/m}^2\cdot\text{K}$$

Two independent expressions for the heat flux may be written based upon application of Newton's law of cooling at the tube exit and an overall energy balance.

$$q'' = h(T_{s,o} - T_{m,o}) \quad ; \quad q'' = \frac{\dot{m}c_p(T_{m,o} - T_{m,i})}{\pi DL} \qquad (1, 2)$$

Equating Eqs. (1) and (2) yields

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$$T_{m,o} = \left[hT_{s,o} + \frac{\dot{m}c_p}{\pi DL} T_{m,i} \right] \Big/ \left[\frac{\dot{m}c_p}{\pi DL} + h \right]$$

$$= \frac{\left[29.47\,\text{W/m}^2 \cdot \text{K} \times 160°\text{C} + \dfrac{135 \times 10^{-6}\,\text{kg/s} \times 1014\ \text{J/kg} \cdot \text{K}}{\pi \times 0.005\ \text{m} \times 2\ \text{m}} \times 100°\text{C} \right]}{\left[\dfrac{135 \times 10^{-6}\,\text{kg/s} \times 1014\ \text{J/kg} \cdot \text{K}}{\pi \times 0.005\ \text{m} \times 2\ \text{m}} + 24.97\,\text{W/m}^2 \cdot \text{K} \right]}$$

$$= 152.3°\text{C}$$

Hence, the heat rate is

$$q = \dot{m}c_p (T_{m,o} - T_{m,i}) = 135 \times 10^{-6}\,\text{kg/s} \times 1014\text{J/kg} \cdot \text{K} \times 52.3°\text{C} = 7.16\ \text{W} \qquad <$$

(b) If L = 0.2 m, conditions at $x = L$ are not fully developed and the value of the heat transfer coefficient at the tube exit would exceed that of part (a).

(c) If the flow rate is increased by an order of magnitude, the Reynolds number will increase to Re_D = 14,940, and the flow will be turbulent at the tube exit. Since L/D = 2 m / 0.005 m = 400, the turbulent flow at the tube exit will also be fully developed. The heat transfer coefficient at the tube exit would exceed that of part (a).

COMMENTS: In part (b), the local heat transfer coefficient would exceed h = 29.47 W/m^2 at the tube exit and could be estimated using Fig. 8.10a. Specifically, for $Gz^{-1} = (x/D)/(Re_D Pr)$ = (0.2 m/ 0.005 m)/(1494 × 0.690) = 0.039, $Nu_D \approx 4.6$. Hence, h = 29.47 W/m^2 × (4.6/4.36) = 31.1 W/m^2·K. In part (c), the local heat transfer coefficient would exceed h = 29.47 W/m^2 and could be evaluated using the Dittus-Boelter correlation. Specifically, $Nu_D = 0.023 \times (14{,}940)^{4/5} 0.690^{0.4} = 43.3$. Hence, h = 29.47 W/m^2 × (43.3/4.36) = 292.7 W/m^2·K. For $T_{s,o}$ to remain the same, the heat rate associated with either part (b) or part (c) would have to exceed that of part (a).