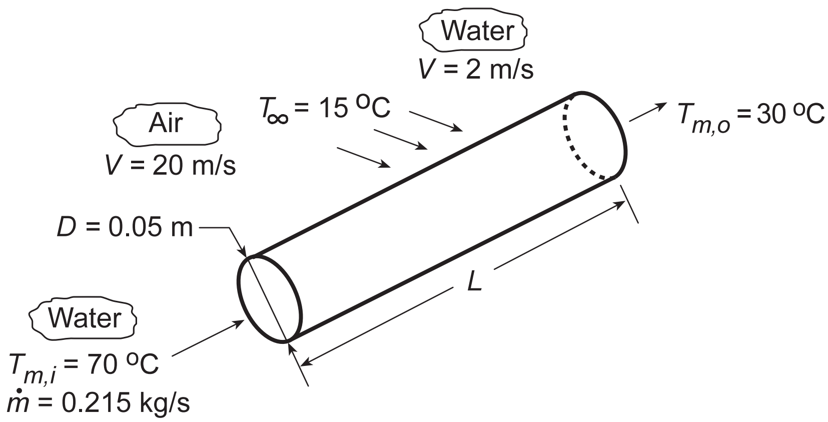
PROBLEM 8.62

KNOWN: Diameter of tube through which water of prescribed flow rate and inlet and outlet temperatures flows. Temperature of fluid in cross flow over the tube.

FIND: (a) Required tube length for air in cross flow at prescribed velocity, (b) Required tube length for water in cross flow at a prescribed velocity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) Negligible tube wall conduction resistance, (4)Water is incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A.6*, water ($\overline{T}_m$ = 50°C = 323 K): c_p = 4181 J/kg·K, $\mu = 548 \times 10^{-6}$ N·s/m^2, k = 0.643 W/m·K, Pr = 3.56. *Table A.4*, air (assume T_f = 300 K): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707. *Table A.6*, water (assume T_f = 300 K): $\nu = 0.858 \times 10^{-6}$ m^2/s, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: The required heat rate may be determined from the overall energy balance,

$$q = \dot{m}c_p\left(T_{m,i} - T_{m,o}\right) = 0.215\,\text{kg/s}\left(4181\,\text{J/kg}\cdot\text{K}\right)40^\circ\text{C} = 35,960\,\text{W}$$

and the required tube length may be determined from the rate equation, Eq. 8.46a,

$$L = \frac{q}{U\pi D\Delta T_{\ell m}}$$

where

$$\Delta T_{\ell m} = \frac{\left(T_{m,i} - T_\infty\right) - \left(T_{m,o} - T_\infty\right)}{\ell n\left(\dfrac{T_{m,i} - T_\infty}{T_{m,o} - T_\infty}\right)} = 30.8^\circ\text{C} \qquad \text{and} \qquad 1/U = 1/h_i + 1/h_o.$$

With

$$Re_{D_i} = 4\dot{m}/\pi D\mu = 0.860\,\text{kg/s}\Big/\pi\left(0.05\,\text{m}\right)548\times10^{-6}\,\text{N}\cdot\text{s}\big/\text{m}^2 = 9991$$

the flow is turbulent and, assuming fully developed flow throughout the tube, the inside convection coefficient is determined from Eq. 8.62

$$Nu_{D_i} = \frac{(f/8)(Re_{D_i} - 1000)\,Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} = \frac{(0.0315/8)(9991 - 1000)3.56}{1 + 12.7(0.0315/8)^{1/2}(3.56^{2/3} - 1)} = 61.1$$

where f = $(0.79\ \ln Re_{Di} = 1.64)^{-2}$ = 0.0315

$$h_i = Nu_{D_i}\,k/D = 61.1\left(0.643\,\text{W/m}\cdot\text{K}\right)\big/0.05\,\text{m} = 786\,\text{W}\big/\text{m}^2\cdot\text{K}$$

Continued...

(a) For air in cross flow at 20 m/s, Re_{D_o} = VD/ν = 20 m/s(0.05 m)/15.89 × 10^{-6} m^2/s = 62,933. From the Churchill/Bernstein correlation, it follows that

$$Nu_{D_o} = 0.3 + \frac{0.62\, Re_{D_o}^{1/2}\, Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_{D_o}}{282,000}\right)^{5/8}\right]^{4/5} = 158.7$$

$$h_o = Nu_{D_o}\, k/D = 158.7\left(0.0263\, W/m\cdot K\right)/0.05\, m = 83.5\, W/m^2\cdot K$$

Hence, U = $\left(1/h_i + 1/h_o\right)^{-1}$ = 75.5 W/m^2·K and

$$L = \frac{35,960\, W}{\left(75.5\, W/m^2\cdot K\right)\pi\left(0.05\, m\right)30.8^\circ C} = 98.5\, m$$ <

(b) For water in cross flow at 2 m/s, Re_{D_o} = 2 m/s(0.05 m)/0.858 × 10^{-6} m^2/s = 116,550, and the correlation yields $Nu_{D_o} = 527.3$. Hence,

$$h_o = Nu_{D_o}\, k/D = 527.3\left(0.613\, W/m\cdot K\right)/0.05\, m = 6,465\, W/m^2\cdot K$$

U = $\left(1/h_i + 1/h_o\right)^{-1}$ = 701 W/m^2·K

Hence,

$$L = \frac{35,960\, W}{\left(701\, W/m^2\cdot K\right)\pi\left(0.05\, m\right)30.8^\circ C} = 10.6\, m$$ <

COMMENTS: The foregoing results clearly indicate the superiority of water (relative to air) as a heat transfer fluid. Note the dominant contribution made by the smaller convection coefficient to the value of U in each of the two cases.