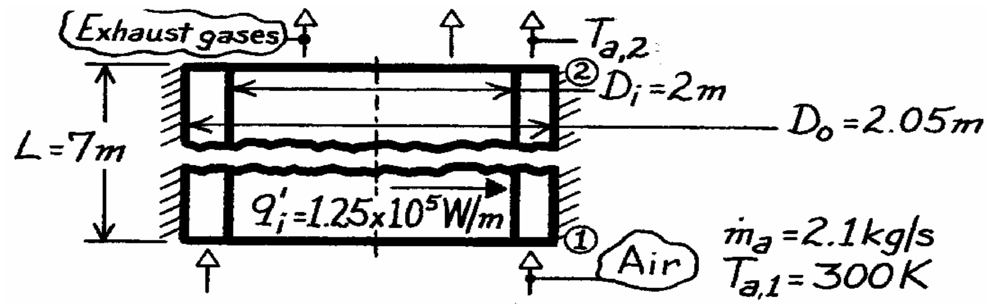
PROBLEM 8.98

KNOWN: Heat rate per unit length at the inner surface of an annular recuperator of prescribed dimensions. Flow rate and inlet temperature of air passing through annular region.

FIND: (a) Temperature of air leaving the recuperator, (b) Inner pipe temperature at inlet and outlet and outer pipe temperature at inlet.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heating of recuperator inner surface, (4) Adiabatic outer surface, (5)Air is ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed air flow throughout.

PROPERTIES: *Table A-4*, Air (given): c_p = 1030 J/kg·K, μ = 270 × 10^{-7} N·s/m^2, k = 0.041 W/m·K, Pr = 0.68.

ANALYSIS: (a) From an energy balance on the air

$$q_i' \; L = \dot{m}_a \; c_{p,a} \left(T_{a,2} - T_{a,1}\right)$$

$$T_{a,2} = T_{a,1} + \frac{q_i' \; L}{\dot{m}_a \; c_{p,a}} = 300 \text{ K} + \frac{1.25 \times 10^5 \text{W/m} \times 7\text{m}}{2.1 \text{ kg/s} \times 1030 \text{ J/kg} \cdot \text{K}} = 704.5 \text{ K}. \qquad <$$

(b) The surface temperatures may be evaluated from Eqs. 8.67 and 8.68 with

$$\text{Re}_D = \frac{\rho \, u_m D_h}{\mu} = \frac{\dot{m}_a \; \left(D_o - D_i\right)}{(\pi/4) \; \left(D_o^2 - D_i^2\right)\mu} = \frac{4 \; \dot{m}_a}{\pi \left(D_o + D_i\right)\mu} = \frac{4\left(2.1 \text{ kg/s}\right)}{\pi\left(4.05\text{m}\right) \; 270 \times 10^{-7} \text{N} \cdot \text{s/m}^2}$$

$$\text{Re}_D = 24,452$$

the flow is turbulent and from Eq. 8.60

$$h_i \approx h_o \approx \frac{k}{D_h} 0.023 \; \text{Re}_D^{4/5} \, \text{Pr}^{0.4} = \frac{0.041 \text{ W/m} \cdot \text{K}}{0.05 \text{ m}} 0.023\left(24,452\right)^{4/5} \left(0.68\right)^{0.4} = 52 \text{ W/m}^2 \cdot \text{K}.$$

With $\quad q_i'' = q_i' / \pi \; D_i = 1.25 \times 10^5 \text{W/m}/\pi \times 2\text{m} = 19{,}900 \text{ W/m}^2$

Eq. 8.67 gives

$$\left(T_{s,i} - T_m\right) = q_i'' / h_i = 19,900 \text{ W/m}^2 / 52 \text{ W/m}^2 \cdot \text{K} = 383\text{K}$$

$$T_{s,i,1} = 683\text{K} \qquad T_{s,i,2} = 1087\text{K}. \qquad <$$

From Eq. 8.68, with $q_o'' = 0$, $\left(T_{s,o} - T_m\right) = 0$. Hence

$$T_{s,o,1} = T_{a,1} = 300\text{K}. \qquad <$$