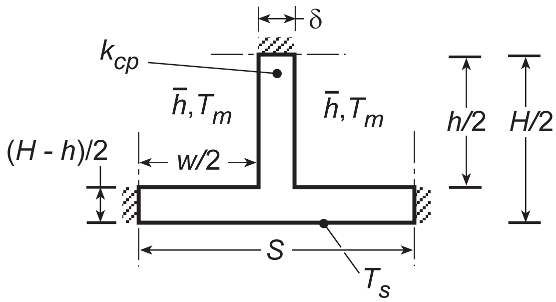
PROBLEM 8.82

KNOWN: Dimensions, surface temperature and thermal conductivity of a *cold plate*. Velocity, inlet temperature, and properties of coolant.

FIND: (a) Model for determining the heat rate q and outlet temperature, $T_{m,o}$, (b) Values of q and $T_{m,o}$ for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible liquid with negligible viscous dissipation, (3) Constant properties, (4) Symmetry about the midplane (horizontal) of the cold plate and the midplane (vertical) of each cooling channel, (5) Negligible heat transfer at sidewalls of cold plate, (6) One-dimensional conduction from outer surface of cold plate to base surface of channel and within the channel side walls, which act as extended surfaces.

PROPERTIES: Water (prescribed): $\rho = 984\ kg/m^3$, $c_p = 4184\ J/kg\cdot K$, $\mu = 489 \times 10^{-6}\ N\cdot s/m^2$, $k = 0.65\ W/m\cdot K$, $Pr = 3.15$.

ANALYSIS: (a) The outlet temperature, $T_{m,o}$, may be determined from the energy balance prescribed by Eq. 8.45b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}_1 c_p R_{tot}}\right)$$

where $\dot{m}_1 = \rho u_m A_c$ is the flowrate for a single channel and R_{tot} is the total resistance to heat transfer between the cold plate surface and the coolant for a particular channel. This resistance may be determined from the symmetrical section shown schematically, which represents one-half of the cell associated with a full channel. With the number of channels (and cells) corresponding to N = W/S, there are 2N = 2(W/S) symmetrical sections, and the total resistance R_{tot} of a cell is one-half that of a symmetrical section. Hence, $R_{tot} = R_{ss}/2$, where the resistance of the symmetrical section includes the effect of conduction through the outer wall of the cold plate and convection from the inner surfaces. Hence,

$$R_{ss} = \frac{(H-h)/2}{k_{cp}(SW)} + \frac{1}{\eta_o \bar{h} A_t}$$

where $A_t = A_f + A_b = 2(h/2 \times W) + (w \times W)$, $\bar{h}$ is the average convection coefficient for the channel flow, and η_o is the overall surface efficiency.

$$\eta_o = 1 - \frac{A_f}{A_t}(1 - \eta_f)$$

Continued...

PROBLEM 8.82 (Cont.)

The efficiency η_f corresponds to that of a straight, rectangular fin with an adiabatic tip, Eq. 3.92, and L_c = w/2. With $D_h = 4A_c/P = 4w^2/4w = w = 0.006\,m$, $Re_{D_h} = \rho u_m D_h/\mu$ = 984 kg/m^3 × 2 m/s × 0.006 m/489 × 10^{-6} N·s/m^2 = 24,150 and the channel flow is turbulent. Assuming fully-developed flow throughout the channel, the Dittus-Boelter correlation, Eq. 8.60, may therefore be used to evaluate $\overline{h}$, where

$$\overline{Nu}_D \approx Nu_{D,fd} = 0.023 Re_D^{4/5} Pr^{0.4}$$

The total heat rate for the cold plate may be expressed as

$$q = Nq_1 = N\dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right)$$

(b) For the prescribed conditions,

$$\dot{m}_1 = \rho u_m A_c = 984\,kg/m^3\,(2\,m/s)(0.006\,m)^2 = 0.0708\,kg/s$$

$$\overline{Nu}_D = 0.023(24{,}150)^{4/5}(3.15)^{0.4} = 116.8$$

$$\overline{h} = 116.8\,k/D_h = 116.8(0.65\,W/m\cdot K)/(0.006\,m) = 12{,}650\,W/m^2\cdot K$$

$$A_f = 2(h/2 \times W) = 2(0.003\,m \times 0.1\,m) = 6\times10^{-4}\,m^2$$

$$A_t = A_f + A_b = 6\times10^{-4}\,m^2 + (0.006\,m \times 0.1\,m) = 1.2\times10^{-3}\,m^2$$

With m = $\left(\overline{h}P_f/k_{cp}A_{cf}\right)^{1/2} = \left[\overline{h}\left(2\delta + 2W\right)/k_{cp}\left(\delta W\right)\right]^{1/2}$ = [12,650 W/m^2·K(0.008 + 0.200)m/400 W/m·K(0.004 × 0.100)m^2]$^{1/2}$ = 128.2 m^{-1}.

$$\eta_f = \frac{\tanh m(h/2)}{m(h/2)} = \frac{\tanh(128.2\times0.003)}{128.2\times0.003} = \frac{0.366}{0.385} = 0.952$$

$$\eta_o = 1 - 0.5(1 - 0.952) = 0.976$$

$$R_{ss} = \frac{(0.010 - 0.006)\,m/2}{400\,W/m\cdot K\,(0.01\,m \times 0.1\,m)} + \frac{1}{0.976\left(12650\,W/m^2\cdot K\right)1.2\times10^{-3}\,m^2}$$

$$R_{ss} = (0.005 + 0.0675)\,K/W = 0.0725\,K/W$$

With $R_{tot} = R_{ss}/2$ = 0.0362 K/W,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{1}{0.0708\,kg/s \times 4184\,J/kg\cdot K \times 0.0362\,K/W}\right) = 0.911$$

$$T_{m,o} = T_s - 0.911(T_s - T_{m,i}) = 360\,K - 0.911(360 - 300)\,K = 305.3\,K \qquad <$$

The total heat rate is

$$q = N\dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right) = 10 \times 0.0708\,kg/s \times 4184\,J/kg\cdot K\,(305.3 - 300)\,K = 15{,}700\,W \qquad <$$

COMMENTS: The prescribed properties correspond to a value of $\overline{T}_m$ which significantly exceeds that obtained from the foregoing solution ($\overline{T}_m$ = 302.6 K). Hence, the calculations should be repeated using more appropriate thermophysical properties (see next problem). From Eq. 3.90, the effectiveness of the extended surface is

$$\varepsilon = R_{t,b}/R_{t,f} = \left(\overline{h}\delta W\right)^{-1}/\left(\overline{h}A_f\eta_f\right)^{-1} = \left(A_f\eta_f/\delta W\right) = \left(6\times10^{-4}\,m^2 \times 0.954\right)/\left(0.004\,m \times 0.10\,m\right) = 1.43.$$

Hence, the ribs are only marginally effective in enhancing heat transfer to the coolant.