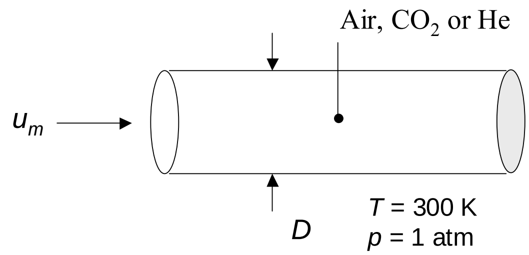
PROBLEM 8.110

KNOWN: Temperature and pressure of a gas flowing in a circular tube.

FIND: The critical tube diameter, D_c, below which incompressible turbulent flow cannot exist for (*a*) air (*b*) CO_2, and (*c*) He.

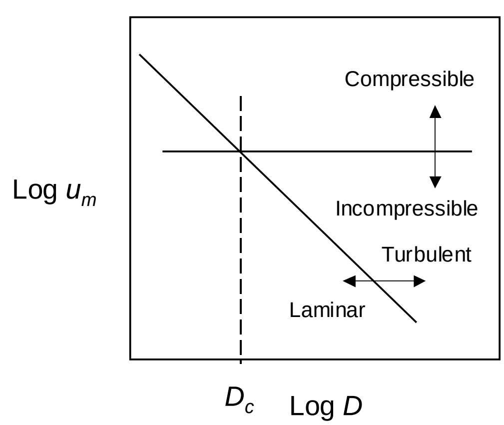
SCHEMATIC:



ASSUMPTIONS: (1) Ideal gas behavior. (2) Fully-developed flow.

PROPERTIES: Table A.4 (T = 300 K): Air; c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2. CO_2; c_p = 851 J/kg·K, $\mu = 149 \times 10^{-7}$ N·s/m^2. He; c_p = 5193 J/kg·K, $\mu = 199 \times 10^{-7}$ N·s/m^2. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol. CO_2; $\mathcal{M}$ = 44.01 kg/kmol He; $\mathcal{M}$ = 4.003 kg/kmol.

ANALYSIS: The relationship $Re_{D,c} = u_m D/\nu \approx 2300$ may be plotted on a *log-log* scale, as shown in the figure below. Laminar flow occurs to the left of the sloped line, while turbulent flow occurs to the right of the line. The critical Mach number $Ma_c = u_m/a \approx 0.3$ is drawn as the horizontal line that separates regions of incompressible flow (below the line) and compressible flow (above the line). It is evident that below a critical diameter, D_c, turbulent incompressible flow and heat transfer cannot exist.



(*a*) From the ideal gas equation of state,

$$\rho = p/RT \tag{1}$$

and from Section 6.4.2 the speed of sound is

$$a = \sqrt{\gamma RT} \tag{2}$$

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where $\gamma \equiv c_p/c_v$ is the ratio of specific heats. The mean velocity may be related to the Mach number, *Ma*, and is

$$u_m = Ma \cdot a \qquad (3)$$

Combining the preceding equations yields

$$Re = \frac{Ma \cdot p}{\mu}\sqrt{\frac{\gamma}{RT}}D \qquad (4)$$

Specifying $Re = Re_c$ and $Ma = Ma_c$ leads to the following expression for the critical tube diameter

$$D_c = \frac{Re_c}{Ma_c}\sqrt{\frac{RT}{\gamma}}\frac{\mu}{p} \qquad (5)$$

For air, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315\ \text{kJ/kmol}\cdot\text{K}}{28.97\ \text{kg/kmol}} = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.007\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.720\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad \gamma = \frac{c_p}{c_v} = \frac{1.007}{0.720} = 1.399$$

Therefore,

$$D_c = \frac{2300}{0.3}\sqrt{\frac{287\ \text{J/kg}\cdot\text{K}\times 300\ \text{K}}{1.399}} \times \frac{184.6\times10^{-7}\ \text{N}\cdot\text{s/m}^2}{1.0133\times10^5\ \text{N/m}^2} = 346\times10^{-6}\text{m} = 0.346\ \text{mm} \qquad <$$

(*b,c*) The calculations may be repeated for CO_2 and He, yielding the following results.

Gas	R (kJ/kg·K)	c_v (kJ/kg·K)	γ	D_c (mm)	
CO_2	0.189	0.662	1.285	0.237	<
He	2.077	3.116	1.667	0.920	<

COMMENTS: (1) Below the critical diameter, D_c, the effects of compressibility must *always* be accounted for if the flow is turbulent, and are *often* important if the flow is laminar. Because the correlations of Chapter 8 do not account for the effects of compressibility, they may not be applied to situations where turbulence exists and the tube diameter is less than D_c. The correlations must be used with caution if the flow is laminar and $D < D_c$ since compressibility effects might be important. (2) The critical diameter is moderately dependent on the specific gas of interest, for the three gases considered here.