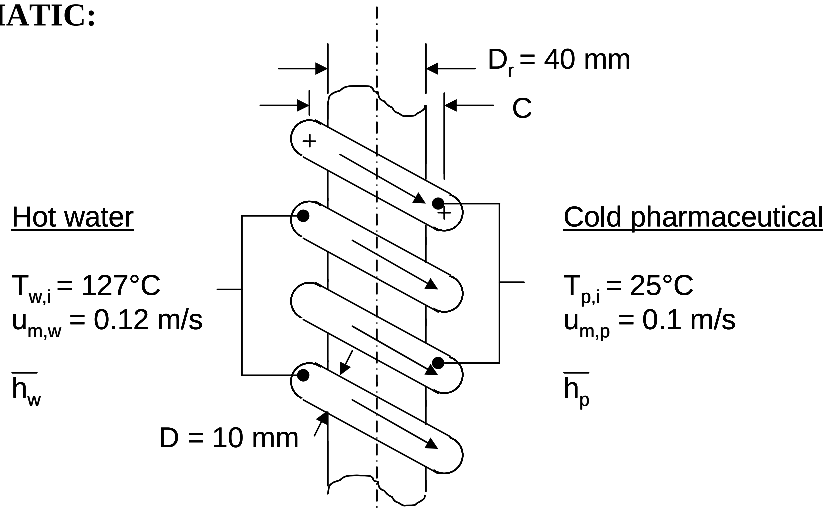
PROBLEM 8.106

KNOWN: Inlet temperatures and flow rates of a pharmaceutical product and pressurized water, tube diameter, coil diameter and number of coils.

FIND: (a) The outlet temperature of the pharmaceutical product, (b) The variation of the pharmaceutical outlet temperature with the pressurized water flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties and steady-state conditions, (2) Incompressible liquid and negligible viscous dissipation, (3) Fully developed flow, (4) Negligible tube wall thermal resistance, (5) Negligible heat loss to surroundings and ambient.

PROPERTIES: Table A.6, water: ($\overline{T}_m$ = 380 K): k = 0.683 W/m·K, c_p = 4226 J/kg·K, $\mu = 260 \times 10^{-6}$ N·s/m^2, Pr = 1.61, ρ = 953.3 kg/m^3. Pharmaceutical (given): k = 0.80 W/m·K, c_p = 4000 J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, Pr = 10, ρ = 1000 kg/m^3.

ANALYSIS: For the water,

$$\dot{m}_w = \frac{\rho u_w \pi D^2}{4} = \frac{953.3\text{kg/m}^3 \times 0.12\text{m/s} \times \pi \times (0.01\text{m})^2}{4} = 0.00899\text{kg/s}$$

$$Re_{D,w} = \frac{4\dot{m}_w}{\pi D \mu} = \frac{4 \times 0.00899\text{kg/s}}{\pi \times 0.01\text{m} \times 260 \times 10^{-6}\text{kg/s}\cdot\text{m}} = 4400$$

For the pharmaceutical,

$$\dot{m}_p = \frac{\rho u_m \pi D^2}{4} = \frac{1000\text{kg/m}^3 \times 0.10\text{m/s} \times \pi \times (0.01\text{m})^2}{4} = 0.00785\text{kg/s}$$

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PROBLEM 8.106 (Cont.)

$$Re_{D,p} = \frac{4\dot{m}_p}{\pi D\mu} = \frac{4\times 0.00785\,kg/s}{\pi\times 0.01m\times 2\times 10^{-3}\,kg/s\cdot m} = 500$$

The flow of the pharmaceutical is laminar ($Re_{D,p} < 2300$). For the coiled tube, $C = D_r + 2(D/2) = 40$ mm + 2 ×5 mm = 50 mm. Using Equation 8.74, $Re_{D,c,h,w} = 2300[1 + 12 \times (10/50)^{0.5}] = 14{,}640$. Therefore, the flow of the pressurized water is laminar ($Re_{D,w} = 4400 < 14{,}640$).

For the pharmaceutical product, $Re_{D,p}(D/C)^{1/2} = 500 \times (10/50)^{1/2} = 223$, while for the water $Re_{D,w}(D/C)^{1/2} = 4400 \times (10/50)^{1/2} = 1967$. For each tube, $C/D = 50/10 = 5 > 3$.

For the pharmaceutical product and water, the overall energy balances are

$$q = \dot{m}_p c_{p,p}(T_{p,o} - T_{p,i}) \quad ; \quad q = \dot{m}_w c_{p,w}(T_{w,i} - T_{w,o}) \qquad (1,2)$$

For the pharmaceutical and water, Equation 8.42 is

$$\frac{T_s - T_{p,o}}{T_s - T_{p,i}} = \exp\left(-\frac{\pi DL}{\dot{m}_p c_{p,p}}\overline{h_p}\right) \quad ; \quad \frac{T_s - T_{w,o}}{T_s - T_{w,i}} = \exp\left(-\frac{\pi DL}{\dot{m}_w c_{p,w}}\overline{h_w}\right) \qquad (3,4)$$

Once we determine $\overline{h_p}$ and $\overline{h_w}$, we may solve Equations (1) through (4) simultaneously for four unknowns: q, $T_{p,o}$, $T_{w,o}$ and T_s. We will use Equation 8.76, but be aware that we are using the correlation outside of its recommended range of applicability for the water. For the pharmaceutical product, Equation 8.77 yields

$$a = \left(1 + \frac{957\times(50/10)}{(500)^2\times 10}\right) = 1.002 \quad ; \quad b = 1 + \frac{0.477}{10} = 1.048$$

Therefore, Equation 8.76 becomes

$$Nu_{D,p} = \left[\left(3.66 + \frac{4.343}{1.002}\right)^3 + 1.158\left(\frac{500(10/50)^{1/2}}{1.048}\right)^{3/2}\right]^{1/3} = 16.03$$

Therefore, $\overline{h}_p = Nu_{D,p}k_p/D = 16.03\times 0.80\,W/m\cdot K/0.01m = 1283\,W/m^2\cdot K$. For the pressurized water, Equation 8.77 yields

$$a = \left(1 + \frac{957\times(50/10)}{(4400)^2\times 1.61}\right) = 1.00 \quad ; \quad b = 1 + \frac{0.477}{1.61} = 1.296$$

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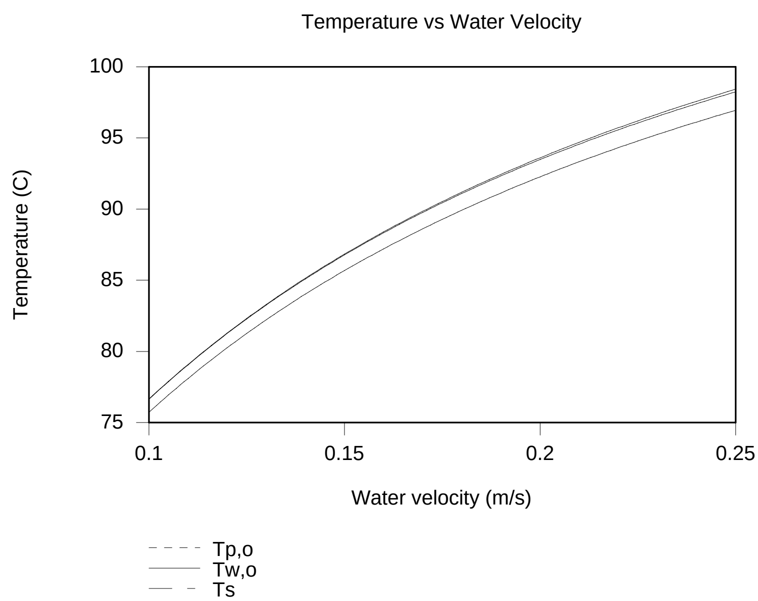
PROBLEM 8.106 (Cont.)

Proceeding as before, we find $Nu_{D,w} = 41.01, \overline{h}_w = 2801 W/m^2 \cdot K$. The tube length is $L = N \times \pi \times D_r = 20 \times \pi \times 0.05$ m = 3.14 m. Substituting values into Equations (1) through (4) and solving simultaneously yields

q = 1736 W, $T_{p,o}$ = 80.25°C, $T_{w,o}$ = 81.28°C, T_s = 81.25°C <

(b) The dependence of the pharmaceutical outlet temperature on the water velocity is shown in the graph below. Note that the pharmaceutical product's outlet temperature can be controlled accurately by modifying the water flow rate.



COMMENTS: (1) The pharmaceutical outlet temperature will be relatively uniform across the diameter of the tube due to mixing associated with secondary flow. (2) Although we have applied Equation 8.76 outside of its range of general applicability, the actual behavior is not expected to be significantly different than predicted. That is, we would still expect the pharmaceutical outlet temperature to be highly controllable by adjusting the water flow rate. Actual outlet temperatures could be easily measured and the water flow rate adjusted to provide the desired thermal response. (3) The average mean water temperature is $\overline{T}_m = (T_{w,i} + T_{w,o})/2$ = (127°C + 81.3°C)/2 = 104°C = 377 K. The assumed mean temperature of 380 K is reasonable.