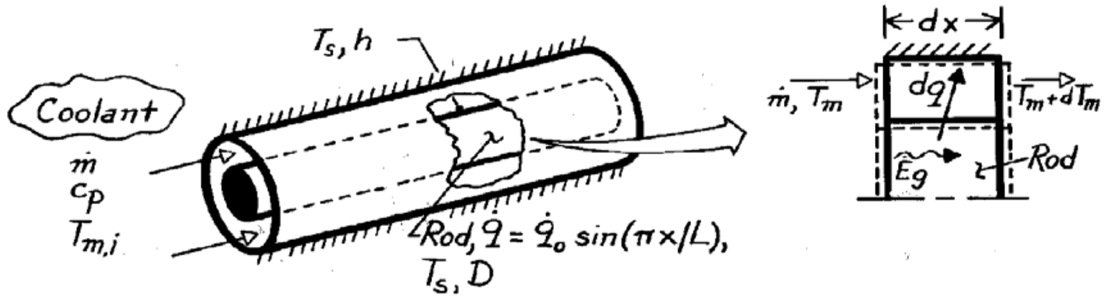
PROBLEM 8.14

KNOWN: Geometry and coolant flow conditions associated with a nuclear fuel rod. Axial variation of heat generation within the rod.

FIND: (a) Axial variation of local heat flux and total heat transfer rate, (b) Axial variation of mean coolant temperature, (c) Axial variation of rod surface temperature and location of maximum temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant fluid properties, (3) Uniform surface convection coefficient, (4) Negligible axial conduction in rod and fluid, (5) Incompressible liquid with negligible viscous dissipation, (6) Outer surface is adiabatic.

ANALYSIS: (a) Performing an energy balance for a control volume about the rod,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad -dq + \dot{E}_g = 0$$

or

$$-q''(\pi D\, dx) + \dot{q}_o \sin(\pi x/L)\left(\pi D^2/4\right) dx = 0 \qquad q'' = \dot{q}_o (D/4)\sin(\pi x/L). \qquad <$$

The total heat transfer rate is then

$$q = \int_0^L q''\, \pi D\, dx = \left(\pi D^2/4\right)\dot{q}_o \int_0^L \sin(\pi x/L)\, dx$$

$$q = \frac{\pi D^2}{4}\dot{q}_o \left(-\frac{L}{\pi}\cos\frac{\pi x}{L}\right)\Bigg|_0^L = \frac{D^2 \dot{q}_o L}{4}(1+1)$$

$$q = \frac{D^2 L}{2}\dot{q}_o. \qquad (1) \qquad <$$

(b) Performing an energy balance for a control volume about the coolant,

$$\dot{m}\, c_p\, T_m + dq = \dot{m}\, c_p\, (T_m + dT_m) = 0.$$

Hence

$$\dot{m}\, c_p\, d\,T_m = dq = (\pi D\, dx)\, q''$$

$$\frac{d\,T_m}{dx} = \frac{\pi D}{\dot{m}\, c_p}\frac{\dot{q}_o D}{4}\sin\left(\frac{\pi x}{L}\right).$$

Continued …

Integrating,

$$T_m(x) - T_{m,i} = \frac{\pi D^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \int_0^x \sin \frac{\pi x}{L} dx$$

$$T_m(x) = T_{m,i} + \frac{L D^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \left[1 - \cos \frac{\pi x}{L}\right] \qquad (2) \quad <$$

(c) From Newton's law of cooling,

$$q'' = h(T_s - T_m).$$

Hence

$$T_s = \frac{q''}{h} + T_m$$

$$T_s = \frac{\dot{q}_o D}{4h} \sin \frac{\pi x}{L} + T_{m,i} + \frac{LD^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \left[1 - \cos \frac{\pi x}{L}\right]. \quad <$$

To determine the location of the maximum surface temperature, evaluate

$$\frac{d T_s}{dx} = 0 = \frac{\dot{q}_o D\pi}{4hL} \cos \frac{\pi x}{L} + \frac{LD^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \frac{\pi}{L} \sin \frac{\pi x}{L}$$

or

$$\frac{1}{hL} \cos \frac{\pi x}{L} + \frac{D}{\dot{m} c_p} \sin \frac{\pi x}{L} = 0.$$

Hence

$$\tan \frac{\pi x}{L} = -\frac{\dot{m} c_p}{D h L}$$

$$x = \frac{L}{\pi} \tan^{-1} \left(-\frac{\dot{m} c_p}{D h L}\right) = x_{max}. \quad <$$

COMMENTS: Note from Eq. (2) that

$$T_{m,o} = T_m(L) = T_{m,i} + \frac{L D^2 \dot{q}_o}{2 \dot{m} c_p}$$

which is equivalent to the result obtained by combining Eq. (1) and Eq. 8.34.