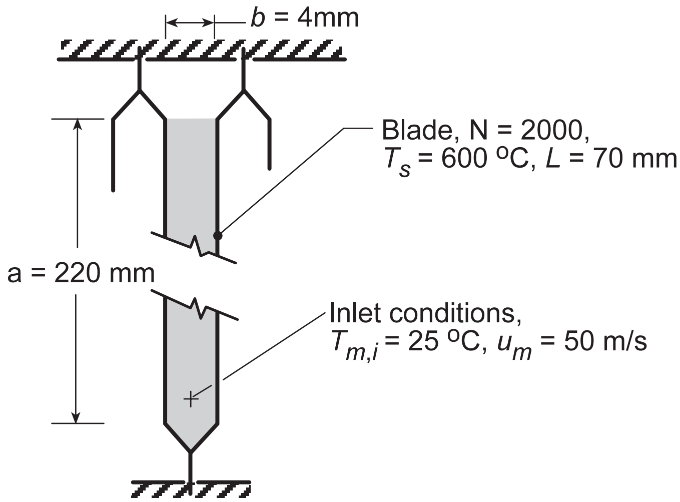
PROBLEM 8.91

KNOWN: Channel formed between metallic blades dissipating heat by internal volumetric generation.

FIND: (a) The heat removal rate per blade for the prescribed thermal conditions and (b) Time required to slow a train of mass 10^6 kg from 120 km/h to 50 km/h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions for channel blades and air flow, (2) The blades form a channel of rectangular (a × b) cross-section and length L, (3) Ideal gas with negligible viscous dissipation, pressure variation, and axial conduction, and (4) Fully developed flow conditions in the channel.

PROPERTIES: *Table A.4*, Air ($\overline{T}_m \approx 350$ K, 1 atm): ρ = 0.995 kg/m^3, c_p = 1009 J/kg ·K, ν = 20.92 × 10^{-6} m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) The heat removal rate by the air from a single channel (one blade) follows from an overall energy balance,

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \tag{1}$$

where the outlet temperature can be determined from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\overline{h}\right) \tag{2}$$

The hydraulic diameter, D_h, follows from Eq. 8.66,

$$D_h = \frac{4A_c}{P} = \frac{4(a \times b)}{2(a+b)} = \frac{4(0.220 \times 0.004)\,m^2}{2(0.220 + 0.004)\,m} = 0.0079\,m \tag{3}$$

Assuming $\overline{T}_m = 350K$, the Reynolds number is

$$Re_{Dh} = \frac{u_m D_h}{\nu} = \frac{50\,m/s \times 0.0079m}{20.92 \times 10^{-6}\,m^2/s} = 18,779 \tag{4}$$

Using the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{Dh} = \frac{\overline{h}D_h}{k} = 0.023 Re_{Dh}^{0.8} P_r^{0.4} = 0.023(18,779)^{0.8}(0.700)^{0.4} = 52.37 \tag{5}$$

Continued...

PROBLEM 8.91 (Cont.)

$$\bar{h} = \frac{0.030\,\mathrm{W/m\cdot K}}{0.0079\mathrm{m}} \times 52.37 = 199\,\mathrm{W/m^2\cdot K}$$

Hence, the outlet temperature is

$$\frac{600 - T_{m,o}}{(600-25)^\circ\mathrm{C}} = \exp\left(-\frac{2(0.220+0.004)\mathrm{m}\times 0.070\mathrm{m}}{0.0438\,\mathrm{kg/s}\times 1009\,\mathrm{J/kg\cdot K}} 199\,\mathrm{W/m^2\cdot K}\right)$$

$$T_{m,o} = 100.7^\circ\mathrm{C}$$

where

$\dot{m} = \rho A_c u_m = 0.995\ \mathrm{kg/m^3} \times (0.220 \times 0.004)\ \mathrm{m^2} \times 50\ \mathrm{m/s} = 0.0438\ \mathrm{kg/s}$

and the rate of heat removal per blade, from Eq. (1), is

$$q = 0.0438\,\mathrm{kg/s}\times 1009\,\mathrm{J/kg\cdot K}\,(100.7-25)^\circ\mathrm{C} = 3.346\,\mathrm{kW}$$ <

(b) From an energy balance on the locomotive over an interval of time, Δt, the heat energy transferred to the air stream results in a change in kinetic energy of the train,

$$-E_{out} = \Delta E = KE_f - KE_i \qquad (6)$$

$$-(q \times N)\times \Delta t = \frac{1}{2} M \left(V_f^2 - V_i^2\right)$$

$$-3346\,\mathrm{W/blade}\times 2000\mathrm{blades}\times \Delta t(\mathrm{s}) = \frac{1}{2}\times 10^6\,\mathrm{kg}\left[\left(\frac{50,000}{3600}\right)^2 - \left(\frac{120,000}{3600}\right)^2\right]\mathrm{m^2/s^2}$$

$$\Delta t = 69\mathrm{s}$$ <

COMMENTS: (1) For the channel, $L/D_h = 0.070\ \mathrm{m}/0.0079\ \mathrm{m} = 8.9 < 10$ so that the assumption of fully developed conditions may not be satisfied. Recognize that the flow at the channel entrance may be highly turbulent because of the upstream fan swirl and ducting.

(2) What benefits could be realized by increasing the heat transfer coefficient? Aside from increasing velocity, what design changes would you make to increase h?

(3) Our assumption for $\bar{T}_m$ = 350 K at which to evaluate properties is reasonable considering T_m = (100.7 + 25)°C/2 = 335 K.