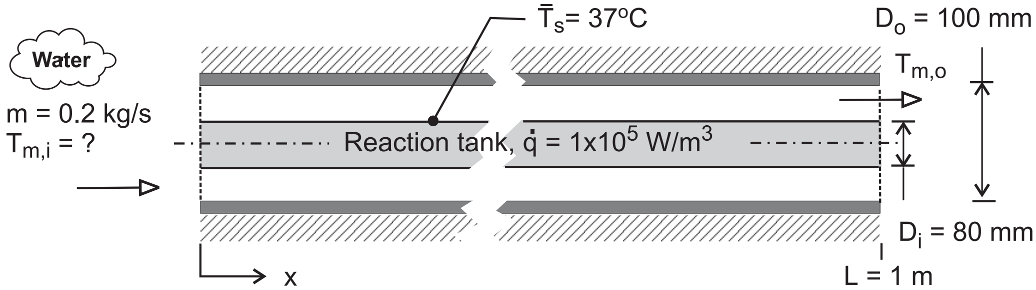
# PROBLEM 8.99

**KNOWN:** A concentric tube arrangement for removing heat generated from a biochemical reaction in a settling tank. Water is supplied to the annular region at rate of 0.2 kg/s.

**FIND:** (a) The inlet temperature of the supply water that will provide for an average tank surface temperature of 37°C; assume and then justify fully developed flow and thermal conditions; and (b) Sketch the water and surface temperatures along the flow direction for two cases: the fully developed conditions of part (a), and when entrance effects are important. Comment on the features of the temperature distributions, with particular attention to the longitudinal gradient on the tank surface. What change to the system or operating conditions would you make to reduce the gradient?

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Fully developed flow and thermal conditions, (2) Inner annulus surface has uniform heat flux, while outer surface is insulated, (3) Constant properties, (4) Incompressible liquid with negligible viscous dissipation.

**PROPERTIES:** *Table A-6*, Water ($T_m$ = 304 K): $\rho$ = 995.6 kg/m$^3$, $c_p$ = 4178 J/kg·K, $\nu$ = 7.987 × 10$^{-7}$ m$^2$/s, k = 0.618 W/m·K, Pr = 5.39.

**ANALYSIS:** (a) The overall energy balance on the fluid passing through the concentric tube is

$$q = \dot{m}\, c_p \left(T_{m,i} - T_{m,o}\right) \tag{1}$$

and from an energy balance on the reaction tank,

$$q = \dot{q}(\pi D_i^2 / 4)/L = 1\times10^5 \mathrm{W/m^3}(\pi(0.08\mathrm{m})^2 / 4)\times 1\mathrm{m} = 503\mathrm{W}. \tag{2}$$

The convection rate equation applied to the inner surface $A_{s,i}$ is

$$q = \overline{h}_i A_{s,i}(\overline{T}_s - \overline{T_m}) = \overline{h}_i \pi D_i L(\overline{T}_s - \overline{T_m}) \tag{3}$$

where $\overline{T}_s$ is the average inner surface temperature and

$$\overline{T_m} = \left(T_{m,i} + T_{m,o}\right)/2. \tag{4}$$

To estimate $\overline{h}$, begin by characterizing the flow with

$$Re_{Dh} = u_m D_h / \nu \qquad D_h = D_o - D_i \qquad \dot{m} = \rho A_c u_m$$

where $A_c = \pi\left(D_o^2 - D_i^4\right)/4$. Substituting numerical values find

$$Re_{Dh} = 1779$$

Assuming fully developed conditions for laminar flow through an annulus, it follows from Table 8.3 and Eq. 8.72 with $D_i/D_o$ = 0.8,

$$\overline{Nu}_i = \overline{h}_i D_h / k = 5.58 \qquad \overline{h}_i = 172\ \mathrm{W/m^2 \cdot K}$$

Continued …

## PROBLEM 8.99 (Cont.)

Using Eq. (3) with $\overline{h}_i$, and $\overline{T}_s = 37°C$, and q from Eq. (2), find

$$\overline{T_m} = 25.4°C$$

From Eqs. (1) and (4), calculate

$$T_{m,i} = 25.1°C \qquad T_{m,o} = 25.7°C$$ <

For this annulus, the thermal entry length from Eq. 8.23 is

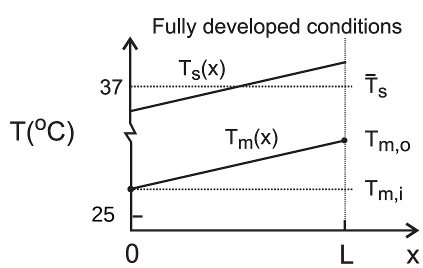
$$x_{fd,t} = D_h \times 0.05\, Re_{Dh}\, Pr$$

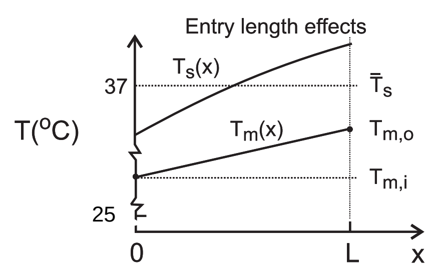
$$x_{fd,t} = (0.100 - 0.080)\text{m} \times 0.05 \times 1779 \times 5.39 = 9.59 \text{ m}$$

Since L = 1 m, we conclude that entry length effects are significant, and the fully developed flow assumption is approximate.

(b) Since the fluid is being heated by flow over a surface with uniform heat flux, the mean fluid temperature, $T_m(x)$, will increase linearly with longitudinal distance x. Assuming fully developed conditions, the surface temperature $T_s(x)$ will likewise increase linearly with distance as shown in the schematic below. Note that the longitudinal temperature difference is about 0.6°C, and that the inlet mean temperature is 25.1°C.

Considering now entrance length effects, the convection coefficient is no longer uniform, and will be largest near the entrance, and larger than for the fully developed flow everywhere. Hence, we expect the surface temperature near the entrance to be closer to the mean fluid temperature than elsewhere. We also expect the average mean temperature of the fluid will be higher so that the average surface temperature, $\overline{T}_s$, remains at 37°C. However, the rise in temperature of the fluid ($T_{m,o} - T_{m,i}$) will remain the same, about 0.6°C, since the heat removal rate is the same. Increasing the flow rate will tend to minimize the longitudinal gradient by reducing ($T_{m,o} - T_{m,i}$) and increasing h(x). The graph below illustrates the distinctive features of the fully developed flow and entrance length effects.




**COMMENTS:** The thermophysical properties required in the convection correlation and the energy equations should be evaluated at $T_m = (T_{m,i} + T_{m,o})/2 \approx 298$ K.