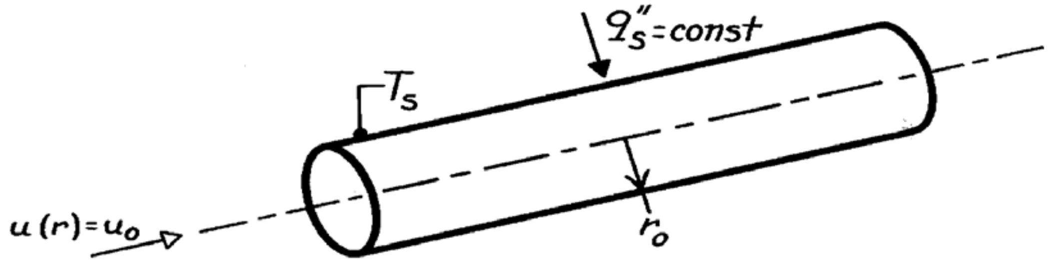
PROBLEM 8.21

KNOWN: Laminar, slug flow in a circular tube with uniform surface heat flux.

FIND: Temperature distribution and Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, incompressible flow, with negligible viscous dissipation, (2) Constant properties, (3) Fully developed, laminar flow, (4) Uniform surface heat flux.

ANALYSIS: With v = 0 for fully developed flow and $\partial T/\partial x = dT_m/dx$ = const, from Eqs. 8.32 and 8.39, the energy equation, Eq. 8.48, reduces to

$$u_o \frac{dT_m}{dx} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right).$$

Integrating twice, it follows that

$$T(r) = \frac{u_o}{\alpha} \frac{dT_m}{dx} \frac{r^2}{4} + C_1 \ell n(r) + C_2.$$

Since T(0) must remain finite, $C_1 = 0$. Hence, with $T(r_o) = T_s$

$$C_2 = T_s - \frac{u_o}{\alpha} \frac{dT_m}{dx} \frac{r_o^2}{4} \qquad T(r) = T_s - \frac{u_o}{4\alpha} \frac{dT_m}{dx}\left(r_o^2 - r^2\right).$$ <

From Eq. 8.26, with $u_m = u_o$,

$$T_m = \frac{2}{r_o^2} \int_0^{r_o} Tr\, dr = \frac{2}{r_o^2} \int_0^{r_o} \left[T_s r - \frac{u_o}{4\alpha} \frac{dT_m}{dx}\left(r r_o^2 - r^3\right)\right] dr$$

$$T_m = \frac{2}{r_o^2}\left[T_s \frac{r_o^2}{2} - \frac{u_o}{4\alpha} \frac{dT_m}{dx}\left(\frac{r_o^4}{2} - \frac{r_o^4}{4}\right)\right] = T_s - \frac{u_o r_o^2}{8\alpha} \frac{dT_m}{dx}.$$

From Eq. 8.27 and Fourier's law,

$$h = \frac{q_s''}{T_s - T_m} = \frac{k \frac{\partial T}{\partial r}\Big|_{r_o}}{T_s - T_m}$$

hence,

$$h = \frac{k\left(\frac{u_o r_o}{2\alpha}\right) \frac{dT_m}{dx}}{\frac{u_o r_o^2}{8\alpha} \frac{dT_m}{dx}} = \frac{4k}{r_o} = \frac{8k}{D} \qquad \overline{Nu}_D = \frac{hD}{k} = 8.$$ <