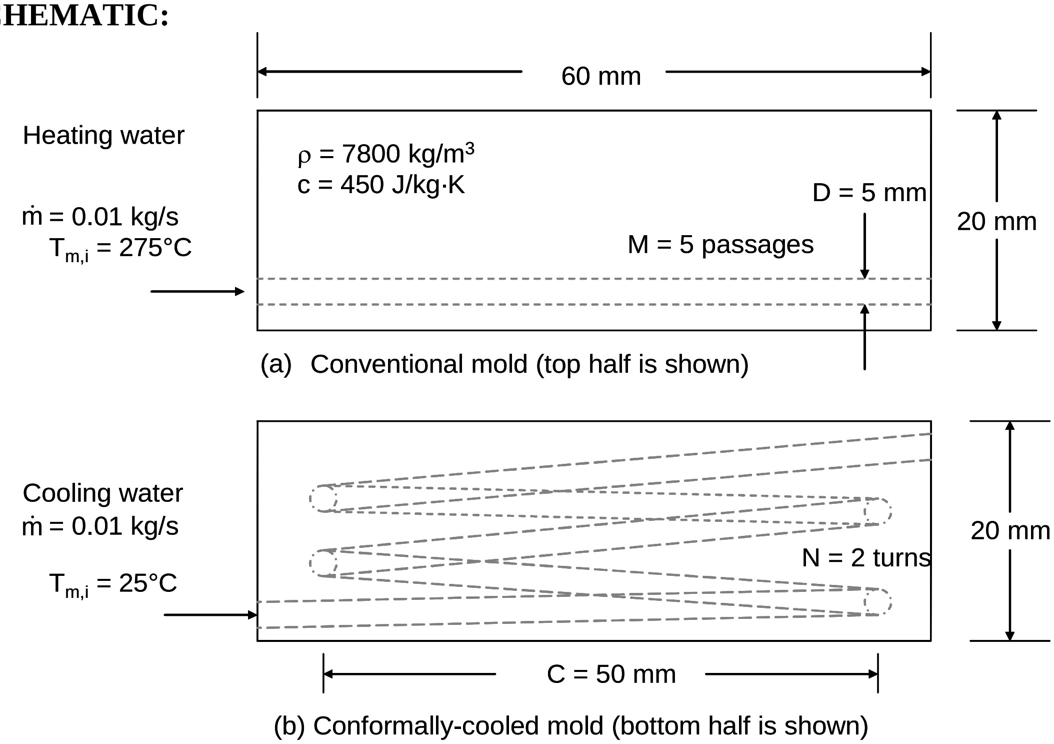
PROBLEM 8.105

KNOWN: Pressurized water inlet temperature and total mass flow rate for mold cooling and heating. Water channel dimensions for conventional and conformally-cooled mold. Initial hot and cold mold temperatures, mold dimensions and mold properties.

FIND: (a) Initial heating rate of a cold (100°C) mold, initial cooling rate of a hot (200°C) mold for straight water channels with D = 50 mm, (b) Initial heating rate of a cold (100°C) mold, initial cooling rate of a hot (200°C) mold for a conformally-cooled mold with water channels of diameter D = 50 mm, (c) Surface areas of cooling/heating channels for both molds and determination of which mold will enable production of more parts per day.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Incompressible liquid and negligible viscous dissipation, (3) Fully developed hydrodynamic conditions at the entrance, (4) Negligible part mass, (5) Water sufficiently pressurized to prevent boiling, (6) Negligible heat transfer in short straight sections of the channel for the conformally-cooled case.

PROPERTIES: Table A.6, water: ($\overline{T}_m$ = 260°C, assumed): k = 0.6038 W/m·K, c_p = 4989 J/kg·K, μ = 103.1 × 10^{-6} N·s/m^2, Pr = 0.853. ($\overline{T}_m$ = 40°C, assumed): k = 0.6316 W/m·K, c_p = 4179 J/kg·K, μ = 656.6 × 10^{-6} N·s/m^2, Pr = 4.344. (T_s = 200°C): μ_s = 133.9 × 10^{-6} N·s/m^2.

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ANALYSIS: (a) Heating, $\overline{T_m} = 260°C$. The Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{(4\times 0.01\,kg/s)/5}{\pi\times 5\times 10^{-3}\,m\times 103.1\times 10^{-6}\,N\cdot s/m^2} = 4940$$

From the Gnielinski correlation, with f = $(0.790\ln Re_D - 1.64)^{-2} = (0.790\ln 4940 - 1.64)^{-2} = 38.8 \times 10^{-3}$,

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} = \frac{(38.8\times 10^{-3}/8)\times(4940 - 1000)\times 0.853}{1 + 12.7(38.8\times 10^{-3}/8)^{1/2}(0.853^{2/3} - 1)} = 17.89$$

Therefore, $h_D = Nu_D k/D = 17.89 \times 0.6038$ W/m·K/5 × 10^{-3}m = 2161 W/m^2·K. For P = πD = π × 5 × 10^{-3} m = 15.7 × 10^{-3} m, L = 60 × 10^{-3} m, $\dot{m}$ = 0.01 kg/s/5 = 0.002 kg/s, Equation 8.42 is written

$$\frac{100 - T_{m,o}}{100 - 275} = \exp\left(-\frac{15.7\times 10^{-3}\,m\times 60\times 10^{-3}\,m\times 2161\,W/m^2\cdot K}{0.002\,kg/s\times 4989\,J/kg\cdot K}\right)$$

from which $T_{m,o}$ = 243°C. Therefore,
$q_w = \dot{m}c_p(T_{m,o} - T_{m,i}) = 0.002\,kg/s\times 4989\,J/kg\cdot K\times(243°C - 275°C) = 319\,W/channel$ and, for the entire mold, $q_h = -q_w \times M \times 2$ = 319W × 5 × 2 = 3190 W <

Cooling, $\overline{T_m} = 40°C$. The Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{(4\times 0.01\,kg/s)/5}{\pi\times 5\times 10^{-3}\,m\times 656.6\times 10^{-6}\,N\cdot s/m^2} = 776$$

Using Equation 8.56,

$$Nu_D = 3.66 + \frac{0.0668(D/L)Re_D\,Pr}{1 + 0.04[(D/L)Re_D\,Pr]^{2/3}} = 3.66 + \frac{0.0668(5/60)\times 776\times 4.344}{1 + 0.04[(5/60)\times 776\times 4.344]^{2/3}} = 10.57$$

Therefore, $h_D = Nu_D k/D = 10.57\times 0.6316$ W/m·K/5 × 10^{-3}m = 1335 W/m^2·K. Equation 8.42 yields

$$\frac{200 - T_{m,o}}{100 - 25} = \exp\left(-\frac{15.7\times 10^{-3}\,m\times 60\times 10^{-3}\,m\times 1335\,W/m^2\cdot K}{0.002\,kg/s\times 4179\,J/kg\cdot K}\right)$$

from which $T_{m,o}$ = 49.4°C. Therefore,
$q_w = \dot{m}c_p(T_{m,o} - T_{m,i}) = 0.002\,kg/s\times 4179\,J/kg\cdot K\times(49.4°C - 25°C) = 203.9\,W/channel$ and, for the entire mold, $q_c = -q_w \times M \times 2$ = -203.9W × 5 × 2 = -2039 W <

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PROBLEM 8.105 (Cont.)

(b) Heating, $\overline{T_m} = 260°C.$ The critical Reynolds number is

$$Re_{D,c,h} = Re_{D,c}\left[1+12(D/C)^{0.5}\right] = 2300\times\left[1+12(5/50)^{0.5}\right] = 11030$$. The actual

Reynolds number is $Re_D = \dfrac{4\dot{m}}{\pi D\mu} = \dfrac{4\times 0.01\text{kg/s}}{\pi\times 5\times 10^{-3}\text{m}\times 103.1\times 10^{-6}\text{N}\cdot\text{s/m}^2} = 24700$ and the flow is turbulent. Using the Gnielinski correlation, with f = $(0.790\ln Re_D - 1.64)^{-2} = (0.790 \ln 24700 - 1.64)^{-2} = 24.8\times 10^{-3}$,

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1+12.7(f/8)^{1/2}(Pr^{2/3}-1)} = \frac{\left(24.8\times 10^{-3}/8\right)\times\left(24700-1000\right)\times 0.853}{1+12.7(24.8\times 10^{-3}/8)^{1/2}(0.853^{2/3}-1)} = 62.67$$

Therefore, $h_D = Nu_D k/D = 62.67 \times 0.6038$ W/m·K/5 × 10^{-3}m = 7570 W/m^2·K. For P = 15.7 × 10^{-3} m, L = 2πC = 2 × π × 50 × 10^{-3} m = 0.314 m, and $\dot{m}$ =0.01 kg/s, Equation 8.42 is written as

$$\frac{100 - T_{m,o}}{100-275} = \exp\left(-\frac{15.7\times 10^{-3}\text{m}\times 0.314\text{m}\times 7570\text{W/m}^2\cdot\text{K}}{0.01\text{kg/s}\times 4989\text{J/kg}\cdot\text{K}}\right)$$

from which $T_{m,o}$ = 182.8°C. Then, q_h = 0.02 kg/s × 4989 J/kg·K × (182.8°C - 275°C) = 9197 W $<$

Cooling, $\overline{T_m} = 40°C.$ The Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{\left(4\times 0.01\text{kg/s}\right)}{\pi\times 5\times 10^{-3}\text{m}\times 656.6\times 10^{-6}\text{N}\cdot\text{s/m}^2} = 3880$$

Since $Re_D < Re_{D,c,h}$, the flow is laminar and $Re_D(D/C)^{1/2} = 3880 \times(5/50)^{1/2} = 1227$. The values of a and b for use in Equation 8.77 are

$$a = \left(\frac{1+957(C/D)}{Re_D^2\,Pr}\right) = \left(\frac{1+957\times(50/5)}{3880^2\times 4.344}\right) = 146\times 10^{-3};\ b = 1+\frac{0.477}{Pr} = -1+\frac{0.477}{4.344} = 1.11$$

Equation 8.76 is rearranged to yield

$$h_D = \frac{0.6316\text{W/m}\cdot\text{K}}{5\times 10^{-3}\text{m}}\left[\left(3.66+\frac{4.343}{146\times 10^{-3}}\right)^3 + 1.158\times\left(\frac{1227}{1.11}\right)^{3/2}\right]^{1/3}\left(\frac{656}{133.9}\right)^{0.14} = 6794\text{W/m}^2\cdot\text{K}$$

Equation 8.42 is written

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PROBLEM 8.105 (Cont.)

$$\frac{200 - T_{m,o}}{100 - 25} = \exp\left(-\frac{15.7 \times 10^{-3}\,\text{m} \times 0.314\,\text{m} \times 6794\,\text{W/m}^2 \cdot \text{K}}{0.01\,\text{kg/s} \times 4179\,\text{J/kg} \cdot \text{K}} \right)$$

from which $T_{m,o}$ = 121.5°C. Therefore,

$$q_c = \dot{m}c_p(T_{m,o} - T_{m,i}) = 0.02\,\text{kg/s} \times 4179\,\text{J/kg} \cdot \text{K} \times (25°\text{C} - 121.5°\text{C}) = -8065\,\text{W}$$ <

(c) For the conventional mold,

$A_{cp} = 2M\pi DL = 2 \times 5 \times \pi \times 5 \times 10^{-3}$ m $\times 60 \times 10^{-3}$ m $= 9.42 \times 10^{-3}$ m^2. For the conformally-cooled mold, $A_{cc} = 2N\pi C\pi D = 2 \times 2 \times \pi^2 \times 50 \times 10^{-3}$ m $\times 5 \times 10^{-3}$ m $= 9.87 \times 10^{-3}$ m^2.

The time rate of change of the mold temperature is

$$\frac{dT}{dt} = \frac{q}{V\rho C} = \frac{q}{(60 \times 10^{-3}\,\text{m})^2 \times 40 \times 10^{-3}\,\text{m} \times 7800\,\text{kg/m}^3 \times 450\,\text{J/kg} \cdot \text{K}} = \frac{q}{505.4\,\text{W} \cdot \text{s/K}}$$

The results are summarized in the following table.

Mold Type	q (W)	Flow Regime	dT/dt (K/s)
Conventional heating	3190	turbulent	6.51
Conventional cooling	-2039	laminar	4.03
Conformal heating	9197	turbulent	18.20
Conformal cooling	-8065	laminar enhanced	15.96

The conformally-cooled mold will increase production by a factor of 3 to 4 times, using the same cooling area.

COMMENTS: (1) The average mean temperature for heating is 258.8°C and 230°C for the conventional and conformally-cooled molds, respectively. The assumed average mean temperature (260°C) is very good for the conventional mold case. A more accurate solution would be obtained by re-calculating the answer for the conformally-cooled case based upon a better estimate of the average mean temperature. (2) The average mean temperature for cooling is 37.2°C and 73.3°C for the conventional and conformally-cooled molds, respectively. The assumed average mean temperature for cooling (40°C) is very good for the conventional mold case. A more accurate solution would be obtained by re-calculating the answer for the conformally-cooled case based upon a better estimate of the average mean temperature. (3) The conformally-cooled mold offers enhanced performance due to higher mean velocity in the case of heating, and enhanced laminar flow due to curvature in the case of cooling. (4) Equation 8.76 has been extended slightly beyond its range of recommended application. Care should be taken in using the predictions.