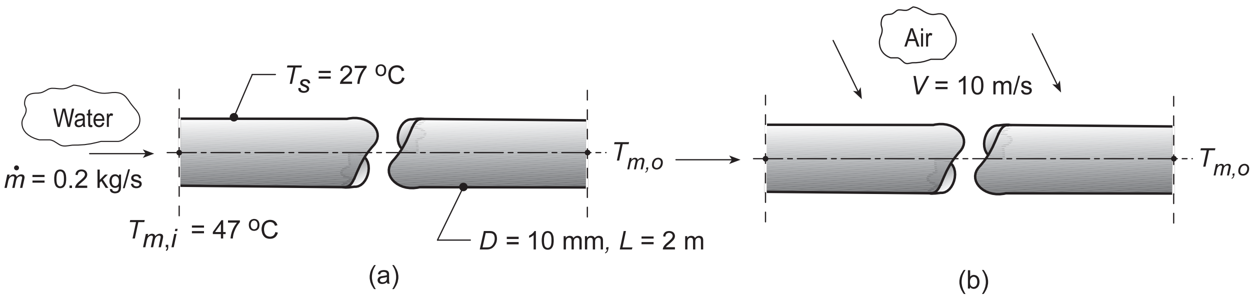
PROBLEM 8.61

KNOWN: Thin walled tube of prescribed diameter and length. Water inlet temperature and flow rate.

FIND: (a) Outlet temperature of the water when the tube surface is maintained at a uniform temperature T_s = 27°C assuming $\overline{T}_m$ = 300 K for evaluating water properties, (b) Outlet temperature of the water when the tube is heated by cross flow of air with V = 10 m/s and T_∞ = 100°C assuming $\overline{T}_f$ = 350 K for evaluating air properties, and (c) Outlet temperature of the water for the conditions of part (b) using properly evaluated properties.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible liquid with negligible viscous dissipation and negligible axial conduction, (3) Fully developed flow and thermal conditions for internal flow, and (4) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = 300 K): ρ = 997 kg/m^3, c_p = 4179 J/kg·K, μ = 855 × 10^{-6} N·s/m^2, k = 0.613 W/m·K, Pr = 5.83; *Table A.4*, Air ($\overline{T}_f$ = 350 K, 1 atm): ν = 20.92 × 10^{-6} m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) For the constant wall temperature cooling process, T_s = 27°C, the water outlet temperature can be determined from Eq. 8.41b, with P = πD,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\overline{h}_i\right) \tag{1}$$

To estimate the convection coefficient, characterize the flow evaluating properties at $\overline{T}_m$ = 300 K

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.2\,\text{kg/s}}{\pi\times 0.010\,\text{m}\times 855\times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 29,783$$

Hence, the flow is turbulent and assuming fully developed (L/D = 200), and using the Dittus-Boelter correlation, Eq. 8.60, find $\overline{h}_i$,

$$Nu_D = \frac{\overline{h}_i D}{k} = 0.023\,Re_D^{0.8}\,Pr^{0.3} \qquad \overline{h}_i = \frac{0.613\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}0.023(29,783)^{0.8}(5.83)^{0.3} = 9080\,\text{W/m}^2\cdot\text{K} \tag{2}$$

Substituting this value for $\overline{h}_i$ into Eq. (1), find

$$\frac{(27 - T_{m,o})}{(27-47)^\circ\text{C}} = \exp\left(-\frac{\pi\times 0.010\,\text{m}\times 2\,\text{m}}{0.2\,\text{kg/s}\times 4179\,\text{J/kg}\cdot\text{K}}\times 9080\,\text{W/m}^2\cdot\text{K}\right) \qquad T_{m,o} = 37.1^\circ\text{C} \quad <$$

(b) For the air heating process, T_∞ = 100°C, the water outlet temperature follows from Eq. 8.45a,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{U}\right) \tag{3}$$

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where the overall coefficient is $\qquad \overline{U} = \left(1/\overline{h}_i + 1/\overline{h}_o\right) \qquad (4)$

To estimate $\overline{h}_o$, use the Churchill-Bernstein correlation, Eq. 7.46, for cross flow over a cylinder using properties evaluated at $\overline{T}_f = 350$ K.

$$Re_D = \frac{VD}{\nu} = \frac{10\,\text{m/s} \times 0.010\,\text{m}}{20.92 \times 10^{-6}\,\text{m}^2/\text{s}} = 4780 \qquad (5)$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad (6)$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(4780)^{1/2}(0.700)^{1/3}}{\left[1 + (0.4/0.700)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{4780}{282{,}000}\right)^{5/8}\right]^{4/5} = 35.76$$

$$\overline{h}_o = \frac{\overline{Nu}_D k}{D} = \frac{0.030\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}} \times 35.76 = 107\,\text{W/m}^2\cdot\text{K}$$

The value of $\overline{h}_i$ can be recalculated for heating conditions:

$$Nu_D = \frac{\overline{h}_i D}{k} = 0.023\,Re_D^{0.8}\,Pr^{0.4} \qquad \overline{h}_i = \frac{0.613\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}\,0.023(29{,}783)^{0.8}(5.83)^{0.4} = 10{,}800\,\text{W/m}^2\cdot\text{K}$$

Next, find $\overline{U}$ then $T_{m,o}$,

$$\overline{U} = (1/10{,}800 + 1/107)^{-1}\,\text{W/m}^2\cdot\text{K} = 106\,\text{W/m}^2\cdot\text{K}$$

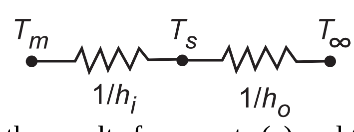
$$\frac{100 - T_{m,o}}{(100-47)^\circ\text{C}} = \exp\left(-\frac{\pi \times 0.010\,\text{m} \times 2\,\text{m}}{0.2\,\text{kg/s} \times 4179\,\text{J/kg}\cdot\text{K}} \times 106\,\text{W/m}^2\cdot\text{K}\right) \qquad T_{m,o} = 47.4^\circ\text{C} \quad <$$

(c) Using the *IHT Correlation Tools* for *Internal Flow* (*Turbulent Flow*) and *External Flow* (over a *Cylinder*) the analyses of part (b) were performed considering the appropriate temperatures to evaluate the thermophysical properties. For internal and external flow, respectively,

$$\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 \qquad \overline{T}_f = \left(\overline{T}_s + T_\infty\right)/2 \qquad (7,8)$$

where the average tube wall temperature is evaluated from the thermal circuit,

$$\frac{\overline{T}_m - \overline{T}_s}{1/\overline{h}_i} = \frac{\overline{T}_s - T_\infty}{1/\overline{h}_o} \qquad (9)$$



The results of the analyses are summarized in the table along with the results from parts (a) and (b),

Condition	$\overline{T}_m$ (K)	$\overline{h}_i$ (W/m^2·K)	$\overline{T}_f$ (K)	$\overline{h}_o$ (W/m^2·K)	$\overline{U}$ (W/m^2·K)	$T_{m,o}$ (°C)
T_s = 27°C	300	9080	---	---	---	37.1°C
T_∞ = 100 °C, T_f = 350°C	300	10,800	350	107	106	47.4°C
Exact solution	320	13,000	347	107.3	106.3	47.4°C

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PROBLEM 8.61 (Cont.)

Note that since $\overline{h}_o << \overline{h}_i$, $\overline{U}$ is controlled by the value of $\overline{h}_o$ which was evaluated near 350 K for both parts (b) and (c). Hence, it follows that $T_{m,o}$ is not very sensitive to $\overline{h}_i$ which, as seen above, is sensitive to the value of $\overline{T}_m$.