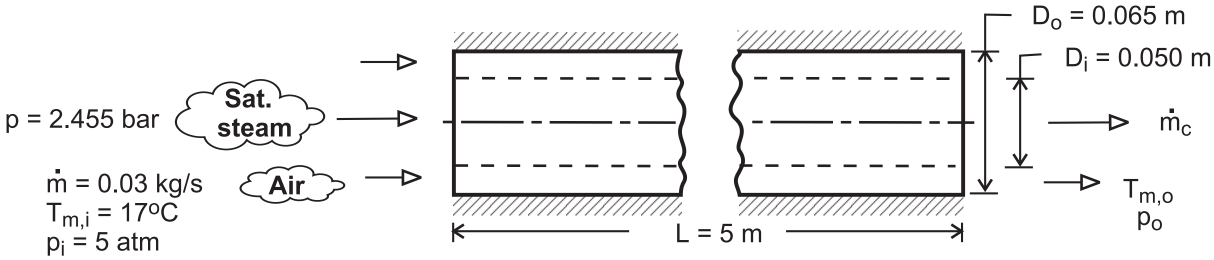
PROBLEM 8.96

KNOWN: Inlet temperature, pressure and flow rate of air. Annulus length and tube diameters. Pressure of saturated steam.

FIND: Outlet temperature and pressure drop of air. Mass rate of steam condensation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Outer surface of annulus is adiabatic, (3) Air is ideal gas with negligible viscous dissipation and pressure variation, (4) Fully developed flow throughout annulus, (5) Smooth annulus surfaces, (6) Constant properties.

PROPERTIES: *Table A-4*, air ($\overline{T}_m \approx 325$ K, p = 5 atm): $\rho = 5 \times \rho\,(1\text{ atm}) = 5.391\text{ kg/m}^3$, $c_p = 1008$ J/kg·K, $\mu = 196.4 \times 10^{-7}\text{ N·s/m}^2$, k = 0.0281 W/m·K, Pr = 0.703. *Table A-6*, sat. steam (p = 2.455 bars): $T_s = 400$ K, $h_{fg} = 2183$ kJ/kg.

ANALYSIS: With a uniform surface temperature, the air outlet temperature is

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi D_i L}{\dot{m}\,c_p}\,\overline{h}\right)$$

With $A_c = \pi\left(D_o^2 - D_i^2\right)/4 = 1.355\times10^{-3}\text{m}^2$, $D_h = D_o - D_i = 0.015\text{m}$ and $Re_D = \rho u_m D_h/\mu$ $= \dot{m}D_h/A_c\mu = 16,900$, the flow is turbulent and the Dittus-Boelter correlation yields

$$\overline{h} \approx h_{fd} = \left(\frac{k}{D_h}\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.0281\,\text{W/m}\cdot\text{K}}{0.015\text{m}}\right)0.023\left(16,900\right)^{4/5}\left(0.703\right)^{0.4} = 90.3\,\text{W/m}^2\cdot\text{K}$$

$$T_{m,o} = 127°\text{C} - \left(110°\text{C}\right)\exp\left(-\frac{\pi\times0.05\text{m}\times5\text{m}\times90.3\,\text{W/m}^2\cdot\text{K}}{0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}}\right) = 116.5°\text{C} \quad <$$

The pressure drop is $\Delta p = f\left(\rho u_m^2/2D_h\right)L$, where, with $u_m = \dot{m}/\rho A_c = 0.03\,\text{kg/s}/$ $\left(5.391\,\text{kg/m}^3\times1.355\times10^{-3}\text{m}^2\right) = 4.11\,\text{m/s}$, and with $Re_D = 16,900$, Eq. 8.21 yields $f = [0.790\ln(Re_D) - 1.64]^{-2} = 0.027$. Hence,

$$\Delta p \approx 0.027\times5.391\,\text{kg/m}^3\,\frac{\left(4.11\,\text{m/s}\right)^2 5\text{m}}{2\times0.015\text{m}} = 415\,\text{N/m}^2 = 4.1\times10^{-3}\text{ atm} \quad <$$

The rate of heat transfer to the air is

$$q = \dot{m}\,c_p\left(T_{m,o} - T_{m,i}\right) = 0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}\left(99.5°\text{C}\right) = 3009\,\text{W}$$

and the rate of condensation is then

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{3009\,\text{W}}{2.183\times10^6\,\text{J/kg}} = 1.38\times10^{-3}\,\text{kg/s} \quad <$$

COMMENTS: (1) With $\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = 340$ K, the initial estimate of 325 K is too low and an iterative solution should be obtained, (2) For a steam flow rate of 0.01 kg/s, approximately 14% of the outflow would be in the form of saturated liquid, (3) With $L/D_h = 333$, the assumption of fully developed flow throughout the tube is excellent.