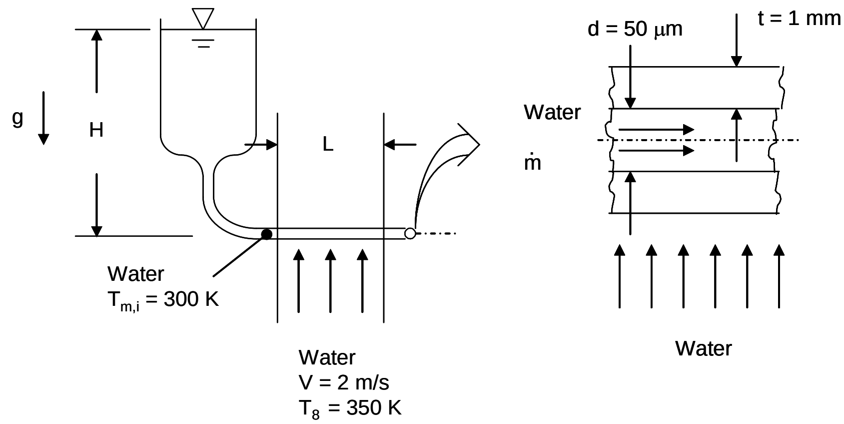
PROBLEM 8.115

KNOWN: Inner diameter of microscale tube, wall thickness of tube, temperature of water inside the tube, and temperature of water in cross flow over the tube.

FIND: (a) Required tube length at Re_D = 2000, (b) Water outlet temperature, (c) Pressure drop associated with the flow of water inside the tube, (d) Height of water column needed to supply the required inlet pressure and time needed to collect 0.1 liter of water. Discuss measurement of outlet water temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties and steady-state conditions, (2) Incompressible liquid and negligible viscous dissipation, (3) Negligible microscale or nanoscale effects.

PROPERTIES: Table A.6, water: ($\overline{T}_m$ = 305 K): k = 0.620 W/m·K, c_p = 4178 J/kg·K, μ = 769 × 10^{-6} N·s/m^2, Pr = 5.2, ρ = 995 kg/m^3: ($\overline{T}$ = 330 K): k = 0.650 W/m·K, c_p = 4194 J/kg·K, μ = 489 × 10^{-6} N·s/m^2, Pr = 3.15, ρ = 984 kg/m^3. Table A.3 glass: k = 1.4 W/m·K.

ANALYSIS: (a) At Re_D = 2000, Equation 8.3 yields $x_{fd,h} = 0.05Re_DPrD$ = 0.05 × 2000 × 5.2 × 50 × 10^{-6} m = 26 × 10^{-3} m. Therefore, L = $2x_{fd,h}$ = 2 × 26 × 10^{-3} m = 52 × 10^{-6} m = 52 mm. <

(b) Equation 8.45a is

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right) \qquad (1)$$

where we will use $\overline{U} = \overline{U_i}$, $A_s = A_{s,i}$. Note that $Re_D = 4\dot{m}/(\pi D\mu)$ so that $\dot{m} = Re_D\,\pi D\mu/4$ = 2000 × π × 50 × 10^{-6} m × 769 × 10^{-6} N·s/m^2/4 = 60.4 × 10^{-6} kg/s. Therefore, u_m = $\dot{m}/(\rho A_c)$ = 60.4 × 10^{-6} kg/s × 4/ (995 kg/m^3× π × (50 × 10^{-6} m)^2) = 31 m/s. From Equation 3.36,

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$$\overline{U_i} = \frac{1}{\frac{1}{h_i} + \frac{d/2}{k_g}\ln\left[\frac{(d/2+t)}{d/2}\right] + \frac{d/2}{(d/2+t)}\frac{1}{h_o}} \tag{2}$$

$A_{s,i} = \pi dL = \pi \times 50 \times 10^{-6}$ m $\times$ 52 $\times 10^{-3}$ m = 8.17 $\times 10^{-6}$ m^2. From Equation 8.56,

$$\overline{Nu}_D = 3.66 + \frac{0.0668(50\times10^{-6}/53\times10^{-3})\times2000\times5.3}{1+0.04\left[(50\times10^{-6}/53\times10^{-3})\times2000\times5.3\right]^{2/3}} = 4.371$$

and $\bar{h}_D = h_i = \overline{Nu}_D \dfrac{k}{D} = 4.371\times0.620\,W/m\cdot K/50\times10^{-6}\,m = 54.2\times10^3\,W/m^2\cdot K$.

For the cross flow of water over the tube, $Re_D = VD\rho/\mu$ = 2 m/s $\times$ (50 $\times 10^{-6}$ m + 2 $\times$ 1 $\times 10^{-3}$ m)(984 kg/m^3)/489 $\times 10^{-6}$ N·s/m^2 = 8253. From Equation 7.46,

$$\overline{Nu}_D = 0.3 + \frac{0.62(8253)^{1/2}(3.15)^{1/3}}{\left[1+(0.4/3.15)^{2/3}\right]^{1/4}}\left[1+\left(\frac{8253}{282,000}\right)^{5/8}\right]^{4/5} = 85.14$$

and

$$\bar{h}_D = h_o = \overline{Nu}_D k/(d+2t) = 85.14\times0.65\,W/m\cdot K/(50\times10^{-6}\,m+2\times1\times10^{-3}\,m) = 27.0\times10^3\,W/m^2\cdot K$$

Therefore,

$$\overline{U}_i = \frac{1}{\left[\begin{array}{l}\frac{1}{54.2\times10^3\,W/m\cdot K} + \frac{50\times10^{-6}\,m/2}{1.4\,W/m\cdot K}\ln\left[\frac{(50\times10^{-6}\,m/2+1\times10^{-3}\,m)}{50\times10^{-6}\,m/2}\right] \\ +\frac{50\times10^{-6}\,m/2}{(50\times10^{-6}\,m/2+1\times10^{-3}\,m)}\times\frac{1}{27.0\times10^3\,W/m^2\cdot K}\end{array}\right]} = 11.7\times10^3\,W/m^2\cdot K$$

Equation (1) becomes

$$\frac{350K - T_{m,o}}{350K-300K} = \exp\left(-\frac{11.7\times10^3\,W/m^2\cdot K\times8.17\times10^{-6}\,m^2}{60.4\times10^{-6}\,kg/s\cdot4194\,J/kg\cdot K}\right)$$

or, $T_{m,o}$ = 316 K $\quad$ <

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(c) For laminar flow, Equation 8.19 yields $f = 64/Re_D = 64/2000 = 32 \times 10^{-3}$. Equation 8.22a yields

$$\Delta p = f\,\frac{\rho u_m^2}{2D}L = \frac{32\times10^{-3}\times995\,\text{kg/m}^3\times(31\,\text{m/s})^2\times52\times10^{-3}\,\text{m}}{2(50\times10^{-6}\,\text{m})} = 15.9\times10^6\,\text{Pa} \qquad <$$

(d) The pressure generated by the water column must offset the pressure drop in the tube. Therefore,

$$\rho g H = \Delta p \quad \text{or} \quad H = \Delta p/\rho g = 15.9\times10^6\,\text{N/m}^2/(995\,\text{kg/m}^3\times9.8\,\text{m/s}^2) = 1630\,\text{m} = 1.63\,\text{km} \qquad <$$

The time required for a particular volume of water to flow through the system is

$$t = \frac{V\rho}{m} = \frac{0.1\times\dfrac{1\,\text{m}^3}{1000\,\text{ml}}\times995\,\text{kg/m}^3}{60.4\times10^{-6}\,\text{kg/s}} = 1650\,\text{s} \qquad <$$

COMMENTS: (1) Microscale experimentation is often very difficult to perform. In addition to the difficulty in measuring the water outlet temperature, establishing a constant flow rate with such a large inlet pressure would be very difficult. (2) Turbulent conditions in microscale systems are rare in nature, and are difficult to achieve experimentally. (3) The glass tube wall is relatively thick. Therefore, conduction in the axial direction is likely to be significant. (4) The average mean water temperature inside the tube is $\overline{T}_m = (T_{m,i} + T_{m,o})/2 = (300\text{ K} + 316\text{ K})/2 = 308\text{ K}$. The assumed mean temperature of 305 K is good.