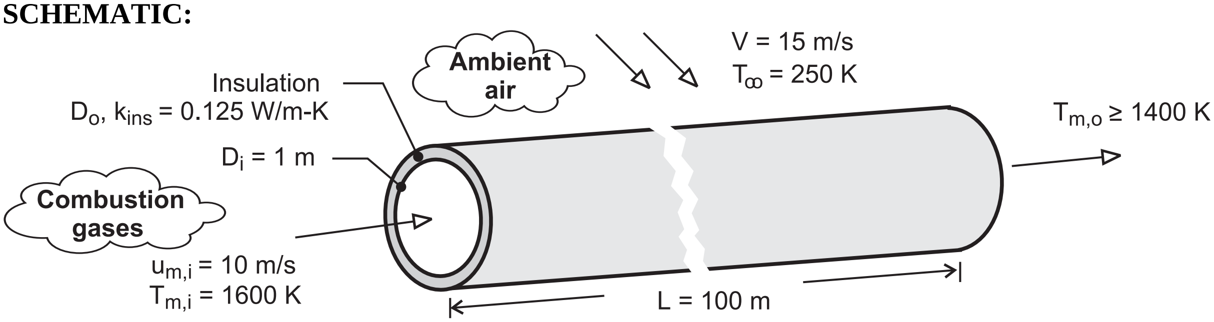
PROBLEM 8.44

KNOWN: Duct diameter and length. Thermal conductivity of insulation. Gas inlet temperature and velocity and minimum allowable outlet temperature. Temperature and velocity of air in cross flow.

FIND: Minimum allowable insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1)Combustion gases are ideal with negligible viscous dissipation and pressure variation, (2) Fully developed flow throughout duct, (3) Negligible duct wall conduction resistance, (4) Negligible effect of insulation thickness on outer convection coefficient and thermal resistance, (5) Properties of gas may be approximated as those of air.

PROPERTIES: *Table A-4*, air (p = 1 atm). $T_{m,i}$ = 1600K: (ρ_i = 0.218 kg/m^3). $\bar{T}_m = (T_{m,i} + T_{m,o})/2$ = 1500K: (ρ = 0.232 kg/m^3, c_p = 1230 J/kg·K, $\mu = 557 \times 10^{-7}$ N·s/m^2, k = 0.100 W/m·K, Pr = 0.685). $T_f \approx$ 300K (assumed): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: From Eqs. (8.45a) and (3.19),

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \frac{-1150\,\text{K}}{-1350\,\text{K}} = 0.852 = \exp\left(-\frac{\bar{U}A_s}{\dot{m}\,c_p}\right) = \exp\left(-\frac{1}{R_{tot}\dot{m}\,c_p}\right)$$

Hence, with $\dot{m} = (\rho\, u_m A_c)_i = 0.218\,\text{kg/m}^3 \times 10\,\text{m/s} \times \pi(1\text{m})^2/4 = 1.712\,\text{kg/s}$,

$$R_{tot} = -\left[\dot{m}\,c_p \ln(0.852)\right]^{-1} = -\left[1.712\,\text{kg/s} \times 1230\,\text{J/kg}\cdot\text{K} \times (-0.160)\right]^{-1} = 2.96 \times 10^{-3}\,\text{K/W}$$

The total thermal resistance is

$$R_{tot} = R_{conv,i} + R_{cond,ins} + R_{conv,o} = (h_i \pi D_i L)^{-1} + \frac{\ln(D_o/D_i)}{2\pi k_{ins} L} + (h_o \pi D_o L)^{-1} \qquad (1)$$

With $Re_{D,i} = 4\dot{m}/\pi D_i \mu = (4 \times 1.712\,\text{kg/s}) / \left(\pi \times 1\text{m} \times 557 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2\right) = 39{,}130$, the Dittus-Boelter correlation yields

$$h_i = \left(\frac{k}{D}\right) 0.023\,Re_D^{4/5}\,Pr^{0.3} = \left(\frac{0.100\,\text{W/m}\cdot\text{K}}{1\text{m}}\right) 0.023(39{,}130)^{4/5}(0.685)^{0.3} = 9.69\,\text{W/m}^2\cdot\text{K}$$

The internal resistance is then

$$R_{conv,i} = (h_i \pi D_i L)^{-1} = \left(9.69\,\text{W/m}^2\cdot\text{K} \times \pi \times 1\text{m} \times 100\text{m}\right)^{-1} = 3.28 \times 10^{-4}\,\text{K/W}$$

With $Re_D \approx VD_i/\nu$ = 15 m/s × 1m/15.89 × 10^{-6} m^2/s = 9.44 × 10^5, the Churchill-Bernstein correlation yields

Continued …

$$h_o \approx \left(\frac{k}{D}\right)\left\{0.3+\frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}\right\}=30.9\,W/m^2\cdot K$$

$$R_{conv,o} \approx \left(h_o \pi D_i L\right)^{-1} = \left(30.9\,W/m^2\cdot K\times\pi\times 1m\times 100m\right)^{-1} = 1.03\times10^{-4}\,K/W$$

Hence, from Eq. (1)

$$\frac{\ln\left(D_o/D_i\right)}{2\pi\,k_{ins}\,L} = \left(2.96\times10^{-3}-3.33\times10^{-4}-1.03\times10^{-4}\right)K/W = 2.53\times10^{-3}\,K/W$$

$$D_o = D_i\,\exp\left(2\pi k_{ins} L\times 2.53\times10^{-3}\,K/W\right) = 1m\times\exp\left(1.59\times10^{-2}\,K/W\times 0.125\,W/m\cdot K\times 100m\right) = 1.22m$$

Hence, the minimum insulation thickness is

$$t_{min} = \left(D_o - D_i\right)/2 = 0.11m \qquad <$$

COMMENTS: With D_o = 1.22m, use of D_i = 1m to evaluate the outer convection coefficient and thermal resistance is a reasonable approximation. However, improved accuracy may be obtained by using the calculated value of D_o to determine conditions at the outer surface and iterating on the solution.