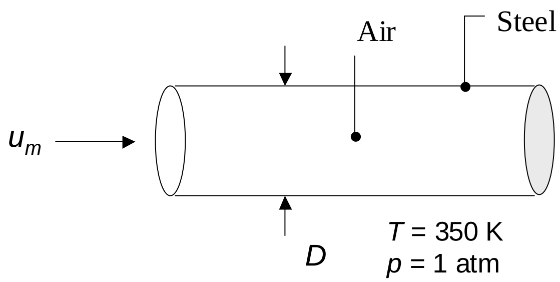
PROBLEM 8.114

KNOWN: Temperature and pressure of a gas flowing in a circular tube of known diameter with constant surface heat flux. Thermal and momentum accommodation coefficients. Fully developed laminar flow.

FIND: Graph of the Nusselt number for tube diameters of 1 μm ≤ D ≤ 1 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Ideal gas behavior. (2) Fully-developed laminar flow.

PROPERTIES: Table A.4 (T = 350 K): Air; c_p = 1009 J/kg·K, k = 0.030 W/m·K, Pr = 0.70. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m.

ANALYSIS: The ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315\ \text{kJ/kmol}\cdot\text{K}}{28.97\ \text{kg/kmol}} = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.009\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.722\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad \gamma = \frac{c_p}{c_v} = \frac{1.009}{0.722} = 1.398$$

From Equation 2.11 the mean free path of air is

$$\lambda_{\text{mfp}} = \frac{k_B T}{\sqrt{2}\pi d^2 p} = \frac{1.381\times10^{-23}\ \text{J/K}\times 350\ \text{K}}{\sqrt{2}\pi\left(0.372\times10^{-9}\ \text{m}\right)^2\left(1.0133\times10^5\ \text{N/m}^2\right)} = 77.6\times10^{-9}\ \text{m} = 77.6\ \text{nm}$$

From Equation 8.78 the Nusselt number may be expressed as

$$Nu_D = \frac{hD}{k} = \frac{48}{11 - 6\zeta + \zeta^2 + 48\Gamma_t} \qquad (1)$$

where

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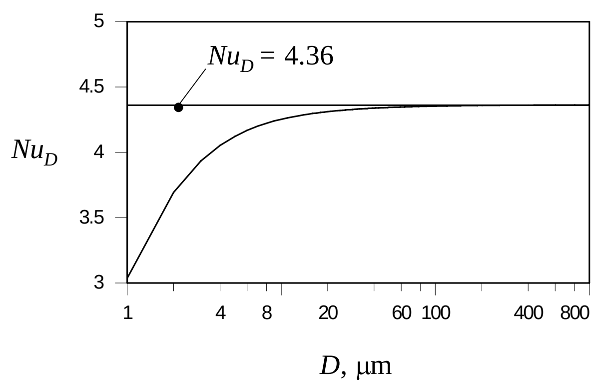
$$\Gamma_t = \frac{2-\alpha_t}{\alpha_t}\frac{2\gamma}{\gamma+1}\left[\frac{\lambda_{\text{mfp}}}{PrD}\right] = \frac{2-0.92}{0.92}\cdot\frac{2\times1.398}{1.398+1}\left[\frac{77.6\times10^{-9}\ \text{m}}{0.700D}\right] = \frac{152\times10^{-9}\ \text{m}}{D} \qquad (2)$$

$$\Gamma_p = \frac{2-\alpha_p}{\alpha_p}\left[\frac{\lambda_{\text{mfp}}}{D}\right] = \frac{2-0.87}{0.87}\cdot\left[\frac{77.6\times10^{-9}\ \text{m}}{D}\right] = \frac{101\times10^{-9}\ \text{m}}{D} \qquad (3)$$

$$\zeta = \frac{8\Gamma_p}{(1+8\Gamma_p)} \qquad (4)$$

Equations 1 through 4 may be combined to yield the following graph that shows the variation of the Nusselt number over the tube diameter range 1 μm ≤ D ≤ 1000 μm.



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COMMENTS: (1) The Nusselt number begins to be affected by the tube dimension at a tube diameter of $D \approx 100$ µm. (2) Equation 8.78 is associated with constant heat flux conditions. We would expect a similar reduction in Nusselt numbers for constant temperature wall conditions.