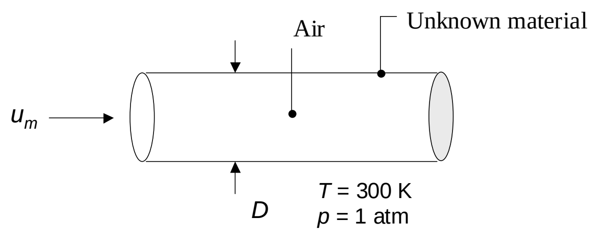
PROBLEM 8.112

KNOWN: Temperature and pressure of air flowing in a circular tube of known diameter. Thermal and momentum accommodation coefficients. Fully developed laminar flow with constant heat flux.

FIND: Graph of the Nusselt number versus tube diameter for 1 μm ≤ D ≤ 1 mm and (*a*) $\alpha_t = 1$, $\alpha_p = 1$, (*b*) $\alpha_t = 0.1$, $\alpha_p = 0.1$, (*c*) $\alpha_t = 1$, $\alpha_p = 0.1$ and (*d*) $\alpha_t = 0.1$, $\alpha_p = 1$.

SCHEMATIC:



ASSUMPTIONS: (1) Ideal gas behavior. (2) Fully-developed laminar flow.

PROPERTIES: Table A.4 (T = 300 K): Air; c_p = 1007 J/kg·K, Pr = 0.707. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m.

ANALYSIS: The ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315\ \text{kJ/kmol}\cdot\text{K}}{28.97\ \text{kg/kmol}} = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.007\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.720\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad \gamma = \frac{c_p}{c_v} = \frac{1.007}{0.720} = 1.399$$

From Equation 2.11 the mean free path of air is

$$\lambda_{\text{mfp}} = \frac{k_B T}{\sqrt{2}\pi d^2 p} = \frac{1.381\times10^{-23}\ \text{J/K}\times 300\ \text{K}}{\sqrt{2}\pi\left(0.372\times10^{-9}\ \text{m}\right)^2\left(1.0133\times10^5\ \text{N/m}^2\right)} = 66.5\times10^{-9}\ \text{m} = 66.5\ \text{nm}$$

From Equation 8.78, the Nusselt number may be expressed as

$$Nu_D = \frac{hD}{k} = \frac{48}{11 - 6\zeta + \zeta^2 + 48\Gamma_t} \qquad (1)$$

where

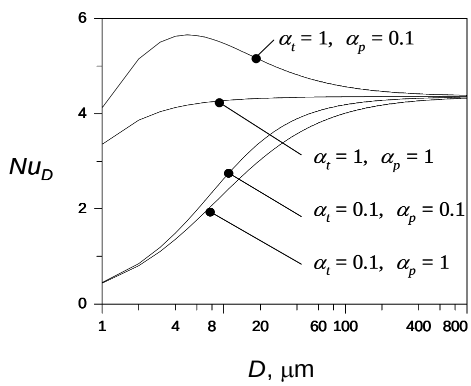
$$\Gamma_t = \frac{2-\alpha_t}{\alpha_t}\frac{2\gamma}{\gamma+1}\left[\frac{\lambda_{\text{mfp}}}{PrD}\right] \qquad (2)$$

Continued…

$$\Gamma_p = \frac{2-\alpha_p}{\alpha_p}\left[\frac{\lambda_{\text{mfp}}}{D}\right] \tag{3}$$

$$\zeta = \frac{8\Gamma_p}{\left(1+8\Gamma_p\right)} \tag{4}$$

Equations 1 through 4 may be combined to yield the following graph that shows the variation of the Nusselt number over the tube diameter range 1 μm $\leq D \leq$ 1000 μm.



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The accommodation coefficients begin to influence the Nusselt number (and hence the convection heat transfer coefficient) at diameters less than approximately 400 μm. <

The Nusselt number is least sensitive to changes in the tube diameter for $\alpha_t = \alpha_p = 1$. <

The Nusselt number can exceed 4.36 when the momentum accommodation coefficient is small and the thermal accommodation coefficient is large. <

Small values of the thermal accommodation coefficient in conjunction with large values of the momentum accommodation coefficient result in the most significant reductions in the Nusselt number. <

The Nusselt number can increase or decrease relative to the value associated with conventional flows, and the change in the Nusselt number can be quite large. Hence prediction of convection heat transfer coefficients in nano- and some microscale devices involving gas flow is typically subject to a high degree of uncertainty. <

Comment: Thermal accommodation coefficients can be of very small value, as discussed in Chapter 3.