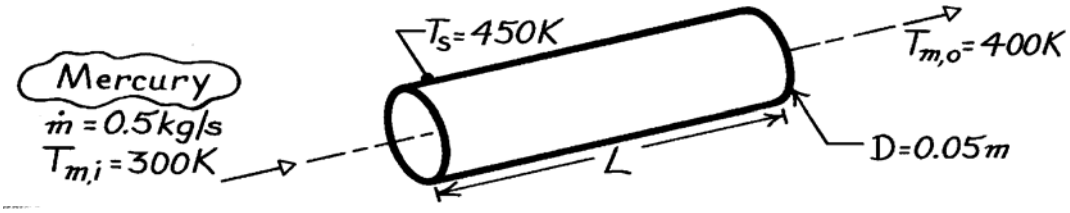
PROBLEM 8.45

KNOWN: Flow rate, inlet temperature and desired outlet temperature of liquid mercury flowing through a tube of prescribed diameter and surface temperature.

FIND: Required tube length and error associated with use of a correlation for moderate to large Pr fluids.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3)Incompressible liquid with negligible viscous dissipation, (4) Fully developed flow.

PROPERTIES: *Table A-5*, Mercury $\left(\overline{T}_m = 350\text{K}\right)$: c_p = 137.7 J/kg·K, $\mu = 0.1309 \times 10^{-2}$ N·s/m^2, k = 9.18 W/m·K, Pr = 0.0196.

ANALYSIS: The Reynolds and Peclet numbers are

$$\text{Re}_D = \frac{4\dot{m}}{\pi\, D\mu} = \frac{4 \times 0.5 \text{ kg/s}}{\pi\left(0.05\text{m}\right) 0.1309 \times 10^{-2} \text{ N}\cdot\text{s/m}^2} = 9727$$

$$\text{Pe}_D = \text{Re}_D\ \text{Pr} = 9727\left(0.0196\right) = 191.$$

Hence, assuming fully developed turbulent flow throughout the tube, it follows from Eq. 8.65 that

$$\overline{h} = \frac{k}{D}\left(5.0 + 0.025\ \text{Pe}_D^{0.8}\right) = \frac{9.18 \text{ W/m}\cdot\text{K}}{0.05 \text{ m}}\left(5.0 + 0.025 \times 191^{0.8}\right) = 1224 \text{ W/m}^2\cdot\text{K}.$$

From Eq. 8.41a, it follows that

$$L = -\frac{\dot{m}\, c_p}{\pi\, D\overline{h}} \ell\text{n}\frac{\Delta T_o}{\Delta T_i} = -\frac{\left(0.5 \text{ kg/s}\right) 137.7 \text{ J/kg}\cdot\text{K}}{\pi\left(0.05 \text{ m}\right) 1224 \text{ W/m}^2\cdot\text{K}} \ell\text{n}\frac{450 - 400}{450 - 300} = 0.39 \text{ m.} \qquad <$$

If the Dittus-Boelter correlation, Eq. 8.60, is used in place of Eq. 8.65,

$$\overline{h} = \frac{k}{D} 0.023\ \text{Re}_D^{4/5}\ \text{Pr}^{0.4} = \frac{9.18 \text{ W/m}^2\cdot\text{K}}{0.05 \text{ m}} 0.023\left(9727\right)^{4/5}\left(0.0196\right)^{0.4} = 1358 \text{ W/m}^2\cdot\text{K}$$

and the required tube length is

$$L = -\frac{\dot{m}\, c_p}{\pi\, D\overline{h}} \ell\text{n}\frac{\Delta T_o}{\Delta T_i} = -\frac{\left(0.5 \text{ kg/s}\right) 137.7 \text{ J/kg}\cdot\text{K}}{\pi\left(0.05 \text{ m}\right) 1358 \text{ W/m}^2\cdot\text{K}} \ell\text{n}\frac{450 - 400}{450 - 300} = 0.35 \text{ m.} \qquad <$$

COMMENTS: (1) Such good agreement between results does not occur in general. For example, if $\text{Re}_D = 2 \times 10^4$, $\overline{h}$ = 1463 from Eq. 8.65 and 2417 from Eq. 8.60. Large errors are usually associated with using conventional (moderate to large Pr) correlations with liquid metals. (2) The Dittus-Boelter correlation is recommended for $\text{Re}_D \gtrsim 10{,}000$, which is not quite satisfied here.