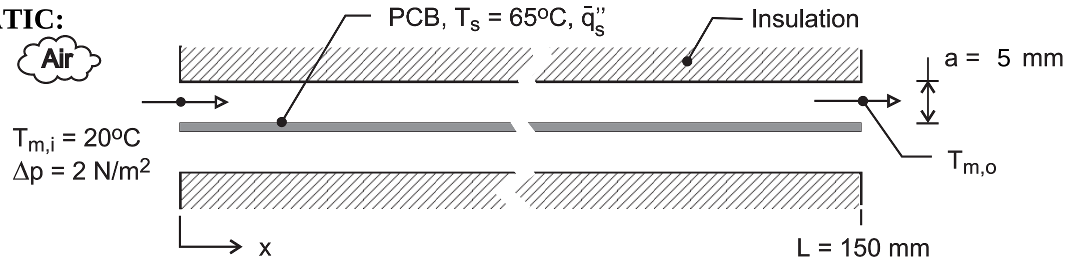
PROBLEM 8.92

KNOWN: Printed-circuit board (PCB) with uniform temperature T_s cooled by laminar, fully developed flow in a parallel-plate channel. The air flow with an inlet temperature of $T_{m,i}$ is driven by a pressure difference, Δp.

FIND: The average heat removal rate per unit area, $\bar{q}''_s \left(W/m^2\right)$, from the PCB.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar, fully developed flow, (2) Upper and lower walls of the channel are insulated and of infinite extent in the transverse direction, (3) PCB has uniform surface temperature, (4) Constant properties, (5) Ideal gas with negligible viscous dissipation.

PROPERTIES: *Table A-4*, Air (T_m = 293 K, 1 atm): ρ = 1.192 kg/m^3, c_p = 1007 J/kg·K, ν = 1.531 × 10^{-5} m^2/s, k = 0.0258 W/m·K, Pr = 0.709.

ANALYSIS: The energy equations for determining the heat rate from one surface of the board are Eqs. 8.34 and 8.41b

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = \bar{q}''_s A_s \qquad (1)$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{P_h L \bar{h}}{\dot{m}\, c_p}\right) \qquad (2)$$

where A_s = Lw and P = w, since heat transfer is only from one surface, where w is the width in the transverse direction. For the fully developed flow condition, the velocity is estimated from the friction pressure drop relation, Eq. 8.22a,

$$\Delta p = f\left(\rho u_m^2 / 2\right)\left(L / D_h\right) \qquad (3)$$

where the hydraulic diameter for the channel cross section is

$$D_h = \frac{4 A_c}{P} = \frac{4(w\,a)}{2(w+a)} = 2a \qquad a << w$$

The friction factor f from Table 8.1 for the cross section b/a = ∞ is

$$f \cdot Re_{Dh} = 96 \qquad (4)$$

where the Reynolds number is

$$Re_{Dh} = u_m D_h / \nu \qquad (5)$$

Continued …

PROBLEM 8.92 (Cont.)

and the flow rate through one channel is

$$\dot{m} = \rho A_c u_m = \rho(wa)u_m \tag{6}$$

For fully developed laminar flow from Table 8.1.

$$\overline{Nu}_D = \bar{h} D_h / k = 4.86 \tag{7}$$

Substituting Eqs. (4) and (5) into Eq. (3) and solving for u_m yields

$$u_m = \Delta p D_h^2 / 48\nu\rho L = 2 N/m^2 \times (0.01 m)^2 / 48 \times 1.531 \times 10^{-5} m^2/s \times 1.192 kg/m^3 \times 0.15 m = 1.52 m/s$$

$$Re = u_m D_h / \nu = 1.52 m/s \times 0.01 m / 1.531 \times 10^{-5} m^2/s = 994$$

Thus the flow is laminar, as assumed. From Eqs. (6), (7), and (2), $\dot{m}/w = \rho u_m a$ = 1.192kg/m^3× 1.52 m/s × 0.005 m = 0.00907 kg/s·m. $\bar{h} = \overline{Nu}_D k / D_h$ = 4.86 × 0.0258 W/m·K/0.01m = 12.5 W/m^2·K. $T_{m,o} = T_s - (T_s - T_{m,i})\exp(-L\bar{h}/(\dot{m}/w)c_p)$ = 65°C - 45°Cexp(-0.15m × 12.5W/m^2·K/0.00907kg/s·m× 1007 J/kg·K) = 28.4°C.

From Eq. (1)

$$q' = \frac{\dot{m}}{w} c_p (T_{m,o} - T_{m,i}) = 0.00907 kg/m \cdot s \times 1007 J/kg \cdot K \times (28.4 - 20)°C = 76.5 W/m$$ <

$$q'' = q'/L = 510 W/m^2$$

COMMENTS: (1) The thermophysical properties of the air are evaluated at the average mean temperature, $\bar{T}_m = (T_{m,i} + T_{m,o})/2$.

(2) The fully developed flow length, $x_{fd,t}$, for the channel follows from Eq. 8.23,

$$x_{fd,t} = D_h \times 0.05 Re_{Dh} Pr$$

$$x_{fd,t} = 2 \times 0.010 \text{ m} \times 0.05 \times 7954 \times 0.709 = 5.6 \text{ m}$$

Since L << $x_{fd,t}$, we conclude that the flow is not likely to be fully developed.