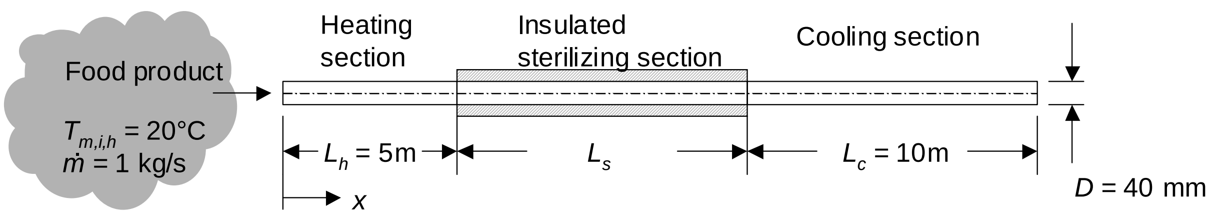
PROBLEM 8.34

KNOWN: Initial food temperature and mass flow rate. Length of heating and cooling sections in a food sterilizer. Diameter of sterilizer tube. Time-at-temperature constraint, and constraint on local maximum food temperature.

FIND: (a) Heat flux in the heating section. (b) Maximum local product temperature and its location. (c) Minimum required sterilizing section length. (d) Sketch of the axial distributions of the mean, surface, and centerline food temperatures from entrance to exit of sterilizer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible viscous dissipation.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = 330 K, p = 1 atm): $\mu = 489\times10^{-6}$ N·s/m^2, Pr = 3.15, k = 0.65 W/m·K, c_p = 4184 J/kg·K, ρ = 984 kg/m^3.

ANALYSIS: (a) An energy balance applied to the heating section yields

$$q = q''A = q''\pi DL_h = \dot{m}c_p(T_{m,o,h} - T_{m,i,h})$$

which may be rearranged to provide the expression

$$q'' = \frac{\dot{m}c_p(T_{m,o,h} - T_{m,i,h})}{\pi DL_h} = \frac{1\text{kg/s}\times 4184\text{J/kg}\cdot\text{K}\times(90-20)°\text{C}}{\pi\times 0.04\text{m}\times 5\text{m}} = 466{,}000\text{W/m}^2 = 466\text{kW/m}^2 \quad <$$

(b) The maximum local product temperature occurs at the tube wall at the end of the heating section. The Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 1\text{kg/s}}{\pi\times 0.04\text{m}\times 489\times10^{-6}\text{N}\cdot\text{s/m}^2} = 65{,}090$$

Hence, the flow is turbulent. Since L_h/D = 5m/0.04m = 125, the flow is fully-developed. Using the Dittus-Boelter correlation,

$$h = \frac{k}{D}\left[0.023Re_D^{4/5}Pr^{0.4}\right] = \frac{0.65\text{W/m}\cdot\text{K}}{0.04\text{m}}\left[0.023\times 65{,}090^{4/5}\times 3.15^{0.4}\right] = 4190\text{W/m}^2\cdot\text{K}$$

From Newton's law of cooling,

$$T_s(x = L_h = 5\text{m}) = T_{m,o,h} + \frac{q''}{h} = 90°\text{C} + \frac{466{,}000\text{W/m}^2}{4190\text{W/m}^2\cdot\text{K}} = 201°\text{C} \quad <$$

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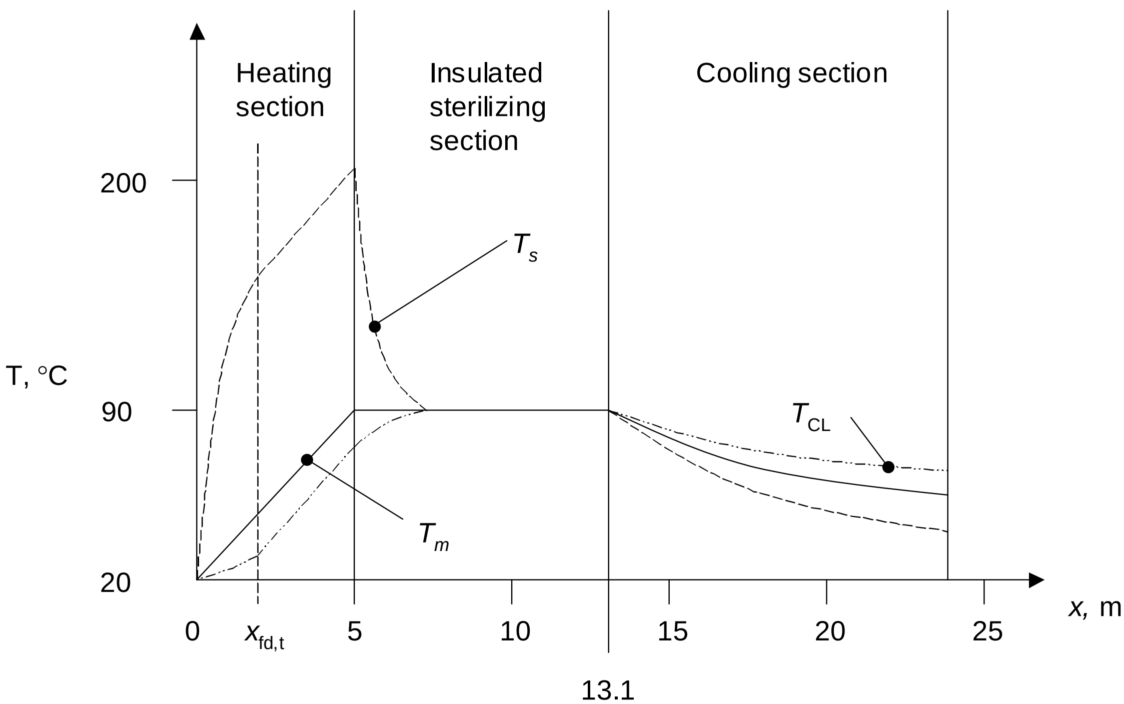
The second constraint is satisfied. <

(c) The minimum length of the sterilizing section is

$$L_s = u_m t_s = \frac{4\dot{m}}{\rho \pi D^2} t_s = \frac{4 \times 1 \text{ kg/s}}{984 \text{ kg/m}^3 \times \pi \times (0.04 \text{ m})^2} \times 10 \text{ s} = 8.1\text{m}$$ <

(d) The axial distributions of the mean, surface, and centerline temperatures are shown below.



Important features of the temperature distribution are as follows.

$0 \le x \le x_{fd,t}$: Near the tube entrance, the heat transfer coefficient is theoretically infinite, and all three temperatures are nearly the same value.

$x_{fd,t} \le x \le L_h$: The flow is fully-developed; the shape of the radial temperature distribution does not change down the tube length. Therefore, the three temperature distributions are parallel.

$L_h \le x \le L_h + L_s$: The heat transfer coefficient is zero. However, temperature differences exist in the fluid, with warm temperatures adjacent to the tube wall and cool temperatures near the centerline of the tube. As the flow progresses down the insulated sterilizing section, the temperatures equilibrate by way of diffusion and turbulent mixing. The equilibration takes place over a distance approximately equal to $x_{fd,t}$.

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$L_h + L_s \le x \le L_h + L_s + L_c$: The fluid is cooled. Hence, the warmest temperature fluid is at the centerline, and the coolest fluid is adjacent to the tube wall. Since the fluid is cooled by exposure of the tube to the environment, the cooling rate is expected to be smaller than the heating rate in the heating section. Hence, the radial temperature differences in the cooling section are smaller than the radial temperature differences in the heating section.

COMMENTS: (1) The velocity of the fluid at the centerline exceeds velocities at any other radial location. Hence, the fluid at the centerline of the tube will not satisfy the time-at-temperature criterion. Therefore, use of a coiled tube or other heat transfer enhancement devices (Section 8.7) would be appropriate in this application. (2) The insulation thickness in the sterilizing section should be much greater than the critical insulation thickness.