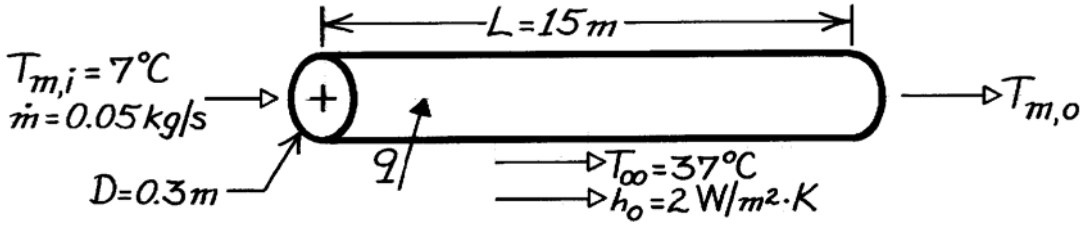
PROBLEM 8.72

KNOWN: Length and diameter of air conditioning duct. Inlet temperature of chilled air. Temperature and convection coefficient associated with outer air. Chilled air flowrate.

FIND: Chilled air exit temperature and heat flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible tube wall conduction resistance, (3) Ideal gas with negligible viscous dissipation, pressure variation, and axial conduction.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ kg/s·m, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: The exit temperature may be obtained from Eq. 8.45a, where

$$\bar{U} = \left(h_i^{-1} + h_o^{-1}\right)^{-1}$$

With $$Re_D = \left(4\dot{m}/\pi\, D\mu\right) = \frac{4\left(0.05\ \text{kg/s}\right)}{\pi\left(0.3\ \text{m}\right)184.6\times10^{-7}\ \text{kg/s}\cdot\text{m}} = 11,495$$

the flow is turbulent and, assuming fully developed conditions over the entire length, the Dittus-Boelter correlation yields

$$Nu_D = 0.023 Re_D^{4/5}\, Pr^{0.4} = 0.023\left(11,495\right)^{4/5}\left(0.707\right)^{0.4} = 35.5$$

$$h_i = Nu_D\left(k/D\right) = 35.5\left(0.0263\ \text{W/m}\cdot\text{K}/0.3\ \text{m}\right) = 3.11\ \text{W/m}^2\cdot\text{K}$$

and $$\bar{U} = \left(3.11^{-1} + 2.0^{-1}\right)^{-1}\left(\text{W/m}^2\cdot\text{K}\right) = 1.22\ \text{W/m}^2\cdot\text{K}.$$

Eq. 8.45a yields $$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)\exp\left[-\left(\pi\ DL/\dot{m}\ c_p\right)\bar{U}\right]$$

$$T_{m,o} = 37^\circ\text{C} - 30^\circ\text{C}\,\exp\left[-\frac{\pi\left(0.3\ \text{m}\right)15\ \text{m}\left(1.22\ \text{W/m}^2\cdot\text{K}\right)}{0.05\ \text{kg/s}\left(1007\ \text{J/kg}\cdot\text{K}\right)}\right] = 15.7^\circ\text{C} \quad <$$

and the heat rate is

$$q = \dot{m}\, c_p\left(T_{m,o} - T_{m,i}\right) = 0.05\ \text{kg/s}\left(1007\ \text{J/kg}\cdot\text{K}\right)\left(8.7^\circ\text{C}\right) = 438\ \text{W}. \quad <$$

COMMENTS: (1) The temperature rise of the chilled air is excessive, and the outer surface of the duct should be insulated to reduce $\bar{U}$ and thereby $T_{m,o}$ and q. (2) The temperature selected for evaluating air properties was not very accurate. Air properties should be evaluated at $\bar{T}_m = (T_{m,o} + T_{m,i})/2 \approx 285$ K.