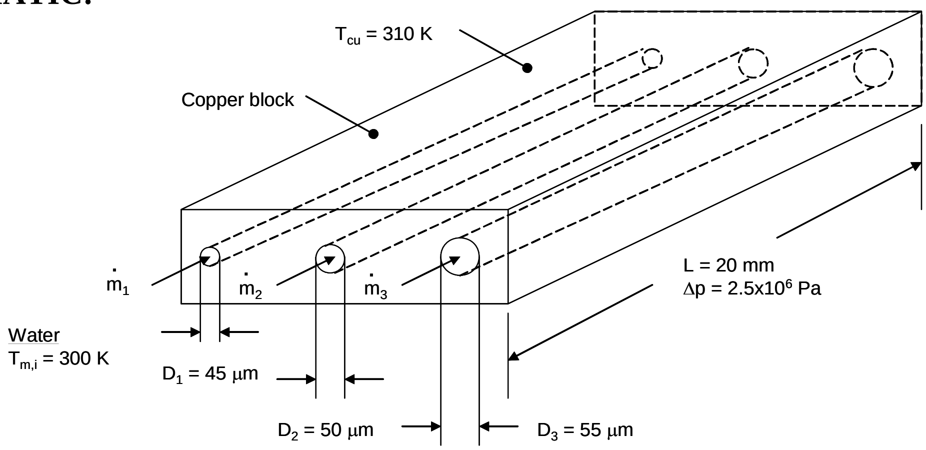
PROBLEM 8.117

KNOWN: Diameters and length of three microchannels machined in a copper block. Inlet temperature of water flowing through the channels, copper block temperature, pressure difference from inlet to outlet of the channels.

FIND: (a) Mass flow rate and outlet temperature in each channel, (b) Average flow rate through each channel and average, mixed temperature of water collected from all three channels, (c) Comparison between average flow rates and average heat transfer rates based upon experiment to that calculated based upon a single microchannel diameter of 50 μm.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties and steady-state conditions, (2) Incompressible liquid and negligible viscous dissipation, (3) Negligible microscale or nanoscale effects, (4) Negligible entrance or exit losses in the microchannels, (5) Fully developed flow for purposes of calculating the mass flow rate in each channel, (6) Isothermal copper block.

PROPERTIES: Table A.6, water: ($\overline{T}_m$ = 305 K): k = 0.620 W/m·K, c_p = 4178 J/kg·K, μ = 769 × 10^{-6} N·s/m^2, ν = 7.728 × 10^{-7} m^2/s, Pr = 5.2, ρ = 995 kg/m^3.

ANALYSIS: (a) For the D = 50 μm channel, from Equation 8.22a,

$$\Delta p = f\rho u_m^2 L/2D = f \times 995\text{kg/m}^3 \times u_m^2 \times 20\times10^{-3}\text{m}/(2\times50\times10^{-6}\text{m}) \qquad (1)$$

where the friction factor may be evaluated using the Petukhov expression,

$$f = (0.790\ln Re_D - 1.64)^{-2} \qquad (2)$$

The Reynolds number may be expressed as

$$Re_D = \frac{u_m D}{\nu} = \frac{u_m \times 50\times10^{-6}\text{m}}{7.728\times10^{-7}\text{m}^2/\text{s}} \qquad (3)$$

Continued…

Simultaneous solution of Equations (1) through (3) yields, for the D = 50 μm channel, Re_D = 845, u_m = 13.06 m/s. The mass flow rate is

$$\dot{m} = \rho u_m \pi D^2/4 = 995\text{kg/m}^3 \times 13.06\text{m/s} \times \pi \times (50\times10^{-6}\text{m})^2/4 = 2.55\times10^{-5}\text{kg/s}$$ <

The thermal entrance length is $x_{fd,t} = 0.05Re_D Pr D$ = 0.05 × 845 × 5.2 × 50 × 10^{-6} m = 11.0× 10^{-3} m = 11.0 mm. From the Hausen correlation,

$$\overline{Nu}_D = 3.66 + \frac{0.0668\times(50\times10^{-6}\text{m}/20\times10^{-3}\text{m})\times845\times5.2}{1+0.04\times\left[(50\times10^{-6}\text{m}/20\times10^{-3}\text{m})\times845\times5.2\right]^{2/3}} = 4.27$$

Hence,

$$\bar{h} = \frac{\overline{Nu}_D k}{D} = \frac{4.27\times0.62\text{W/m}\cdot\text{K}}{50\times10^{-6}\text{m}} = 5.29\times10^4\text{W/m}^2\cdot\text{K}$$

From Equation 8.42,

$$\begin{aligned}T_m(x=L) &= T_s - \left[T_s - T_{m,i}\right]\exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right)\\ &= 310\text{K} - [310\text{K}-300\text{K}]\exp\left(-\frac{\pi\times50\times10^{-6}\text{m}}{2.55\times10^{-5}\text{kg/s}\times4178\text{J/kg}\cdot\text{K}}\times5.29\times10^4\text{W/m}^2\cdot\text{K}\right)\\ &= 307.9\text{K} = 34.9°\text{C} = T_{m,o}\end{aligned}$$

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Results for the three different channels are shown in the table below. <

	D = 45 μm (case 1)	D = 50 μm (case 2)	D = 55 μm (case 3)
Re_D	690	845	1012
u_m (m/s)	11.85	13.06	14.23
$\dot{m}$ (kg/s)	1.88 × 10^{-5}	2.55 × 10^{-5}	3.36 × 10^{-5}
$x_{fd,t}$ (mm)	8.1	11.0	14.5
$\overline{Nu}_D$	4.12	4.27	4.44
$\bar{h}(\text{W/m}^2\cdot\text{K})$	5.68 × 10^4	5.29 × 10^4	5.01 × 10^4
$T_{m,o}$ (K)	308.7	307.9	307.1

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PROBLEM 8.117 (Cont.)

(b) The average mass flow rate is

$$\bar{\dot{m}} = (\dot{m}_1 + \dot{m}_2 + \dot{m}_3)/3 = \left[(1.88\times10^{-5} + 2.55\times10^{-5} + 3.36\times10^{-5})\text{kg/s}\right]/3 = 2.60\times10^{-5}\,\text{kg/s} \quad <$$

(c) The average, mixed outlet temperature is

$$T_{m,o} = (\dot{m}_1 T_{m,o,1} + \dot{m}_2 T_{m,o,2} + \dot{m}_3 T_{m,o,3})/(\dot{m}_1 + \dot{m}_2 + \dot{m}_3)$$

$$= \frac{(1.88\times10^{-5}\,\text{kg/s}\times308.7\text{K} + 2.55\times10^{-5}\,\text{kg/s}\times307.9\text{K} + 3.36\times10^{-5}\,\text{kg/s}\times307.1\text{K})}{(1.88\times10^{-5} + 2.55\times10^{-5} + 3.36\times10^{-5})\text{kg/s}} = 307.7\text{K}$$

(d) Equation 8.42 may be re-arranged to

$$\bar{h} = -\frac{\dot{m}c_p}{PL}\ln\left(\frac{T_s - T_{m,o}}{T_s - T_{m,L}}\right) = -\frac{2.60\times10^{-5}\,\text{kg/s}\times4178\,\text{J/kg}\cdot\text{K}}{\pi\times50\times10^{-6}\,\text{m}\times20\times10^{-3}\,\text{m}}\ln\left(\frac{310-307.7}{310-300}\right) = 50{,}800\,\text{W/m}^2\cdot\text{K}$$

Thus, the inferred value of the mass flow rate is 2% greater than the predicted value for a 50 μm diameter channel. The inferred value of the convection coefficient (50,800 W/m2·K) is 4% less than the predicted value for a 50 μm diameter channel. The experimenter must carefully assess his or her claims since the differences are small and might be attributed to variations in the channel dimensions that occur during their manufacture.

COMMENTS: (1) Experimentation at the microscale is challenging. Misinterpretation of the experimental results might occur unless the experimental system is designed very carefully. For example, the diameters of the channels might need to be measured after their manufacture. (2) When boring holes, the hole diameter is always greater than the diameter of the tool. If the experimentalist assumes that the actual hole size is the same as the tool size, what (inappropriate) conclusions might he or she make regarding possible microscale fluid flow and heat transfer effects when analyzing the measured results?