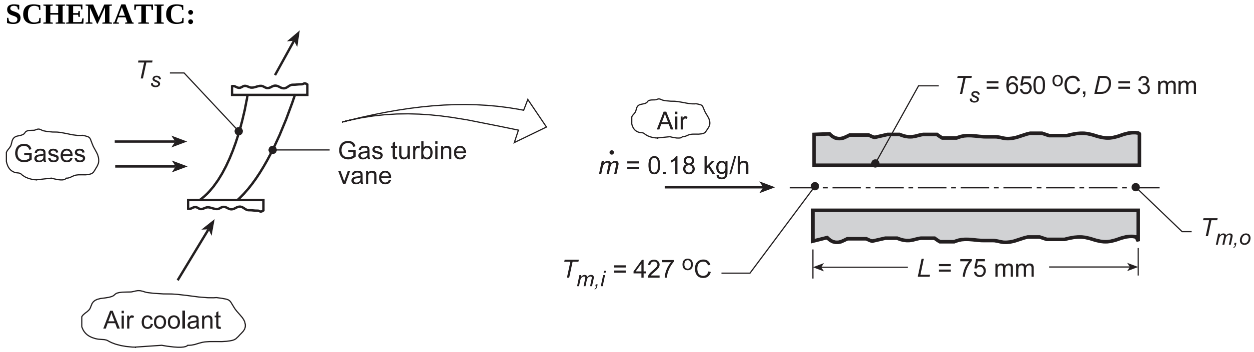
PROBLEM 8.50

KNOWN: Gas turbine vane approximated as a tube of prescribed diameter and length maintained at a known surface temperature. Air inlet temperature and flowrate.

FIND: (a) Outlet temperature of the air coolant for the prescribed conditions and (b) Compute and plot the air outlet temperature $T_{m,o}$ as a function of flow rate, $0.1 \le \dot{m} \le 0.6$ kg/h. Compare this result with those for vanes having passage diameters of 2 and 4 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation.

PROPERTIES: *Table A.4*, Air (assume $\overline{T}_m$ = 780 K, 1 atm): c_p = 1094 J/kg·K, k = 0.0563 W/m·K, μ = 363.7×10^{-7} N·s/m^2, Pr = 0.706.

ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m}c_p}\right) \tag{1}$$

where P = πD. For flow in a circular passage,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 0.18\,\mathrm{kg/h}\,(1/3600\,\mathrm{s/h})}{\pi(0.003\,\mathrm{m})363.7 \times 10^{-7}\,\mathrm{N \cdot s/m^2}} = 584\,. \tag{2}$$

The flow is laminar, and from Eq. 8.3, $x_{fd,h} = 0.05Re_D D$ = 88 mm. Thus, the flow is in the combined entry length. From Eq. 8.56, $Gz_D = (D/L)Re_D Pr$ = 16.5 and from Eq. 8.58,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \frac{\dfrac{3.66}{\tanh[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}]} + 0.0499 Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\,Pr^{1/6}\,Gz_D^{-1/6})} \tag{3}$$

$$\bar{h} = \frac{0.0563\,\mathrm{W/m \cdot K}}{0.003\,\mathrm{m}}\left(\frac{\dfrac{3.66}{\tanh[2.264 \times 16.5^{-1/3} + 1.7 \times 16.5^{-2/3}]} + 0.0499 \times 16.5 \tanh(16.5^{-1})}{\tanh(2.432 \times 0.706^{1/6} \times 16.5^{-1/6})}\right) = 95.0\,\mathrm{W/m^2 \cdot K}$$

Hence, the air outlet temperature is

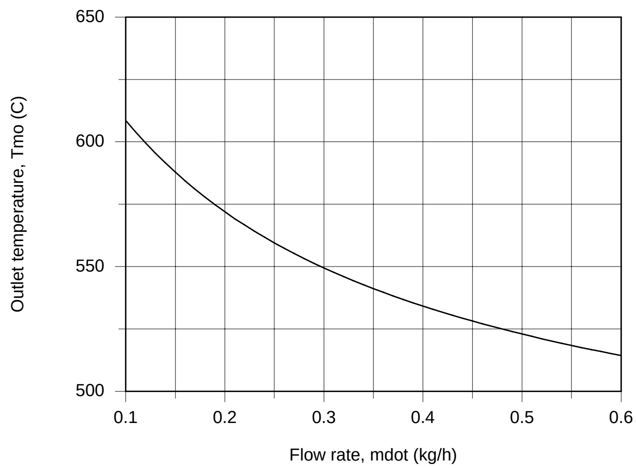
$$\frac{650 - T_{m,o}}{(650 - 427)^\circ\mathrm{C}} = \exp\left(-\frac{\pi(0.003\,\mathrm{m}) \times 0.075\,\mathrm{m} \times 95.0\,\mathrm{W/m^2 \cdot K}}{(0.18/3600)\,\mathrm{kg/s} \times 1094\ \mathrm{J/kg \cdot K}}\right)$$

$$T_{m,o} = 585^\circ\mathrm{C}$$ <

Continued...

PROBLEM 8.50 (Cont.)

(b) Using the *IHT Correlations Tool*, *Internal Flow*, for *Laminar Flow* with *combined entry length*, along with the energy balance and rate equations above, the outlet temperature $T_{m,o}$ was calculated as a function of flow rate for diameters of D = 2, 3 and 4 mm. The plot below shows that $T_{m,o}$ decreases strongly with increasing flow rate, but is independent of passage diameter.



COMMENTS: (1) Based upon the calculation for $T_{m,o}$ = 585°C, $\overline{T}_m$ = 779 K which is in good agreement with our assumption to evaluate the thermophysical properties. (2) Why is $T_{m,o}$ independent of D? Since Re_D varies inversely with D, Gz_D is independent of D, and so is Nu_D. From Eq. (3), note that $\overline{h}$ is inversely proportional to D, $\overline{h} \sim D^{-1}$. From Eq. (1), note that on the right-hand side the product $P \cdot \overline{h}$ will be independent of D. Hence, $T_{m,o}$ will depend only on the mass flow rate. This is, of course, a consequence of the laminar flow condition and will not be the same for turbulent flow.