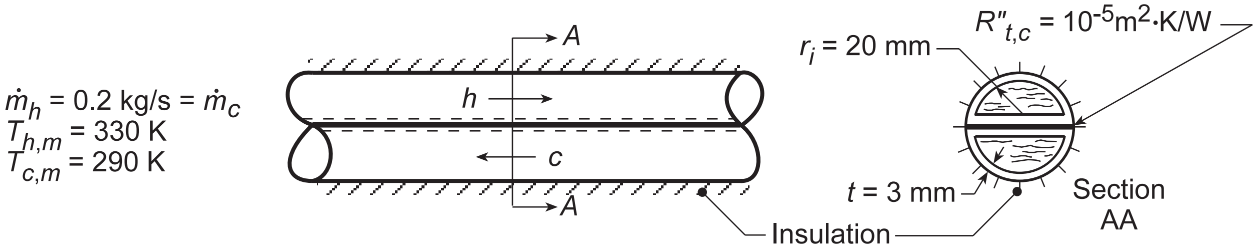
PROBLEM 8.86

KNOWN: Dimensions of semi-circular copper tubes in contact at plane surfaces. Thermal contact resistance. Tube flow conditions.

FIND: (a) Heat rate per unit tube length, and (b) The effect on the heat rate when the fluids are ethylene glycol, the exchanger tube is fabricated from an aluminum alloy, or the exchanger tube thickness is increased.

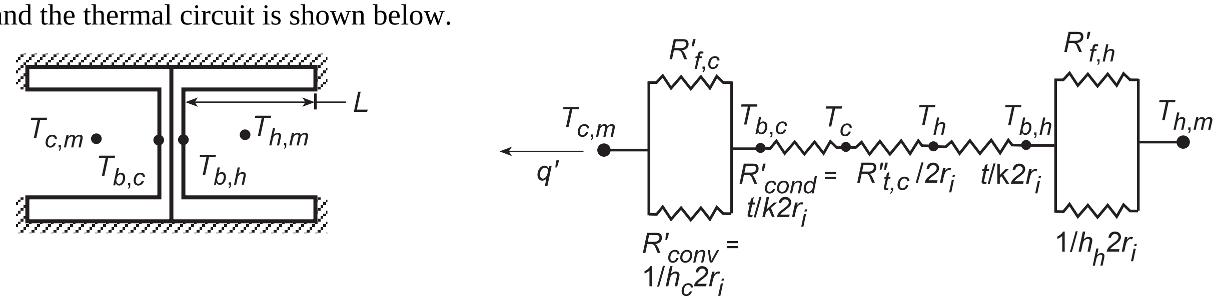
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Adiabatic outer surface, (4) Fully developed flow, (5) Negligible heat loss to surroundings.

PROPERTIES: *Table A.1*, Copper ($T \approx 300$ K): $k = 400$ W/m·K; Water (given): $\mu = 800 \times 10^{-6}$ kg/s·m, $k = 0.625$ W/m·K, $Pr = 5.35$.

ANALYSIS: (a,b) Heat transfer from the hot to cold fluids is *enhanced* by conduction through the semi-circular portions of the tube walls. The walls may be approximated as straight fins with an insulated tip, and the thermal circuit is shown below.



Note that, since each semi-circular surface is insulated on one side, surfaces may be combined to yield a *single* fin of thickness 2t with convection on both sides. Also, due to the equivalent geometry and the assumption of constant properties, there is symmetry on opposite sides of the contact resistance. From the thermal circuit, the heat rate is

$$q' = \frac{T_{h,m} - T_{c,m}}{R'_{tot}} \tag{1}$$

For flow through the semi-circular tube,

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{4\dot{m} A_c}{A_c P \mu} = \frac{4\dot{m}}{P\mu} = \frac{4\dot{m}}{\left(2r_i + \pi r_i\right)\mu} \tag{2}$$

$$Re_D = \frac{4 \times 0.2\,\text{kg/s}}{\left(2+\pi\right)0.02\,\text{m} \times 800 \times 10^{-6}\,\text{kg/s}\cdot\text{m}} = 9725$$

the flow is turbulent. Using the Gnielinski correlation, since $Re_D < 10{,}000$

$$Nu_D = \frac{(f/8)(Re_D - 1000)\,Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} = 69.9 \tag{3}$$

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where f = $(0.79\ln(Re_D)-1.64)^{-2} = 0.0317$

$$D_h = \frac{4A_c}{P} = \frac{4\left(\pi r_i^2/2\right)}{(\pi+2)r_i} = \frac{2\pi}{\pi+2}0.02\,\mathrm{m} = 0.0244\,\mathrm{m} \tag{4}$$

$$h = Nu_D\frac{k}{D_h} = 69.9\frac{0.625}{0.0244} = 1790\,\mathrm{W/m^2\cdot K}. \tag{5}$$

Find now values for the thermal resistance of the circuit.

$$R'_{conv} = \frac{1}{2r_i h} = \frac{1}{(0.04\,\mathrm{m})1790\,\mathrm{W/m^2\cdot K}} = 0.0140\,\mathrm{m\cdot K/W} \tag{6}$$

$$R'_{fin} = \frac{\theta_b}{q'_f} = \frac{1}{\left(hP'kA'_c\right)^{1/2}\tanh\left(hP/kA_c\right)L} \tag{7}$$

$$L = \pi r_i/2 = \pi(0.01\,\mathrm{m}) = 0.0314\,\mathrm{m} \qquad A_c = 2t\cdot 1\,\mathrm{m} = 0.006\,\mathrm{m^2} \qquad P \approx 2.1\ \mathrm{m} \quad (8,9,10)$$

$$\left(hP'kA'_c\right)^{1/2} = \left(1790\,\mathrm{W/m^2\cdot K}\times 2\,\mathrm{m/m}\times 400\,\mathrm{W/m\cdot K}\times 0.006\,\mathrm{m^2/s}\right)^{1/2} = 92.7\,\mathrm{W/K\cdot m}$$

$$\left(hP/kA_c\right)^{1/2}L = \left(1790\,\mathrm{W/m^2\cdot K}\times 2\,\mathrm{m}/400\,\mathrm{W/m\cdot K}\times 0.006\,\mathrm{m^2}\right)^{1/2}0.0314\,\mathrm{m} = 1.21$$

$$R'_{fin} = \frac{1}{92.7\,\mathrm{W/m\cdot K}(0.838)} = 0.0129\,\mathrm{m\cdot K/W} \tag{11}$$

$$R'_{cond} = \frac{t}{2kr_i} = \frac{0.003\,\mathrm{m}}{2(400\,\mathrm{W/m\cdot K})(0.02\,\mathrm{m})} = 1.875\times 10^{-4}\,\mathrm{m\cdot K/W} \tag{12}$$

$$R'_{t,c} = \frac{R''_{t,c}}{2r_i} = \frac{10^{-5}\,\mathrm{m^2\cdot K/W}}{2(0.02\,\mathrm{m})} = 2.5\times 10^{-4}\,\mathrm{m\cdot K/W} \tag{13}$$

The equivalent resistance of the parallel circuit is

$$R'_{eq} = \left(R'^{-1}_{fin} + R'^{-1}_{conv}\right)^{-1} = \left(77.6\,\mathrm{W/m\cdot K} + 71.5\,\mathrm{W/m\cdot K}\right)^{-1} = 6.70\times 10^{-3}\,\mathrm{m\cdot K/W} \tag{14}$$

Hence

$$R'_{tot} = 2\left(R'_{eq} + R'_{cond}\right) + R'_{t,c} \tag{15}$$

$$R'_{tot} = \left[2\left(6.70\times 10^{-3} + 1.875\times 10^{-4}\right) + 2.50\times 10^{-4}\right]\mathrm{m\cdot K/W} = 0.0140\,\mathrm{m\cdot K/W}$$

$$q' = \frac{(330-290)\,\mathrm{K}}{0.0140\,\mathrm{m\cdot K/W}} = 2850\,\mathrm{W/m}. \qquad <$$

(c) Using the *IHT Workspace* with the foregoing equations, analyses were performed and the results summarized in the table below. The “Conditions” are described below; the “Change” is relative to the base case condition.

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PROBLEM 8.86 (Cont.)

Condition*	$R'_{conv} \times 10^4$ (m·K/W)	$R'_{fin} \times 10^4$ (m·K/W)	$R'_{cond} \times 10^4$ (m·K/W)	$R'_{tot} \times 10^4$ (m·K/W)	$R'_{eq} \times 10^4$ (m·K/W)	q' (W/m)	Change (%)
Base case	140	129	1.88	140	67.0	2850	--
Ethylene glycol	6550	4210	1.88	5130	2560	77.9	-97
Aluminum alloy	140	171	4.24	165	76.9	2430	-15
Thicker tube	140	120	2.50	136	64.4	2930	+2.8

*Conditions: change from base case

Base case - water, copper (k = 400 W/m·K), t = 3 mm
Ethylene glycol - ethylene glycol instead of water, Re_D = 727, laminar, Nu_D = 3.66 estimated
Aluminum alloy - alloy (k = 177 W/m·K) instead of copper
Thicker tube - t = 4 mm instead of 3 mm

As expected, using ethylene glycol as the working fluid would decrease the heat rate, especially because the flow becomes laminar. Note that R'_{conv} is the dominate resistance since the convection coefficient is considerably reduced compared to that with water. Using aluminum alloy, rather than copper, as the tube material reduces the heat rate by 14%. Conduction-convection (fin) in the tube wall is important as can be seen by examining the change in R'_{fin} relative to the base condition. Increasing the tube wall thickness for the copper tube exchanger from 3 to 4 mm had only a marginal positive effect on the heat rate.

COMMENTS: A more accurate calculation would account for the absence of symmetry about the contact plane. Evaluation of water properties at $T_{h,m}$ = 330 K and $T_{c,m}$ = 290 K yields h_h = 1930 W/m^2·K and h_c = 1470 W/m^2·K.