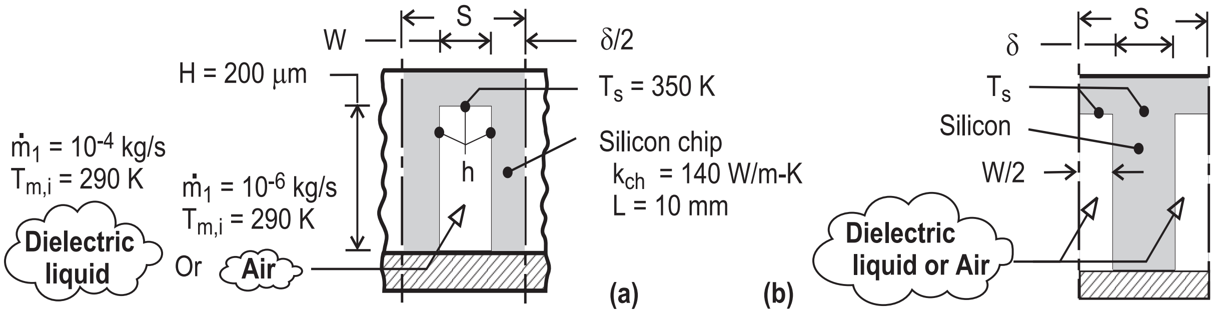
PROBLEM 8.111

KNOWN: Chip and cooling channel dimensions. Channel flow rate and inlet temperature. Temperature of chip at base of channel.

FIND: (a) Outlet temperature and chip power dissipation for dielectric liquid, (b) Outlet temperature and chip power dissipation for air.

SCHEMATIC:



ASSUMPTIONS: (1)Applicability of Eq. 8.34, (2) Flow may be approximated as fully developed and channel walls as isothermal for purposes of estimating the convection coefficient, (3) One-dimensional conduction along the channel side walls, (4) Adiabatic condition at end of side walls, (5) Heat dissipation is exclusively through fluid flow in channels, (6) Constant properties.

PROPERTIES: Prescribed. Dielectric liquid: $c_p = 1050$ J/kg·K, $k = 0.065$ W/m·K, $\mu = 0.0012$ N·s/m^2, Pr = 15. Air: $c_p = 1007$ J/kg·K, $k = 0.0263$ W/m·K, $\mu = 185 \times 10^{-7}$ N·s/m^2, Pr = 0.707.

ANALYSIS: (a) The channel side walls act as fins, and a *unit* channel/sidewall combination is shown in schematic (a), where $\delta = S - W = 150$ μm. Alternatively, the unit cell may be represented in terms of a single fin of thickness δ, as shown in schematic (b). The thermal resistance of the unit cell may be obtained from the expression for a fin array, Eq. (3.108), $R_{t,o} = (\eta_o \, h \, A_t)^{-1}$, where $A_t = A_f + A_b = L\,(2H + W) = 4.5 \times 10^{-6}$ m^2. With $A_c = H \times W = 10^{-8}$ m^2 and $D_h = 4\,A_c/2(H + W) = 8 \times 10^{-5}$m, the Reynolds number is $Re_D = \rho u_m D_h/\mu = \dot{m}_1 \, D_h/A_c\mu = 667$. Hence, the flow is laminar, and assuming fully developed conditions throughout a channel with uniform surface temperature, Table 8.1 yields $Nu_D = 4.44$. Hence,

$$h = \frac{k}{D_h} Nu_D = \frac{0.065\,\text{W/m}\cdot\text{K} \times 4.44}{8 \times 10^{-5}\,\text{m}} = 3608\,\text{W/m}^2\cdot\text{K}$$

With $m = (2\,h/k_{ch}\,\delta)^{1/2} = 586$ m^{-1} and $mH = 0.117$, the fin efficiency is

$$\eta_f = \frac{\tanh mH}{mH} = \frac{0.1167}{0.117} = 0.995$$

and the overall surface efficiency is

$$\eta_o = 1 - \frac{A_f}{A_t}(1 - \eta_f) = 1 - \frac{4.0 \times 10^{-6}}{4.5 \times 10^{-6}}(1 - 0.995) = 0.996.$$

The thermal resistance of the unit cell is then

$$R_{t,o} = (\eta_o h\,A_t)^{-1} = \left(0.996 \times 3608\,\text{W/m}^2\cdot\text{K} \times 4.5 \times 10^{-6}\,\text{m}^2\right)^{-1} = 61.9\,\text{K/W}$$

The outlet temperature follows from Eq. (8.45b),

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{1}{\dot{m}_1\,c_p\,R_{t,o}}\right) = 350\text{K}$$

Continued …

PROBLEM 8.111 (Cont.)

$$-(60\text{K})\exp\left(-\frac{1}{10^{-4}\,\text{kg/s}\times 1050\,\text{J/kg}\cdot\text{K}\times 61.9\,\text{K/W}}\right) = 298.6\text{K}$$ <

The heat rate per channel is then

$$q_1 = \dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right) = 10^{-4}\,\text{kg/s}\times 1050\,\text{J/kg}\cdot\text{K}\times 8.6\,\text{K} = 0.899\,\text{W}$$

and the chip power dissipation is

$$q = Nq_1 = 50\times 0.899\,\text{W} = 45.0\,\text{W}$$ <

(b) With $\dot{m}_1 = 10^{-6}$ kg/s, $Re_D = \dot{m}_1 D_h / A_c\mu = 432$ and the flow is laminar. Hence, with $Nu_D = 4.44$,

$$h = \frac{k}{D_h} Nu_D = \frac{0.0263\,\text{W/m}\cdot\text{K}\times 4.44}{8\times 10^{-5}\text{m}} = 1460\,\text{W/m}^2\cdot\text{K}$$

With $m = (2\,h/k_{ch}\delta)^{1/2} = 373\ \text{m}^{-1}$ and $mH = 0.0746$, the fin efficiency is

$$\eta_f = \frac{\tanh mH}{mH} = \frac{0.0744}{0.0746} = 0.998$$

and the overall surface efficiency is

$$\eta_o = 1 - \frac{A_f}{A_t}(1-\eta_f) = 1 - \frac{4.0\times 10^{-6}}{4.5\times 10^{-6}}(1-0.998) = 0.998$$

Hence, $$R_{t,o} = \left(\eta_o h A_t\right)^{-1} = \left(0.998\times 1460\,\text{W/m}^2\cdot\text{K}\times 4.5\times 10^{-6}\text{m}^2\right)^{-1} = 153\,\text{K/W}$$

The outlet temperature is then

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{1}{\dot{m}_1 c_p R_{t,o}}\right) = 350\text{K}$$

$$-(60\text{K})\exp\left(-\frac{1}{10^{-6}\,\text{kg/s}\times 1007\,\text{J/kg}\cdot\text{K}\times 153\,\text{K/W}}\right) = 349.9\,\text{K}$$ <

$$q_1 = \dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right) = 10^{-6}\,\text{kg/s}\times 1007\,\text{J/kg}\cdot\text{K}\times 59.9\,\text{K} = 0.060\,\text{W}$$

$$q = Nq_1 = 3.02\,\text{W}$$ <

COMMENTS: (1) For laminar flow in the channels, there is a clear advantage to using the dielectric liquid instead of air. (2) The prescribed channel geometry is by no means optimized, and the number of fins should be increased by reducing S. Also, channel dimensions and/or flow rates could be increased to achieve turbulent flow and hence much larger values of h. (3) With $L/D_h = 125$ and $L/Dh)_{fd} \approx 0.05\,Re_D\,Pr = 500$ for the dielectric liquid, fully developed flow is not achieved and its assumption yields a conservative (under) estimate of the convection coefficient. The coefficient is also underestimated by using a Nusselt number that presumes heat transfer from all four (rather than three) surfaces of a channel.