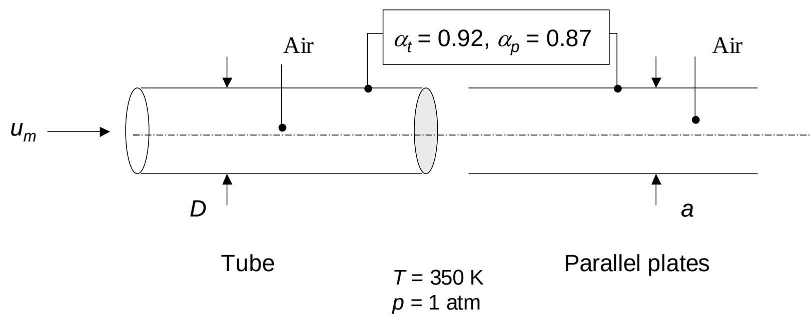
PROBLEM 8.116

KNOWN: Temperature and pressure of air flowing in a circular tube or between parallel plates. Thermal and momentum accommodation coefficients.

FIND: Tube diameter D and plate spacing a that correspond to a 10 percent reduction in the Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Ideal gas behavior. (2) Fully-developed laminar flow.

PROPERTIES: Table A.4 (T = 350 K): Air; c_p = 1009 J/kg·K, Pr = 0.70. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m.

ANALYSIS: The ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol}\cdot\text{K}}{28.97 \text{ kg/kmol}} = 0.287 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.009\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.722\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{1.009}{0.722} = 1.398$$

From Equation 2.11 the mean free path of air is

$$\lambda_{\text{mfp}} = \frac{k_B T}{\sqrt{2}\pi d^2 p} = \frac{1.381\times10^{-23} \text{ J/K}\times 350 \text{ K}}{\sqrt{2}\pi\left(0.372 \times 10^{-9} \text{ m}\right)^2\left(1.0133\times10^5 \text{ N/m}^2\right)} = 77.6\times10^{-9} \text{ m} = 77.6 \text{ nm}$$

From Equation 8.78 the Nusselt number for the tube may be expressed as

$$Nu_D = \frac{48}{11 - 6\zeta + \zeta^2 + 48\Gamma_t} = 0.9\times4.36 = 3.92 \qquad (1)$$

where

$$\Gamma_t = \frac{2-\alpha_t}{\alpha_t}\frac{2\gamma}{\gamma+1}\left[\frac{\lambda_{\text{mfp}}}{PrD}\right] = \frac{2-0.92}{0.92}\cdot\frac{2\times1.398}{1.398+1}\left[\frac{77.6\times10^{-9} \text{ m}}{0.700D}\right] = \frac{152\times10^{-9} \text{ m}}{D} \qquad (2)$$

Continued…

PROBLEM 8.116 (Cont.)

$$\Gamma_p = \frac{2-\alpha_p}{\alpha_p}\left[\frac{\lambda_{\text{mfp}}}{D}\right] = \frac{2-0.87}{0.87}\cdot\left[\frac{77.6\times10^{-9}\text{ m}}{D}\right] = \frac{101\times10^{-9}\text{ m}}{D} \tag{3}$$

$$\zeta = \frac{8\Gamma_p}{\left(1+8\Gamma_p\right)} \tag{4}$$

Equations 1 through 4 may be solved by trial-and-error to yield $D = 2.94 \times 10^{-6}$ m = 2.94 μm. <

From Equation 8.79 the Nusselt number for the parallel plate configuration may be expressed as

$$Nu_D = \frac{140}{17-6\zeta+(2/3)\zeta^2+70\Gamma_t} = 0.9\times8.23 = 7.41 \tag{5}$$

where

$$\Gamma_t = \frac{2-\alpha_t}{\alpha_t}\frac{2\gamma}{\gamma+1}\left[\frac{\lambda_{\text{mfp}}}{PrD_h}\right] = \frac{2-0.92}{0.92}\cdot\frac{2\times1.398}{1.398+1}\left[\frac{77.6\times10^{-9}\text{ m}}{0.700D_h}\right] = \frac{152\times10^{-9}\text{ m}}{D_h} \tag{6}$$

$$\Gamma_p = \frac{2-\alpha_p}{\alpha_p}\left[\frac{\lambda_{\text{mfp}}}{D_h}\right] = \frac{2-0.87}{0.87}\cdot\left[\frac{77.6\times10^{-9}\text{ m}}{D_h}\right] = \frac{101\times10^{-9}\text{ m}}{D_h} \tag{7}$$

$$\zeta = \frac{6\Gamma_p}{\left(1+6\Gamma_p\right)} \tag{8}$$

Equations 5 through 8 may be solved by trial-and-error to yield $D_h = 3.97 \times 10^{-6}$ m = 3.97 μm. The plate spacing $a = D_h/2$ = 3.97 μm/2 = 1.99 μm. <

COMMENTS: The tube diameter and plate spacing required to reduce the Nusselt number by 10 percent are quite small. In situations involving characteristic dimensions that are not extremely small, the effect of the molecule-wall interaction can typically be neglected.