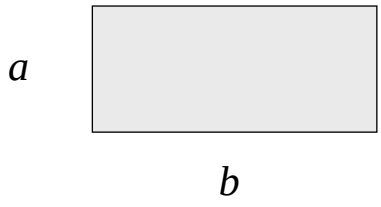
PROBLEM 8.87

KNOWN: Rectangular channel with constant surface temperature. Aspect ratio.

FIND: Which aspect ratio channel provides the largest heat transfer rate. Whether this is greater than, equal to, or less than the heat transfer rate for a circular tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Incompressible flow, (3) Laminar, (4) Fully-developed.

ANALYSIS: The heat transfer rate is given by $q_s = \dot{m}c_p(T_{m,o} - T_{m,i})$, where from Eq. 8.41b with constant heat transfer coefficient,

$$T_{m,o} - T_{m,i} = (T_s - T_{m,i})\left[1 - \exp\left(-\frac{hPL}{\dot{m}c_p}\right)\right]$$

Thus, the heat transfer rate increases with increasing values of

$$\frac{hPL}{\dot{m}c_p} = \frac{NukPL}{\dot{m}c_p D_h} = \frac{NukP^2L}{4\dot{m}c_p A_c}$$

For fixed mass flow rate and length, and assuming the same properties, the relevant parameter that determines the heat transfer rate is therefore NuP^2/A_c. For a rectangular channel, $P^2/A_c = 4(a+b)^2/ab = 4(1+b/a)^2/(b/a)$, whereas for a circular tube, $P^2/A_c = 4\pi$. The table below compares values of NuP^2/A_c for the three different aspect ratio rectangular channels and a circular tube.

b/a	***Nu***	P^2/A_c	NuP^2/A_c
1.0	2.98	16	47.7
1.43	3.08	16.5	50.9
2.0	3.39	18	61.0
Circular tube	3.66	12.6	46.0

The rectangular channel with b/a = 2.0 provides the largest heat transfer rate, which is larger than for a circular tube. <

COMMENTS: The Nusselt numbers for the rectangular channel are all less than 3.66 for the circular tube, but their convective heat transfer rates are larger than that of the circular tube because their P^2/A_c values are larger.