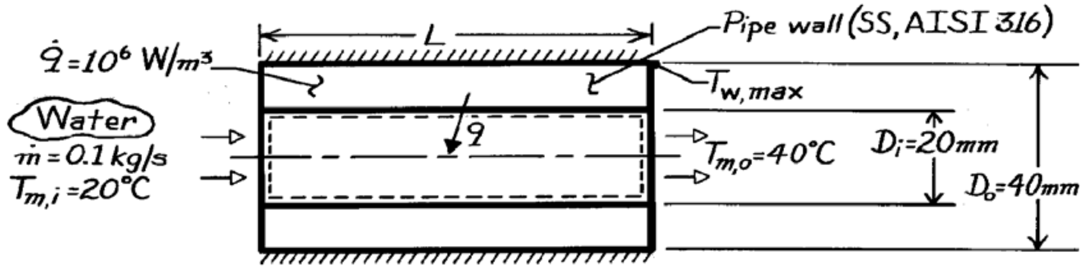
PROBLEM 8.37

KNOWN: Inner and outer diameter of a steel pipe insulated on the outside and experiencing uniform heat generation. Flow rate and inlet temperature of water flowing through the pipe.

FIND: (a) Pipe length required to achieve desired outlet temperature, (b) Location and value of maximum pipe temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Incompressible liquid with negligible viscous dissipation, (4) One-dimensional radial conduction in pipe wall, (5) Outer surface is adiabatic.

PROPERTIES: *Table A-1*, Stainless steel 316 ($T \approx 400K$): $k = 15$ W/m·K; *Table A-6*, Water $\left(\overline{T}_m = 303K\right)$: $c_p = 4178$ J/kg·K, $k = 0.617$ W/m·K, $\mu = 803 \times 10^{-6}$ N·s/m^2, Pr = 5.45.

ANALYSIS: (a) Performing an energy balance for a control volume about the inner tube, it follows that

$$\dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = q = \dot{q}\left(\pi/4\right)\left(D_o^2 - D_i^2\right)L$$

$$L = \frac{\dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right)}{\dot{q}\left(\pi/4\right)\left(D_o^2 - D_i^2\right)} = \frac{\left(0.1\ \text{kg/s}\right)4178\left(\text{J/kg}\cdot\text{K}\right)20^\circ\text{C}}{10^6\ \text{W/m}^3\left(\pi/4\right)\left[\left(0.04\text{m}\right)^2 - \left(0.02\text{m}\right)^2\right]}$$

$$L = 8.87\text{m}.$$ <

(b) The maximum wall temperature exists at the pipe exit ($x = L$) and the insulated surface ($r = r_o$). From Eq. 3.56, the radial temperature distribution in the wall is of the form

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ell n\, r + C_2.$$

Considering the boundary conditions;

$$r = r_o: \quad \left.\frac{dT}{dr}\right)_{r=r_o} = 0 = -\frac{\dot{q}}{2k}\, r_o + \frac{C_1}{r_o} \qquad\qquad C_1 = \frac{\dot{q}r_o^2}{2k}$$

Continued …

$$r = r_i: \qquad T(r_i) = T_s = -\frac{\dot{q}}{4k} r_i^2 + \frac{\dot{q}\, r_o^2}{2k} \ell n\, r_i + C_2 \qquad C_2 = \frac{\dot{q}}{4k} r_i^2 - \frac{\dot{q}\, r_o^2}{2k} \ell n\, r_i + T_s.$$

The temperature distribution and the maximum wall temperature ($r = r_o$) are

$$T(r) = -\frac{\dot{q}}{4k}\left(r^2 - r_i^2\right) + \frac{\dot{q}\, r_o^2}{2k} \ell n\, \frac{r}{r_i} + T_s$$

$$T_{w,max} = T(r_o) = -\frac{\dot{q}}{4k}\left(r_o^2 - r_i^2\right) + \frac{\dot{q}\, r_o^2}{2k} \ell n\, \frac{r_o}{r_i} + T_s$$

where T_s, the inner surface temperature of the wall at the exit, follows from

$$q_s'' = \frac{\dot{q}(\pi/4)\left(D_o^2 - D_i^2\right)L}{\pi\, D_i L} = \frac{\dot{q}\left(D_o^2 - D_i^2\right)}{4\, D_i} = h\left(T_s - T_{m,o}\right)$$

where h is the local convection coefficient at the exit. With

$$Re_D = \frac{4\,\dot{m}}{\pi\, D_i \mu} = \frac{4 \times 0.1\ \text{kg/s}}{\pi(0.02\text{m})803 \times 10^{-6}\ \text{N}\cdot\text{s/m}^2} = 7928$$

the flow is turbulent and, with $(L/D_i) = (8.87\ \text{m}/0.02\text{m}) = 444 >> (x_{fd}/D) \approx 10$, it is also fully developed. Hence, from the Gnielinski correlation, Eq. 8.62,

$$h = \frac{k}{D_i}\left[\frac{(f/8)(Re_D - 1000)\,Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}\right]$$

$$= \frac{0.617\ \text{W/m}\cdot\text{K}}{0.02\ \text{m}}\left[\frac{(0.033618)(7928 - 1000)5.45}{1 + 12.7(0.033618)^{1/2}(5.45^{2/3} - 1)}\right] = 1796\ \text{W/m}^2\cdot\text{K}$$

where from Eq. 8.21, $f = (0.790 \ln Re_D - 1.64)^{-2} = 0.0336$. Hence, the inner surface temperature of the wall at the exit is

$$T_s = \frac{\dot{q}\left(D_o^2 - D_i^2\right)}{4\, h\, D_i} + T_{m,o} = \frac{10^6\ \text{W/m}^3\left[(0.04\text{m})^2 - (0.02\text{m})^2\right]}{4 \times 1796\ \text{W/m}^2\cdot\text{K}(0.02\text{m})} + 40^\circ\text{C} = 48.4^\circ\text{C}$$

and $$T_{w,max} = -\frac{10^6\ \text{W/m}^3}{4 \times 15\ \text{W/m}\cdot\text{K}}\left[(0.02\text{m})^2 - (0.01\text{m})^2\right]$$

$$+\frac{10^6\ \text{W/m}^3(0.02\text{m})^2}{2 \times 15\ \text{W/m}\cdot\text{K}} \ell n \frac{0.02}{0.01} + 48.4^\circ\text{C} = 52.6^\circ\text{C}. \qquad <$$

COMMENTS: The physical situation corresponds to a uniform surface heat flux, and T_m increases linearly with x. In the fully developed region, T_s also increases linearly with x.