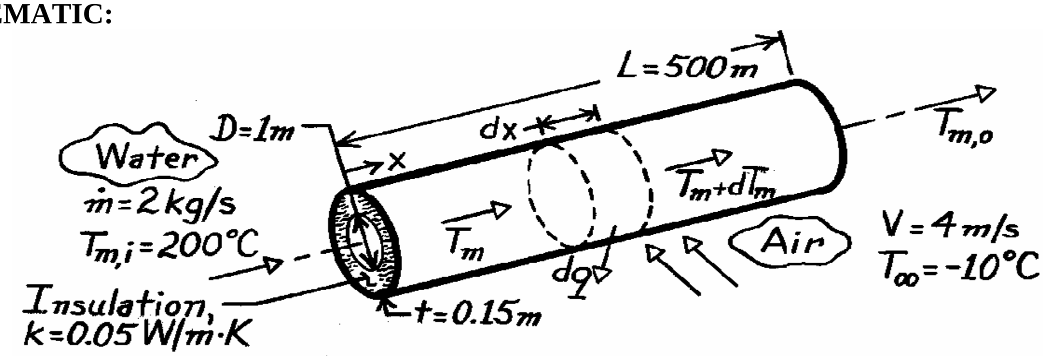
PROBLEM 8.73

KNOWN: Flow conditions associated with water passing through a pipe and air flowing over the pipe.

FIND: (a) Differential equation which determines the variation of the mixed-mean temperature of the water, (b) Heat transfer per unit length of pipe at the inlet and outlet temperature of the water.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature drop across the pipe wall, (2) Negligible radiation exchange between outer surface of insulation and surroundings, (3) Fully developed flow throughout pipe, (4) Water is incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A-6*, Water ($T_{m,i}$ = 200°C): $c_{p,w}$ = 4500 J/kg·K, $\mu_w = 134 \times 10^{-6}$ N·s/m^2, k_w = 0.665 W/m·K, Pr_w = 0.91; *Table A-4*, Air (T_∞ = -10°C): $\nu_a = 12.6 \times 10^{-6}$ m^2/s, k_a = 0.023 W/m·K, Pr_a = 0.71, $Pr_s \approx 0.7$.

ANALYSIS: (a) Following the development of Section 8.3.1 and applying Eq. 1.12e to a differential element in the water, we obtain

$$dq = -\dot{m}\, c_{p,w}\, dT_m$$

where $$dq = U_i dA_i \left(T_m - T_\infty\right) = U_i \pi\, D\, dx \left(T_m - T_\infty\right).$$

Substituting into the energy balance, it follows that

$$\frac{d\,T_m}{dx} = -\frac{U_i \pi\, D}{\dot{m}\, c_p}\left(T_m - T_\infty\right). \qquad (1)$$ <

The overall heat transfer coefficient based on the inside surface area may be evaluated from Eq. 3.36 which, for the present conditions, reduces to

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{D}{2k}\ln\left(\dfrac{D+2t}{D}\right) + \dfrac{D}{D+2t}\,\dfrac{1}{h_o}}. \qquad (2)$$

For the *inner water flow*, Eq. 8.6 gives

$$Re_D = \frac{4\,\dot{m}}{\pi\, D\mu_w} = \frac{4\times 2\ \text{kg/s}}{\pi\left(1\ \text{m}\right)\times 134\times 10^{-6}\ \text{kg/s}\cdot\text{m}} = 19{,}004.$$

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Hence, the flow is turbulent. With the assumption of fully developed conditions, it follows from Eq. 8.60 that

$$h_i = \frac{k_w}{D} \times 0.023\, Re_D^{4/5}\, Pr_w^{0.3}. \quad (3)$$

For the *external air flow*

$$Re_D = \frac{V(D+2t)}{\nu} = \frac{4\ m/s(1.3m)}{12.6\times10^{-6}\ m^2/s} = 4.13\times10^5.$$

Using Eq. 7.45 to obtain the outside convection coefficient,

$$h_o = \frac{k_a}{(D+2t)} \times 0.076\, Re_D^{0.7}\, Pr_a^{0.37}\, (Pr_a / Pr_s)^{1/4}. \quad (4)$$

(b) The heat transfer per unit length of pipe at the inlet is

$$q' = \pi\, D\, U_i (T_{m,i} - T_\infty). \quad (5)$$

From Eqs. (3 and 4),

$$h_i = \frac{0.665\ W/m\cdot K}{1\ m} \times 0.023(19{,}004)^{4/5}\ (0.91)^{0.3} = 39.4\ W/m^2\cdot K$$

$$h_o = \frac{0.023\ W/m\cdot K}{(1.3\ m)} \times 0.076\left(4.13\times10^5\right)^{0.7} (0.71)^{0.37} (1)^{1/4} = 10.1\ W/m^2\cdot K.$$

Hence, from Eq. (2)

$$U_i = \left[\frac{1}{39.4\ W/m^2\cdot K} + \frac{1\ m}{0.1\ W/m\cdot K}\ell n\left(\frac{1.3}{1}\right) + \frac{1}{1.3} \times \frac{1}{10.1\ W/m^2\cdot K}\right]^{-1} = 0.37\ W/m^2\cdot K$$

and from Eq. (5)

$$q' = \pi(1\ m)\ \left(0.37\ W/m^2\cdot K\right)\ (200+10)^\circ C = 244\ W/m.$$ <

Since U_i is a constant, independent of x, Eq. (1) may be integrated from x = 0 to x = L. The result is Eq. 8.45a.

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi\ DL}{\dot{m}\ c_{p,w}} U_i\right) = \exp\left(-\frac{\pi\times 1m\times 500m}{2\ kg/s\times 4500\ J/kg\cdot K}\times 0.37\ W/m^2\cdot K\right)$$

Hence $$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = 0.937.$$

$$T_{m,o} = T_\infty + 0.937\left(T_{m,i} - T_\infty\right) = 187^\circ C.$$ <

COMMENTS: The largest contribution to the denominator on the right-hand side of Eq. (2) is made by the conduction term (the insulation provides 96% of the total resistance to heat transfer). For this reason the assumption of fully developed conditions throughout the pipe has a negligible effect on the calculations. Since the reduction in T_m is small (13°C), little error is incurred by evaluating all properties of water at $T_{m,i}$.