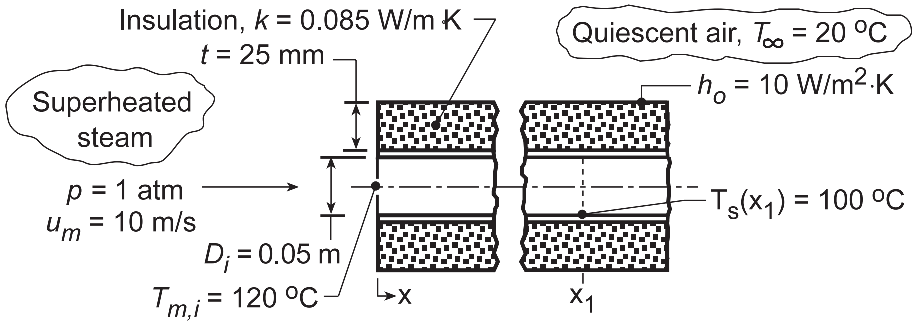
PROBLEM 8.71

KNOWN: Superheated steam passing through thin-walled pipe covered with insulation and suspended in a quiescent air.

FIND: Point along pipe surface where steam will begin condensing (x_1).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Steam is ideal gas with negligible viscous dissipation and pressure variation, (3) Steam properties may be approximated as those corresponding to saturated conditions.

PROPERTIES: *Table A.6*, Saturated steam ($\overline{T}_m$ = (100 + 120)°C/2 = 110°C ≈ 385 K): $\rho_g = 0.876$ kg/m^3, $c_{p,g} = 2080$ J/kg·K, $\mu_g = 12.49 \times 10^{-6}$ N·S/m^2, $k_g = 0.0258$ W/m·K, $Pr_g = 1.004$.

ANALYSIS: From Eq. 8.45a, where $T_{m,x}$ is the mean temperature at any distance x,

$$\frac{T_\infty - T_{m,x}}{T_\infty - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}U\right) \tag{1}$$

The mass flow rate, with $A_c = \pi D^2/4$, is

$$\dot{m} = \rho_g A_c u_m = 0.876\,\mathrm{kg/m^3}\left(\pi(0.050\,\mathrm{m})^2/4\right)\times 10\,\mathrm{m/s} = 0.0172\,\mathrm{kg/s}$$

and for the internal flow,

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4\times 0.0172\,\mathrm{kg/s}}{\pi(0.050\,\mathrm{m})\times 12.49\times 10^{-6}\,\mathrm{N\cdot s/m^2}} = 35,068.$$

Assuming the flow is fully developed, the Dittus-Boelter correlation yields

$$Nu_D = \frac{h_i D}{k} = 0.023(35,068)^{4/5}(1.004)^{0.3} = 99.58$$

$$h_i = \frac{0.0258\,\mathrm{W/m\cdot K}}{0.050\,\mathrm{m}}\times 99.58 = 51.4\,\mathrm{W/m^2\cdot K}$$

Hence, from Eq. 3.36, the overall coefficient for the inner surface is

$$U_i = \left[\frac{1}{h_i} + \frac{D_i \ln(D_o/D_i)}{2k} + \frac{D_i}{D_o}\frac{1}{h_o}\right]^{-1} = \left[\frac{1}{51.4} + \frac{(0.050)\ln(0.100/0.050)}{2\times 0.085} + \frac{0.050}{0.100}\frac{1}{10}\right]^{-1}\mathrm{W/m^2\cdot K}$$

$$U_i = \left[1.946\times 10^{-2} + 2.039\times 10^{-1} + 5.000\times 10^{-2}\right]^{-1} = 3.66\,\mathrm{W/m^2\cdot K}.$$

Continued...

With condensation occurring when the surface temperature reaches 100°C, the corresponding value of T_m may be determined from the local ($x = x_1$) requirement that $U_i(\pi D_i)\left[T_m(x_1) - T_\infty\right] = h_i(\pi D_i)\left[T_m(x_1) - T_s\right]$. Hence,

$$T_m(x_1) = \frac{T_\infty - (h_i/U_i)T_s}{1 - (h_i/U_i)} = \frac{20 - (51.4/3.66)100^\circ\mathrm{C}}{1 - (51.4/3.66)} = 106^\circ\mathrm{C}$$

The distance at which the mean steam temperature is 106°C can then be estimated from Eq. (1), where $P = \pi D_i$ and $U = U_i$,

$$\frac{(20 - 106)^\circ\mathrm{C}}{(20 - 120)^\circ\mathrm{C}} = \exp\left(-\frac{\pi(0.050\,\mathrm{m})3.66\,\mathrm{W/m^2\cdot K}(x_1)}{0.0172\,\mathrm{kg/s} \times 2080\,\mathrm{J/kg\cdot K}}\right)$$

$x_1 = 9.3$ m $<$

COMMENTS: Note that condensation first occurs at the location for which the surface, and not the mean, temperature reaches 100°C.