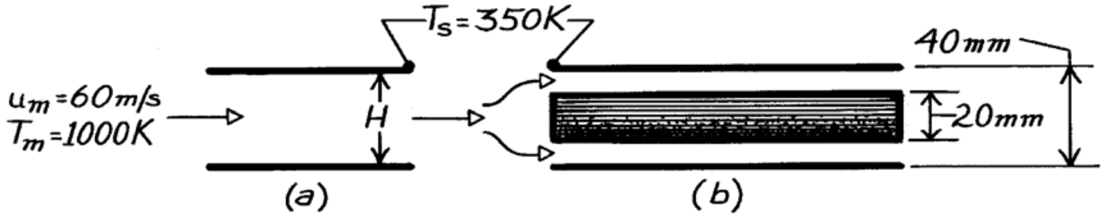
PROBLEM 8.84

KNOWN: Temperature and velocity of gas flow between parallel plates of prescribed surface temperature and separation. Thickness and location of plate insert.

FIND: Heat flux to the plates (a) without and (b) with the insert.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Gas has properties of atmospheric air, (4) Plates are of infinite width W, (5) Fully developed flow.

PROPERTIES: *Table A-4*, Air (1 atm, T_m = 1000 K): $\rho = 0.348$ kg/m^3, $\mu = 424.4 \times 10^{-7}$ kg/s·m, k = 0.0667 W/m·K, Pr = 0.726.

ANALYSIS: (a) Based upon the hydraulic diameter D_h, the Reynolds number is

$$D_h = 4\,A_c/P = 4(H\cdot W)/2(H+W) = 2H = 80\text{ mm}$$

$$Re_{D_h} = \frac{\rho\,u_m\,D_h}{\mu} = \frac{0.348\text{ kg/m}^3(60\text{ m/s})0.08\text{ m}}{424.4\times10^{-7}\text{ kg/s}\cdot\text{m}} = 39,360.$$

Since the flow is fully developed and turbulent, use the Dittus-Boelter correlation,

$$Nu_D = 0.023\,Re_D^{4/5}\,Pr^{0.3} = 0.023(39,360)^{4/5}(0.726)^{0.3} = 99.1$$

$$h = \frac{k}{D_h}Nu_D = \frac{0.0667\text{ W/m}\cdot\text{K}}{0.08\text{ m}}99.1 = 82.6\text{ W/m}^2\cdot\text{K}$$

$$q'' = h(T_m - T_s) = 82.6\text{ W/m}^2\cdot\text{K}(1000-350)\text{K} = 53,700\text{ W/m}^2.$$ <

(b) From continuity,

$$\dot{m} = (\rho\,u_m A)_a = (\rho\,u_m A)_b \qquad u_m)_b = u_m)_a\,(\rho A)_a\,/\,(\rho A)_b = 60\text{ m/s}(40/20) = 120\text{ m/s}.$$

For each of the resulting channels, D_h = 0.02 m and

$$Re_{D_h} = \frac{\rho\,u_m\,D_h}{\mu} = \frac{0.348\text{ kg/m}^3(120\text{ m/s})0.02\text{ m}}{424.4\times10^{-7}\text{kg/s}\cdot\text{m}} = 19,680.$$

Since the flow is still turbulent,

$$Nu_D = 0.023(19,680)^{4/5}(0.726)^{0.3} = 56.9 \qquad h = \frac{56.9(0.0667\text{ W/m}\cdot\text{K})}{0.02\text{ m}} = 189.8\text{ W/m}^2\cdot\text{K}$$

$$q'' = 189.8\text{ W/m}^2\cdot\text{K}\,(1000-350)\text{K} = 123,400\text{ W/m}^2.$$ <

COMMENTS: From the Dittus-Boelter equation,

$$h_b/h_a = (u_{m,b}/u_{m,a})^{0.8}(D_{h,a}/D_{h,b})^{0.2} = (2)^{0.8}(4)^{0.2} = 1.74\times1.32 = 2.30.$$

Hence, heat transfer enhancement due to the insert is primarily a result of the increase in u_m and secondarily a result of the decrease in D_h.