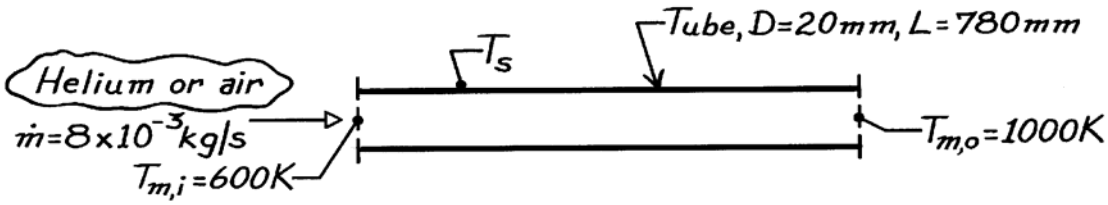
PROBLEM 8.51

KNOWN: Gas-cooled nuclear reactor tube of 20 mm diameter and 780 mm length with helium heated from 600 K to 1000 K at 8×10^{-3} kg/s.

FIND: (a) Uniform tube wall temperature required to heat the helium, (b) Outlet temperature and required flow rate to achieve same removal rate and wall temperature if the coolant gas is air.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation, (3) Fully developed conditions.

PROPERTIES: *Table A-4*, Helium $\left(\overline{T}_m = 800\text{K, 1 atm}\right)$: $\rho = 0.06272$ kg/m^3, $c_p = 5193$ J/kg·K, k = 0.304 W/m·K, $\mu = 382 \times 10^{-7}$ N·s/m^2, $\nu = 6.39 \times 10^{-4}$ m^2/s, Pr = 0.654; Air $\left(\overline{T}_m = 800\text{K, 1 atm}\right)$: ρ = 0.4354 kg/m^3, $c_p = 1099$ J/kg·K, k = 57.3×10^{-3} W/m·K, $\nu = 84.93 \times 10^{-6}$ m^2/s, Pr = 0.709.

ANALYSIS: (a) For helium and a constant wall temperature, from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\overline{h}}{\dot{m}\, c_p}\right)$$

where P = πD. For the circular tube,

$$Re_D = \frac{4\dot{m}}{\pi\, D\mu} = \frac{4 \times 8 \times 10^{-3}\ \text{kg/s}}{\pi \times 0.020\ \text{m} \times 382 \times 10^{-7}\,\text{N} \cdot \text{s/m}^2} = 1.333 \times 10^4$$

and using the Dittus-Boelter correlation for turbulent, fully developed flow,

$$Nu = 0.023\ Re_D^{4/5}\ Pr^{0.4} = 0.023\left(1.333 \times 10^4\right)^{4/5}\left(0.654\right)^{0.4} = 38.7$$

$$h = Nu \cdot k/D = 38.7 \times 0.304\ \text{W/m} \cdot \text{K/0.02 m} = 588\ \text{W/m}^2 \cdot \text{K}.$$

Hence, the surface temperature is

$$\frac{T_s - 1000\ \text{K}}{T_s - 600\ \text{K}} = \exp\left[-\frac{\pi\left(0.020\ \text{m}\right) \times 0.780\ \text{m} \times 588\ \text{W/m}^2 \cdot \text{K}}{8 \times 10^{-3}\ \text{kg/s} \times 5193\ \text{J/kg} \cdot \text{K}}\right] = 0.500$$

$$T_s = 1400\ \text{K}.$$ <

The heat rate with helium coolant is

$$q = \dot{m}\, c_p\left(T_{m,o} - T_{m,i}\right) = 8 \times 10^{-3}\ \text{kg/s} \times 5193\ \text{J/kg} \cdot \text{K}\left(1000 - 600\right)\text{K} = 16.62\ \text{kW}.$$

Continued …

(b) For the same heat removal rate (q) and wall temperature (T_s) with air supplied at $T_{m,i}$, the relevant relations are

$$q = 16{,}620\ \text{W} = \dot{m}_a\ c_p \left(T_{m,o} - T_{m,i}\right) \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left[-\frac{PL\bar{h}_a}{\dot{m}_a\ c_p}\right] \tag{2}$$

$$Re = \frac{4\dot{m}_a}{\pi\ D\mu} \qquad \frac{\bar{h}D}{k} = 0.023\ Re_D^{4/5}\ Pr^{0.4} \tag{3,4}$$

where $T_{m,o}$ and $\dot{m}$ are unknown. An iterative solution is required: assume a value of $T_{m,o}$ and find $\dot{m}$ from Eq. (1); use $\dot{m}$ in Eqs. (3) and (4) to find $\bar{h}$ and then Eq. (2) to evaluate $T_{m,o}$; compare results and iterate. Using thermophysical properties of air evaluated at $\bar{T}_m$ = 800K, the above relations, written in the order they would be used in the iteration, become

$$\dot{m}_a = \frac{15.1}{T_{m,o} - 600} \tag{5}$$

$$\bar{h}_a = 5600\dot{m}_a^{4/5} \tag{6}$$

$$T_{m,o} = 1400\ \text{K} - 800\ \text{K} \times \exp\left[-4.459 \times 10^{-5}\left(\bar{h}_a / \dot{m}_a\right)\right] \tag{7}$$

Results of the iterative solution are

Trial	$T_{m,o}$ (K) (Assumed)	$\dot{m}$ (kg/s) Eq. (5)	$\bar{h}_a$ $\left(\text{W/m}^2 \cdot \text{K}\right)$ Eq. (6)	$T_{m,o}$ (K) Eq. (7)
1	1000	3.781×10^{-2}	407	905
2	950	4.321×10^{-2}	453	899
3	900	5.041×10^{-2}	513	891
4	890	5.215×10^{-2}	527	890

Hence, we find

$$\dot{m}_a = 5.22 \times 10^{-2}\ \text{kg/s} \qquad T_{m,o} = 890\ \text{K}. \qquad <$$

COMMENTS: To achieve the same cooling rate with air, the required mass rate is 6.5 times that obtained with helium.