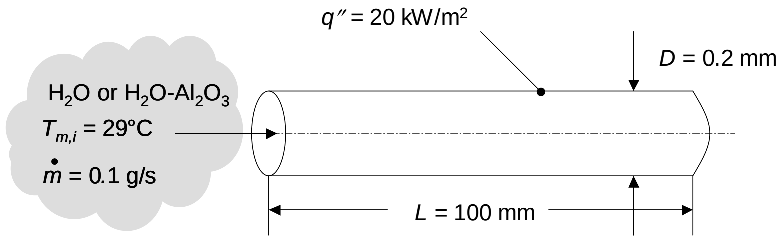
PROBLEM 8.66

KNOWN: Dimensions of circular tube, applied constant heat flux, inlet temperature, mass flow rate, and expression for nanofluid viscosity.

FIND: Tube wall temperature at the tube exit for pure water and for a water-Al_2O_3 nanofluid.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties.

PROPERTIES: *Table A.4*, water (300 K): $\mu_{\text{bf}} = 855 \times 10^{-6}$ m²/s, $k_{\text{bf}} = 0.613$ W/m·K, $c_{p,\text{bf}} = 4179$ J/kg·K, $Pr_{\text{bf}} = 5.83$. Example 2.2, nanofluid (300 K): $\mu_{\text{nf}} = 962 \times 10^{-6}$ m²/s, $k_{\text{nf}} = 0.705$ W/m·K, $c_{p,\text{nf}} = 3587$ J/kg·K, $Pr_{\text{nf}} = \mu_{\text{nf}}\, c_{p,\text{nf}}/ k_{\text{nf}} = 4.91$.

ANALYSIS: The Reynolds number for the pure water is $Re_D = 4\dot{m}/\pi D\mu_{\text{bf}} = \left[4\times(0.1/1000\text{ kg/s})\right]/\left(\pi\times 0.0002\text{ m}\times 855\times 10^{-6}\text{ N}\cdot\text{s/m}^2\right) = 745$ and the flow is laminar. Similarly, the Reynolds number for the nanofluid is $Re_{D,\text{nf}} = 662$. The hydrodynamic entrance length for the pure water is $x_{fd,h} = 0.05 Re_D D = 0.05 \times 745 \times 0.2/1000\text{ m} = 7.45 \times 10^{-3}\text{ m} = 7.45$ mm and the flow at the tube exit is hydrodynamically fully developed. Similarly, the hydrodynamic entrance length for the nanofluid is $x_{fd,h,\text{nf}} = 6.62 \times 10^{-3}$ m = 6.62 mm and the flow at the tube exit is also hydrodynamically fully developed. For the pure water, the thermal entrance length is $x_{fd,t} = x_{fd,h}Pr_{\text{bf}} =$ 7.45 mm × 5.83 = 43.4 mm, while for the nanofluid $x_{fd,t,\text{nf}} = x_{fd,h,\text{nf}}Pr_{\text{nf}} =$ 6.62 mm × 4.91 = 32.5 mm and the flow is also thermally fully-developed at the tube exit for both fluids.

For constant heat flux conditions, the local Nusselt number in the fully-developed region is $Nu_D =$ 4.36. Therefore, the local heat transfer coefficient at the tube exit is:

Pure fluid: $h_{\text{bf}} = Nu_D k_{\text{bf}}/D = 4.36 \times 0.613\text{ W/m}\cdot\text{K}/(0.2/1000\text{m}) = = 13{,}360\text{ W/m}^2\cdot\text{K}$.
Nanofluid: $h_{\text{nf}} = Nu_D k_{\text{nf}}/D = 4.36 \times 0.705\text{ W/m}\cdot\text{K}/(0.2/1000\text{m}) = = 15{,}370\text{ W/m}^2\cdot\text{K}$.

Applying Eq. (8.40) to the pure fluid yields

$$T_{m,o} = T_{m,i} + \frac{q''\pi D}{\dot{m}c_{p,\text{bf}}}L = 29°\text{C} + \frac{20{,}000\text{ W/m}^2\pi(0.2/1000\text{ m})}{(0.1/1000\text{ kg/s})\times(4179\text{ J/kg}\cdot\text{K})}0.1\text{ m} = 29°\text{C} + 3.00°\text{C} = 32.00°\text{C}$$

whereas applying Eq. (8.40) to the nanofluid results in

$$T_{m,o,\text{nf}} = T_{m,i} + \frac{q''\pi D}{\dot{m}c_{p,\text{nf}}}L = 29°\text{C} + \frac{20{,}000\text{ W/m}^2\pi(0.2/1000\text{ m})}{(0.1/1000\text{ kg/s})\times(3587\text{ J/kg}\cdot\text{K})}0.1\text{ m} = 29°\text{C} + 3.50°\text{C} = 32.50°\text{C}$$

Continued…

From Eq. (8.27) the wall temperature at the outlet of the tube carrying the pure water is,

$$T_s(x = L) = T_{m,o} + q'' / h_{\text{bf}} = 32.00°\text{C} + 20{,}000\,\text{W/m}^2 / 13{,}360\ \text{W/m}^2 \cdot \text{K} = 32°\text{C}\ + 1.50°\text{C} = 33.50°\text{C}$$ <

Similarly for the nanofluid,

$$T_{s,\text{nf}}(x = L) = T_{m,o,\text{nf}} + q'' / h_{\text{nf}}$$
$$= 32.50°\text{C} + 20{,}000\,\text{W/m}^2 / 15{,}370\ \text{W/m}^2 \cdot \text{K} = 32.50°\text{C}\ + 1.30°\text{C} = 33.80°\text{C}$$ <

COMMENTS: Although the nanofluid provides a larger thermal conductivity and, in turn, a larger convective heat transfer coefficient relative to the pure water, the wall temperature at the tube outlet with the nanofluid exceeds that of the wall temperature using pure water. This is due to the reduction of the specific heat upon addition of the nanoparticles to the pure water and the associated increase in the outlet mean temperature. Hence, careful consideration of the flow conditions must be made in order to determine whether wall temperatures will decrease or increase with use of the nanofluid.