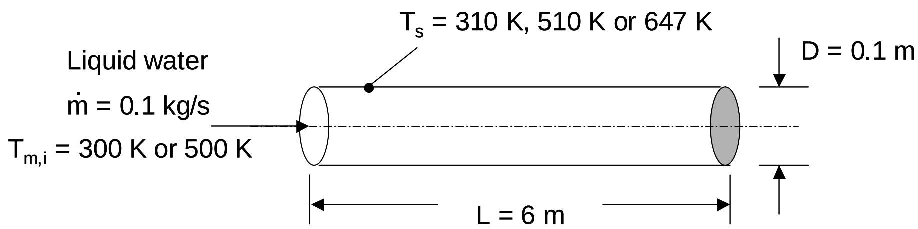
PROBLEM 8.48

KNOWN: Diameter and length of circular tube, liquid water flow rate, liquid water entrance temperatures and tube surface temperatures.

FIND: Water outlet temperatures for (a) $T_{m,i}$ = 500 K, T_s = 510 K and (b) $T_{m,i}$ = 300 K, T_s = 310 K. (c) Discuss whether the flow is laminar or turbulent for $T_{m,i}$ = 300 K, T_s = 647 K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties in parts (a) and (b), (3) Negligible viscous dissipation.

PROPERTIES: *Table A.6*, liquid water ($\overline{T}_m$ = 505 K, assumed): $\mu = 115.5\times10^{-6}$ N·s/m^2, Pr = 0.855, k = 0.635 W/m·K, c_p = 4700 J/kg·K. Liquid water ($\overline{T}_m$ = 305 K, assumed): $\mu = 769\times10^{-6}$ N·s/m^2, Pr = 5.20, k = 0.620 W/m·K, c_p = 4178 J/kg·K.

ANALYSIS: (a) We begin by calculating the Reynolds number

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times0.1\text{ kg/s}}{\pi\times0.1\text{m}\times115.5\times10^{-6}\text{N}\cdot\text{s/m}^2} = 11{,}014$$

Therefore, the flow is in a fully turbulent condition. Since L/D = 6m/0.1m = 60, we conclude that entrance effects are not important. We may use Dittus-Boelter (Eq. 8.60) to determine the average heat transfer coefficient and the mean outlet temperature may be found from Eq. (8.41b).

$$\overline{h} = \frac{k}{D}\left[0.023Re_{\text{D}}^{4/5}Pr^{0.4}\right] = \frac{0.635\text{W/m}\cdot\text{K}}{0.1\text{m}}\left[0.023\times11{,}014^{4/5}0.855^{0.4}\right] = 235\text{ W/m}^2\cdot\text{K}$$

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{PL}{\dot{m}c_p}\overline{h}\right)$$

$$= 510\text{ K} - 10\text{ K}\times\exp\left(-\frac{\pi\times0.1\text{ m}\times6\text{ m}}{0.1\text{ kg/s}\times4700\text{ J/kg}\cdot\text{K}}235\text{ W/m}^2\cdot\text{K}\right) = 506.1\text{ K}$$ <

(b) The Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times0.1\text{ kg/s}}{\pi\times0.1\text{m}\times769\times10^{-6}\text{N}\cdot\text{s/m}^2} = 1655$$

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Therefore, the flow is laminar. The thermal entrance length is $x_{fd,t} = 0.05 \times D \times Re_D \times Pr = 0.05 \times 0.1\text{m} \times 1655 \times 5.20 = 43.0\text{ m} > L$. Therefore, we expect entrance effects to be significant. With $Pr > 5$, we may use Eq. (8.57) with Eq. (8.56) for the Graetz number, to estimate the value of $\bar{h}$.

$$h = \frac{k}{D}\left\{3.66 + \frac{0.0668(D/L)Re_D Pr}{1+0.04\left[(D/L)Re_D Pr\right]^{2/3}}\right\}$$

$$= \frac{0.620\text{W/m}\cdot\text{K}}{0.1\text{ m}}\left\{3.66 + \frac{0.0668(0.1\text{ m}/6\text{ m})\times 1655\times 5.20}{1+0.04\left[(0.1\text{ m}/6.0\text{ m})\times 1655\times 5.20\right]^{2/3}}\right\} = 51.0\text{ W/m}^2\cdot\text{K}$$

Using Eq. (8.41b)

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right)$$

$$= 310\text{ K} - 10\text{ K}\times\exp\left(-\frac{\pi\times 0.1\text{ m}\times 6\text{ m}}{0.1\text{ kg/s}\times 4178\text{ J/kg}\cdot\text{K}}51\text{ W/m}^2\cdot\text{K}\right) = 302.1\text{ K} \qquad <$$

(c) The temperature variations within the water are very large. Therefore, properties are expected to vary significantly from location to location. Near the entrance of the tube, average temperatures will be low, and the flow is expected to be laminar. However, as the boundary layer regions grow, higher temperatures will exist in a greater portion of the liquid and viscosities may drop to very low values. Hence, the flow may trip into turbulent conditions at a location between the tube entrance and the tube exit. The assumption of constant properties under the conditions of part (c) may not be appropriate.

COMMENTS: Even though entrance effects are important for the laminar flow conditions of part (b), the heat transfer coefficient is small relative to that associated with the turbulent conditions of part (a).