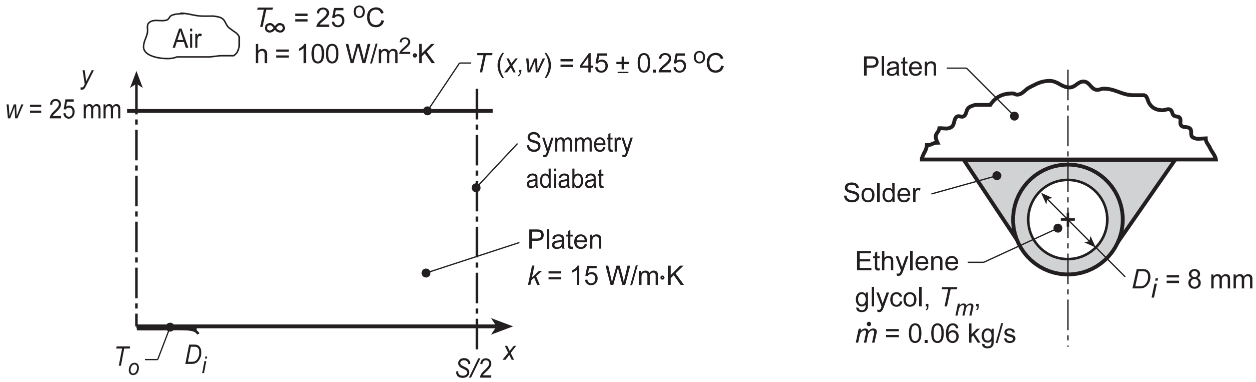
PROBLEM 8.76

KNOWN: Platen heated by hot ethylene glycol flowing through tubing arrangement with spacing S soldered to lower surface. Top surface exposed to convection process.

FIND: Tube spacing S and heating fluid temperature T_m which will maintain the top surface at 45 ± 0.25°C.

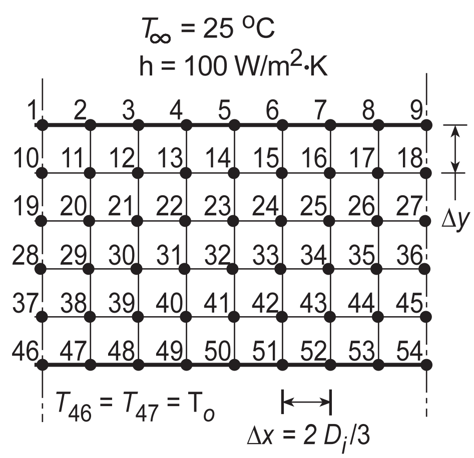
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions; (2) Lower surface is insulated, all heat transfer from hot fluid is into platen; (3) Copper tube is thick-walled such that interface between solder and platen is isothermal; (4) Fully developed flow conditions in tube.

PROPERTIES: *Table A.4*, Ethylene glycol (T_m = 60°C): $\mu = 0.00522\ N\cdot s/m^2$, k = 0.2603 W/m·K.

ANALYSIS: Begin the analysis by setting up a nodal mesh (9 ×6) to represent the platen experiencing convection on the top surface (T_∞, h) while the two side boundaries are symmetry adiabats. On the lower surface, nodes 46 and 47 represent the isothermal platen-solder interface maintained at T_o by the hot fluid. The remaining nodes (49-54) are insulated on their lower boundary.



The heat rate supplied by the tube to the platen can be expressed as

$$q'_{cv} = 0.5 h_o (\pi D_i)(T_m - T_o) \tag{1}$$

From energy balances about nodes 46 and 47, the heat rate into the platen by conduction can be expressed as

$$q'_{cd} = q'_a + q'_b + q'_c \tag{2}$$

$$q'_a = k(\Delta x/2)(T_{46} - T_{37})/\Delta y \tag{3}$$

Continued...

$$q'_b = k(\Delta x)(T_{47} - T_{38})/\Delta y \tag{4}$$

$$q'_c = k(\Delta y/2)(T_{47} - T_{48})/\Delta x \tag{5}$$

and we require that

$$q'_{cd} = q'_{cv} \tag{6}$$

The convection coefficient for internal flow can be estimated from a correlation assuming fully developed flow. First, characterize the flow with

$$Re_D = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4 \times 0.06\,\mathrm{kg/s}}{\pi(0.008\,\mathrm{m})0.00522\,\mathrm{N \cdot s/m^2}} = 1829$$

and since it is laminar,

$$Nu_D = \frac{h_o D_i}{k} = 3.66$$

$$h_o = 3.66 \times 0.2603\,\mathrm{W/m \cdot K}/0.008\,\mathrm{m} = 119.1\,\mathrm{W/m \cdot K}$$

where properties are evaluated at T_m. Using the *IHT Finite-Difference Tool* for *Two-Dimensional Steady-State Conditions* and the *Properties Tool* for *Ethylene Glycol*, along with the foregoing rate equations and energy balances, Eqs. (1-6), a model was developed to solve for the temperature distribution in the platen. In the solution, we determined what hot fluid temperature was required to maintain T_1 = 45°C. Two trials were run. In the first, the nodal arrangement was as shown above (9 × 6) for which S/2 = (9 - 1)Δx = 42.67 mm with $\Delta x = 2D_i/3$ = 5.33 mm and Δy = w/5 = 5 mm. In the second trial, we repositioned the right-hand symmetry adiabat to pass vertically through the nodes 6-51 so that now the nodal mesh is (6 × 6) and S/2 = (6 - 1)Δx = 26.65 mm with Δx and Δy remaining the same. The results of the trials are tabulated below.

Trial	Mesh	T_1 (°C)	T_6 (°C)	T_9 (°C)	T_m (°C)	q'_{cv} (W/m)
1	9 × 6	45.0	43.5	43.0	105	80.5
2	6 × 6	45.0	44.5	---	85	52.6

From the trial 2 results, the surface temperature uniformity is (T_1 - T_6) = 0.5°C which satisfies the ±0.25°C requirement. So that suitable tube spacing and fluid temperature are

S = 53 mm $\qquad$ T_m = 85°C $\qquad$ <

COMMENTS: (1) Recognize that the grid spacing is quite coarse and good practice demands that we repeat the analysis decreasing the nodal spacing until no further changes are seen in T_m.

(2) In the first trial, note that T_m = 105°C which of course, is not possible.