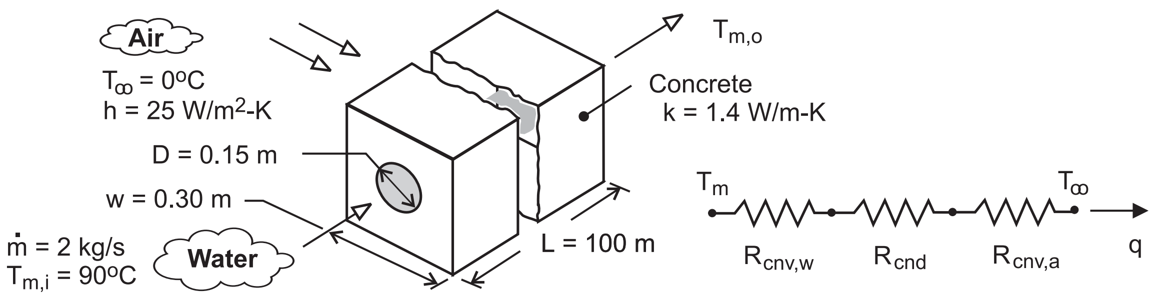
PROBLEM 8.40

KNOWN: Dimensions and thermal conductivity of concrete duct. Convection conditions of ambient air. Flow rate and inlet temperature of water flow through duct.

FIND: (a) Outlet temperature, (b) Pressure drop and pump power requirement, (c) Effect of flow rate and pipe diameter on outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Fully developed flow throughout duct, (3) Negligible pipe wall conduction resistance, (4)Water is incompressible liquid with negligible viscous dissipation, (5) Constant properties.

PROPERTIES: *Table A-6*, water $\left(\overline{T}_m \approx 360\,K\right)$: $\rho = 967\,kg/m^3$, $c_p = 4203\,J/kg\cdot K$, $\mu = 324\times10^{-6}$ $N\cdot s/m^2$, $k_w = 0.674\,W/m\cdot K$, $Pr = 2.02$.

ANALYSIS: (a) The outlet temperature is given by

$$T_{m,o} = T_\infty + \left(T_{m,i} - T_\infty\right)\exp\left(-UA/\dot{m}\,c_p\right)$$

where

$$UA = \left(R_{tot}\right)^{-1} = \left(R_{cnv,w} + R_{cnd} + R_{cnv,a}\right)^{-1}$$

From Table 4.1, Case 6,

$$R_{cnd} = \frac{\ln\left(1.08\,w/D\right)}{2\pi kL} = \frac{\ln\left(1.08\times0.30m/0.15m\right)}{2\pi\left(1.4\,W/m\cdot K\right)100m} = 8.75\times10^{-4}\ K/W$$

$$R_{cnv,a} = \left(4\,w\,L\,h\right)^{-1} = \left(4\times0.3m\times100m\times25\,W/m^2\cdot K\right)^{-1} = 3.33\times10^{-4}\,K/W$$

With $Re_D = 4\dot{m}/\pi D\mu = \left(4\times2\,kg/s\right)/\left(\pi\times0.15m\times324\times10^{-6}\,N\cdot s/m^2\right) = 52,400,$

$$\overline{h}_w \approx h_{fd} = \frac{k_w}{D}0.023\,Re_D^{4/5}\,Pr^{0.3} = \frac{0.674\,W/m\cdot K\times0.023}{0.15m}\left(52,400\right)^{4/5}\left(2.02\right)^{0.3} = 761\,W/m^2\cdot K$$

$$R_{cnv,w} = \left(\pi D L \overline{h}_w\right)^{-1} = \left(\pi\times0.15m\times100m\times761\,W/m^2\cdot K\right)^{-1} = 2.79\times10^{-5}\,K/W$$

$$UA = \left[\left(2.79\times10^{-5} + 8.75\times10^{-4} + 3.33\times10^{-4}\right)K/W\right]^{-1} = 809\,W/K$$

$$T_{m,o} = 0°C + 90°C\ \exp\left(-\frac{809\,W/K}{2\,kg/s\times4203\,J/kg\cdot K}\right) = 81.7°C$$

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Continued …

PROBLEM 8.40 (Cont.)

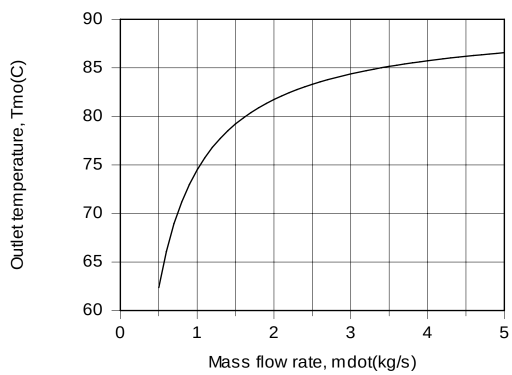
(b) From Eq. 8.21, f = [0.790ln(Re_D) – 1.64]$^{-2}$ = 0.0207 and $u_m = \dot{m}/\rho\pi D^2/4 = 0.117\,\text{m/s}$,

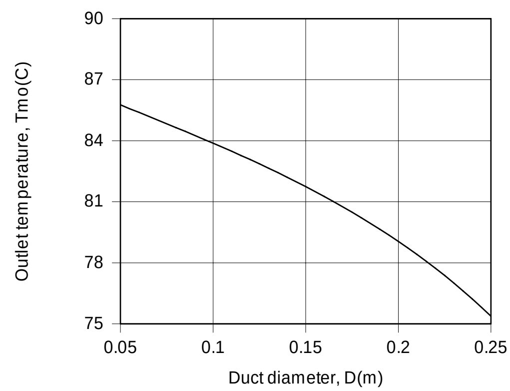
$$\Delta p = f\,\frac{\rho u_m^2}{2D}L = 0.0207\,\frac{967\,\text{kg/m}^3\,(0.117\,\text{m/s})^2}{2\times 0.15\text{m}}100\text{m} = 91\,\text{N/m}^2 = 9.1\times 10^{-4}\,\text{bars}$$ <

With $\dot{\forall} = \dot{m}/\rho = 2.07\times 10^{-3}\,\text{m}^3/\text{s}$, the pump power requirement is

$$P = \Delta p\dot{\forall} = \left(91\,\text{N/m}^2\right)2.07\times 10^{-3}\,\text{m}^3/\text{s} = 0.19\,\text{W}$$ <

(c) The effects of varying the flowrate and duct diameter were assessed using the IHT software, and results are shown below.





Although $R_{cnv,w}$, and hence R_{tot}, decreases with increasing $\dot{m}$, thereby increasing UA, the effect is significantly less than that of $\dot{m}$ to the first power, causing the exponential term, $\exp\left(-UA/\dot{m}c_p\right)$, to approach unity and $T_{m,o}$ to approach $T_{m,i}$. The effect can alternatively be attributed to a reduction in the residence time of the water in the pipe (u_m increases with increasing $\dot{m}$ for fixed D). With increasing D for fixed $\dot{m}$ and w, $T_{m,o}$ decreases due to an increase in the residence time, as well as a reduction in the conduction resistance, R_{cnd}.

COMMENTS: (1) Use of $\overline{T}_m = 360\,\text{K}$ to evaluate properties of the water for Parts (a) and (b) is reasonable, and iteration is not necessary. (2) The pressure drop and pump power requirement are small.