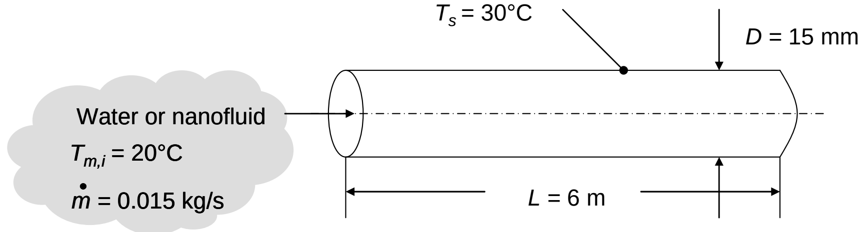
PROBLEM 8.19

KNOWN: Tube length, diameter and surface temperature. Mass flow rate and inlet temperature of fluid.

FIND: (a) Heat transfer rate if the fluid is water. (b) Heat transfer rate for the nanofluid of Example 2.2.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible viscous dissipation.

PROPERTIES: *Table A.4*, water (300 K): $\mu_{\mathrm{bf}} = 855 \times 10^{-6}$ m^2/s, $k_{\mathrm{bf}} = 0.613$ W/m·K, $c_{p,\mathrm{bf}} = 4179$ J/kg·K, $Pr_{\mathrm{bf}} = 5.83$. Example 2.2, nanofluid (300 K): $\rho_{\mathrm{nf}} = 1146$ kg/m^3, $\mu_{\mathrm{nf}} = 962 \times 10^{-6}$ m^2/s, $\nu_{\mathrm{nf}} = \mu_{\mathrm{nf}}/\rho_{\mathrm{nf}} = 839 \times 10^{-9}$ m^2/s, $k_{\mathrm{nf}} = 0.705$ W/m·K, $c_{p,\mathrm{nf}} = 3587$ J/kg·K, $\alpha_{\mathrm{nf}} = 171 \times 10^{-9}$ m^2/s, $Pr_{\mathrm{nf}} = \nu_{\mathrm{nf}}/\alpha_{\mathrm{nf}} = 4.91$.

ANALYSIS: (a) The Reynolds number is

$$Re_D = 4\dot{m}/\pi D\mu_{\mathrm{bf}} = 4 \times 0.015\ \mathrm{kg/s}/\left[\pi \times 0.015\mathrm{m} \times 855 \times 10^{-6}\mathrm{m}^2/\mathrm{s}\right] = 1489$$

Therefore the flow is laminar. The hydrodynamic and thermal entry lengths are

$$x_{fd,h} = 0.05 D Re_D = 0.05 \times 0.015\ \mathrm{m} \times 1489 = 1.12\ \mathrm{m}$$
$$x_{fd,t} = 0.05 D Re_D Pr_{\mathrm{bf}} = 0.05 \times 0.015\ \mathrm{m} \times 1489 \times 5.83 = 6.51\ \mathrm{m}$$

Since the tube length is L = 6 m, the temperature is still developing. The hydrodynamic entry length is less than the tube length, but perhaps not sufficiently shorter to consider the velocity to be fully developed through the entire tube. With $Pr_{\mathrm{bf}} > 5$, the Hausen correlation, Equation 8.57, could be used as an approximation. However the nanofluid Prandtl number is less than 5. To compare the two fluids on an equal basis, we will use the combined entry correlation, Equation 8.58, for both. With $Gz_D = (D/L) Re_D Pr_{\mathrm{bf}} = (0.015\ \mathrm{m}/6\ \mathrm{m}) \times 1489 \times 5.83 = 21.7$, Equation 8.58 is

$$\overline{Nu}_D = \frac{\dfrac{3.66}{\tanh\left[2.264 \times 21.7^{-1/3} + 1.7 \times 21.7^{-2/3}\right]} + 0.0499 \times 21.7 \times \tanh\left(21.7^{-1}\right)}{\tanh\left(2.432 \times 5.83^{1/6} \times 21.7^{-1/6}\right)} = 4.98$$

Therefore $\bar{h} = \overline{Nu}_D k_{\mathrm{bf}} / D = 4.98 \times 0.613\ \mathrm{W/m \cdot K} / 0.015\ \mathrm{m} = 203\ \mathrm{W/m^2 \cdot K}$. From Equation 8.41b

Continued…

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{\pi DL}{\dot{m}c_{p,\mathrm{bf}}}\bar{h}\right)$$

$$= 30°\mathrm{C} - 10°\mathrm{C}\exp\left(-\frac{\pi \times 0.015\ \mathrm{m} \times 6\mathrm{m}}{0.015\ \mathrm{kg/s} \times 4179\ \mathrm{J/kg\cdot K}} 203\ \mathrm{W/m^2\cdot K}\right) = 26.0°\mathrm{C}$$

Therefore the heat transfer rate to the water is

$$q = \dot{m}c_{p,\mathrm{bf}}(T_{m,o} - T_{m,i}) = 0.015\ \mathrm{kg/s} \times 4179\ \mathrm{J/kg\cdot K} \times (26.0°\mathrm{C} - 20°\mathrm{C}) = 376\ \mathrm{W} \qquad <$$

(b) The preceding calculations may be repeated for the nanofluid. The results are:

$Re_D = 1324$, $x_{fd,h} = 0.99$ m, $x_{fd,t} = 4.87$ m

The combined entry solution is again appropriate. The remaining results are:

$Gz_D = 16.2$, $\overline{Nu}_D = 4.68$ $\bar{h} = 220\ \mathrm{W/m^2\cdot K}$, $T_{m,o} = 26.9°\mathrm{C}$, and $q = 369$ W $\qquad <$

COMMENTS: (1) The nanofluid of Example 2.2 is water containing Al_2O_3 nanoparticles. The thermal conductivity of the nanofluid is 15% greater than that of the base fluid (water). In addition, the convection heat transfer coefficient of the nanofluid is 8% greater than that of the water, and the temperature increase of the nanofluid is 15% higher than for the water. *However, less heat is transferred to the nanofluid than to the water*. This is because the nanofluid suffers from a reduced specific heat relative to the pure water. Any claim that a nanofluid is a better heat transfer medium than its corresponding base fluid because of its larger thermal conductivity is suspect. In this problem, the pure water is the preferred heat transfer fluid if the objective is to maximize the heat transfer rate. In addition, the nanofluid is more costly to produce, and because of its larger viscosity, would suffer from larger pressure drops and higher pumping costs. (2) Use of the Hausen correlation, Equation 8.57 yields q = 366 W and 362 W for the water and nanofluid, respectively. Hence, the predictions of the Hausen correlation are within 3% of the predictions using the correlation of Baehr and Stephan. Use of the Hausen correlation also predicts less heat transfer for the nanofluid than for the pure water.