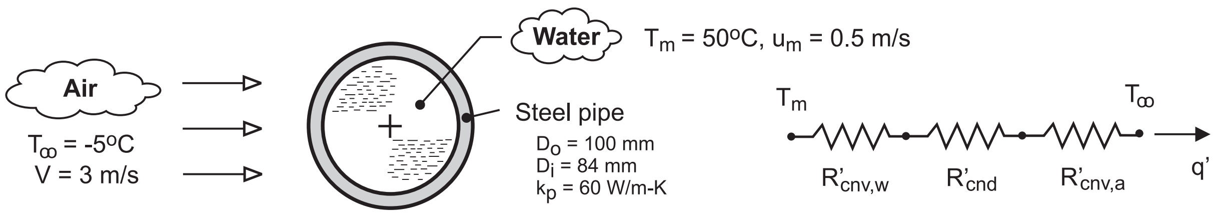
PROBLEM 8.36

KNOWN: Diameters and thermal conductivity of steel pipe. Temperature and velocity of water flow in pipe. Temperature and velocity of air in cross flow over pipe. Cost of producing hot water.

FIND: Daily cost of heat loss per unit length of pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Constant properties, (3) Negligible radiation from outer surface, (4) Fully-developed flow in pipe.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 300K$): $k_a = 0.0263$ W/m·K, $\nu_a = 15.89 \times 10^{-6}$ m^2/s, $Pr_a = 0.707$. *Table A-6*, water (T_m = 323 K): $\rho_w = 988$ kg/m^3, $\mu_w = 548 \times 10^{-6}$ N·s/m^2, $k_w = 0.643$ W/m·K, $Pr_w = 3.56$.

ANALYSIS: The heat loss per unit length of pipe is

$$q' = \frac{T_m - T_\infty}{R'_{cnv,w} + R'_{cnd} + R'_{cnv,a}} = \frac{T_m - T_\infty}{\left(h_w \pi D_i\right)^{-1} + \frac{\ln\left(D_o / D_i\right)}{2\pi k_p} + \left(h_a \pi D_o\right)^{-1}}$$

With $Re_{D,w} = \rho_w u_m D_i / \mu_w = 988\,kg/m^3 \times 0.5\,m/s \times 0.084\,m / 548 \times 10^{-6}\,N \cdot s/m^2 = 75,700$, flow is turbulent, and for fully developed conditions, the Dittus-Boelter correlation yields

$$h_w = \frac{k_w}{D_i} 0.023 Re_{D_w}^{0.8}\ Pr_w^{0.3} = 0.023 \frac{0.643\,W/m\cdot K}{0.084\,m}(75,700)^{0.8}(3.56)^{0.3} = 2060\,W/m^2\cdot K$$

With $Re_{D,a} = VD_o / \nu_a = 3\,m/s \times (0.1m) / 15.89 \times 10^{-6}\,m^2/s = 18,880$, the Churchill-Bernstein correlation yields

$$h_a = \bar{h} = \frac{k_a}{D_o}\left\{0.3 + \frac{0.62 Re_{D,a}^{1/2}\ Pr_a^{1/3}}{\left[1 + \left(0.4/Pr_a\right)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_{D,w}}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 20.1\,W/m^2\cdot K$$

Hence,

$$q' = \frac{50°C - (-5°C)}{\left(1.84 \times 10^{-3} + 0.46 \times 10^{-3} + 158.3610^{-3}\right)K/W} = 342\,W/m = 0.342\,kW/m$$

The daily energy loss is then

$$Q' = 0.346\,kW/m \times 24\,h/d = 8.22\,kW\cdot h/d\cdot m$$

and the associated cost is

$$C' = \left(8.22\,kW\cdot h/d\cdot m\right)\left(\$0.05/kW\cdot h\right) = \$0.411/m\cdot d \qquad <$$

COMMENTS: Because $R'_{cnv,a} >> R'_{cnv,w}$, the convection resistance for the water side of the pipe could have been neglected, with negligible error. The implication is that the temperature of the pipe's inner surface closely approximates that of the water. If $R'_{cnv,w}$ is neglected, the heat loss is $q' = 346\,W/m$.