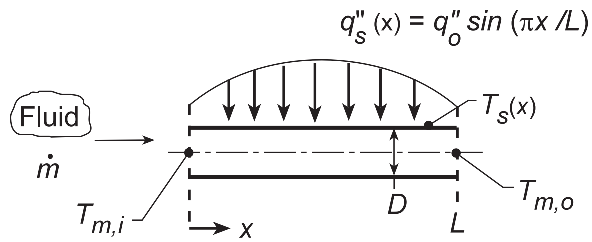
PROBLEM 8.23

KNOWN: Thin-walled tube experiences sinusoidal heat flux distribution on the wall.

FIND: (a) Total rate of heat transfer from the tube to the fluid, q, (b) Fluid outlet temperature, $T_{m,o}$, (c) Axial distribution of the wall temperature $T_s(x)$ and (d) Magnitude and position of the highest wall temperature, and (e) For prescribed conditions, calculate and plot the mean fluid and surface temperatures, $T_m(x)$ and $T_s(x)$, respectively, as a function of distance along the tube; identify features of the distributions; explore the effect of ±25% changes in the convection coefficient on the distributions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Applicability of Eq. 8.34, (3) Turbulent, fully developed flow.

ANALYSIS: (a) The total rate of heat transfer from the tube to the fluid is

$$q = \int_0^L q''_s P dx = q''_o \pi D \int_0^D \sin(\pi x/L) dx = q''_o \pi D (L/\pi)\left[-\cos(\pi x/L)\right]_0^L = 2DLq''_o \quad (1) <$$

(b) The fluid outlet temperature follows from the overall energy balance with knowledge of the total heat rate,

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 2DLq''_o \qquad T_{m,o} = T_{m,i} + \left(2DLq''_o/\dot{m}c_p\right) \quad (2) <$$

(c) The axial distribution of the wall temperature can be determined from the rate equation

$$q''_s = h\left[T_s(x) - T_m(x)\right] \qquad T_{s,x} = T_{m,x}(x) + q''_s/h \quad (3)$$

where, by combining expressions of parts (a) and (b), $T_{m,x}(x)$ is

$$\int_o^x q''_s P dx = \dot{m}c_p\left(T_{m,x} - T_{m,i}\right)$$

$$T_{m,x} = T_{m,i} + \frac{q''_o \pi D}{\dot{m}c_p}\int_0^x \sin(\pi x/L) dx = T_{m,i} + \frac{DLq''_o}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] \quad (4)$$

Hence, substituting Eq. (4) into (3), find

$$T_s(x) = T_{m,i} + \frac{DLq''_o}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] + \frac{q''_o}{h}\sin(\pi x/L) \quad (5) <$$

(d) To determine the location of the maximum wall temperature x' where $T_x(x') = T_{s,max}$, set

$$\frac{dT_s(x)}{dx} = 0 = \frac{d}{dx}\left\{\frac{DLq''_o}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] + \frac{q''_o}{h}\sin(\pi x/L)\right\}$$

$$\frac{DLq''_o}{\dot{m}c_p}\cdot\frac{\pi}{L}\cdot\sin(\pi x'/L) + \frac{q''_o}{h}\cdot\frac{\pi}{L}\cdot\cos(\pi x'/L) = 0 \qquad \tan(\pi x'/L) = -\frac{q''_o/h}{DLq''_o/\dot{m}c_p} = -\frac{\dot{m}c_p}{DLh}$$

Continued...

PROBLEM 8.23 (Cont.)

$$x' = \frac{L}{\pi}\tan^{-1}\left(-\dot{m}c_p / DLh\right) \qquad (6) <$$

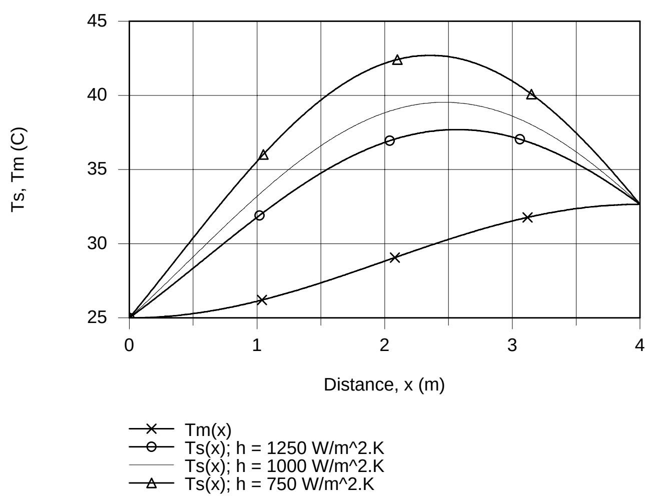
At this location, the wall temperature is

$$T_{s,max} = T_s(x') = T_{m,i} + \frac{DLq_o''}{\dot{m}c_p}\left[1-\cos\left(\pi x'/L\right)\right] + \frac{q_o''}{h}\sin\left(\pi x'/L\right) \qquad (7) <$$

(e) Consider the prescribed conditions for which to compute and plot $T_m(x)$ and $T_s(x)$,

$D = 40$ mm	$\dot{m} = 0.025$ kg/s	$h = 1000\ \mathrm{W/m^2}$	$q_o'' = 10{,}000\ \mathrm{W/m^2}$
$L = 4$ m	$c_p = 4180$ J/kg·K	$T_{m,i} = 25°C$	

Using Eqs. (4) and (5) in IHT, the results are plotted below.



The effect of a lower convection coefficient is to increase the wall temperature. The position of the maximum temperature, $T_{s,max}$, moves away from the tube exit with decreasing convection coefficient.

COMMENTS: (1) Because the flow is fully developed and turbulent, assuming h is constant along the entire length of the tube is reasonable.

(2) To determine whether the $T_x(x)$ distribution has a maximum (rather than a minimum), you should evaluate $d^2T_s(x)/dx^2$ to show the value is indeed negative.