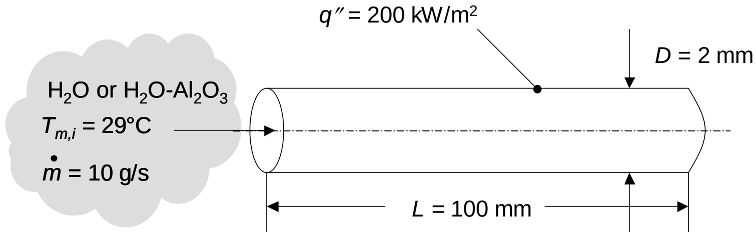
PROBLEM 8.67

KNOWN: Dimensions of circular tube, applied constant heat flux, inlet temperature, mass flow rate.

FIND: Tube wall temperature at the tube exit for pure water and for a water-Al_2O_3 nanofluid.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties.

PROPERTIES: *Table A.4*, water (300 K): $\mu_{\rm bf} = 855 \times 10^{-6}$ m^2/s, $k_{\rm bf}$ = 0.613 W/m·K, $c_{p,\rm bf}$ = 4179 J/kg·K, $Pr_{\rm bf}$ = 5.83. Example 2.2, nanofluid (300 K): $\mu_{\rm nf} = 962 \times 10^{-6}$ m^2/s, $k_{\rm nf}$ = 0.705 W/m·K, $c_{p,\rm nf}$ = 3587 J/kg·K, $Pr_{\rm nf}$ = 4.91.

ANALYSIS: The Reynolds number for the pure water is $Re_D = 4\dot{m}/\pi D\mu_{\rm bf} = [4\times(10/1000 \text{ kg/s})]/\left(\pi\times 0.002 \text{ m}\times 855\times10^{-6} \text{ N}\cdot\text{s/m}^2\right) = 7450$ and the flow is turbulent. Similarly, the Reynolds number for the nanofluid is $Re_{D,\rm nf}$ = 6620. Since L/D = 100/2 = 50, the flow is fully-developed at the tube exit for both fluids.

The local Nusselt number is evaluated using the Gnielinski correlation. For pure water, Eq. (8.21) yields, $f_{\rm bf} = (0.790\ln(7450) - 1.64)^{-2} = 0.0342$ while for the nanofluid, $f_{\rm nf} = (0.790\ln(6620) - 1.64)^{-2}$ = 0.0355. The Gnielinski correlation yields, for the pure fluid

$$Nu_{D,\rm bf} = \frac{(0.0342/8)(7450-1000)5.83}{1+12.7(0.0342/8)^{1/2}\left(5.83^{2/3}-1\right)} = 56.24$$

while for the nanofluid,

$$Nu_{D,\rm nf} = \frac{(0.0355/8)(6620-1000)4.91}{1+12.7(0.0355/8)^{1/2}\left(4.91^{2/3}-1\right)} = 47.08$$

Hence, $h_{\rm bf} = Nu_{D,\rm bf}k_{\rm bf}/D$ = 56.24(0.613 W/m·K)/0.002 m = 17,240 W/m^2·K and $h_{\rm nf} = Nu_{D,\rm bf}k_{\rm nf}/D$ = 47.08(0.705 W/m·K)/0.002 m = 16,600 W/m^2·K.

Applying Eq. (8.40) to the pure fluid yields

$$T_{m,o} = T_{m,i} + \frac{q''\pi D}{\dot{m}c_{p,\rm bf}}L = 29°\text{C} + \frac{200{,}000 \text{ W/m}^2\pi(2/1000 \text{ m})}{(10/1000 \text{ kg/s})\times(4179 \text{ J/kg}\cdot\text{K})}0.1 \text{ m} = 29°\text{C}+3.00°\text{C} = 32.00°\text{C}$$

whereas applying Eq. (8.40) to the nanofluid results in

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PROBLEM 8.67 (Cont.)

$$T_{m,o,\text{nf}} = T_{m,i} + \frac{q''\pi D}{\dot{m}c_{p,\text{nf}}}L = 29°\text{C} + \frac{200,000\ \text{W/m}^2\pi(2/1000\ \text{m})}{(10/1000\ \text{kg/s})\times(3587\ \text{J/kg}\cdot\text{K})}0.1\ \text{m} = 29°\text{C} + 3.50°\text{C} = 32.50°\text{C}$$

From Eq. (8.27) the wall temperature at the outlet of the tube carrying the pure water is,

$$T_s(x = L) = T_{m,o} + q''/h_{\text{bf}}$$

$$= 32.00°\text{C} + 200,000\text{W/m}^2/17,240\ \text{W/m}^2\cdot\text{K} = 32.00°\text{C} + 11.61°\text{C} = 43.61°\text{C}$$ $<$

Similarly for the nanofluid,

$$T_{s,\text{nf}}(x = L) = T_{m,o,\text{nf}} + q''/h_{\text{nf}}$$

$$= 32.50°\text{C} + 200,000\text{W/m}^2/16,600\ \text{W/m}^2\cdot\text{K} = 32.50°\text{C} + 12.05°\text{C} = 44.55°\text{C}$$ $<$

COMMENT: The nanofluid provides a larger thermal conductivity but a smaller convective heat transfer coefficient relative to the pure water. If the objective is to minimize the wall temperature at the outlet of the tube, the nanofluid is *not* the appropriate selection. The wall temperature at the tube outlet may be greater than, less than, or equal to the wall temperature associated with use of pure water, depending on the tube geometry and flow rate.