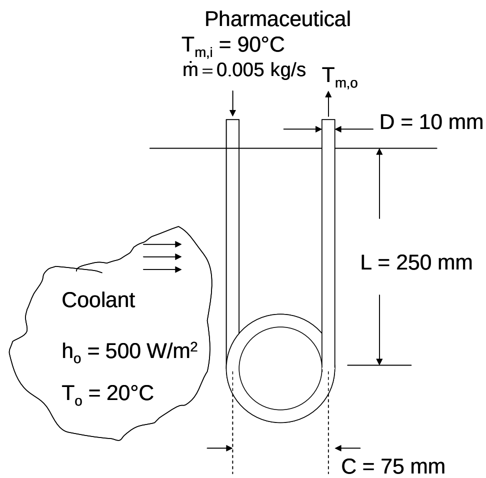
PROBLEM 8.104

KNOWN: Geometry and dimensions of a tube with straight and coiled sections. Temperature and convection coefficient of coolant flowing outside the tube. Inlet temperature, mass flow rate, and properties of pharmaceutical fluid in tube.

FIND: (a) Outlet temperature of pharmaceutical, (b) Outlet temperature with inner heat transfer coefficient doubled in straight sections, (c) Effect of left- or right-handed spiral.

SCHEMATIC:



ASSUMPTIONS: (1) Tube wall thermal resistance is negligible. (2) Flow is fully-developed in coiled section. (3) Flow in last straight section is unaffected by swirl introduced in coiled section. (4) Constant properties.

PROPERTIES: Pharmaceutical fluid (given): $\rho = 1200\ \text{kg/m}^3$, $\mu = 4 \times 10^{-3}\ \text{N·s/m}^2$, $c_p = 2000\ \text{J/kg·K}$, $k = 0.5\ \text{W/m·K}$, $Pr = \mu c_p/k = 16$.

ANALYSIS:
(a) The Reynolds number is

$$Re_D = \frac{4\,\dot{m}}{\pi D \mu} = \frac{4 \times 0.005\ \text{kg/s}}{\pi \times 0.01\ \text{m} \times 4 \times 10^{-3}\ \text{N}\cdot\text{s/m}^2} = 159$$

Thus the flow is laminar.

<u>1st Straight Section.</u> The development length in the straight section is

$$x_{fd,h} = 0.05\,Re_D\,D = 0.05 \times 159 \times 0.01\ \text{m} = 0.08\ \text{m}$$

$$x_{fd,t} = x_{fd,h} \cdot Pr = 0.08\ \text{m} \times 16 = 1.3\ \text{m}$$

The flow is thermally developing. With Pr > 5, we can use Equation 8.57 with Equation 8.56,

$$\overline{Nu}_D = 3.66 + \frac{0.0668\,(D/L)\,Re_D\,Pr}{1 + 0.04\left[(D/L)\,Re_D\,Pr\right]^{2/3}} = 7.29$$

Thus $\quad h_i = \overline{Nu}_D k/D = 7.29 \times 0.5\ \text{W/m}\cdot\text{K}/0.01\ \text{m} = 365\ \text{W/m}^2\cdot\text{K}$.

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PROBLEM 8.104 (Cont.)

The mean temperature at the end of the first straight section can be found from Equation 8.45a,

$$T_{m,o1} = T_\infty + (T_{m,i} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

where $\overline{U} = \left[1/h_i + 1/h_o\right]^{-1} = \left[1/365\ \text{W/m}^2\cdot\text{K} + 1/500\ \text{W/m}^2\cdot\text{K}\right]^{-1} = 211\ \text{W/m}^2\cdot\text{K}$.

Thus $\quad T_{m,o1} = 20°\text{C} + (90°\text{C} - 20°\text{C})\exp\left(-\frac{211\ \text{W/m}^2\cdot\text{K}\times\pi\times 0.01\ \text{m}\times 0.25\ \text{m}}{0.005\ \text{kg/s}\times 2000\ \text{J/kg}\cdot\text{K}}\right) = 79.3°\text{C}$

Coiled Section. The critical Reynolds number in the coiled section is given by Equation 8.74,

$$Re_{D,C,h} = Re_{D,C}\left[1 + 12(D/C)^{0.5}\right]$$

where $Re_{D,C}$ = 2300. Since this must be greater than 2300, the flow in the coiled section, with Re_D = 159, is still laminar. The length of the coiled section is 6.5 π C = 6.5 π (0.075 m) = 1.53 m. Since development lengths are 20 to 50% shorter in coiled tubes than in straight tubes the flow can be approximated as fully developed. The Nusselt number is given by Equation 8.76, with

$$a = \left[1 + \frac{957\,(C/D)}{Re_D^2\ Pr}\right] = \left[1 + \frac{957\,(75\ \text{mm}/10\ \text{mm})}{(159)^2\times 16}\right] = 1.018$$

and b = 1 + 0.477/Pr = 1+ 0.477/16 = 1.030. Note that $Re_D\,(D/C)^{1/2} = 58$, therefore the criteria for using Equations 8.76 and 8.77 are satisfied. Thus assuming $\mu_s = \mu$,

$$Nu_D = \left[\left(3.66 + \frac{4.343}{a}\right)^3 + 1.158\left(\frac{Re_D\ (D/C)^{1/2}}{b}\right)^{3/2}\right]^{1/3}$$

$$= \left[\left(3.66 + \frac{4.343}{1.018}\right)^3 + 1.158\left(\frac{159\ (10\ \text{mm}/75\ \text{mm})^{1/2}}{1.030}\right)^{3/2}\right]^{1/3} = 9.96$$

and $h_i = Nu_D k/D = 498\ \text{W/m}^2\cdot\text{K}$.

Then $\quad \overline{U} = \left[1/h_i + 1/h_o\right]^{-1} = \left[1/498\ \text{W/m}^2\cdot\text{K} + 1/500\ \text{W/m}^2\cdot\text{K}\right]^{-1} = 250\ \text{W/m}^2\cdot\text{K}$.

The outlet temperature of the coiled section can be found from Equation 8.45a, with $A_s = (\pi D)(6.5\pi C) = 0.048\ \text{m}^2$, and the inlet temperature is the outlet temperature of the straight section:

$$T_{m,o2} = T_\infty + (T_{m,o1} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

$$T_{m,o2} = 20°\text{C} + (79.3°\text{C} - 20°\text{C})\exp\left(-\frac{250\ \text{W/m}^2\cdot\text{K}\times 0.048\ \text{m}^2}{0.005\ \text{kg/s}\times 2000\ \text{J/kg}\cdot\text{K}}\right) = 37.9°\text{C}$$

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2nd Straight Section. The overall heat transfer coefficient would be the same as in the 1st straight section. The outlet temperature can be calculated from Equation 8.45a with the inlet temperature equal to the outlet temperature of the coiled section.

$$T_{m,o3} = T_\infty + (T_{m,o2} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

$$T_{m,o3} = 20°C + (37.9°C - 20°C)\exp\left(-\frac{211\ W/m^2\cdot K \times \pi \times 0.01\ m \times 0.25\ m^2}{0.005\ kg/s \times 2000\ J/kg\cdot K}\right)$$

$$T_{m,o3} = 35.1°C$$ <

(b) Repeating the calculations with h_i in the straight sections doubled, in the 1st straight section:

$$\overline{U} = \left[1/730\ W/m^2\cdot K + 1/500\ W/m^2\cdot K\right]^{-1} = 297\ W/m^2\cdot K$$

$$T_{m,o1} = 75.4°C$$

In the coiled section, $\overline{U}$ is unchanged, and

$$T_{m,o2} = 36.7°C$$

In the 2nd straight section, $\overline{U} = 297\ W/m^2\cdot K$ and

$$T_{m,o3} = 33.2°C$$

(c) Yes, the orientation of the springs could have an effect, because they introduce swirl that interacts with the swirl introduced in the coiled section. However, the effect is probably small.

COMMENTS: The analysis is only approximate. In particular, the flow in the last section would be affected by the swirl introduced in the coiled section, which would in turn affect the heat transfer.