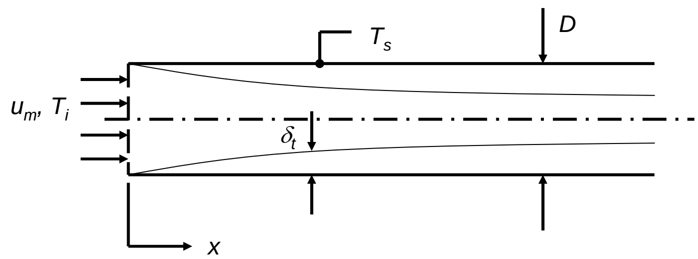
PROBLEM 8.15

KNOWN: Laminar boundary layer development in a tube entrance.

FIND: (a) Expression for Nu_D in terms of Gz_D^{-1} and Pr. Plot of Nu_D versus Gz_D^{-1} for Pr = 0.7. (b) Expression for $\overline{Nu}_D$ in terms of Gz_D^{-1} and Pr. Comparison to combined entrance length correlation in the limit of small x.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties. (2) Laminar conditions.

ANALYSIS: (a) From Equation 7.21, the Nusselt number based upon the streamwise coordinate x is

$$Nu_x = \frac{hx}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \tag{1}$$

Multiplying both sides of Equation 1 by D/x and substituting $Re_x = Re_D x/D$ yields

$$Nu_D = \frac{hD}{k} = 0.332\left[Re_x\left(\frac{x}{D}\right)\right]^{1/2}\left[\frac{D}{x}\right]Pr^{1/3} = 0.332\left[Re_x\left(\frac{D}{x}\right)\right]^{1/2} Pr^{1/3} \tag{2}$$

Substituting Gz_D^{-1} = $(x/D)/(Re_D Pr)$ into Equation 2 and noting that $Pr^{1/3} = Pr^{1/2} \cdot Pr^{-1/6}$ yields

$$Nu_D = 0.332\,[Gz_D^{-1}]^{-1/2}\, Pr^{-1/6}$$ <

The expression for the local Nusselt number, Nu_D with Pr = 0.7 is plotted below.

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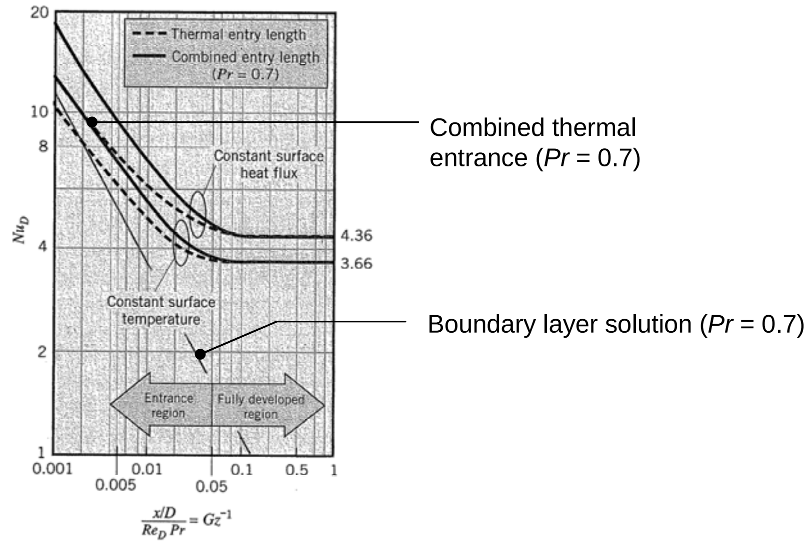
PROBLEM 8.15 (Cont.)



(b) Equation 7.25 gives the following for the average Nusselt number:

$$\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$$

Following the same steps as in part (a), this can be rewritten as

$$\overline{Nu}_D = 0.664[Gz_D^{-1}]^{-1/2} Pr^{-1/6} \tag{3}$$

The average Nusselt number for the combined entrance length is given as

$$\overline{Nu}_D = \frac{\dfrac{3.66}{\tanh\left[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}\right]} + 0.0499 Gz_D \tanh\left(Gz_D^{-1}\right)}{\tanh\left(2.432 Pr^{1/6} Gz_D^{-1/6}\right)}$$

In the limit of small *x*, Gz_D^{-1} is also small. Furthermore, $Gz_D^{-2/3} << Gz_D^{-1/3}$ Noting that tanh(ε) $\rightarrow \varepsilon$ as $\varepsilon \rightarrow 0$, we find

$$\overline{Nu}_D = \frac{\dfrac{3.66}{\tanh\left[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}\right]} + 0.0499 Gz_D \tanh\left(Gz_D^{-1}\right)}{\tanh\left(2.432 Pr^{1/6} Gz_D^{-1/6}\right)}$$

$$\rightarrow \frac{\dfrac{3.66}{2.264 Gz_D^{-1/3}} + 0.0499 Gz_D Gz_D^{-1}}{2.432 Pr^{1/6} Gz_D^{-1/6}} \rightarrow \frac{3.66}{2.264 \times 2.432 Gz_D^{-1/6} Gz_D^{-1/3} Pr^{1/6}} = 0.665[Gz_D^{-1}]^{-1/2} Pr^{-1/6}$$

This is in excellent agreement with Eq. (3).

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PROBLEM 8.15 (Cont.)

COMMENT: The combined thermal entrance length solution and the boundary layer solution based upon the results of Chapter 7 exhibit asymptotic behavior at small inverse Graetz numbers. Small values of Gz_D^{-1} correspond to the locations where the boundary layer is very thin.