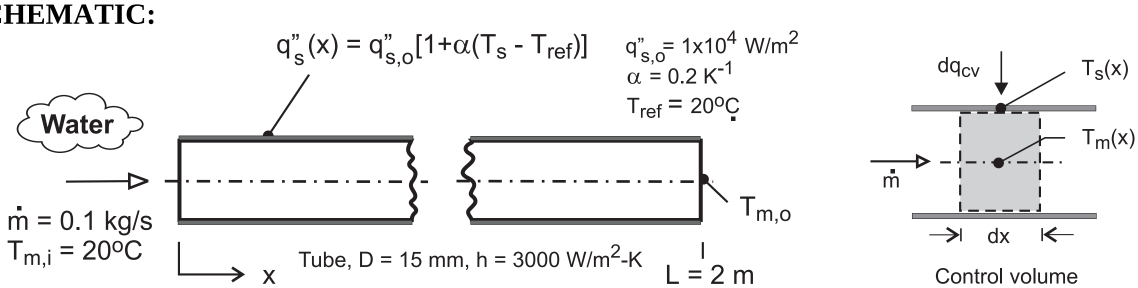
PROBLEM 8.24

KNOWN: Water is heated in a tube having a wall flux that is dependent upon the wall temperature.

FIND: (a) Beginning with a properly defined differential control volume in the tube, derive expressions that can be used to obtain the temperatures for the water and the wall surface as a function of distance from the inlet, $T_m(x)$ and $T_s(x)$, respectively; (b) Using a numerical integration scheme, calculate and plot the temperature distributions, $T_m(x)$ and $T_s(x)$, on the same graph. Identify and comment on the main features of the distributions; and (c) Calculate the total heat transfer rate to the water.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed flow and thermal conditions, (3) No losses to the outer surface of the tube, (3) Constant properties, and (4) Incompressible liquid with negligible viscous dissipation .

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = 300\ K\right)$: c_p = 4179 J/kg·K

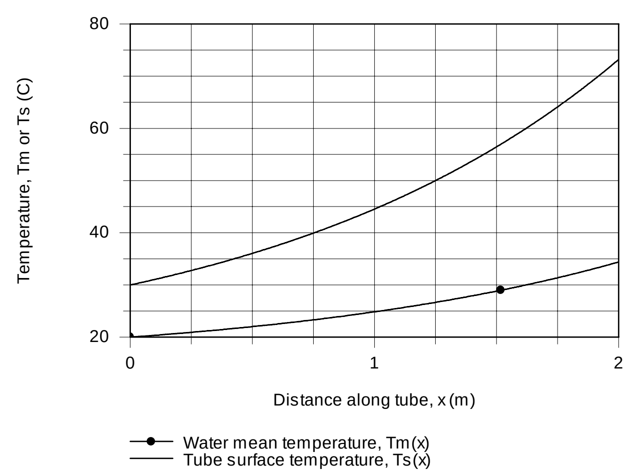
ANALYSIS: (a) The properly defined control volume of perimeter P = πD shown in the above schematic follows from Fig. 8.6. The energy balance on the CV includes advection, convection at the inner tube surface, and the heat flux dissipated in the tube wall. (See Eq. 8.37).

$$\dot{m}\,c_p \frac{dT_m}{dx} = q_s''(x)P = h\,P\left[T_s(x) - T_m(x)\right] \qquad (1,2)$$

where $q_s''(x)$ is dependent upon $T_s(x)$ according to the relation

$$q_s''(x) = q_{s,o}''\left[1 + \alpha\left(T_s(x) - T_{ref}\right)\right] \qquad (3)$$

(b) Eqs. (1 and 2) with Eq. (3) can be solved by numerical integration using the Der function in *IHT* as shown in Comment 1. The temperature distributions for the water and wall surface are plotted below.



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(c) The total heat transfer to the water can be evaluated from an overall energy balance on the water,

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \qquad (4)$$

$$q = 0.1 \text{ kg/s} \times 4179 \text{ J/kg}\cdot\text{K}\left(34.4 - 20\right)\text{K} = 6018 \text{ W} \qquad <$$

Alternatively, the heat rate can be evaluated by integration of the heat flux from the tube surface over the length of the tube,

$$q = \int_0^L q''_s\left(x\right) P dx \qquad (5)$$

where $q''_s\left(x\right)$ is given by Eq. (3), and $T_s(x)$ and $T_m(x)$ are determined from the differential form of the energy equation, Eqs. (1) and (2). The result as shown in the *IHT* code below is 6005 W.

COMMENTS: (1) Note that $T_m(x)$ increases with distance greater than linearly, as expected since $q''_s(x)$ does. Also as expected, the difference, $T_s(x) - T_m(x)$, likewise increases with distance greater than linearly.

(2) In the foregoing analysis, c_p is evaluated at the mean fluid temperature $T_m = (T_{m,i} + T_{m,o})/2$.

(3) The *IHT* code representing the foregoing equations to calculate and plot the temperature distribution and to calculate the total heat rate to the water is shown below.

```
/* Results: integration for distributions; conditions at x = 2 m
F_xTs  Ts      q'      q"s_x     x       Tm
11.64  73.18   5483    1.164E5   2       34.39
3      30      1414    3E4       0       20   */

/* Results: heat rate by energy balances on fluid and tube surface
q_eb   q_hf
6018   6005    */

/* Results: for evaluating cp at Tm
Ts     cp      q"s_x     x       Tm
73.31  4179    1.166E5   2       34.44
30     4179    3E4       0       20      */

// Energy balances
mdot * cp * der(Tm,x) = q'                // Energy balance, Eq. 8.37
q' = q"s_x * P
q"s_x = q"o * F_xTs
q' = h * P * (Ts - Tm)                    // Convection rate equation
P = pi * D

// Surface heat flux specification
F_xTs = (1 + alpha * (Ts -Tref))
alpha = 0.2
Tref = 20

// Overall heat rate
// Energy balance on the fluid
q_eb = mdot * cp * (Tmo - Tmi)
Tmi = 20
Tmo = 34.4                  // From initial solve
// Integration of the surface heat flux
q_hf =  q"o * P * INTEGRAL(F_xTs, x)

// Input variables
mdot = 0.1
D = 0.015
h = 3000
q"o = 1.0e4
// L = 2                    // Limit of integration over x
// Tmi = 20                 // Initial condition for integration

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xx = 0              // Quality (0=sat liquid or 1=sat vapor)
cp = cp_Tx("Water",Tmm,xx)          // Specific heat, J/kg·K
Tmm = (20 + 34.4) / 2 + 273
```