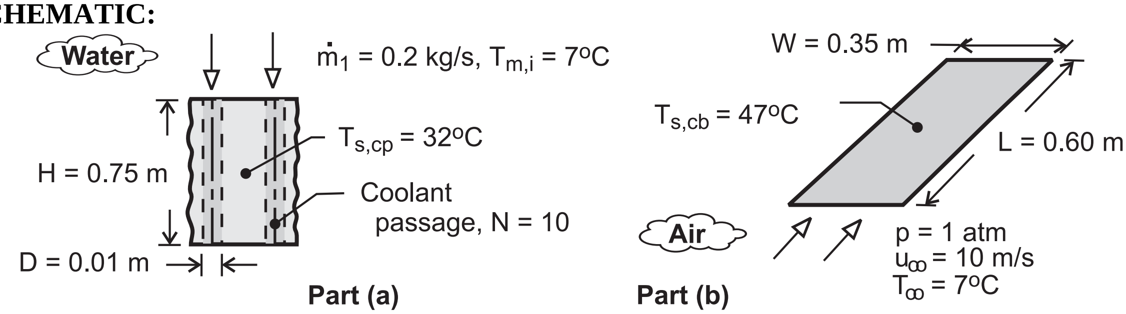
PROBLEM 8.56

KNOWN: Cold plate geometry and temperature. Inlet temperature and flow rate of water. Number of circuit boards and temperature and velocity of air in parallel flow over boards.

FIND: (a) Heat dissipation by cold plates, (b) Heat dissipation by air flow.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal cold plate, (2) All heated generated by circuit boards is dissipated by cold plates (Part (a)), (3) Circuit boards may be represented as isothermal at an average surface temperature, (4) Air flow over circuit boards approximates that over a flat plate in parallel flow, (5) Steady operation, (6) Constant properties, (7) Water is incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m \approx 290\text{K}\right)$: c_p = 4184 J/kg·K, $\mu = 1080 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2$, k=0.598 W/m·K, Pr = 7.56. *Table A-4*, Air (p = 1 atm, T_f = 300K): $\nu = 15.89 \times 10^{-6}\ \text{m}^2/\text{s}$, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) With $Re_D = 4\dot{m}_1/\pi D\mu = 4 \times 0.2\,\text{kg/s}/\pi \times 0.01\text{m} \times 1080 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2 = 23{,}600$, the flow is turbulent, and from Eq. (8.60),

$$h = \frac{k}{D} Nu_D = 0.023 \frac{k}{D} Re_D^{4/5} Pr^{0.4} = \frac{0.023 \times 0.598\,\text{W/m} \cdot \text{K}}{0.01\text{m}} (23{,}600)^{4/5} (7.56)^{0.4} = 9{,}730\,\text{W/m}^2 \cdot \text{K}$$

With H/D = 0.75/0.01 = 75, it is reasonable to assume fully developed flow throughout the tube. Hence, from Eqs. (8.41b) and (8.34)

$$\frac{T_{s,cp} - T_{m,o}}{T_{s,cp} - T_{m,i}} = \exp\left(-\frac{\pi D H}{\dot{m}_1 c_p} h\right) = \exp\left(-\frac{\pi \times 0.01\text{m} \times 0.75\text{m} \times 9730\,\text{W/m}^2 \cdot \text{K}}{0.2\,\text{kg/s} \times 4184\,\text{J/kg} \cdot \text{K}}\right) = 0.760$$

$$T_{m,o} = T_{s,cp} - 0.76\left(T_{s,cp} - T_{m,i}\right) = 13°\text{C}$$

$$q_1 = \dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right) = 0.2\,\text{kg/s} \times 4184\,\text{J/kg} \cdot \text{K} \times 6°\text{C} = 5021\,\text{W}$$

With a total of 2N = 20 passages, the total heat dissipation is

$$q = 2Nq_1 = 20 \times 5021\,\text{W} = 100\,\text{kW}$$ <

(b) For the air flow, $Re_D = u_\infty L/\nu = 10\,\text{m/s} \times 0.60\text{m}/15.89 \times 10^{-6}\,\text{m}^2\text{s} = 378{,}000$, and the flow is laminar. From Eq. (7.25),

$$\overline{h} = \frac{k}{L} \overline{Nu}_L = 0.664 \frac{k}{L} Re_L^{1/2} Pr^{1/3} = \frac{0.664 \times 0.0263\,\text{W/m} \cdot \text{K}}{0.60\text{m}} (378{,}000)^{1/2} (0.707)^{1/3} = 15.9\,\text{W/m}^2 \cdot \text{K}$$

Heat dissipation to the air from both sides of 10 circuit boards is then

$$q = 2N_{cb}\overline{h}(WL)\left(T_{s,cb} - T_\infty\right) = 20 \times 15.9\,\text{W/m}^2 \cdot \text{K} \times 0.21\text{m}^2 \times 40°\text{C} = 2{,}670\,\text{W}$$ <

COMMENTS: The cooling capacity of the cold plates far exceeds that of the air flow. However, the challenge would be one of efficiently transferring such a large amount of energy to the cold plates without incurring excessive temperatures on the circuit boards.