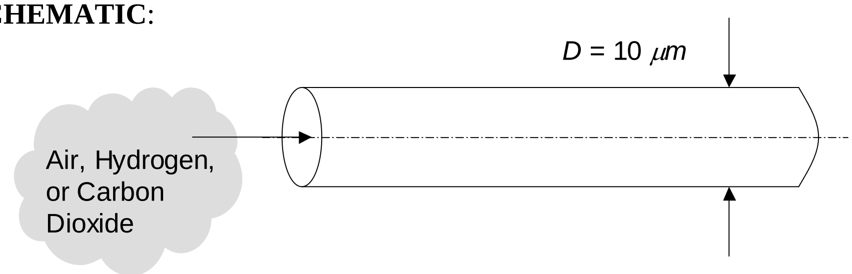
PROBLEM 8.108

KNOWN: Flow of an ideal gas through a small diameter tube.

FIND: Expression for the transition density, below which microscale effects become important. Value of the transition density for hydrogen, air and carbon dioxide.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Ideal gas.

PROPERTIES: Figure 2.8 and *Table A.4* (p = 1 atm, T = 300 K) Air: $\mathcal{M}$ = 28.97 kmol/kg, d = 0.372 nm, ρ = 1.161 kg/m^3, H_2: $\mathcal{M}$ = 2.016 kmol/kg, d = 0.274 nm, ρ = 0.0808 kg/m^3, CO_2: $\mathcal{M}$ = 44.01 kmol/kg, d = 0.464 nm, ρ = 1.773 kg/m^3.

ANALYSIS: From Eq. 2.11 the mean free path is

$$\lambda_{\text{mfp}} = \frac{k_B T}{\sqrt{2}\pi d^2 p}$$

and from the ideal gas equation of state, $p = \rho \mathcal{R} T/\mathcal{M}$. Microscale effects become important at $\lambda_{\text{mfp}}/D \approx$ 0.01. Therefore,

$$\frac{\lambda_{\text{mfp}}}{D} = 0.01 = \frac{k_B \mathcal{M}}{\sqrt{2}\pi d^2 \rho_c \mathcal{R} D} \quad \text{or} \quad \rho_c = \frac{100 k_B \mathcal{M}}{\sqrt{2}\pi d^2 \mathcal{R} D}$$

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For air,

$$\rho_{c,\text{Air}} = \frac{100 k_B \mathcal{M}}{\sqrt{2}\pi d^2 \mathcal{R} D} = \frac{100 \times 1.381 \times 10^{-23}\,\text{J/K} \times 28.97\ \text{kmol/kg}}{\sqrt{2}\pi \times \left(0.372 \times 10^{-9}\,\text{m}\right)^2 \times 8315\ \text{J/kmol} \cdot \text{K} \times 10 \times 10^{-6}\ \text{m}} = 0.783\ \text{kg/m}^3$$ <

Repeating the calculation for hydrogen and CO_2 yields

$$\rho_{c,\text{H2}} = 0.100\ \text{kg/m}^3;\ \rho_{c,\text{CO2}} = 0.764\ \text{kg/m}^3.$$ <

The ratios of the transition to molecular density at p = 1 atm, T = 300 K, for the three gases are:

Gas	Ratio
Air	0.783/1.161 = 0.674
H_2	0.100/0.0808 = 1.24
CO_2	0.764/1.773 = 0.431

COMMENT: Microscale effects could be important, especially for hydrogen at atmospheric pressure and T = 300 K.