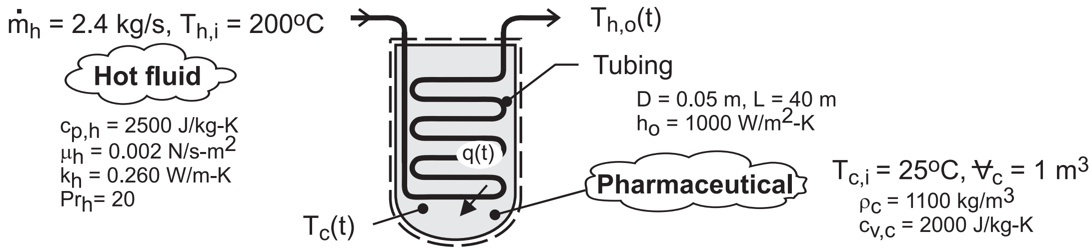
PROBLEM 8.32

KNOWN: Inlet temperature, flow rate and properties of hot fluid. Initial temperature, volume and properties of pharmaceutical. Heat transfer coefficient at outer surface and dimensions of coil.

FIND: (a) Expressions for $T_c(t)$ and $T_{h,o}(t)$, (b) Plots of $T_c(t)$ and $T_{h,o}(t)$ for prescribed conditions. Effect of flow rate on time for pharmaceutical to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible heat loss from vessel to surroundings, (3) Pharmaceutical is isothermal, (4) Negligible work due to stirring, (5) Negligible thermal energy generation (or absorption) due to chemical reactions associated with the batch process, (6) Hot fluid is an incompressible liquid with negligible viscous dissipation, (7) Negligible tube wall conduction resistance.

ANALYSIS: (a) Performing an energy balance for a control surface about the stirred liquid, it follows that

$$\frac{dU_c}{dt} = \frac{d}{dt}\left(\rho_c V_c\, c_{v,c}\, T_c\right) = \rho_c V_c\, c_{v,c}\frac{dT_c}{dt} = q(t) \tag{1}$$

where, $$q(t) = \dot{m}_h\, c_{p,h}\left(T_{h,i} - T_{h,o}\right) \tag{2}$$

or, $$q(t) = UA_s\, \Delta T_{\ell m} \tag{3a}$$

where

$$\Delta T_{\ell m} = \frac{\left(T_{h,i} - T_c\right) - \left(T_{h,o} - T_c\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} = \frac{\left(T_{h,i} - T_{h,o}\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} \tag{3b}$$

Substituting (3b) into (3a) and equating to (2),

$$\dot{m}_h\, c_{p,h}\left(T_{h,i} - T_{h,o}\right) = UA_s \frac{\left(T_{h,i} - T_{h,o}\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)}$$

Hence, $$\ell n\left(\frac{T_{h,i} - T_c}{T_{h,o} - T_c}\right) = \frac{UA_s}{\dot{m}_h\, c_{p,h}}$$

or, $$T_{h,o}(t) = T_c + \left(T_{h,i} - T_c\right)\exp\left(-UA_s / \dot{m}_h\, c_{p,h}\right) \tag{4}$$ <

Substituting Eqs. (2) and (4) into Eq. (1),

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$$\rho_c V_c c_{v,c} \frac{dT_c}{dt} = \dot{m}_h c_{p,h}\left[T_{h,i} - T_c - \left(T_{h,i} - T_c\right)\exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right]$$

$$\frac{dT_c}{dt} = \frac{\dot{m}_h c_{p,h}\left(T_{h,i} - T_c\right)}{\rho_c V_c c_{v,c}}\left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right]$$

$$-\int_{T_{c,i}}^{T_c(t)} \frac{dT_c}{\left(T_c - T_{h,i}\right)} = \frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}}\left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right]\int_o^t dt$$

$$-\ell n\left(\frac{T_c - T_{h,i}}{T_{c,i} - T_{h,i}}\right) = \frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}}\left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right]t$$

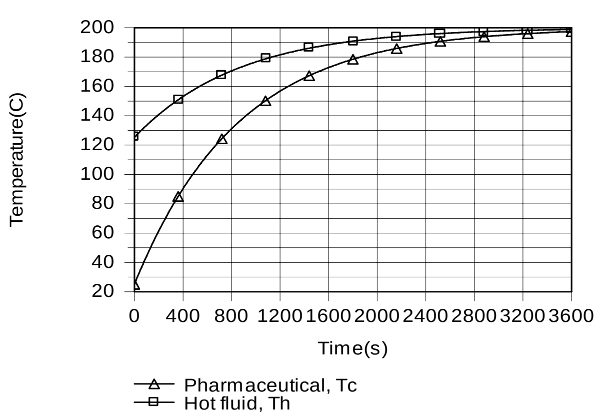
$$T_c(t) = T_{h,i} - \left(T_{h,i} - T_{c,i}\right)\exp\left\{-\frac{\dot{m}_h c_{p,h}\left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right]t}{\rho_c V_c c_{v,c}}\right\} \qquad (5) <$$

Eq. (5) may be used to determine $T_c(t)$ and the result used with (4) to determine $T_{h,o}(t)$.

(b) To evaluate the temperature histories, the overall heat transfer coefficient, $U = \left(h_o^{-1} + h_i^{-1}\right)^{-1}$, must first be determined. With $Re_D = 4\dot{m} / \pi D\mu = 4 \times 2.4\,kg/s / \pi\left(0.05m\right)0.002\,N \cdot s/m^2 = 30,600$, the flow is turbulent and

$$h_i = \frac{k}{D} Nu_D = \frac{0.260\,W/m \cdot K}{0.05m}\left[0.023(30,600)^{4/5}(20)^{0.3}\right] = 1140\,W/m^2 \cdot K$$

Hence, $U = \left[(1000)^{-1} + (1140)^{-1}\right]^{-1} W/m^2 \cdot K = 532\,W/m^2 \cdot K$. As shown below, the temperature of the pharmaceuticals increases with time due to heat transfer from the hot fluid, approaching the inlet temperature of the hot fluid (and its maximum possible temperature of 200°C) at t = 3600s.



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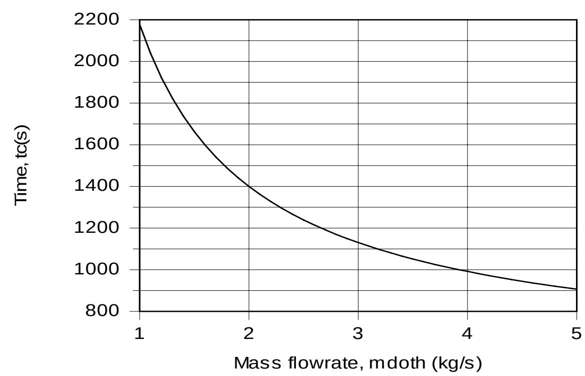
With increasing T_c, the rate of heat transfer from the hot fluid decreases (from 4.49×10^5 W at t = 0 to 6760 W at 3600s), in which case $T_{h,o}$ increases (from 125.2°C at t = 0 to 198.9°C at 3600s). The time required for the pharmaceuticals to reach a temperature of T_c = 160°C is

$$t_c = 1266\text{s}$$ <

With increasing $\dot{m}_h$, the overall heat transfer coefficient increases due to increasing h_i and the hot fluid maintains a higher temperature as it flows through the tube. Both effects enhance heat transfer to the pharmaceutical, thereby reducing the time to reach 160°C from 2178s for $\dot{m}_h = 1\,\text{kg/s}$ to 906s at 5 kg/s.



For $1 \le \dot{m}_h \le 5\,\text{kg/s}$, $12{,}700 \le Re_D \le 63{,}700$ and $565 \le h_i \le 2050\,\text{W/m}^2 \cdot \text{K}$.

COMMENTS: (1) Although design changes involving the length and diameter of the coil can be used to alter the heating rate, process control parameters are limited to $T_{h,i}$ and $\dot{m}_h$. (2) Coiling the tube can increase the inside heat transfer coefficient, as will be seen in Section 8.7.