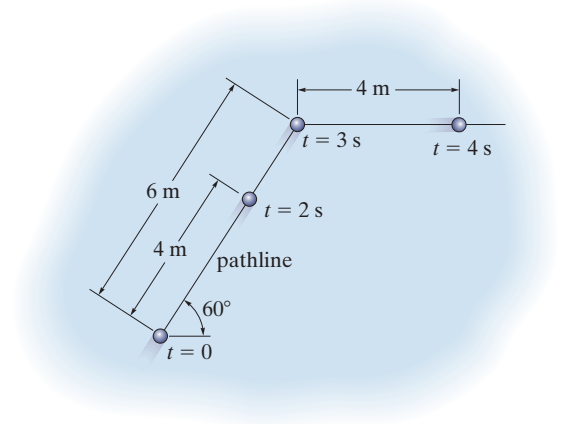


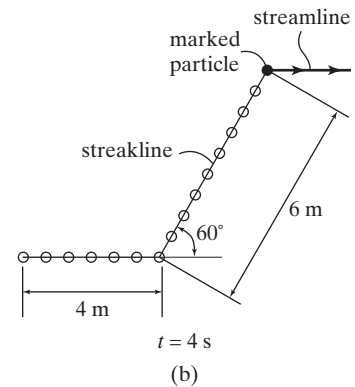
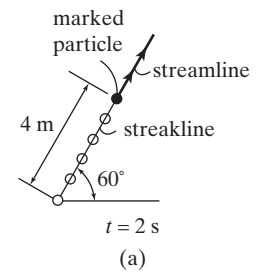
**3-1.** A marked particle is released into a flow when  $t = 0$ , and the pathline for a particle is shown. Draw the streakline, and the streamline for the particle when  $t = 2$  s and  $t = 4$  s.



## SOLUTION

Since the streamlines have a constant direction for the time interval  $0 \leq t < 3$  s, the pathline and streakline coincide with the streamline when  $t = 2$  s as shown in Fig. *a*.

The pathline and streakline will coincide with the streamline until  $t = 3$  s, after which the streamline makes a sudden change in direction. Thus, the streamline of the marked particle and the streakline when  $t = 4$  s will be as shown in Fig. *b*.

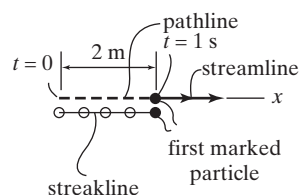


**3-2.** The flow of a liquid is originally along the positive  $x$  axis at 2 m/s for 3 s. If it then suddenly changes to 4 m/s along the positive  $y$  axis for  $t > 3$  s, draw the pathline and streamline for the first marked particle when  $t = 1$  s and  $t = 4$  s. Also, draw the streaklines at these two times.

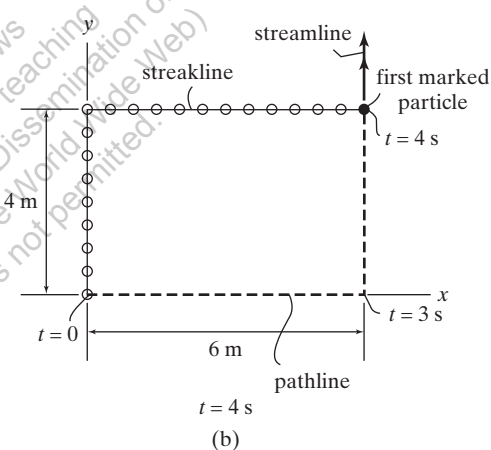
## SOLUTION

Since the streamlines have a constant direction along the positive  $x$  axis for the time interval  $0 \leq t < 3$  s, the pathline and streakline coincide with the streamline when  $t = 1$  s as shown in Fig. *a*.

The pathline and streakline will coincide with the streamline until  $t = 3$  s, after which the streamline makes a sudden change in direction. Thus, the streamline and pathline of the first marked particle and the streakline when  $t = 4$  s will be as shown in Fig. *b*.



(a)



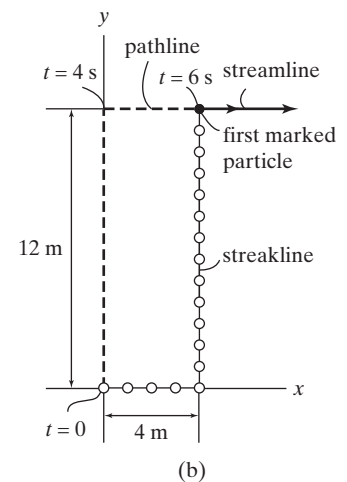
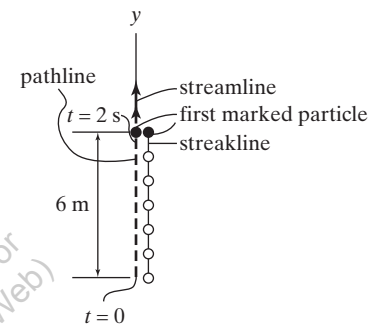
(b)

**3–3.** The flow of a liquid is originally along the positive  $y$  axis at 3 m/s for 4 s. If it then suddenly changes to 2 m/s along the positive  $x$  axis for  $t > 4$  s, draw the pathline and streamline for the first marked particle when  $t = 2$  s and  $t = 6$  s. Also, draw the streakline at these two times.

## SOLUTION

Since the streamlines have a constant direction along the positive  $y$  axis,  $0 \leq t < 4$  s, the pathline and streakline coincide with the streamline when  $t = 2$  s as shown in Fig. *a*.

The pathline and streakline will coincide with the streamline until  $t = 4$  s, when the streamline makes a sudden change in direction. The pathline, streamline, and streakline are shown in Fig. *b*.



**\*3–4.** A two-dimensional flow field for a fluid can be described by  $\mathbf{V} = \{(2x + 1)\mathbf{i} - (y + 3x)\mathbf{j}\}$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity of a particle located at (2 m, 3 m), and its direction measured counterclockwise from the  $x$  axis.

## SOLUTION

The velocity vector for a particle at  $x = 2$  m and  $y = 3$  m is

$$\begin{aligned}\mathbf{V} &= \{(2x + 1)\mathbf{i} - (y + 3x)\mathbf{j}\} \text{ m/s} \\ &= [2(2) + 1]\mathbf{i} - [3 + 3(2)]\mathbf{j} \\ &= \{5\mathbf{i} - 9\mathbf{j}\} \text{ m/s}\end{aligned}$$

The magnitude of  $\mathbf{V}$  is

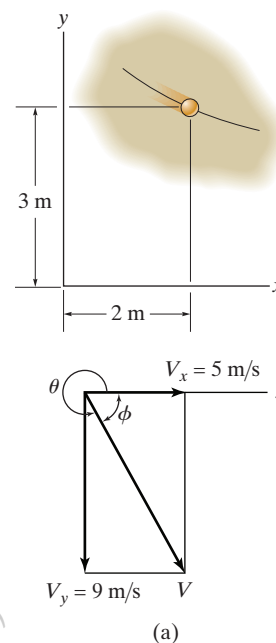
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(5 \text{ m/s})^2 + (-9 \text{ m/s})^2} = 10.3 \text{ m/s} \quad \text{Ans.}$$

As indicated in Fig. *a*, the direction of  $\mathbf{V}$  is defined by  $\theta = 360^\circ - \phi$ , where

$$\phi = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{9 \text{ m/s}}{5 \text{ m/s}}\right) = 60.95^\circ$$

Thus,

$$\theta = 360^\circ - 60.95^\circ = 299^\circ \quad \text{Ans.}$$



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**3–5.** A two-dimensional flow field for a liquid can be described by  $\mathbf{V} = \{(5y^2 - x)\mathbf{i} + (3x + y)\mathbf{j}\}$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity of a particle located at (5 m, -2 m), and its direction measured counterclockwise from the  $x$  axis.

## SOLUTION

The velocity vector of a particle at  $x = 5$  m and  $y = -2$  m is

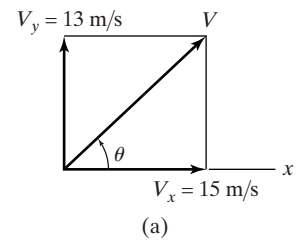
$$\begin{aligned}\mathbf{V} &= \{(5y^2 - x)\mathbf{i} + (3x + y)\mathbf{j}\} \text{ m/s} \\ &= [5(-2)^2 - 5]\mathbf{i} + [3(5) + (-2)]\mathbf{j} \\ &= \{15\mathbf{i} + 13\mathbf{j}\} \text{ m/s}\end{aligned}$$

The magnitude of  $\mathbf{V}$  is

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(15 \text{ m/s})^2 + (13 \text{ m/s})^2} = 19.8 \text{ m/s} \quad \textbf{Ans.}$$

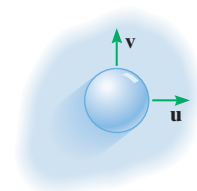
As indicated in Fig. *a*, the direction of  $\mathbf{V}$  is defined by

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{13 \text{ m/s}}{15 \text{ m/s}}\right) = 40.9^\circ \quad \textbf{Ans.}$$



**Ans:**  
 $V = 19.8 \text{ m/s}$   
 $\theta = 40.9^\circ$

**3–6.** The soap bubble is released in the air and rises with a velocity of  $\mathbf{V} = \{(0.8x)\mathbf{i} + (0.06t^2)\mathbf{j}\}$  m/s, where  $x$  is meters and  $t$  is in seconds. Determine the magnitude of the bubble's velocity, and its direction measured counterclockwise from the  $x$  axis, when  $t = 5$  s, at which time  $x = 2$  m and  $y = 3$  m. Draw its streamline at this instant.



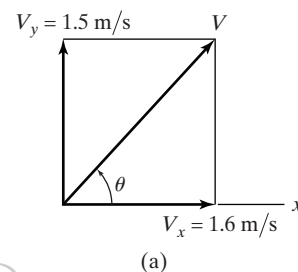
## SOLUTION

The velocity vector of a particle at  $x = 2$  m and the corresponding time  $t = 5$  s is

$$\begin{aligned}\mathbf{V} &= \{(0.8x)\mathbf{i} + (0.06t^2)\mathbf{j}\} \text{ m/s} \\ &= [0.8(2)\mathbf{i} + 0.06(5)^2\mathbf{j}] \\ &= \{1.6\mathbf{i} + 1.5\mathbf{j}\} \text{ m/s}\end{aligned}$$

The magnitude of  $\mathbf{V}$  is

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(1.6 \text{ m/s})^2 + (1.5 \text{ m/s})^2} = 2.19 \text{ m/s} \quad \text{Ans.}$$



As indicated in Fig. *a*, the direction of  $\mathbf{V}$  is defined by

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{1.5 \text{ m/s}}{1.6 \text{ m/s}}\right) = 43.2^\circ$$

Using the definition of the slope of the streamline and initial condition at  $x = 2$  m,  $y = 3$  m.

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{0.06t^2}{0.8x}$$

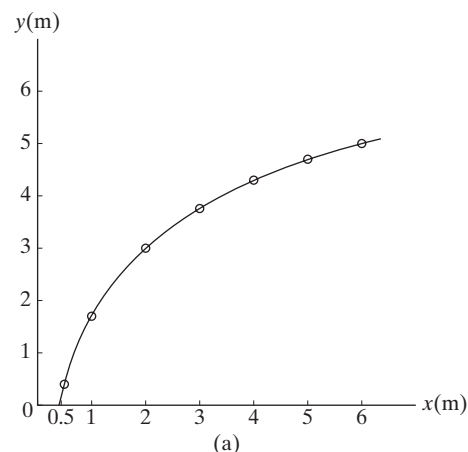
Note that since we are finding the streamline, which represents a single instant in time,  $t = 5$  s,  $t$  is a constant.

$$\begin{aligned}\int_{3 \text{ m}}^y \frac{dy}{t^2} &= \int_{2 \text{ m}}^x \frac{0.075 dx}{x} \\ \frac{1}{t^2}(y - 3) &= 0.075 \ln \frac{x}{2} \\ y &= \left(0.075t^2 \ln \frac{x}{2} + 3\right) \text{ m}\end{aligned}$$

When  $t = 5$  s,

$$\begin{aligned}y &= 0.075(5^2) \ln \left(\frac{x}{2}\right) + 3 \\ y &= \left[1.875 \ln \left(\frac{x}{2}\right) + 3\right] \text{ m}\end{aligned}$$

$x(\text{m})$	0.5	1	2	3	4	5	6
$y(\text{m})$	0.401	1.700	3	3.760	4.300	4.718	5.060



The plot of the streamline is shown in Fig. *a*

**Ans:**  
 $V = 2.19 \text{ m/s}$   
 $\theta = 43.2^\circ$

**3–7.** A flow field for a fluid is described by  $u = (2 + y)$  m/s and  $v = (2y)$  m/s, where  $y$  is in meters. Determine the equation of the streamline that passes through point (3 m, 2 m), and find the velocity of a particle located at this point. Draw this streamline.

## SOLUTION

As indicated in Fig *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \tan \theta \\ \frac{dy}{dx} &= \frac{v}{u} \\ \frac{dy}{dx} &= \frac{2y}{2 + y} \\ \int \frac{2 + y}{2y} dy &= \int dx \\ \ln y + \frac{1}{2}y &= x + C\end{aligned}$$

At point (3 m, 2 m), we obtain

$$\begin{aligned}\ln(2) + \frac{1}{2}(2) &= 3 + C \\ C &= -1.31\end{aligned}$$

Thus,

$$\begin{aligned}\ln y + \frac{1}{2}y &= x - 1.31 \\ \ln y^2 + y &= 2x - 2.61\end{aligned}$$

At point (3 m, 2 m)

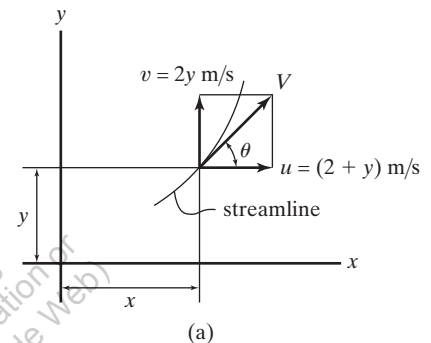
$$\begin{aligned}u &= (2 + 2) \text{ m/s} = 4 \text{ m/s} \rightarrow \\ v &= 2(2) = 4 \text{ m/s} \uparrow\end{aligned}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(4 \text{ m/s})^2 + (4 \text{ m/s})^2} = 5.66 \text{ m/s}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{4 \text{ m/s}}{4 \text{ m/s}}\right) = 45^\circ \nearrow$$



**Ans.**

**Ans.**

**Ans.**

**Ans:**

$$\ln y^2 + y = 2x - 2.61$$

$$V = 5.66 \text{ m/s}$$

$$\theta = 45^\circ \nearrow$$

**\*3-8.** A flow field is described by  $u = (x^2 + 5)$  m/s and  $v = (-6xy)$  m/s. Determine the equation of the streamline that passes through point (5 m, 1 m), and find the velocity of a particle located at this point. Draw this streamline.

## SOLUTION

As indicated in Fig *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \tan \theta \\ \frac{dy}{dx} &= \frac{v}{u} = \frac{-6xy}{x^2 + 5} \\ \int \frac{dy}{y} &= -6 \int \frac{x}{x^2 + 5} dx \\ \ln y &= -3 \ln(x^2 + 5) + C\end{aligned}$$

At  $x = 5$  m,  $y = 1$  m. Then,

$$\begin{aligned}\ln 1 &= -3 \ln[(5)^2 + 5] + C \\ C &= 3 \ln 30\end{aligned}$$

Thus

$$\begin{aligned}\ln y &= -3 \ln(x^2 + 5) + 3 \ln 30 \\ \ln y + \ln(x^2 + 5)^3 &= 3 \ln 30 \\ \ln[y(x^2 + 5)^3] &= \ln 30^3 \\ y(x^2 + 5)^3 &= 30^3 \\ y &= \frac{27(10^3)}{(x^2 + 5)^3}\end{aligned}$$

At point (5 m, 1m),

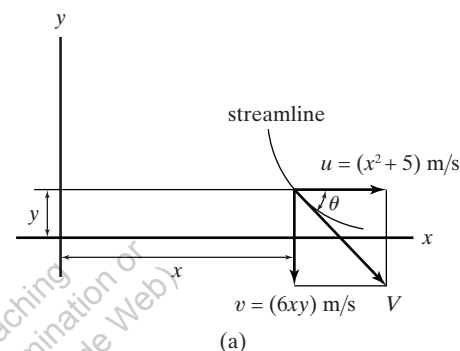
$$\begin{aligned}u &= (5^2 + 5) \text{ m/s} = 30 \text{ m/s} \rightarrow \\ v &= -6(5)(1) = -30 \text{ m/s} = 30 \text{ m/s} \downarrow\end{aligned}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(30 \text{ m/s})^2 + (30 \text{ m/s})^2} = 42.4 \text{ m/s} \quad \text{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{30 \text{ m/s}}{30 \text{ m/s}}\right) = 45^\circ \quad \text{Ans.}$$





**3-9.** Particles travel within a flow field defined by  $\mathbf{V} = \{2y^2\mathbf{i} + 4\mathbf{j}\}$  m/s, where  $x$  and  $y$  are in meters. Determine the equation of the streamline passing through point (1 m, 2 m), and find the velocity of a particle located at this point. Draw this streamline.

## SOLUTION

As indicated in Fig. *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \tan \theta \\ \frac{dy}{dx} &= \frac{v}{u} = \frac{4}{2y^2} \\ \int y^2 dy &= \int 2 dx\end{aligned}$$

$$\frac{1}{3}y^3 = 2x + C$$

At  $x = 1$  m,  $y = 2$  m. Then

$$\frac{1}{3}(2)^3 = 2(1) + C$$

$$C = \frac{2}{3}$$

Thus,

$$\begin{aligned}\frac{1}{3}y^3 &= 2x + \frac{2}{3} \\ y^3 &= 6x + 2\end{aligned}$$

At point (1 m, 2 m)

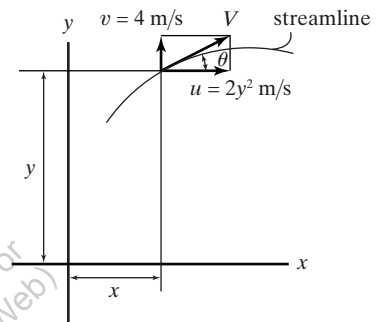
$$\begin{aligned}u &= 2(2^2) = 8 \text{ m/s} \rightarrow \\ v &= 4 \text{ m/s} \uparrow\end{aligned}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(8 \text{ m/s})^2 + (4 \text{ m/s})^2} = 8.94 \text{ m/s}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{4}{8}\right) = 26.6^\circ \nearrow$$



(a)

**Ans.**

**Ans.**

**Ans.**

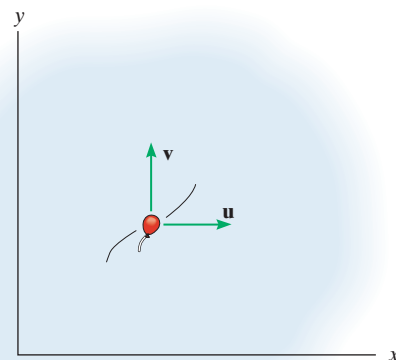
**Ans:**

$$y^3 = 6x + 2$$

$$V = 8.94 \text{ m/s}$$

$$\theta = 26.6^\circ \nearrow$$

**3–10.** A balloon is released into the air from the origin and carried along by the wind, which blows at a constant rate of  $u = 0.5$  m/s. Also, buoyancy and thermal winds cause the balloon to rise at a rate of  $v = (0.8 + 0.6y)$  m/s. Determine the equation of the streamline for the balloon, and draw this streamline.



## SOLUTION

As indicated in Fig. *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan \theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{0.8 + 0.6y}{0.5} = 1.6 + 1.2y$$

Since the balloon starts at  $y = 0, x = 0$ , using these values,

$$\int_0^y \frac{dy}{1.6 + 1.2y} = \int_0^x dx$$

$$\frac{1}{1.2} \ln(1.6 + 1.2y) \Big|_0^y = x$$

$$\ln\left(\frac{1.6 + 1.2y}{1.6}\right) = 1.2x$$

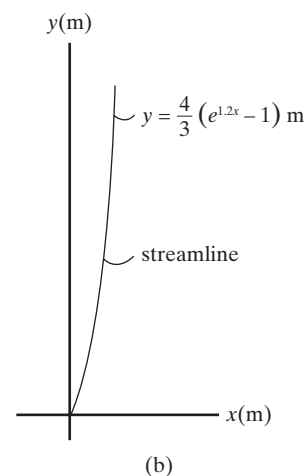
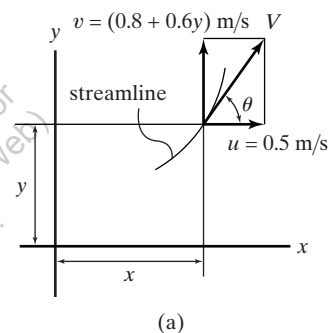
$$\ln\left(1 + \frac{3}{4}y\right) = 1.2x$$

$$1 + \frac{3}{4}y = e^{1.2x}$$

$$y = \frac{4}{3}(e^{1.2x} - 1) \text{ m}$$

**Ans.**

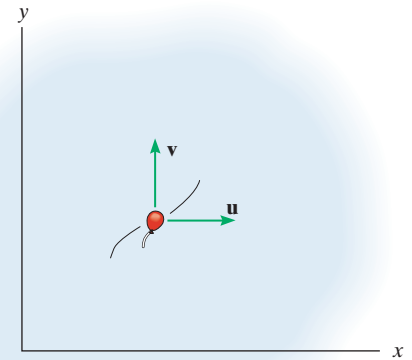
Using this result, the streamline is shown in Fig. *b*.



**Ans:**

$$y = \frac{4}{3}(e^{1.2x} - 1)$$

**3–11.** A balloon is released into the air from point (1 m, 0) and carried along by the wind, which blows at a rate of  $u = (0.8x)$  m/s, where  $x$  is in meters. Also, buoyancy and thermal winds cause the balloon to rise at a rate of  $v = (1.6 + 0.4y)$  m/s, where  $y$  is in meters. Determine the equation of the streamline for the balloon, and draw this streamline.



## SOLUTION

As indicated in Fig. *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan \theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{1.6 + 0.4y}{0.8x}$$

The balloon starts at point (1 m, 0).

$$\int_0^y \frac{dy}{1.6 + 0.4y} = \int_1^x \frac{dx}{0.8x}$$

$$\frac{1}{0.4} \ln(1.6 + 0.4y) \Big|_0^y = \frac{1}{0.8} \ln x \Big|_1^x$$

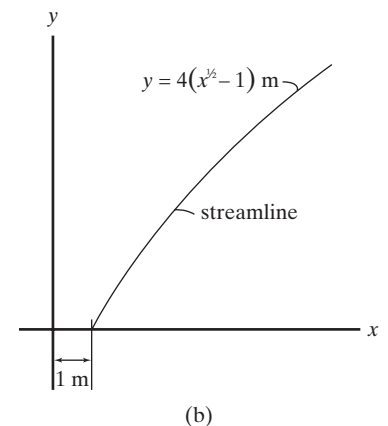
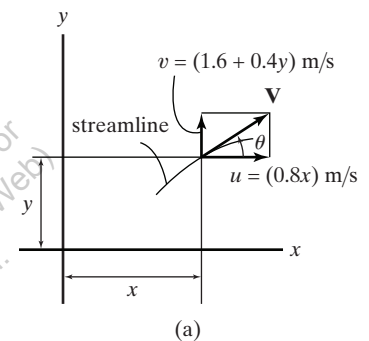
$$\frac{1}{0.4} \ln\left(\frac{1.6 + 0.4y}{1.6}\right) = \frac{1}{0.8} \ln x$$

$$\ln\left(1 + \frac{1}{4}y\right)^2 = \ln x$$

$$\left(1 + \frac{1}{4}y\right)^2 = x$$

$$y = 4(x^{1/2} - 1) \text{ m}$$

Using this result, the streamline is shown in Fig. *b*.



**Ans.**

**Ans:**  
 $y = 4(x^{1/2} - 1)$

**\*3–12.** A flow field is defined by  $u = (8y)$  m/s and  $v = (6x)$  m/s, where  $x$  and  $y$  are in meters. Determine the equation of the streamline that passes through point (1 m, 2 m). Draw this streamline.

## SOLUTION

As indicated in Fig. *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \tan \theta \\ \frac{dy}{dx} &= \frac{v}{u} = \frac{6x}{8y} \\ \int 8y \, dy &= \int 6x \, dx \\ 4y^2 &= 3x^2 + C\end{aligned}$$

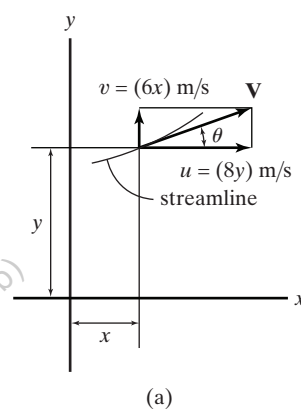
At  $x = 1$  m,  $y = 2$  m. Then

$$\begin{aligned}4(2)^2 &= 3(1)^2 + C \\ C &= 13\end{aligned}$$

Thus

$$4y^2 = 3x^2 + 13$$

**Ans.**



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**3–13.** A flow field is defined by  $u = (3x)$  ft/s and  $v = (6y)$  ft/s, where  $x$  and  $y$  are in feet. Determine the equation of the streamline passing through point (3 ft, 1 ft). Draw this streamline.

## SOLUTION

As indicated in Fig. *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \tan \theta \\ \frac{dy}{dx} &= \frac{v}{u} = \frac{6y}{3x} \\ \int \frac{dy}{2y} &= \int \frac{dx}{x} \\ \frac{1}{2} \ln y &= \ln x + C\end{aligned}$$

At  $x = 3$  ft,  $y = 1$  ft. Then

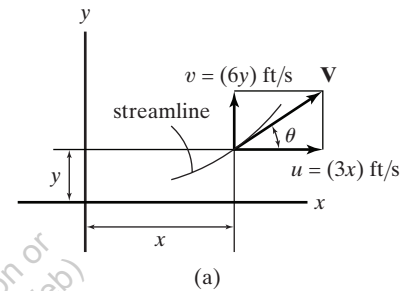
$$\frac{1}{2} \ln y = \ln x - \ln 3$$

$$\frac{1}{2} \ln y = \ln \frac{x}{3}$$

$$\ln y = \ln \left( \frac{x}{3} \right)^2$$

$$y = \frac{x^2}{9}$$

**Ans.**



**Ans:**  
 $y = x^2/9$

**3–14.** A flow of water is defined by  $u = 5 \text{ m/s}$  and  $v = 8 \text{ m/s}$ . If metal flakes are released into the flow at the origin  $(0, 0)$ , draw the streamline and pathline for these particles.

## SOLUTION

Since the velocity  $\mathbf{V}$  is constant, Fig. *a*, the streamline will be a straight line with a slope.

$$\begin{aligned}\frac{dy}{dx} &= \tan \theta \\ \frac{dy}{dx} &= \frac{v}{u} = \frac{8}{5} \\ y &= 1.6x + C\end{aligned}$$

At  $x = 0, y = 0$ . Then

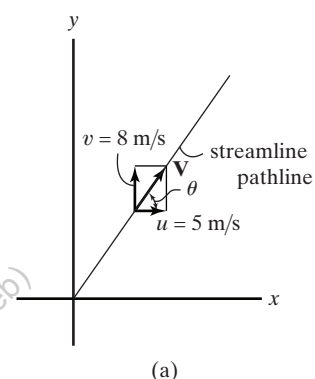
$$C = 0$$

Thus

$$y = 1.6x$$

**Ans.**

Since the direction of velocity  $\mathbf{V}$  remains constant so does the streamline, and the flow is steady. Therefore, the pathline coincides with the streamline and shares the same equation.



**Ans:**  
 $y = 1.6x$

**3–15.** A flow field is defined by  $u = [8x/(x^2 + y^2)]$  m/s and  $v = [8y/(x^2 + y^2)]$  m/s, where  $x$  and  $y$  are in meters. Determine the equation of the streamline passing through point (1 m, 1 m). Draw this streamline.

## SOLUTION

As indicated in Fig. *a*, the velocity  $\mathbf{V}$  of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan \theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{8y/(x^2 + y^2)}{8x/(x^2 + y^2)} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln \frac{y}{x} = C$$

$$\frac{y}{x} = C'$$

At  $x = 1$  m,  $y = 1$  m. Then

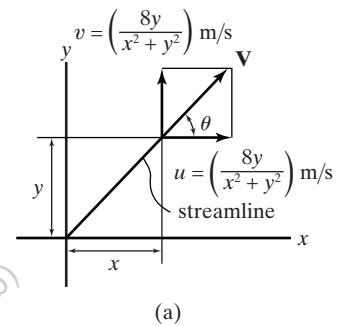
$$C' = 1$$

Thus,

$$\frac{y}{x} = 1$$

$$y = x$$

**Ans.**



**Ans:**  
 $y = x$

**\*3-16.** A fluid has velocity components of  $u = [30/(2x + 1)]$  m/s and  $v = (2ty)$  m/s, where  $x$  and  $y$  are in meters and  $t$  is in seconds. Determine the pathline that passes through point (2 m, 6 m) at time  $t = 2$  s. Plot this pathline for  $0 \leq x \leq 4$  m.

## SOLUTION

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform flow. Because we are finding a pathline,  $t$  is not a constant but a variable. We must first find equations relating  $x$  to  $t$  and  $y$  to  $t$ , and then eliminate  $t$ . Using the definition of velocity

$$\frac{dx}{dt} = u = \frac{30}{2x + 1}; \quad \int_{2 \text{ m}}^x (2x + 1) dx = 30 \int_{2 \text{ s}}^t dt$$

$$(x^2 + x) \Big|_{2 \text{ m}}^x = 30 t \Big|_{2 \text{ s}}^t$$

$$x^2 + x - 6 = 30(t - 2)$$

$$t = \frac{1}{30}(x^2 + x + 54) \quad (1)$$

$$\frac{dy}{dt} = v = 2ty; \quad \int_{6 \text{ m}}^y \frac{dy}{y} = 2 \int_{2 \text{ s}}^t t dt$$

$$\ln y \Big|_{6 \text{ m}}^y = t^2 \Big|_{2 \text{ s}}^t$$

$$\ln \frac{y}{6} = t^2 - 4$$

$$\frac{y}{6} = e^{t^2 - 4}$$

$$y = 6e^{t^2 - 4}$$

Substitute Eq. (1) into Eq. (2),

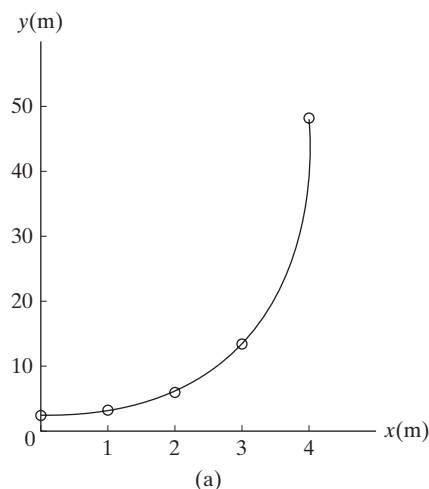
$$y = 6e^{\frac{1}{900}(x^2 + x + 54)^2 - 4}$$

The plot of the pathline is shown in Fig. a.

$x(\text{m})$	0	1	2	3	4
$y(\text{m})$	2.81	3.58	6.00	13.90	48.24

(2)

Ans.





**3-17.** A fluid has velocity components of  $u = [30/(2x + 1)]$  m/s and  $v = (2ty)$  m/s where  $x$  and  $y$  are in meters and  $t$  is in seconds. Determine the streamlines that passes through point (1 m, 4 m) at times  $t = 1$  s,  $t = 2$  s, and  $t = 3$  s. Plot each of these streamlines for  $0 \leq x \leq 4$  m.

## SOLUTION

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}, \quad \frac{dy}{dx} = \frac{2ty}{30/(2x + 1)} = \frac{1}{15}ty(2x + 1)$$

$$\int_{4 \text{ m}}^y \frac{dy}{y} = \frac{1}{15}t \int_{1 \text{ m}}^x (2x + 1)dx$$

$$\ln y \Big|_{4 \text{ m}}^y = \frac{1}{15}t(x^2 + x) \Big|_{1 \text{ m}}^x$$

$$\ln \frac{y}{4} = \frac{1}{15}t(x^2 + x - 2)$$

$$y = 4e^{t(x^2+x-2)/15}$$

For  $t = 1$  s,

$$y = 4e^{(x^2+x-2)/15}$$

**Ans.**

For  $t = 2$  s,

$$y = 4e^{2(x^2+x-2)/15}$$

**Ans.**

For  $t = 3$  s,

$$y = 4e^{3(x^2+x-2)/15}$$

**Ans.**

The plot of these streamlines are shown in Fig. a

For  $t = 1$  s

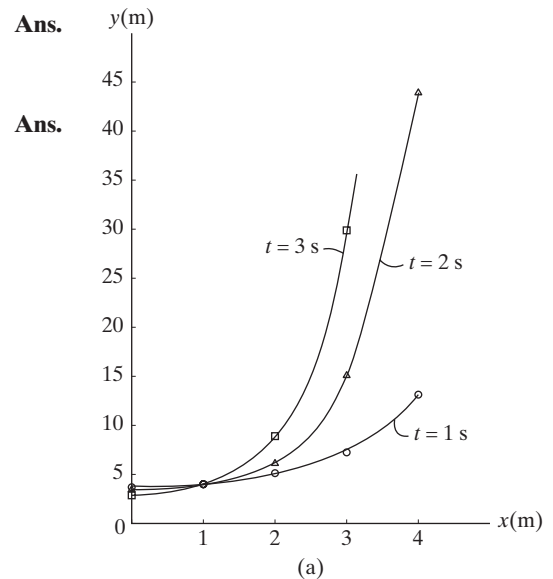
$x(\text{m})$	0	1	2	3	4
$y(\text{m})$	3.50	4	5.22	7.79	13.3

For  $t = 2$  s

$x(\text{m})$	0	1	2	3	4
$y(\text{m})$	3.06	4	6.82	15.2	44.1

For  $t = 3$  s

$x(\text{m})$	0	1	2	3	4
$y(\text{m})$	2.68	4	8.90	29.6	146



**Ans:**

$$\text{For } t = 1 \text{ s, } y = 4e^{(x^2+x-2)/15}$$

$$\text{For } t = 2 \text{ s, } y = 4e^{2(x^2+x-2)/15}$$

$$\text{For } t = 3 \text{ s, } y = 4e^{3(x^2+x-2)/15}$$

**3-18.** A fluid has velocity components of  $u = [30/(2x + 1)]$  m/s and  $v = (2ty)$  m/s, where  $x$  and  $y$  are in meters and  $t$  is in seconds. Determine the streamlines that pass through point (2 m, 6 m) at times  $t = 2$  s and  $t = 5$  s. Plot these streamlines for  $0 \leq x \leq 4$  m.

## SOLUTION

For  $t = 2$  s

$x(\text{m})$	0	1	2	3	4
$y(\text{m})$	2.70	3.52	6.00	13.35	38.80

For  $t = 5$  s

$x(\text{m})$	0	1	2	3	4
$y(\text{m})$	0.812	1.58	6.00	44.33	638.06

Since the velocity components are a function of time and position the flow can be classified as unsteady nonuniform. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{2ty}{30/(2x + 1)} = \frac{1}{15}ty(2x + 1)$$

Note that since we are finding the streamline, which represents a single instant in time, either  $t = 2$  s or  $t = 5$  s,  $t$  is a constant.

$$\int_{6 \text{ m}}^y \frac{dy}{y} = \frac{1}{15}t \int_{2 \text{ m}}^x (2x + 1)dx$$

$$\ln y \Big|_{6 \text{ m}}^y = \frac{1}{15}t(x^2 + x) \Big|_{2 \text{ m}}^x$$

$$\ln \frac{y}{6} = \frac{1}{15}t(x^2 + x - 6)$$

$$y = 6e^{\frac{1}{15}t(x^2 + x - 6)}$$

For  $t = 2$  s,

$$y = 6e^{\frac{2}{15}(x^2 + x - 6)}$$

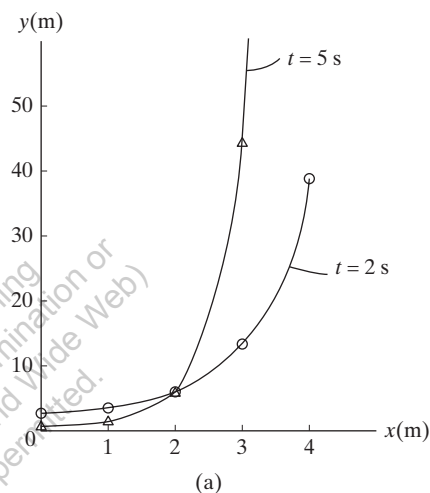
**Ans.**

For  $t = 5$  s,

$$y = 6e^{\frac{1}{3}(x^2 + x - 6)}$$

**Ans.**

The plots of these two streamlines are show in Fig. *a*.



**Ans:**

$$\text{For } t = 2 \text{ s, } y = 6e^{2(x^2 + x - 6)/15}$$

$$\text{For } t = 5 \text{ s, } y = 6e^{(x^2 + x - 6)/3}$$

**3–19.** A particle travels along a streamline defined by  $y^3 = 8x - 12$ . If its speed is 5 m/s when it is at  $x = 1$  m, determine the two components of its velocity at this point. Sketch the velocity on the streamline.

## SOLUTION

$x(\text{m})$	0	1	1.5	2	3	4	5
$y(\text{m})$	-2.29	-1.59	0	1.59	2.29	2.71	3.04

The plot of the streamline is shown in Fig. *a*. Taking the derivative of the streamline equation,

$$3y^2 \frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \tan \theta = \frac{8}{3y^2}$$

When  $x = 1$  m,

$$y^3 = 8(1) - 12; \quad y = -1.5874$$

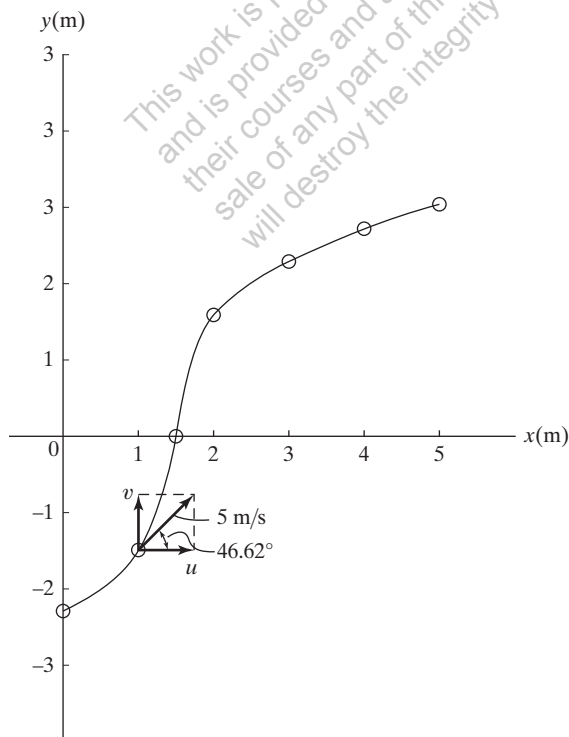
Then

$$\left. \frac{dy}{dx} \right|_{x=1 \text{ m}} = \tan \theta \Big|_{x=1 \text{ m}} = \frac{8}{3(-1.5874)^2}; \quad \theta \Big|_{x=1 \text{ m}} = 46.62^\circ$$

Therefore, the horizontal and vertical components of the velocity are

$$u = (5 \text{ m/s}) \cos 46.62^\circ = 3.43 \text{ m/s} \quad \text{Ans.}$$

$$v = (5 \text{ m/s}) \sin 46.62^\circ = 3.63 \text{ m/s} \quad \text{Ans.}$$



(a)

$$\begin{aligned} \text{Ans:} \\ u &= 3.43 \text{ m/s} \\ v &= 3.63 \text{ m/s} \end{aligned}$$

**\*3–20.** A flow field is defined by  $u = (0.8t)$  m/s and  $v = 0.4$  m/s, where  $t$  is in seconds. Plot the pathline for a particle that passes through the origin when  $t = 0$ . Also, draw the streamline for the particle when  $t = 4$  s.

## SOLUTION

Here,  $u = \frac{dx}{dt}$ . Then,

$$dx = u dt$$

Using  $x = 0$  when  $t = 0$  as the integration limit,

$$\int_0^x dx = \int_0^t [(0.8t) \text{ m/s}] dt$$

$$x = 0.4t^2 \quad (1)$$

Also,  $v = \frac{dy}{dt}$ . Then

$$dy = v dt$$

Using  $y = 0$  when  $t = 0$  as the integration limit,

$$\int_0^y dy = \int_0^t (0.4 \text{ m/s}) dt$$

$$y = 0.4t \quad (2)$$

Eliminating  $t$  from Eqs. (1) and (2)

$$y^2 = 0.4x$$

This equation represents the pathline of the particle. The  $x$  and  $y$  values of the pathline for the first five seconds are tabulated below.

$t$	$x$	$y$
1	0.4	0.4
2	1.6	0.8
3	3.6	1.2
4	6.4	1.6
5	10	2

A plot of the pathline is shown in Fig. *a*.

From Eqs. (1) and (2), when  $t = 4$  s,

$$x = 0.4(4^2) = 6.4 \text{ m} \quad y = 0.4(4) = 1.6 \text{ m}$$

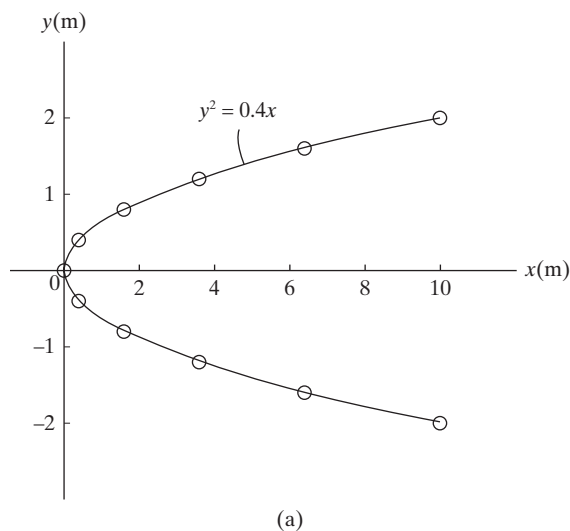
Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{0.4}{0.8t}$$

$$t \int_{1.6 \text{ m}}^y dy = \frac{1}{2} \int_{6.4 \text{ m}}^x dx$$

$$t(y - 1.6) = \frac{1}{2}(x - 6.4)$$

$$y = \left[ \frac{1}{2t}(x - 6.4) + 1.6 \right] \text{ m}$$



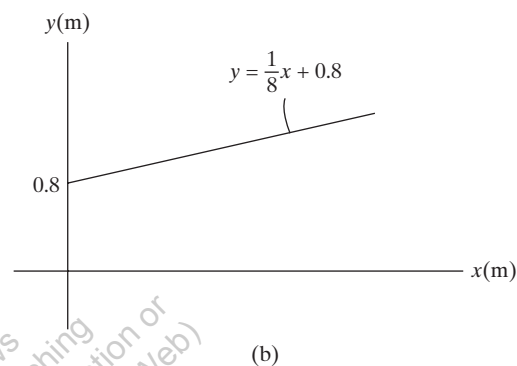
**\*3–20. (continued)**

When  $t = 4$  s,

$$y = \frac{1}{2(4)}(x - 6.4) + 1.6$$

$$y = \frac{1}{8}x + 0.8$$

The plot of the streamline is shown in Fig. *b*.



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**3–21.** The velocity for an oil flow is defined by  $\mathbf{V} = \{3y^2\mathbf{i} + 8\mathbf{j}\}$  m/s, where  $y$  is in meters. What is the equation of the streamline that passes through point (2 m, 1 m)? If a particle is at this point when  $t = 0$ , at what point is it located when  $t = 1$  s?

## SOLUTION

Since the velocity components are a function of position only, the flow can be classified as steady nonuniform. Here,  $u = (3y^2)$  m/s and  $v = 8$  m/s. The slope of the streamline is defined by

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{8}{3y^2}$$

$$\int_{1 \text{ m}}^y 3y^2 dy = 8 \int_{2 \text{ m}}^x dx$$

$$y^3 \Big|_{1 \text{ m}}^y = 8x \Big|_{2 \text{ m}}^x$$

$$y^3 - 1 = 8x - 16$$

$$y^3 = 8x - 15 \quad (1)$$

**Ans.**

From the definition of velocity

$$\frac{dy}{dt} = 8$$

$$\int_{1 \text{ m}}^y dy = \int_0^{1 \text{ s}} 8 dt$$

$$y \Big|_{1 \text{ m}}^y = 8t \Big|_0^{1 \text{ s}}$$

$$y - 1 = 8$$

$$y = 9 \text{ m}$$

**Ans.**

Substituting this result into Eq. (1)

$$9^3 = 8x - 15$$

$$x = 93 \text{ m}$$

**Ans.**

**Ans:**

$$y^3 = 8x - 15, y = 9 \text{ m}$$

$$x = 93 \text{ m}$$

**3–22.** The circulation of a fluid is defined by the velocity field  $u = (6 - 3x)$  m/s and  $v = 2$  m/s, where  $x$  is in meters. Plot the streamline that passes through the origin for  $0 \leq x < 2$  m.

### SOLUTION

$x(\text{m})$	0	0.25	0.5	0.75	1
$y(\text{m})$	0	0.089	0.192	0.313	0.462
$x(\text{m})$	1.25	1.5	1.75	2	
$y(\text{m})$	0.654	0.924	1.386	$\infty$	

Since the velocity component is a function of position only, the flow can be classified as steady nonuniform. Using the definition of the slope of a streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{2}{6 - 3x}$$

$$\int_0^y dy = 2 \int_0^x \frac{dx}{6 - 3x}$$

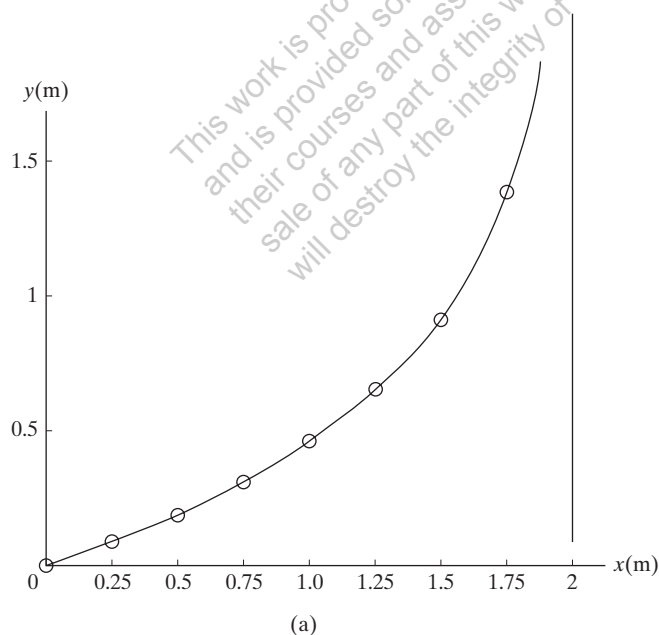
$$y = -\frac{2}{3} \ln(6 - 3x) \Big|_0^x$$

$$y = -\frac{2}{3} \ln\left(\frac{6 - 3x}{6}\right)$$

$$y = \frac{2}{3} \ln\left(\frac{2}{2 - x}\right)$$

**Ans.**

The plot of this streamline is shown in Fig. *a*



**Ans:**

$$y = \frac{2}{3} \ln\left(\frac{2}{2 - x}\right)$$

**3–23.** A stream of water has velocity components of  $u = -2$  m/s,  $v = 3$  m/s for  $0 \leq t < 10$  s; and  $u = 5$  m/s,  $v = -2$  m/s for  $10 \text{ s} < t \leq 15$  s. Plot the pathline and streamline for a particle released at point  $(0, 0)$  when  $t = 0$  s.

## SOLUTION

Using the definition of velocity, for  $0 \leq t < 10$  s

$$\begin{aligned}\frac{dx}{dt} &= u; & \frac{dx}{dt} &= -2 \\ \int_0^x dx &= -2 \int_0^t dt \\ x &= (-2t) \text{ m}\end{aligned}\quad (1)$$

When  $t = 10$  s,  $x = -2(10) = -20$  m

$$\begin{aligned}\frac{dy}{dt} &= v; & \frac{dy}{dt} &= 3 \\ \int_0^y dy &= 3 \int_0^t dt \\ y &= (3t) \text{ m}\end{aligned}\quad (2)$$

When  $t = 10$  s,  $y = 3(10) = 30$  m

The equation of the streamline can be determined by eliminating  $t$  from Eq. (1) and (2).

$$y = -\frac{3}{2}x \quad \text{Ans.}$$

For  $10 < t \leq 15$  s.

$$\begin{aligned}\frac{dx}{dt} &= u; & \frac{dx}{dt} &= 5 \\ \int_{-20 \text{ m}}^x dx &= 5 \int_{10 \text{ s}}^t dt \\ x - (-20) &= 5(t - 10) \\ x &= (5t - 70) \text{ m}\end{aligned}\quad (3)$$

At  $t = 15$  s,  $x = 5(15) - 70 = 5$  m

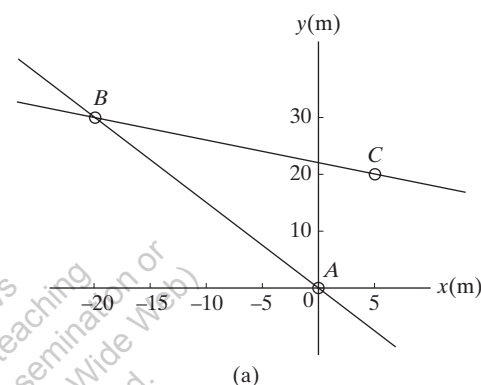
$$\begin{aligned}\frac{dy}{dt} &= v; & \frac{dy}{dt} &= -2 \\ \int_{30 \text{ m}}^y dy &= -2 \int_{10 \text{ s}}^t dt \\ y - 30 &= -2(t - 10) \\ y &= (-2t + 50) \text{ m}\end{aligned}\quad (4)$$

When  $t = 15$  s,  $y = -2(15) + 50 = 20$  m

Eliminate  $t$  from Eqs. (3) and (4),

$$y = \left(-\frac{2}{5}x + 22\right) \quad \text{Ans.}$$

The two streamlines intersect at  $(-20, 30)$ , point  $B$  in Fig. (a). The pathline is the path  $ABC$ .





**\*3–24.** A velocity field is defined by  $u = (4x)$  m/s and  $v = (2t)$  m/s, where  $t$  is in seconds and  $x$  is in meters. Determine the equation of the streamline that passes through point (2 m, 6 m) for  $t = 1$  s. Plot this streamline for  $0.25 \text{ m} \leq x \leq 4 \text{ m}$ .

## SOLUTION

$x(\text{m})$	0.25	0.5	0.75	1	2	3	4
$y(\text{m})$	4.96	5.31	5.51	5.65	6	6.20	6.35

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform. Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}, \quad \frac{dy}{dx} = \frac{2t}{4x} = \frac{t}{2x}$$

$$\int_{6 \text{ m}}^y dy = \frac{t}{2} \int_{2 \text{ m}}^x \frac{dx}{x}$$

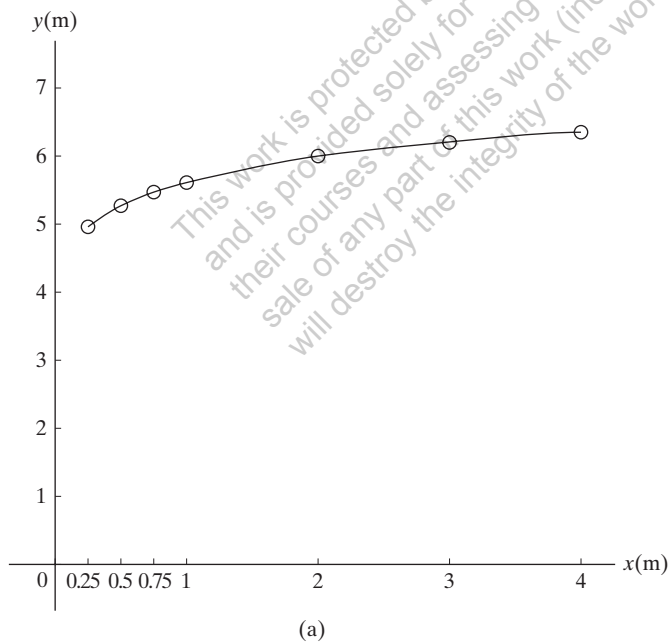
$$y - 6 = \frac{t}{2} \ln \frac{x}{2}$$

$$y = \frac{t}{2} \ln \frac{x}{2} + 6$$

For  $t = 1$  s,

$$y = \left( \frac{1}{2} \ln \frac{x}{2} + 6 \right) \text{ m} \quad \text{Ans.}$$

The plot of this streamline is shown in Fig. *a*.



**3–25.** The velocity field is defined by  $u = (4x) \text{ m/s}$  and  $v = (2t) \text{ m/s}$ , where  $t$  is in seconds and  $x$  is in meters. Determine the pathline that passes through point  $(2 \text{ m}, 6 \text{ m})$  when  $t = 1 \text{ s}$ . Plot this pathline for  $0.25 \text{ m} \leq x \leq 4 \text{ m}$ .

## SOLUTION

$x(\text{m})$	0.25	0.50	0.75	1	2	3	4
$y(\text{m})$	5.23	5.43	5.57	5.68	6	6.21	6.38

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform. Using the definition of velocity,

$$\frac{dx}{dt} = u = 4x; \quad \int_{2 \text{ m}}^x \frac{dx}{4x} = \int_{1 \text{ s}}^t dt$$

$$\frac{1}{4} \ln x \Big|_{2 \text{ m}}^x = t \Big|_{1 \text{ s}}^t$$

$$\frac{1}{4} \ln \frac{x}{2} = t - 1$$

$$t = \frac{1}{4} \ln \frac{x}{2} + 1 \quad (1)$$

$$\frac{dy}{dt} = v = 2t; \quad \int_{6 \text{ m}}^y dy = \int_{1 \text{ s}}^t 2t dt$$

$$y - 6 = t^2 \Big|_{1 \text{ s}}^t$$

$$y = t^2 + 5 \quad (2)$$

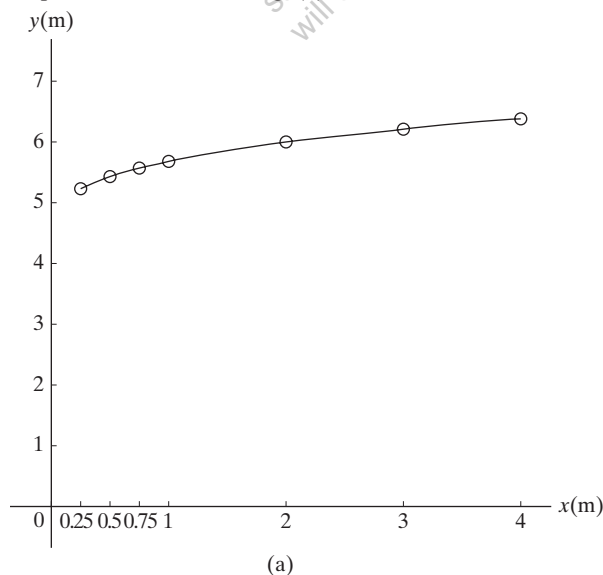
Substitute Eq. (1) into (2),

$$y = \left( \frac{1}{4} \ln \frac{x}{2} + 1 \right)^2 + 5$$

$$y = \left( \frac{1}{16} \ln^2 \frac{x}{2} + \frac{1}{2} \ln \frac{x}{2} + 6 \right)$$

**Ans.**

The plot of this pathline is shown in Fig. (a)



**Ans:**

$$y = \frac{1}{16} \ln^2 \frac{x}{2} + \frac{1}{2} \ln \frac{x}{2} + 6$$

**3–26.** The velocity field of a fluid is defined by  $u = (\frac{1}{2}x)$  m/s,  $v = (\frac{1}{8}y^2)$  m/s for  $0 \leq t < 5$  s and by  $u = (-\frac{1}{4}x^2)$  m/s,  $v = (\frac{1}{4}y)$  m/s for  $5 \leq t \leq 10$  s, where  $x$  and  $y$  are in meters. Plot the streamline and pathline for a particle released at point (1 m, 1 m) when  $t = 0$  s.

## SOLUTION

Using the definition of velocity, for  $0 \leq t < 5$  s,

$$\frac{dx}{dt} = u; \quad \frac{dx}{dt} = \frac{1}{2}x$$

$$\int_{1 \text{ m}}^x \frac{dx}{x} = \int_0^t \frac{1}{2} dt$$

$$\ln x = \frac{1}{2}t$$

$$x = \left(e^{\frac{1}{2}t}\right) \text{ m} \quad (1)$$

When  $t = 5$  s,  $x = e^{\frac{1}{2}(5)} = 12.18$  m

$$\frac{dy}{dt} = v; \quad \frac{dy}{dt} = \frac{1}{8}y^2$$

$$\int_{1 \text{ m}}^y \frac{dy}{y^2} = \int_0^t \frac{1}{8} dt$$

$$-\left(\frac{1}{y}\right)\bigg|_{1 \text{ m}}^y = \frac{1}{8}t$$

$$t - \frac{1}{y} = \frac{1}{8}t$$

$$\frac{y-1}{y} = \frac{1}{8}t$$

$$y\left(1 - \frac{1}{8}t\right) = 1$$

$$y = \left(\frac{8}{8-t}\right) \text{ m} \quad t \neq 8 \text{ s} \quad (2)$$

When  $t = 5$  s,  $y = \frac{8}{8-5} = 2.667$  m

The equation of the streamline and pathline can be determined by eliminating  $t$  from Eqs. (1) and (2)

$$y = \left(\frac{8}{8-2 \ln x}\right) \text{ m}$$

$x(\text{m})$	1	3	5	7	9	11	12.18
$y(\text{m})$	1	1.38	1.67	1.95	2.22	2.50	2.67

For  $5 \leq t \leq 10$  s,

$$\frac{dx}{dt} = u; \quad \frac{dx}{dt} = -\frac{1}{4}x^2$$

$$\int_{12.18 \text{ m}}^x \frac{dx}{x^2} = -\frac{1}{4} \int_{5 \text{ s}}^t dt$$

3-26. (continued)

$$-\left(\frac{1}{x} - \frac{1}{12.18}\right) = -\frac{1}{4}(t - 5)$$

$$\frac{1}{x} = \frac{t}{4} - 1.1679$$

$$x = \left(\frac{4}{t - 4.6717}\right) \text{ m} \quad t \neq 4.6717 \text{ s} \quad (3)$$

When  $t = 10 \text{ s}$ ,  $x = \frac{4}{10 - 4.6717} = 0.751 \text{ m}$

$$\frac{dy}{dt} = v; \quad \frac{dy}{dt} = \frac{1}{4}y$$

$$\int_{2.667 \text{ m}}^y \frac{dy}{y} = \frac{1}{4} \int_{5 \text{ s}}^t dt$$

$$\ln \frac{y}{2.667} = \frac{1}{4}(t - 5)$$

$$\frac{y}{2.667} = e^{\frac{1}{4}(t-5)}$$

$$y = \left[2.667e^{\frac{1}{4}(t-5)}\right] \text{ m} \quad (4)$$

When  $t = 10 \text{ s}$ ,  $y = 2.667e^{\frac{1}{4}(10-5)} = 9.31 \text{ m}$

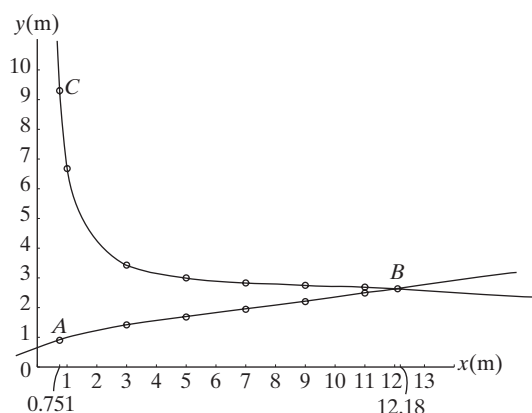
Eliminate  $t$  from Eqs. (3) and (4),

$$y = 2.667e^{\frac{1}{4}\left[4\left(\frac{1}{x} + 1.1679\right) - 5\right]}$$

$$= \left[2.667e^{\left(\frac{1}{x} - 0.08208\right)}\right] \text{ m}$$

$x(\text{m})$	0.751	1	3	5	7	9	11	12.18
$y(\text{m})$	9.31	6.68	3.43	3.00	2.83	2.75	2.69	2.67

The two streamlines intersect at (12.18, 2.67), point  $B$  in Fig. (a). The pathline is the path  $ABC$ .



**Ans:**

$$\text{For } 0 \leq t < 5 \text{ s, } y = \frac{8}{8 - 2 \ln x}$$

$$\text{For } 5 \text{ s} < t \leq 10 \text{ s, } y = 2.67e^{(1/x - 0.0821)}$$

**3–27.** A two-dimensional flow field for a liquid can be described by  $\mathbf{V} = \{(6y^2 - 1)\mathbf{i} + (3x + 2)\mathbf{j}\}$  m/s, where  $x$  and  $y$  are in meters. Find the streamline that passes through point (6 m, 2 m) and determine the velocity at this point. Sketch the velocity on the streamline.

## SOLUTION

We have steady flow since the velocity does not depend upon time.

$$u = 6y^2 - 1$$

$$v = 3x + 2$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{3x + 2}{6y^2 - 1}$$

$$\int_2^y (6y^2 - 1) dy = \int_6^x (3x + 2) dx$$

$$2y^3 - y \Big|_2^y = 1.5x^2 + 2x \Big|_6^x$$

$$2y^3 - y - [2(2)^3 - 2] = 1.5x^2 + 2x - [1.5(6)^2 + 2(6)]$$

$$2y^3 - 1.5x^2 - y - 2x + 52 = 0$$

At (6 m, 2 m)

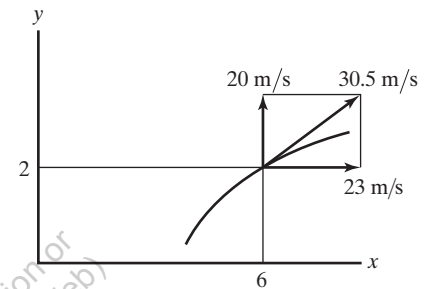
$$u = 6(2)^2 - 1 = 23 \text{ m/s} \rightarrow$$

$$v = 3(6) + 2 = 20 \text{ m/s} \uparrow$$

$$V = \sqrt{(23 \text{ m/s})^2 + (20 \text{ m/s})^2} = 30.5 \text{ m/s}$$

**Ans.**

**Ans.**



**Ans:**

$$2y^3 - 1.5x^2 - y - 2x + 52 = 0$$

$$V = 30.5 \text{ m/s}$$

**\*3–28.** A flow field for a liquid can be described by  $\mathbf{V} = \{(2x + 1)\mathbf{i} - y\mathbf{j}\}$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity of a particle located at points (3 m, 1 m). Sketch the velocity on the streamline.

## SOLUTION

We have steady flow since the velocity does not depend upon time.

$$u = 2x + 1$$

$$v = -y$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-y}{2x + 1}$$

$$-\int \frac{dy}{y} = \int \frac{dx}{(2x + 1)}$$

$$-\ln y = \frac{1}{2} \ln (2x + 1)$$

$$-y \Big|_1^y = (2x + 1)^{\frac{1}{2}} \Big|_3^x$$

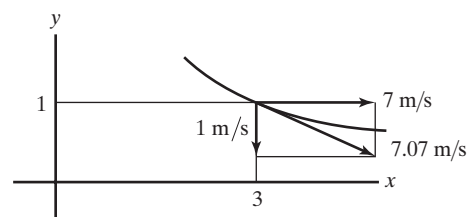
$$-y + 1 = (2x + 1)^{\frac{1}{2}} - [2(3) + 1]^{\frac{1}{2}}$$

$$y = 3.65 - (2x + 1)^{\frac{1}{2}} \quad \text{Ans.}$$

$$u = 2(3) + 1 = 7 \text{ m/s}$$

$$v = -1 \text{ m/s}$$

$$V = \sqrt{(7 \text{ m/s})^2 + (-1 \text{ m/s})^2} = 7.07 \text{ m/s} \quad \text{Ans.}$$



**3–29.** Air flows uniformly through the center of a horizontal duct with a velocity of  $V = (6t^2 + 5) \text{ m/s}$ , where  $t$  is in seconds. Determine the acceleration of the flow when  $t = 2 \text{ s}$ .

### SOLUTION

Since the flow is along the horizontal ( $\bar{x}$  axis)  $v = w = 0$ . Also, the velocity is a function of time  $t$  only. Therefore, the convective acceleration is zero, so that

$$u \frac{\partial V}{\partial x} = 0.$$

$$\begin{aligned} a &= \frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} \\ &= 12t + 0 \\ &= (12t) \text{ m/s}^2 \end{aligned}$$

When  $t = 2 \text{ s}$ ,

$$a = 12(2) = 24 \text{ m/s}^2$$

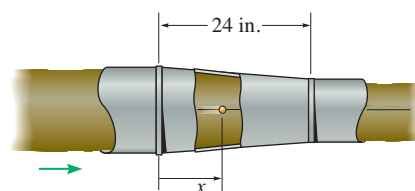
**Ans.**

*Note:* The flow is unsteady since its velocity is a function of time.

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**Ans:**  
 $a = 24 \text{ m/s}^2$

**3–30.** Oil flows through the reducer such that particles along its centerline have a velocity of  $V = (4xt)$  in./s, where  $x$  is in inches and  $t$  is in seconds. Determine the acceleration of the particles at  $x = 16$  in. when  $t = 2$  s.



## SOLUTION

Since the flow is along the  $x$  axis,  $v = w = 0$

$$\begin{aligned} a &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \\ &= 4x + (4xt)(4t) \\ &= 4x + 16xt^2 \\ &= [4x(1 + 4t^2)] \text{ in./s}^2 \end{aligned}$$

When  $t = 2$  s,  $x = 16$  in. Then

$$a = [4(16)[1 + 4(2^2)]] \text{ in./s}^2 = 1088 \text{ in./s}^2$$

*Note:* The flow is unsteady since its velocity is a function of time.

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**Ans:**  
1088 in./s<sup>2</sup>



**3–31.** A fluid has velocity components of  $u = (6y + t)$  ft/s and  $v = (2tx)$  ft/s where  $x$  and  $y$  are in feet and  $t$  is in seconds. Determine the magnitude of the acceleration of a particle passing through the point (1 ft, 2 ft) when  $t = 1$  s.

## SOLUTION

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar component of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 1 + (6y + t)(0) + (2tx)(6) \\ &= (1 + 12tx) \text{ ft/s}^2 \end{aligned}$$

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 2x + (6y + t)(2t) + (2tx)(0) \\ &= (2x + 12ty + 2t^2) \text{ ft/s}^2 \end{aligned}$$

When  $t = 1$  s,  $x = 1$  ft and  $y = 2$  ft, then

$$\begin{aligned} a_x &= [1 + 12(1)(1)] = 13 \text{ ft/s}^2 \\ a_y &= [2(1) + 12(1)(2) + 2(1^2)] = 28 \text{ ft/s}^2 \end{aligned}$$

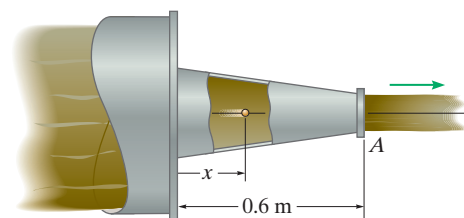
Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(13 \text{ ft/s}^2)^2 + (28 \text{ ft/s}^2)^2} = 30.9 \text{ ft/s}^2$$

**Ans.**

**Ans:**  
30.9 ft/s<sup>2</sup>

**\*3–32.** The velocity for the flow of a gas along the center streamline of the pipe is defined by  $u = (10x^2 + 200t + 6)$  m/s, where  $x$  is in meters and  $t$  is in seconds. Determine the acceleration of a particle when  $t = 0.01$  s and it is at  $A$ , just before leaving the nozzle.



## SOLUTION

$$a = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial t} = 200 \quad \frac{\partial u}{\partial x} = 20x$$

$$a = [200 + (10x^2 + 200t + 6)(20x)] \text{ m/s}^2$$

When  $t = 0.01$  s,  $x = 0.6$  m.

$$\begin{aligned} a &= \{200 + [10(0.6^2) + 200(0.01) + 6][20(0.6)]\} \text{ m/s}^2 \\ &= 339 \text{ m/s}^2 \end{aligned}$$

**Ans.**

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**3–33.** A fluid has velocity components of  $u = (2x^2 - 2y^2 + y)$  m/s and  $v = (y + xy)$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity and acceleration of a particle at point (2 m, 4 m).

## SOLUTION

### Velocity.

At  $x = 2$  m,  $y = 4$  m,

$$u = 2(2^2) - 2(4^2) + 4 = -20 \text{ m/s}$$

$$v = 4 + 2(4) = 12 \text{ m/s}$$

The magnitude of the particle's velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(-20 \text{ m/s})^2 + (12 \text{ m/s})^2} = 23.3 \text{ m/s}$$

**Ans.**

**Acceleration.** The  $x$  and  $y$  components of the particle's acceleration, with  $w = 0$  are

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + (2x^2 - 2y^2 + y)(4x) + (y + xy)(-4y + 1) \end{aligned}$$

At  $x = 2$  m,  $y = 4$  m,

$$a_x = -340 \text{ m/s}^2$$

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (2x^2 - 2y^2 + y)(y) + (y + xy)(1 + x) \end{aligned}$$

At  $x = 2$  m,  $y = 4$  m,

$$a_y = -44 \text{ m/s}^2$$

The magnitude of the particle's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-340 \text{ m/s}^2)^2 + (-44 \text{ m/s}^2)^2} = 343 \text{ m/s}^2$$

**Ans.**

**Ans:**

$$V = 23.3 \text{ m/s}$$

$$a = 343 \text{ m/s}^2$$

**3-34.** A fluid velocity components of  $u = (5y^2 - x)$  m/s and  $v = (4x^2)$  m/s, where  $x$  and  $y$  are in meters. Determine the velocity and acceleration of particles passing through point (2 m, 1 m).

## SOLUTION

Since the velocity components are a function of position only the flow can be classified as steady nonuniform. At point  $x = 2$  m and  $y = 1$  m,

$$u = 5(1^2) - 2 = 3 \text{ m/s}$$

$$v = 4(2^2) = 16 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(3 \text{ m/s})^2 + (16 \text{ m/s})^2} = 16.3 \text{ m/s}$$

**Ans.**

Its direction is

$$\theta_v = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{16 \text{ m/s}}{3 \text{ m/s}}\right) = 79.4^\circ \nearrow$$

**Ans.**

For two dimensional flow, the Eulerian description is

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axis

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + (5y^2 - x)(-1) + 4x^2(10y) \\ &= (x - 5y^2) + 40x^2y \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (5y^2 - x)(8x) + 4x^2(0) \\ &= 8x(5y^2 - x) \end{aligned}$$

At point  $x = 2$  m and  $y = 1$  m,

$$a_x = [2 - 5(1^2)] + 40(2^2)(1) = 157 \text{ m/s}^2$$

$$a_y = 8(2)[5(1^2) - 2] = 48 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(157 \text{ m/s}^2)^2 + (48 \text{ m/s}^2)^2} = 164 \text{ m/s}^2$$

**Ans.**

Its direction is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{48 \text{ m/s}^2}{157 \text{ m/s}^2}\right) = 17.0^\circ \nearrow$$

**Ans.**

**Ans:**

$$V = 16.3 \text{ m/s}$$

$$\theta_v = 79.4^\circ \nearrow$$

$$a = 164 \text{ m/s}^2$$

$$\theta_a = 17.0^\circ \nearrow$$

**3-35.** A fluid has velocity components of  $u = (5y^2)$  m/s and  $v = (4x - 1)$  m/s, where  $x$  and  $y$  are in meters. Determine the equation of the streamline passing through point (1 m, 1 m). Find the components of the acceleration of a particle located at this point and sketch the acceleration on the streamline.

## SOLUTION

Since the velocity components are independent of time but are a function of position, the flow can be classified as steady nonuniform. The slope of the streamline is

$$\begin{aligned}\frac{dy}{dx} &= \frac{v}{u}; \quad \frac{dy}{dx} = \frac{4x - 1}{5y^2} \\ \int_{1 \text{ m}}^y 5y^2 dy &= \int_{1 \text{ m}}^x (4x - 1) dx \\ y^3 &= \frac{1}{5}(6x^2 - 3x + 2) \text{ where } x \text{ is in } m\end{aligned}$$

For two dimensional flow, the Eulerian description is

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along  $x$  and  $y$  axes,

$$\begin{aligned}a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 5y^2(0) + (4x - 1)(10y) \\ &= 40xy - 10y \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + 5y^2(4) + (4x - 1)(0) \\ &= 20y^2\end{aligned}$$

At point  $x = 1 \text{ m}$  and  $y = 1 \text{ m}$ ,

$$a_x = 40(1)(1) - 10(1) = 30 \text{ m/s}^2$$

$$a_y = 20(1^2) = 20 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(30 \text{ m/s}^2)^2 + (20 \text{ m/s}^2)^2} = 36.1 \text{ m/s}^2$$

**Ans.**

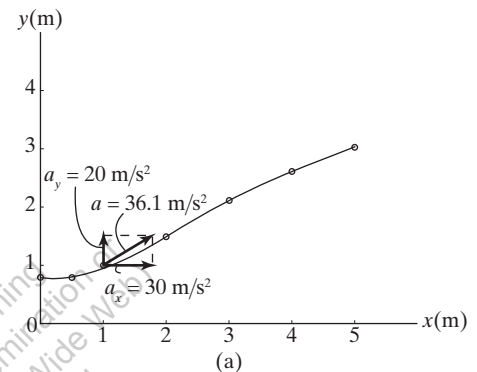
Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{20 \text{ m/s}^2}{30 \text{ m/s}^2}\right) = 33.7^\circ \nearrow$$

**Ans.**

The plot of the streamline and the acceleration on point (1 m, 1 m) is shown in Fig. *a*.

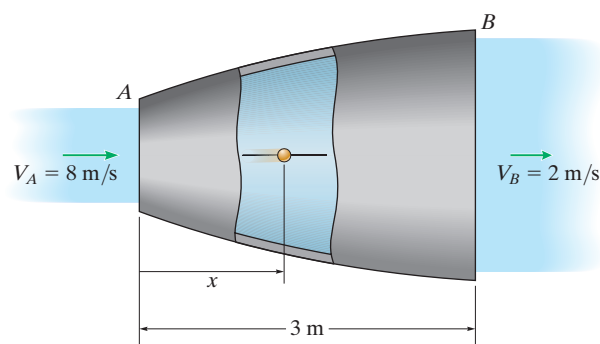
$x(\text{m})$	0	0.5	1	2	3	4	5
$y(\text{m})$	0.737	0.737	1	1.59	2.11	2.58	3.01



**Ans:**

$$\begin{aligned}a &= 36.1 \text{ m/s}^2 \\ \theta &= 33.7^\circ \nearrow\end{aligned}$$

**\*3–36.** Air flowing through the center of the duct has been found to decrease in speed from  $V_A = 8 \text{ m/s}$  to  $V_B = 2 \text{ m/s}$  in a linear manner. Determine the velocity and acceleration of a particle moving horizontally through the duct as a function of its position  $x$ . Also, find the position of the particle as a function of time if  $x = 0$  when  $t = 0$ .



## SOLUTION

Since the velocity is a function of position only, the flow can be classified as steady nonuniform. Since the velocity varies linearly with  $x$ ,

$$V = V_A + \left( \frac{V_B - V_A}{L_{AB}} \right)x = 8 + \left( \frac{2 - 8}{3} \right)x = (8 - 2x) \text{ m/s} \quad \text{Ans.}$$

For one dimensional flow, the Eulerian description gives

$$\begin{aligned} \mathbf{a} &= \frac{\partial \mathbf{V}}{\partial t} + V \frac{\partial \mathbf{V}}{\partial x} \\ &= 0 + (8 - 2x)(-2) \\ &= 4(x - 4) \text{ m/s}^2 \quad \text{Ans.} \end{aligned}$$

using the definition of velocity,

$$\begin{aligned} \frac{dx}{dt} &= V = 8 - 2x; \quad \int_0^x \frac{dx}{8 - 2x} = \int_0^t dt \\ -\frac{1}{2} \ln(8 - 2x) \Big|_0^x &= t \\ \frac{1}{2} \ln \left( \frac{8}{8 - 2x} \right) &= t \\ \ln \left( \frac{8}{8 - 2x} \right) &= 2t \\ \frac{8}{8 - 2x} &= e^{2t} \\ x &= 4(1 - e^{-2t}) \text{ m} \quad \text{Ans.} \end{aligned}$$

**3-37.** A fluid has velocity components of  $u = (8t^2)$  m/s and  $v = (7y + 3x)$  m/s, where  $x$  and  $y$  are in meters and  $t$  is in seconds. Determine the velocity and acceleration of a particle passing through point (1 m, 1 m) when  $t = 2$  s.

## SOLUTION

Since the velocity components are functions of time and position the flow can be classified as unsteady nonuniform. When  $t = 2$  s,  $x = 1$  m and  $y = 1$  m.

$$u = 8(2^2) = 32 \text{ m/s}$$

$$v = 7(1) + 3(1) = 10 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(32 \text{ m/s})^2 + (10 \text{ m/s})^2} = 33.5 \text{ m/s} \quad \text{Ans.}$$

Its direction is

$$\theta_v = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{10 \text{ m/s}}{32 \text{ m/s}}\right) = 17.4^\circ \angle \theta_v \quad \text{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 16t + 8t^2(0) + (7y + 3x)(0) \\ &= (16t) \text{ m/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (8t^2)(3) + (7y + 3x)(7) \\ &= [24t^2 + 7(7y + 3x)] \text{ m/s}^2 \end{aligned}$$

When  $t = 2$  s,  $x = 1$  m and  $y = 1$  m.

$$a_x = 16(2) = 32 \text{ m/s}^2$$

$$a_y = 24(2^2) + 7[7(1) + 3(1)] = 166 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(32 \text{ m/s}^2)^2 + (166 \text{ m/s}^2)^2} = 169 \text{ m/s}^2 \quad \text{Ans.}$$

Its direction is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{166 \text{ m/s}^2}{32 \text{ m/s}^2}\right) = 79.1^\circ \angle \theta_a \quad \text{Ans.}$$

**Ans:**

$$V = 33.5 \text{ m/s}$$

$$\theta_V = 17.4^\circ$$

$$a = 169 \text{ m/s}^2$$

$$\theta_a = 79.1^\circ \angle$$

**3–38.** A fluid has velocity components of  $u = (8x)$  ft/s and  $v = (8y)$  ft/s, where  $x$  and  $y$  are in feet. Determine the equation of the streamline and the acceleration of particles passing through point (2 ft, 1 ft). Also find the acceleration of a particle located at this point. Is the flow steady or unsteady?

## SOLUTION

Since the velocity components are the function of position but not the time, **the flow is steady (Ans.)** but nonuniform. Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{8y}{8x} = \frac{y}{x}$$

$$\int_{1 \text{ ft}}^y \frac{dy}{y} = \int_{2 \text{ ft}}^x \frac{dx}{x}$$

$$\ln y \Big|_{1 \text{ ft}}^y = \ln x \Big|_{2 \text{ ft}}^x$$

$$\ln y = \ln \frac{x}{2}$$

$$y = \frac{1}{2}x$$

**Ans.**

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 8x(8) + 8y(0) \\ &= (64x) \text{ ft/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (8x)(0) + 8y(8) \\ &= (64y) \text{ ft/s}^2 \end{aligned}$$

At  $x = 2$  ft,  $y = 1$  ft. Then

$$a_x = 64(2) = 128 \text{ ft/s}^2 \quad a_y = 64(1) = 64 \text{ ft/s}^2$$

The magnitude of the acceleration is

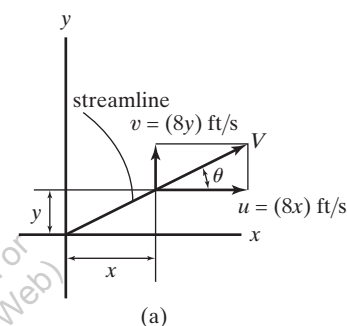
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(128 \text{ ft/s}^2)^2 + (64 \text{ ft/s}^2)^2} = 143 \text{ ft/s}^2$$

**Ans.**

Its direction is

$$\theta = \tan^{-1} \left( \frac{a_y}{a_x} \right) = \tan^{-1} \left( \frac{64 \text{ ft/s}^2}{128 \text{ ft/s}^2} \right) = 26.6^\circ \nearrow \theta$$

**Ans.**



**Ans:**

$$y = x/2, a = 143 \text{ ft/s}^2$$

$$\theta = 26.6^\circ \nearrow$$



**3–39.** A fluid velocity components of  $u = (2y^2)$  m/s and  $v = (8xy)$  m/s, where  $x$  and  $y$  are in meters. Determine the equation of the streamline passing through point (1 m, 2 m). Also, what is the acceleration of a particle at this point? Is the flow steady or unsteady?

## SOLUTION

Since the velocity components are the function of position, not of time, the flow can be classified as **steady (Ans.)** but nonuniform. Using the definition of the slope of the streamline,

$$\begin{aligned}\frac{dy}{dx} &= \frac{v}{u}; & \frac{dy}{dx} &= \frac{8xy}{2y^2} = \frac{4x}{y} \\ \int_{2\text{ m}}^y y \, dy &= \int_{1\text{ m}}^x 4x \, dx \\ \frac{y^2}{2} \Big|_{2\text{ m}}^y &= 2x^2 \Big|_{1\text{ m}}^x \\ \frac{y^2}{2} - 2 &= 2x^2 - 2 \\ y^2 &= 4x^2 \\ y &= 2x\end{aligned}$$

**Ans.**

(Note that  $x = 1$ ,  $y = 2$  is not a solution to  $y = -2x$ .)  
For two dimensional flow, the Eulerian description gives,

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes

$$\begin{aligned}a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 2y^2(0) + 8xy(4y) \\ &= (32xy^2) \text{ m/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + 2y^2(8y) + (8xy)(8x) \\ &= (16y^3 + 64x^2y) \text{ m/s}^2\end{aligned}$$

At point  $x = 1$  m and  $y = 2$  m,

$$\begin{aligned}a_x &= 32(1)(2^2) = 128 \text{ m/s}^2 \\ a_y &= [16(2^3) + 64(1^2)(2)] = 256 \text{ m/s}^2\end{aligned}$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(128 \text{ m/s}^2)^2 + (256 \text{ m/s}^2)^2} = 286 \text{ m/s}^2$$

**Ans.**

Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{256 \text{ m/s}^2}{128 \text{ m/s}^2}\right) = 63.4^\circ \angle \theta$$

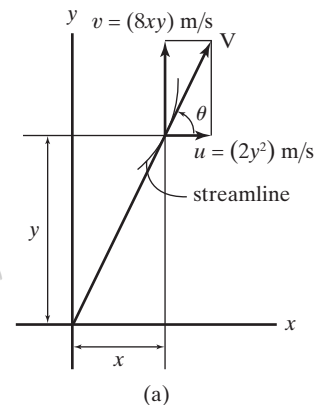
**Ans.**

**Ans:**

$$y = 2x$$

$$a = 286 \text{ m/s}^2$$

$$\theta = 63.4^\circ \angle$$



**\*3–40.** The velocity of a flow field is defined by  $\mathbf{V} = \{4y\mathbf{i} + 2x\mathbf{j}\}$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (2 m, 1 m). Find the equation of the streamline passing through this point, and sketch the velocity and acceleration at the point on this streamline.

## SOLUTION

The flow is steady but nonuniform since the velocity components are a function of position, but not time. At point (2 m, 1 m)

$$u = 4y = 4(1) = 4 \text{ m/s}$$

$$v = 2x = 2(2) = 4 \text{ m/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(4 \text{ m/s})^2 + (4 \text{ m/s})^2} = 5.66 \text{ m/s}$$

**Ans.**

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 4y(0) + (2x)(4) \\ &= (8x) \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + 4y(2) + 2x(0) \\ &= (8y) \text{ m/s}^2 \end{aligned}$$

At point (2 m, 1 m),

$$\begin{aligned} a_x &= 8(2) = 16 \text{ m/s}^2 \\ a_y &= 8(1) = 8 \text{ m/s}^2 \end{aligned}$$

The magnitude of the acceleration is

$$\begin{aligned} a &= \sqrt{a_x^2 + a_y^2} \\ &= \sqrt{(16 \text{ m/s}^2)^2 + (8 \text{ m/s}^2)^2} \\ &= 17.9 \text{ m/s}^2 \end{aligned}$$

**Ans.**

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{2x}{4y} = \frac{x}{2y}$$

$$\int_{1 \text{ m}}^y 2y \, dy = \int_{2 \text{ m}}^x x \, dx$$

$$y^2 \Big|_{1 \text{ m}}^y = \frac{x^2}{2} \Big|_{2 \text{ m}}^x$$

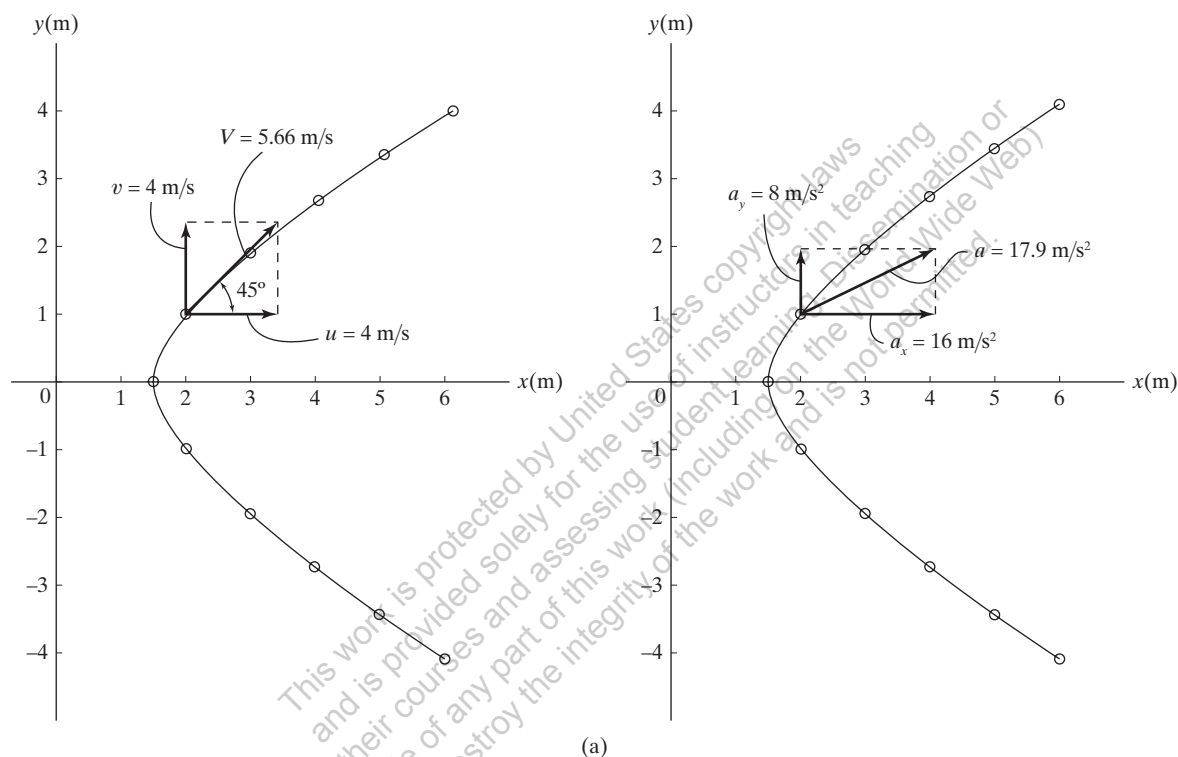
$$y^2 - 1 = \frac{x^2}{2} - 2$$

$$y^2 = \frac{1}{2}x^2 - 1$$

**\*3-40. (continued)**

The plot of this streamline is shown in Fig. *a*

$x(\text{m})$	$\sqrt{2}$	2	3	4	5	6
$y(\text{m})$	0	$\pm 1$	$\pm 1.87$	$\pm 2.65$	$\pm 3.39$	$\pm 4.12$



(a)

**3-41.** The velocity of a flow field is defined by  $\mathbf{V} = \{4x\mathbf{i} + 2\mathbf{j}\}$  m/s, where  $x$  is in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (1 m, 2 m). Find the equation of the streamline passing through this point, and sketch these vectors on this streamline.

## SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady nonuniform. At point (1 m, 2 m),

$$u = 4x = 4(1) = 4 \text{ m/s}$$

$$v = 2 \text{ m/s}$$

The magnitude of velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(4 \text{ m/s})^2 + (2 \text{ m/s})^2} = 4.47 \text{ m/s}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 4x(4) + 2(0) = 16x \end{aligned}$$

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + 4x(0) + 2(0) = 0 \end{aligned}$$

At point (1 m, 2 m),

$$a_x = 16(1) = 16 \text{ m/s}^2, a_y = 0$$

Thus, the magnitude of the acceleration is

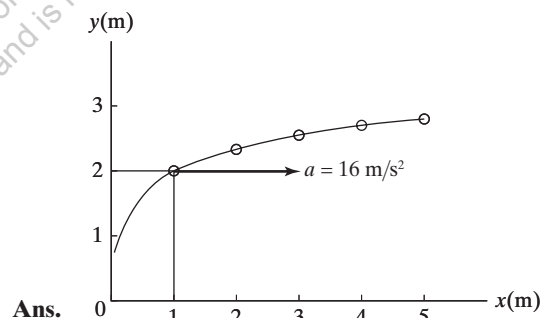
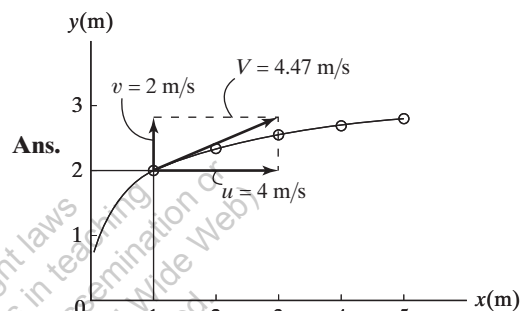
$$a = a_x = 16 \text{ m/s}^2$$

Using the definition of the slope of the streamline,

$$\begin{aligned} \frac{dy}{dx} &= \frac{v}{u}; & \frac{dy}{dx} &= \frac{2}{4x} = \frac{1}{2x} \\ \int_{2 \text{ m}}^y dy &= \frac{1}{2} \int_{1 \text{ m}}^x \frac{dx}{x} \\ y - 2 &= \frac{1}{2} \ln x \\ y &= \left( \frac{1}{2} \ln x + 2 \right) \end{aligned}$$

The plot of this streamline is shown in Fig. *a*

$x(\text{m})$	$e^{-4}$	1	2	3	4	5
$y(\text{m})$	0	2	2.35	2.55	2.69	2.80



**Ans:**

$$V = 4.47 \text{ m/s}, a = 16 \text{ m/s}^2$$

$$y = \frac{1}{2} \ln x + 2$$

**3-42.** The velocity of a flow field is defined by  $u = (2x^2 - y^2)$  m/s and  $v = (-4xy)$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (1 m, 1 m). Find the equation of the streamline passing through this point, and sketch the velocity and acceleration at the point on this streamline.

## SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady but nonuniform. At point (1 m, 1 m),

$$u = 2x^2 - y^2 = 2(1^2) - 1^2 = 1 \text{ m/s}$$

$$v = -4xy = -4(1)(1) = -4 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(1 \text{ m/s})^2 + (-4 \text{ m/s})^2} = 4.12 \text{ m/s}$$

**Ans.**

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + (2x^2 - y^2)(4x) + (-4xy)(-2y) \\ &= 4x(2x^2 - y^2) + 8xy^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (2x^2 - y^2)(-4y) + (-4xy)(-4x) \\ &= -4y(2x^2 - y^2) + 16x^2y \end{aligned}$$

At point (1 m, 1 m),

$$a_x = 4(1)[2(1^2) - 1^2] + 8(1)(1^2) = 12 \text{ m/s}^2$$

$$a_y = -4(1)[2(1^2) - 1^2] + 16(1^2)(1) = 12 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(12 \text{ m/s}^2)^2 + (12 \text{ m/s}^2)^2} = 17.0 \text{ m/s}^2$$

**Ans.**

Using the definition of the slope of the streamline,

$$\begin{aligned} \frac{dy}{dx} &= \frac{v}{u}; & \frac{dy}{dx} &= -\frac{4xy}{2x^2 - y^2} \\ (2x^2 - y^2)dy &= -4xydx \\ 2x^2dy + 4xydx - y^2dy &= 0 \end{aligned}$$

However,  $d(2x^2y) = 2(2xydx + x^2dy) = 2x^2dy + 4xydx$ . Then

$$d(2x^2y) - y^2dy = 0$$

**3-42. (continued)**

Integrating this equation,

$$2x^2y - \frac{y^3}{3} = C$$

with the condition  $y = 1 \text{ m}$  when  $x = 1 \text{ m}$ ,

$$2(1^2)(1) - \frac{1^3}{3} = C$$

$$C = \frac{5}{3}$$

Thus,

$$2x^2y - \frac{y^3}{3} = \frac{5}{3}$$

$$6x^2y - y^3 = 5$$

$$x^2 = \frac{y^3 + 5}{6y}$$

Taking the derivative of this equation with respect to  $y$

$$2x \frac{dx}{dy} = \frac{6y(3y^2) - (y^3 + 5)(6)}{(6y)^2} = \frac{2y^3 - 5}{6y^2}$$

$$\frac{dx}{dy} = \frac{2y^3 - 5}{12xy^2}$$

Set  $\frac{dx}{dy} = 0$ ;

$$2y^3 - 5 = 0$$

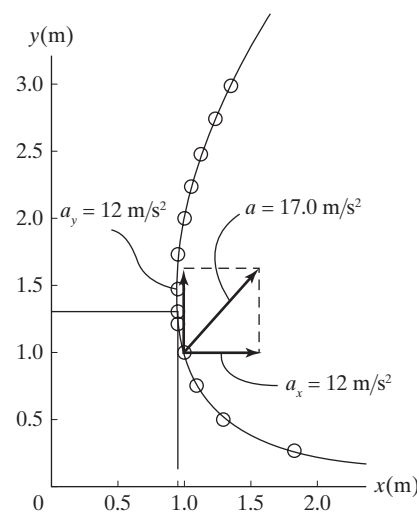
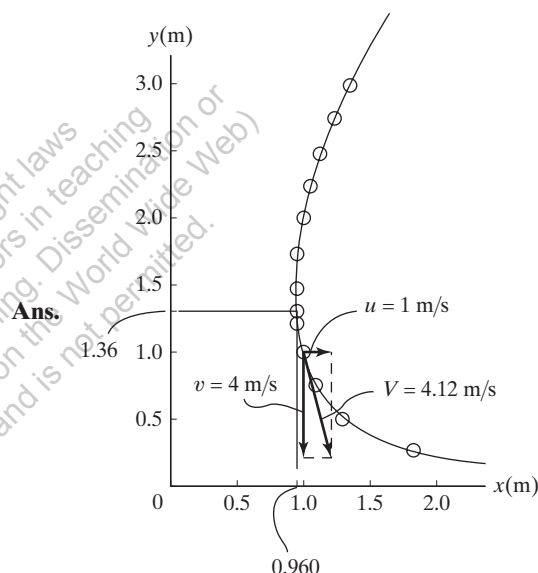
$$y = 1.357 \text{ m}$$

The corresponding  $x$  is

$$x^2 = 1.357^3 + 5$$

$$x = 0.960 \text{ m}$$

$y(\text{m})$	0.25	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
$x(\text{m})$	1.83	1.31	1.10	1.00	0.963	0.965	0.993	1.04	1.10	1.17	1.25	1.33



**Ans:**  
 $V = 4.12 \text{ m/s}$   
 $a = 17.0 \text{ m/s}^2$   
 $x^2 = \frac{y^3 + 5}{6y}$

**3–43.** The velocity of a flow field is defined by  $u = (-y/4) \text{ m/s}$  and  $v = (x/9) \text{ m/s}$ , where  $x$  and  $y$  are in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point  $(3 \text{ m}, 2 \text{ m})$ . Find the equation of the streamline passing through this point, and sketch the velocity and acceleration at the point on this streamline.

## SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady nonuniform. At point  $(3 \text{ m}, 2 \text{ m})$

$$u = \frac{-y}{4} = -\frac{2}{4} = -0.5 \text{ m/s}$$

$$v = \frac{x}{9} = \frac{3}{9} = 0.3333 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(-0.5 \text{ m/s})^2 + (0.3333 \text{ m/s})^2} = 0.601 \text{ m/s} \quad \text{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  are

$$(\rightarrow) a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= 0 + \left(\frac{-y}{4}\right)(0) + \left(\frac{x}{9}\right)\left(-\frac{1}{4}\right)$$

$$= \left(-\frac{1}{36}x\right) \text{ m/s}^2$$

$$(+\uparrow) a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

$$= 0 + \left(\frac{-y}{4}\right)\left(\frac{1}{9}\right) + \left(\frac{x}{9}\right)(0)$$

$$= \left[-\frac{1}{36}y\right] \text{ m/s}^2$$

At point  $(3 \text{ m}, 2 \text{ m})$ ,

$$a_x = -\frac{1}{36}(3) = -0.08333 \text{ m/s}^2$$

$$a_y = -\frac{1}{36}(2) = -0.05556 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{(-0.08333 \text{ m/s}^2)^2 + (-0.05556 \text{ m/s}^2)^2}$$

$$= 0.100 \text{ m/s}^2$$

**Ans.**

3-43. (continued)

Using the definition of slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{x/9}{-y/4} = -\frac{4x}{9y}$$

$$9 \int_{2 \text{ m}}^y y dy = -4 \int_{3 \text{ m}}^x x dx$$

$$\frac{9y^2}{2} \Big|_{2 \text{ m}}^y = -(2x^2) \Big|_{3 \text{ m}}^x$$

$$\frac{9y^2}{2} - 18 = -2x^2 + 18$$

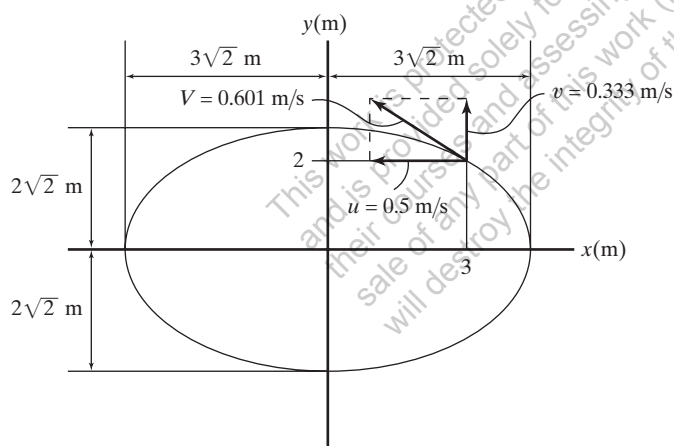
$$9y^2 + 4x^2 = 72$$

$$\frac{x^2}{72/4} + \frac{y^2}{72/9} = 1$$

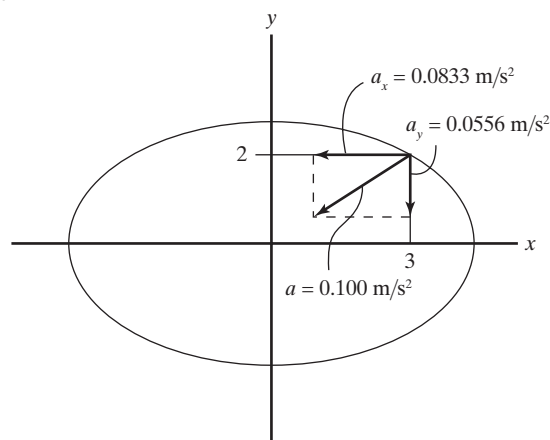
$$\frac{x^2}{(4.24)^2} + \frac{y^2}{(2.83)^2} = 1$$

Ans.

This is an equation of an ellipse with center at (0, 0). The plot of this streamline is shown in Fig. a



(a)



Ans:

$$V = 0.601 \text{ m/s}$$

$$a = 0.100 \text{ m/s}^2$$

$$x^2/(4.24)^2 + y^2/(2.83)^2 = 1$$



**\*3-44.** The velocity of gasoline, along the centerline of a tapered pipe, is given by  $u = (4tx)$  m/s, where  $t$  is in seconds and  $x$  is in meters. Determine the acceleration of a particle when  $t = 0.8$  s if  $u = 0.8$  m/s when  $t = 0.1$  s.

## SOLUTION

The flow is unsteady nonuniform. For one dimensional flow,

$$a = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}$$

Here,  $u = (4tx)$  m/s. Then  $\frac{\partial u}{\partial t} = 4x$  and  $\frac{\partial u}{\partial x} = 4t$ . Thus,

$$a = 4x + (4tx)(4t) = (4x + 16t^2x) \text{ m/s}^2$$

Since  $u = 0.8$  m/s when  $t = 0.1$  s,

$$0.8 = 4(0.1)x \quad x = 2 \text{ m}$$

The position of the particle can be determined from

$$\begin{aligned} \frac{dx}{dt} &= u = 4tx; & \int_{2 \text{ m}}^x \frac{dx}{x} &= 4 \int_{0.15}^t t \, dt \\ \ln x \Big|_{2 \text{ m}}^x &= 2t^2 \Big|_{0.15}^t \\ \ln \frac{x}{2} &= 2t^2 - 0.02 \\ e^{2t^2 - 0.02} &= \frac{x}{2} \\ x &= 2e^{2t^2 - 0.02} \\ x &= 2e^{2(0.8^2) - 0.02} = 7.051 \text{ m} \end{aligned}$$

Thus,  $t = 0.8$  s,

$$\begin{aligned} a &= 4(7.051) + 16(0.8^2)(7.051) \\ &= 100.40 \text{ m/s}^2 \\ &= 100 \text{ m/s}^2 \end{aligned}$$

**Ans.**

**3–45.** The velocity field for a flow of water is defined by  $u = (2x) \text{ m/s}$ ,  $v = (6tx) \text{ m/s}$ , and  $w = (3y) \text{ m/s}$ , where  $t$  is in seconds and  $x, y, z$  are in meters. Determine the acceleration and the position of a particle when  $t = 0.5 \text{ s}$  if this particle is at  $(1 \text{ m}, 0, 0)$  when  $t = 0$ .

## SOLUTION

The flow is unsteady nonuniform. For three dimensional flow,

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y} + w \frac{\partial \mathbf{V}}{\partial z}$$

Thus,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \\ &= 0 + 2x(2) + (6tx)(0) + 3y(0) \\ &= (4x) \text{ m/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \\ &= 6x + 2x(6t) + 6tx(0) + 3y(0) \\ &= (6x + 12tx) \text{ m/s}^2 \\ a_z &= \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \\ &= 0 + 2x(0) + 6tx(3) + 3y(0) \\ &= (18tx) \text{ m/s}^2 \end{aligned}$$

The position of the particle can be determined from

$$\begin{aligned} \frac{dx}{dt} &= u = 2x; & \int_{1 \text{ m}}^x \frac{dx}{x} &= 2 \int_0^t dt \\ \ln x &= 2t \\ x &= (e^{2t}) \text{ m} \\ \frac{dy}{dt} &= v = 6tx = 6te^{2t}; & \int_0^y dy &= 6 \int_0^t te^{2t} dt \\ y &= \frac{3}{2} (2te^{2t} - e^{2t}) \Big|_0^t \\ y &= \frac{3}{2} (2te^{2t} - e^{2t} + 1) \\ \frac{dz}{dt} &= w = 3y = \frac{9}{2} [2te^{2t} - e^{2t} + 1]; \\ \int_0^z dz &= \frac{9}{2} \int_0^t (2te^{2t} - e^{2t} + 1) dt \\ z &= \frac{9}{2} \left[ te^{2t} - \frac{1}{2} e^{2t} - \frac{1}{2} e^{2t} + t \right] \Big|_0^t \\ z &= \frac{9}{2} (te^{2t} - e^{2t} + t + 1) \text{ m} \end{aligned}$$

**3–45. (continued)**

When,  $t = 0.5$  s,

$$x = e^{2(0.5)} = 2.7183 \text{ m} = 2.72 \text{ m}$$

**Ans.**

$$y = \frac{3}{2}[2(0.5)e^{2(0.5)} - e^{2(0.5)} + 1] = 1.5 \text{ m}$$

**Ans.**

$$\text{Thus, } z = \frac{9}{2}[0.5e^{2(0.5)} - e^{2(0.5)} + 0.5 + 1] = 0.6339 \text{ m} = 0.634 \text{ m}$$

**Ans.**

$$a_x = 4(2.7183) = 10.87 \text{ m/s}^2$$

$$a_y = 6(2.7183) + 12(0.5)(2.7183) = 32.62 \text{ m/s}^2$$

$$a_z = 18(0.5)(2.7183) = 24.46 \text{ m/s}^2$$

Then

$$\mathbf{a} = \{10.9\mathbf{i} + 32.6\mathbf{j} + 24.5\mathbf{k}\} \text{ m/s}^2$$

**Ans.**

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**Ans:**

$$x = 2.72 \text{ m}$$

$$y = 1.5 \text{ m}$$

$$z = 0.634 \text{ m}$$

$$\mathbf{a} = \{10.9\mathbf{i} + 32.6\mathbf{j} + 24.5\mathbf{k}\} \text{ m/s}^2$$

**3-46.** A flow field has velocity components of  $u = -(4x + 6)$  m/s and  $v = (10y + 3)$  m/s where  $x$  and  $y$  are in meters. Determine the equation for the streamline that passes through point (1 m, 1 m), and find the acceleration of a particle at this point.

## SOLUTION

Since the velocity components are the function of position but not of time, the flow can be classified as steady but nonuniform. Using the definition of the slope of the streamline,

$$\begin{aligned}\frac{dy}{dx} &= \frac{v}{u}; & \frac{dy}{dx} &= \frac{10y + 3}{-(4x + 6)} \\ \int_{1\text{ m}}^y \frac{dy}{10y + 3} &= - \int_{1\text{ m}}^x \frac{dx}{4x + 6} \\ \frac{1}{10} \ln(10y + 3) \Big|_{1\text{ m}}^y &= -\frac{1}{4} \ln(4x + 6) \Big|_{1\text{ m}}^x \\ \frac{1}{10} \ln\left(\frac{10y + 3}{13}\right) &= \frac{1}{4} \ln\left(\frac{10}{4x + 6}\right) \\ \ln\left(\frac{10y + 3}{13}\right)^{\frac{1}{10}} &= \ln\left(\frac{10}{4x + 6}\right)^{\frac{1}{4}} \\ \left(\frac{10y + 3}{13}\right)^{\frac{1}{10}} &= \left(\frac{10}{4x + 6}\right)^{\frac{1}{4}} \\ \frac{10y + 3}{13} &= \left(\frac{10}{4x + 6}\right)^{\frac{5}{2}} \\ y &= \left[ \frac{411}{(4x + 6)^{5/2}} - 0.3 \right] \text{ m} \quad \text{Ans.}\end{aligned}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\delta \mathbf{V}}{\delta t} + u \frac{\delta \mathbf{V}}{\delta x} + v \frac{\delta \mathbf{V}}{\delta y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned}a_x &= \frac{\delta u}{\delta t} + u \frac{\delta u}{\delta x} + v \frac{\delta u}{\delta y} \\ &= 0 + [-(4x + 6)(-4)] + (10y + 3)(0) \\ &= [4(4x + 6)] \text{ m/s}^2 \\ a_y &= \frac{\delta v}{\delta t} + u \frac{\delta v}{\delta x} + v \frac{\delta v}{\delta y} \\ &= 0 + [-(4x + 6)(0)] + (10y + 3)(10) \\ &= [10(10y + 3)] \text{ m/s}^2\end{aligned}$$

At point (1m, 1m),

$$\begin{aligned}a_x &= 4[4(1) + 6] = 40 \text{ m/s}^2 \rightarrow \\ a_y &= 10[10(1) + 3] = 130 \text{ m/s}^2 \uparrow\end{aligned}$$

**3–46. (continued)**

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(40 \text{ m/s}^2)^2 + (130 \text{ m/s}^2)^2} = 136 \text{ m/s}^2 \quad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{130 \text{ m/s}^2}{40 \text{ m/s}^2}\right) = 72.9^\circ \nearrow \theta \quad \textbf{Ans.}$$

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**Ans:**

$$y = \frac{411}{(4x+6)^{5/2}} - 0.3$$

$$a = 136 \text{ m/s}^2$$

$$\theta = 72.9^\circ \nearrow$$

**3-47.** A velocity field for oil is defined by  $u = (100y)$  m/s,  $v = (0.03 t^2)$  m/s, where  $t$  is in seconds and  $y$  is in meters. Determine the acceleration and the position of a particle when  $t = 0.5$  s. The particle is at the origin when  $t = 0$ .

## SOLUTION

Since the velocity components are a function of both position and time, the flow can be classified as unsteady nonuniform. Using the definition of velocity,

$$\frac{dy}{dt} = v = 0.03t^2; \quad \int_0^y dy = 0.03 \int_0^t t^2 dt$$

$$y = (0.01t^3) \text{ m}$$

When  $t = 0.5$  s,

$$y = 0.01(0.5^3) = 0.00125 \text{ m} = 1.25 \text{ mm}$$

**Ans.**

$$\frac{dx}{dt} = u = 100y = 100(0.01t^3) = t^3; \quad \int_0^x dx = \int_0^t t^3 dt$$

$$x = \left(\frac{1}{4}t^4\right) \text{ m}$$

When  $t = 0.5$  s,

$$x = \frac{1}{4}(0.5^4) = 0.015625 \text{ m} = 15.6 \text{ mm}$$

**Ans.**

For a two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Write the scalar components of this equation along the  $x$  and  $y$  axes,

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= 0 + (100y)(0) + (0.03t^2)(100)$$

$$= (3t^2) \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

$$= 0.06t + (100y)(0) + 0.03t^2(0)$$

$$= (0.06t) \text{ m/s}^2$$

When  $t = 0.5$  s,

$$a_x = 3(0.5^2) = 0.75 \text{ m/s}^2 \rightarrow$$

$$a_y = 0.06(0.5) = 0.03 \text{ m/s}^2 \uparrow$$

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.75 \text{ m/s}^2)^2 + (0.03 \text{ m/s}^2)^2} = 0.751 \text{ m/s}^2$$

**Ans.**

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.03 \text{ m/s}^2}{0.75 \text{ m/s}^2}\right) = 2.29^\circ \nearrow \theta$$

**Ans.**

**Ans:**

$$y = 1.25 \text{ mm}$$

$$x = 15.6 \text{ mm}$$

$$a = 0.751 \text{ m/s}^2$$

$$\theta = 2.29^\circ \nearrow$$

**\*3–48.** If  $u = (2x^2)$  m/s and  $v = (-y)$  m/s where  $x$  and  $y$  are in meters, determine the equation of the streamline that passes through point (2 m, 6 m), and find the acceleration of a particle at this point. Sketch the streamline for  $x > 0$ , and find the equations that define the  $x$  and  $y$  components of acceleration of the particle as a function of time if  $x = 2$  m and  $y = 6$  m when  $t = 0$ .

## SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady but nonuniform. Using the definition of the slope of the streamline,

$$\begin{aligned}\frac{dy}{dx} &= \frac{v}{u}, & \frac{dy}{dx} &= \frac{-y}{2x^2} \\ \int_{6 \text{ m}}^y \frac{dy}{y} &= -\frac{1}{2} \int_{2 \text{ m}}^x \frac{dx}{x^2} \\ \ln y \Big|_{6 \text{ m}}^y &= \frac{1}{2} \left( \frac{1}{x} \right) \Big|_{2 \text{ m}}^x \\ \ln \frac{y}{6} &= \frac{1}{2} \left( \frac{1}{x} - \frac{1}{2} \right) \\ \ln \frac{y}{6} &= \frac{2-x}{4x} \\ \frac{y}{6} &= e^{\left( \frac{2-x}{4x} \right)} \\ y &= \left[ 6e^{\left( \frac{2-x}{4x} \right)} \right] \text{ m}\end{aligned}$$

**Ans.**

The plot of this streamline is shown in Fig. *a*.

For two dimensional flow, the Eulerian description gives,

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned}a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + (2x^2)(4x) + (-y)(0) \\ &= (8x^3) \text{ m/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (2x^2)(0) + (-y)(-1) \\ &= (y) \text{ m/s}^2\end{aligned}$$

At point (2 m, 6 m),

$$\begin{aligned}a_x &= 8(2^3) = 64 \text{ m/s}^2 \rightarrow \\ a_y &= 6 \text{ m/s}^2 \uparrow\end{aligned}$$

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(64 \text{ m/s}^2)^2 + (6 \text{ m/s}^2)^2} = 64.3 \text{ m/s}^2$$

**Ans.**

And its direction is

$$\theta = \tan^{-1} \left( \frac{a_y}{a_x} \right) = \tan^{-1} \left( \frac{6 \text{ m/s}^2}{64 \text{ m/s}^2} \right) = 5.36^\circ \angle \theta$$

**Ans.**

3-48. (continued)

Using the definition of the velocity,

$$\frac{dx}{dt} = u; \quad \frac{dx}{dt} = 2x^2$$

$$\int_{2\text{ m}}^x \frac{dx}{2x^2} = \int_0^t dt$$

$$-\frac{1}{2} \left( \frac{1}{x} \right) \Big|_{2\text{ m}}^x = t$$

$$-\frac{1}{2} \left( \frac{1}{x} - \frac{1}{2} \right) = t$$

$$\frac{x - 2}{4x} = t$$

$$x = \left( \frac{2}{1 - 4t} \right) \text{ m}$$

$$\frac{dy}{dt} = v; \quad \frac{dy}{dt} = -y$$

$$-\int_{6\text{ m}}^y \frac{dy}{y} = \int_0^t dt$$

$$-\ln y \Big|_{6\text{ m}}^y = t$$

$$\ln \frac{6}{y} = t$$

$$\frac{6}{y} = e^t$$

$$y = (6e^{-t}) \text{ m}$$

Thus,

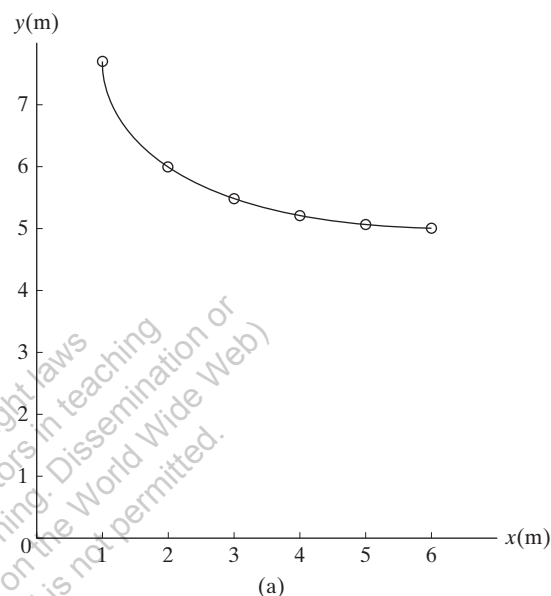
$$u = 2x^2 = 2 \left( \frac{2}{1 - 4t} \right)^2 = \left[ \frac{8}{(1 - 4t)^2} \right] \text{ m/s and } v = -y = (-6e^{-t}) \text{ m/s}$$

Then,

$$a_x = \frac{du}{dt} = -16(1 - 4t)^{-3}(-4) = \left[ \frac{64}{(1 - 4t)^3} \right] \text{ m/s}^2 \quad \text{Ans.}$$

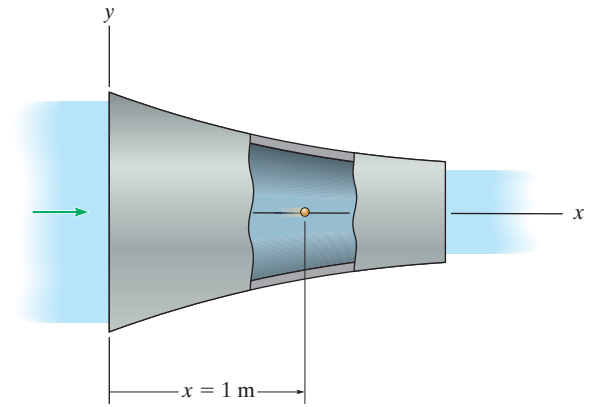
$$a_y = \frac{dv}{dt} = (6e^{-t}) \text{ m/s}^2 \quad \text{Ans.}$$

$x(\text{m})$	0.5	1	2	3	4	5	6
$y(\text{m})$	12.70	7.70	6.00	5.52	5.29	5.16	5.08





**3–49.** Airflow through the duct is defined by the velocity field  $u = (2x^2 + 8) \text{ m/s}$   $v = (-8x) \text{ m/s}$ , where  $x$  is in meters. Determine the acceleration of a fluid particle at the origin  $(0, 0)$  and at point  $(1 \text{ m}, 0)$ . Also, sketch the streamlines that pass through these points.



## SOLUTION

Since the velocity component are a function of position but not time, the flow can be classified as steady but nonuniform. For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= [0 + (2x^2 + 8)(4x) + (-8x)(0)] \\ &= [4x(2x^2 + 8)] \text{ m/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + (2x^2 + 8)(-8) + (-8x)(0) \\ &= [-8(2x^2 + 8)] \text{ m/s}^2 \end{aligned}$$

At point  $(0, 0)$ ,

$$\begin{aligned} a_x &= 4(0)[2(0^2) + 8] = 0 \\ a_y &= -8[2(0^2) + 8] = -64 \text{ m/s}^2 = 64 \text{ m/s}^2 \downarrow \end{aligned}$$

Thus,

$$\mathbf{a} = a_y = 64 \text{ m/s}^2 \downarrow$$

**Ans.**

At point  $(1 \text{ m}, 0)$ ,

$$\begin{aligned} a_x &= 4(1)[2(1^2) + 8] = 40 \text{ m/s}^2 \rightarrow \\ a_y &= -8[2(1^2) + 8] = -80 \text{ m/s}^2 = 80 \text{ m/s}^2 \downarrow \end{aligned}$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(40 \text{ m/s}^2)^2 + (80 \text{ m/s}^2)^2} = 89.4 \text{ m/s}^2$$

**Ans.**

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{80 \text{ m/s}^2}{40 \text{ m/s}^2}\right) = 63.4^\circ \searrow \theta$$

**Ans.**

Using the definition of the slope of the streamline,

$$\begin{aligned} \frac{dy}{dx} &= \frac{v}{u} = \frac{-8x}{2x^2 + 8}; \quad \int dy = -8 \int \frac{x dx}{2x^2 + 8} \\ y &= -2 \ln(2x^2 + 8) + C \end{aligned}$$

**3-49. (continued)**

For the streamline passing through point (0, 0),

$$0 = -2 \ln[2(0^2) + 8] + C \quad C = 2 \ln 8$$

Then  $y = \left[ 2 \ln \left( \frac{8}{2x^2 + 8} \right) \right] \text{ m}$

**Ans.**

For the streamline passing through point (1 m, 0),

$$0 = -2 \ln[2(1^2) + 8] + C \quad C = 2 \ln 10$$

$$y = \left[ 2 \ln \left( \frac{10}{2x^2 + 8} \right) \right] \text{ m}$$

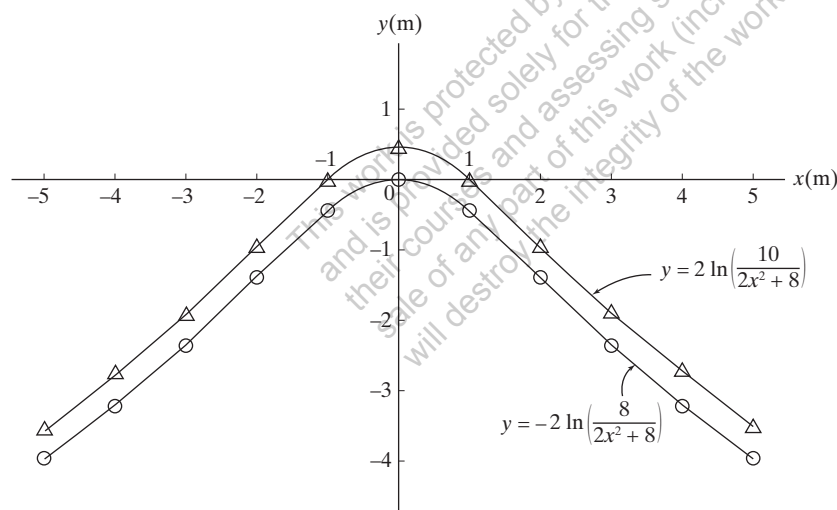
**Ans.**

For point (0, 0)

$x(\text{m})$	0	$\pm 1$	$\pm 2$	$\pm 3$	$\pm 4$	$\pm 5$
$y(\text{m})$	0	-0.446	-1.39	-2.36	-3.22	-3.96

For point (1 m, 0)

$x(\text{m})$	0	$\pm 1$	$\pm 2$	$\pm 3$	$\pm 4$	$\pm 5$
$y(\text{m})$	0.446	0	-0.940	-1.91	-2.77	-3.52



**Ans:**

At point (0, 0),

$$a = 64 \text{ m/s}^2 \downarrow$$

At point (1 m, 0),

$$a = 89.4 \text{ m/s}^2, \theta = 63.4^\circ \quad \swarrow$$

For the streamline passing through point (0, 0),

$$y = \left[ 2 \ln \left( \frac{8}{2x^2 + 8} \right) \right] \text{ m}$$

For the streamline passing through point (1 m, 0),

$$y = \left[ 2 \ln \left( \frac{10}{2x^2 + 8} \right) \right] \text{ m}$$

**3–50.** The velocity field for a fluid is defined by  $u = [y/(x^2 + y^2)] \text{ m/s}$  and  $v = [4x/(x^2 + y^2)] \text{ m/s}$ , where  $x$  and  $y$  are in meters. Determine the acceleration of a particle located at point (2 m, 0) and a particle located at point (4 m, 0). Sketch the equations that define the streamlines that pass through these points.

## SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady but nonuniform. For two dimensional flow, the Eulerian description gives

$$a = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y}$$

Write the scalar components of this equation along  $x$  and  $y$  axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + \left( \frac{y}{x^2 + y^2} \right) \left[ \frac{(x^2 + y^2)(0) - y(2x)}{(x^2 + y^2)^2} \right] + \left( \frac{4x}{x^2 + y^2} \right) \left[ \frac{(x^2 + y^2)(1) - y(2y)}{(x^2 + y^2)^2} \right] \\ &= \left[ \frac{4x^3 - 6xy^2}{(x^2 + y^2)^3} \right] \text{ m/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + \left( \frac{y}{x^2 + y^2} \right) \left[ \frac{(x^2 + y^2)(4) - 4x(2x)}{(x^2 + y^2)^2} \right] + \left( \frac{4x}{x^2 + y^2} \right) \left[ \frac{(x^2 + y^2)(0) - 4x(2y)}{(x^2 + y^2)^2} \right] \\ &= \left[ \frac{4y^3 - 36x^2y}{(x^2 + y^2)^3} \right] \text{ m/s}^2 \end{aligned}$$

At point (2 m, 0)

$$\begin{aligned} a_x &= \frac{4(2^3) - 6(2)(0^2)}{(2^2 + 0^2)^3} = 0.5 \text{ m/s}^2 \rightarrow \\ a_y &= \frac{4(0^3) - 36(2^2)(0)}{(2^2 + 0^2)^3} = 0 \end{aligned}$$

Thus  $a = a_x = 0.5 \text{ m/s}^2 \rightarrow$

**Ans.**

**3–50. (continued)**

At point (4 m, 0)

$$a_x = \frac{4(4^3) - 6(4)(0)}{(4^2 + 0^2)^3} = 0.0625 \text{ m/s}^2 \rightarrow$$

$$a_y = \frac{4(0^3) - 36(4^2)(0)}{(4^2 + 0^2)^3} = 0$$

Thus

$$a = a_x = 0.0625 \text{ m/s}^2 \rightarrow$$

**Ans.**

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{4x/(x^2 + y^2)}{y/(x^2 + y^2)} = \frac{4x}{y}; \quad \int y dy = 4 \int x dx$$

$$\frac{y^2}{2} = 2x^2 + C'$$

$$y^2 = 4x^2 + C$$

For the streamline passing through point (2 m, 0),

$$0^2 = 4(2^2) + C \quad C = -16$$

Then

$$y^2 = 4x^2 - 16$$

$$y = \pm \sqrt{4x^2 - 16} \quad x \geq 2 \text{ m}$$

**Ans.**

For the streamline passes through point (4 m, 0)

$$0^2 = 4(4^2) + C \quad C = -64$$

Then

$$y^2 = 4x^2 - 64$$

$$y = \pm \sqrt{4x^2 - 64} \quad x \geq 4 \text{ m}$$

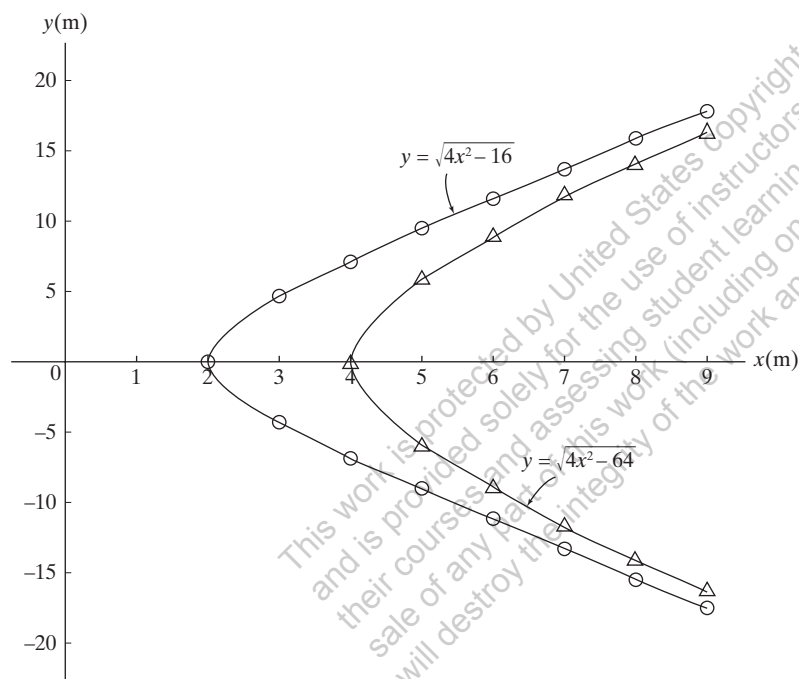
**3-50. (continued)**

For the streamline passing through point (2 m, 0)

$x(\text{m})$	2	3	4	5	6	7	8	9
$y(\text{m})$	0	$\pm 4.47$	$\pm 6.93$	$\pm 9.17$	$\pm 11.31$	$\pm 13.42$	$\pm 15.49$	$\pm 17.55$

For the streamline passing through point (4 m, 0)

$x(\text{m})$	4	5	6	7	8	9
$y(\text{m})$	0	$\pm 6.00$	$\pm 8.94$	$\pm 11.49$	$\pm 13.86$	$\pm 16.12$



**Ans:**

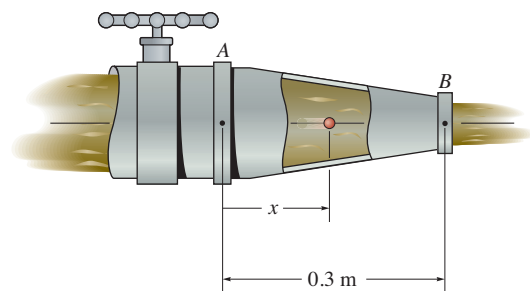
For point (2 m, 0),  
 $a = 0.5 \text{ m/s}^2$

$$y = \pm \sqrt{4x^2 - 16}$$

For point (4 m, 0),  
 $a = 0.0625 \text{ m/s}^2$

$$y = \pm \sqrt{4x^2 - 64}$$

**3-51.** As the valve is closed, oil flows through the nozzle such that along the center streamline it has a velocity of  $V = [6(1 + 0.4x^2)(1 - 0.5t)]$  m/s, where  $x$  is in meters and  $t$  is in seconds. Determine the acceleration of an oil particle at  $x = 0.25$  m when  $t = 1$  s.



## SOLUTION

Here  $V$  only has an  $x$  component, so that  $V = u$ . Since  $V$  is a function of time at each  $x$ , the flow is unsteady. Since  $v = w = 0$ , we have

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \\ &= \frac{\partial}{\partial t} [6(1 + 0.4x^2)(1 - 0.5t)] + [6(1 + 0.4x^2)(1 - 0.5t)] \frac{\partial}{\partial x} [6(1 + 0.4x^2)(1 - 0.5t)] \\ &= [6(1 + 0.4x^2)(0 - 0.5)] + [6(1 + 0.4x^2)(1 - 0.5t)] [6(0 + 0.4(2x))(1 - 0.5t)] \end{aligned}$$

Evaluating this expression at  $x = 0.25$  m,  $t = 1$  s, we get

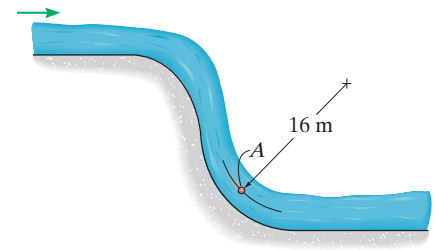
$$\begin{aligned} a_s &= -3.075 \text{ m/s}^2 + 1.845 \text{ m/s}^2 \\ &= -1.23 \text{ m/s}^2 \end{aligned}$$

**Ans.**

Note that the local acceleration component ( $-3.075 \text{ m/s}^2$ ) indicates a deceleration since the valve is being closed to decrease the flow. The convective acceleration ( $1.845 \text{ m/s}^2$ ) is positive since the nozzle constricts as  $x$  increases. The net result causes the particle to decelerate at  $1.23 \text{ m/s}^2$ .

**Ans:**  
 $-1.23 \text{ m/s}^2$

**\*3-52.** As water flows steadily over the spillway, one of its particles follows a streamline that has a radius of curvature of 16 m. If its speed at point *A* is 5 m/s which is increasing at 3 m/s<sup>2</sup>, determine the magnitude of acceleration of the particle.



## SOLUTION

The *n* – *s* coordinate system is established with origin at point *A* as shown in Fig. *a*. Here, the component of the particle's acceleration along the *s* axis is

$$a_s = 3 \text{ m/s}^2$$

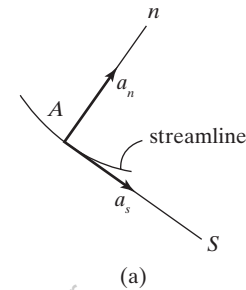
Since the streamline does not rotate, the local acceleration along the *n* axis is zero, so that  $\left(\frac{\partial V}{\partial t}\right)_n = 0$ . Therefore, the component of the particle's acceleration along the *n* axis is

$$\begin{aligned} a_n &= \left(\frac{\partial V}{\partial t}\right)_n + \frac{V^2}{R} \\ &= 0 + \frac{(5 \text{ m/s})^2}{16 \text{ m}} = 1.5625 \text{ m/s}^2 \end{aligned}$$

Thus, the magnitude of the particle's acceleration is

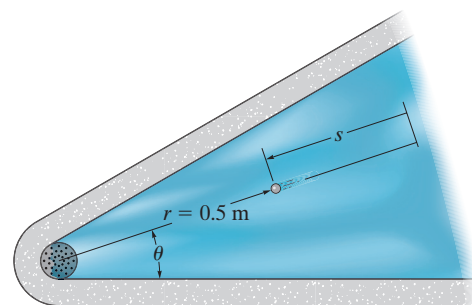
$$\begin{aligned} a &= \sqrt{a_s^2 + a_n^2} = \sqrt{(3 \text{ m/s}^2)^2 + (1.5625 \text{ m/s}^2)^2} \\ &= 3.38 \text{ m/s}^2 \end{aligned}$$

**Ans.**



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**3-53.** Water flows into the drainpipe such that it only has a radial velocity component  $V = (-3/r)$  m/s, where  $r$  is in meters. Determine the acceleration of a particle located at point  $r = 0.5$  m,  $\theta = 20^\circ$ . At  $s = 0$ ,  $r = 1$  m.



## SOLUTION

Fig.  $a$  is based on the initial condition when  $s = 0$ ,  $r = r_D$ . Thus,  $r = 1 - s$ . Then the radial component of velocity is

$$V = -\frac{3}{r} = \left(-\frac{3}{1-s}\right) \text{ m/s}$$

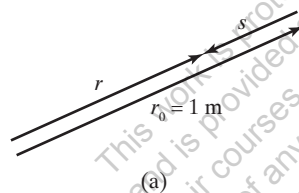
This is one dimensional steady flow since the velocity is along the straight radial line. The Eulerian description gives

$$\begin{aligned} a &= \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial s} \\ &= 0 + \left(-\frac{3}{1-s}\right) \left[-\frac{3}{(1-s)^2}\right] \\ &= \left[\frac{9}{(1-s)^3}\right] \text{ m/s}^2 \end{aligned}$$

When  $1 - s = r = 0.5$  m, this equation gives

$$a = \left(\frac{9}{0.5^3}\right) \text{ m/s}^2 = 72 \text{ m/s}^2 \quad \text{Ans.}$$

The positive sign indicates that  $a$  is directed towards positive  $s$ . Note there is no normal component for motion along a straightline.



**Ans:**  
72 m/s<sup>2</sup>



**3–54.** A particle located at a point within a fluid flow has velocity components of  $u = 4 \text{ m/s}$  and  $v = -3 \text{ m/s}$ , and acceleration components of  $a_x = 2 \text{ m/s}^2$  and  $a_y = 8 \text{ m/s}^2$ . Determine the magnitude of the streamline and normal components of acceleration of the particle.

## SOLUTION

$$V = \sqrt{(4 \text{ m/s})^2 + (-3 \text{ m/s})^2} = 5 \text{ m/s}$$

$$a = \sqrt{(2 \text{ m/s}^2)^2 + (8 \text{ m/s}^2)^2} = 8.246 \text{ m/s}^2$$

$$\mathbf{a} = 2\mathbf{i} + 8\mathbf{j}$$

$$\theta = \tan^{-1} \frac{3}{4} = 36.870^\circ$$

$$u_s = \cos 36.870^\circ \mathbf{i} - \sin 36.870^\circ \mathbf{j}$$

$$= 0.8\mathbf{i} - 0.6\mathbf{j}$$

$$a_s = \mathbf{a} \cdot \mathbf{u}_s = (2\mathbf{i} + 8\mathbf{j}) \cdot (0.8\mathbf{i} - 0.6\mathbf{j})$$

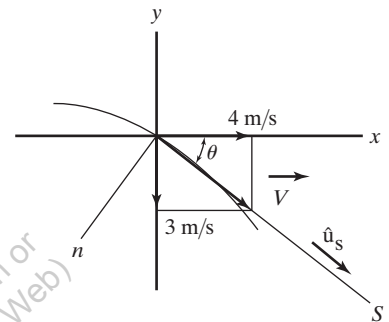
$$a_s = -3.20 \text{ m/s}^2$$

$$a_s = 3.20 \text{ m/s}^2$$

$$a = \sqrt{a_s^2 + a_n^2}$$

$$(8.246)^2 = (3.20)^2 + a_n^2$$

$$a_n = 7.60 \text{ m/s}^2$$



**Ans.**

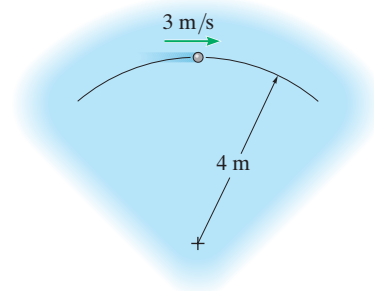
**Ans.**

**Ans:**

$$a_s = 3.20 \text{ m/s}^2$$

$$a_n = 7.60 \text{ m/s}^2$$

**3–55.** A particle moves along the circular streamline, such that it has a velocity of 3 m/s, which is increasing at 3 m/s<sup>2</sup>. Determine the acceleration of the particle, and show the acceleration on the streamline.



## SOLUTION

The normal component of the acceleration is

$$a_n = \frac{V^2}{\rho} = \frac{(3 \text{ m/s})^2}{4 \text{ m}} = 2.25 \text{ m/s}^2$$

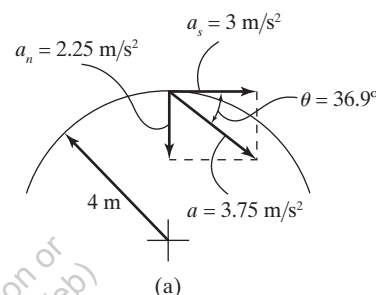
Thus, the magnitude of the acceleration is

$$a = \sqrt{a_s^2 + a_n^2} = \sqrt{(3 \text{ m/s}^2)^2 + (2.25 \text{ m/s}^2)^2} = 3.75 \text{ m/s}^2$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_n}{a_s}\right) = \tan^{-1}\left(\frac{2.25 \text{ m/s}^2}{3 \text{ m/s}^2}\right) = 36.9^\circ$$

The plot of the acceleration on the streamline is shown in Fig. *a*.



**Ans.**

**Ans.**

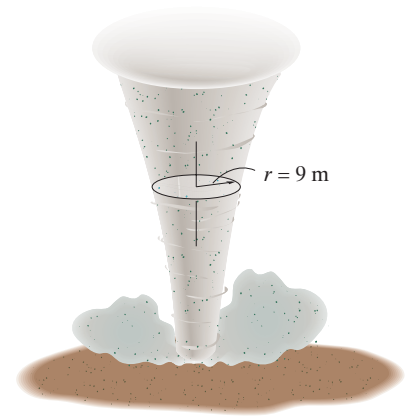
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**Ans:**

$$a = 3.75 \text{ m/s}^2$$

$$\theta = 36.9^\circ$$

**\*3-56.** The motion of a tornado can, in part, be described by a free vortex,  $V = k/r$  where  $k$  is a constant. Consider the steady motion at the radial distance  $r = 3$  m, where  $V = 18$  m/s. Determine the magnitude of the acceleration of a particle traveling on the streamline having a radius of  $r = 9$  m.



## SOLUTION

Using the condition at  $r = 3$  m,  $V = 18$  m/s.

$$V = \frac{k}{r}; \quad 18 \text{ m/s} = \frac{k}{3 \text{ m}} \quad k = 54 \text{ m}^2/\text{s}$$

Then

$$V = \left( \frac{54}{r} \right) \text{ m/s}$$

At  $r = 9$  m,  $V = \left( \frac{54}{9} \right) \text{ m/s} = 6 \text{ m/s}$ . Since the velocity is constant, the streamline component of acceleration is

$$a_s = 0$$

The normal component of acceleration is

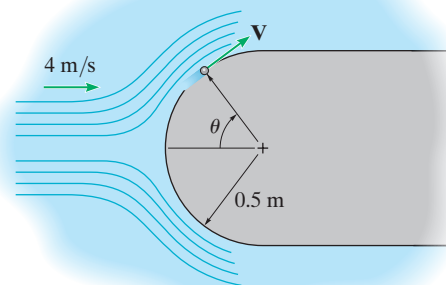
$$a_n = \left( \frac{\partial V}{\partial t} \right)_n + \frac{V^2}{r} = 0 + \frac{(6 \text{ m/s})^2}{9 \text{ m}} = 4 \text{ m/s}^2$$

Thus, the acceleration is

$$a = a_n = 4 \text{ m/s}^2$$

**Ans.**

**3–57.** Air flows around the front circular surface. If the steady-stream velocity is 4 m/s upstream from the surface, and the velocity along the surface is defined by  $V = (16 \sin \theta)$  m/s, determine the magnitude of the streamline and normal components of acceleration of a particle located at  $\theta = 30^\circ$ .



## SOLUTION

The streamline component of acceleration can be determined from

$$a_s = \left( \frac{\partial V}{\partial t} \right)_s + V \frac{\partial V}{\partial s}$$

However,  $s = r\theta$ . Thus,  $\partial s = r \partial \theta = 0.5 \partial \theta$ . Also, the flow is steady,  $\left( \frac{\partial V}{\partial t} \right)_s = 0$  and

$$\frac{\partial V}{\partial s} = \frac{\partial V}{0.5 \partial \theta} = 2 \frac{\partial V}{\partial \theta} = 2(16 \cos \theta) = 32 \cos \theta. \text{ Then}$$

$$a_s = 0 + 16 \sin \theta (32 \cos \theta) = 512 \sin \theta \cos \theta = 256 \sin 2\theta$$

When  $\theta = 30^\circ$ ,

$$a_s = 256 \sin 2(30^\circ) = 221.70 \text{ m/s}^2 = 222 \text{ m/s}^2$$

$$V = (16 \sin 30^\circ) \text{ m/s} = 8 \text{ m/s}$$

**Ans.**

The normal component of acceleration can be determined from

$$a_n = \left( \frac{\partial V}{\partial t} \right)_n + \frac{V^2}{R} = 0 + \frac{(8 \text{ m/s})^2}{0.5 \text{ m}} = 128 \text{ m/s}^2$$

**Ans.**

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**Ans:**

$$a_s = 222 \text{ m/s}^2$$

$$a_n = 128 \text{ m/s}^2$$

**3–58.** Fluid particles have velocity components of  $u = (8y) \text{ m/s}$   $v = (6x) \text{ m/s}$ , where  $x$  and  $y$  are in meters. Determine the magnitude of the streamline and the normal components of acceleration of a particle located at point (1 m, 2 m).

## SOLUTION

At  $x = 1 \text{ m}$ ,  $y = 2 \text{ m}$

$$u = 8(2) = 16 \text{ m/s}$$

$$v = 6(1) = 6 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{6}{16} = 20.56^\circ$$

$$\begin{aligned} u_s &= \cos 20.56^\circ \mathbf{i} + \sin 20.56^\circ \mathbf{j} \\ &= 0.9363 \mathbf{i} + 0.3511 \mathbf{j} \end{aligned}$$

**Acceleration.** With  $w = 0$ , we have

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 8y(0) + 6x(8) = 48x \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + 8y(6) + 6x(0) = 48y \end{aligned}$$

At  $x = 1 \text{ m}$ , and  $y = 2 \text{ m}$ ,

$$\begin{aligned} a_x &= 48(1) = 48 \text{ m/s}^2 \\ a_y &= 48(2) = 96 \text{ m/s}^2 \end{aligned}$$

Therefore, the acceleration is

$$\mathbf{a} = \{48\mathbf{i} + 96\mathbf{j}\} \text{ m/s}^2$$

And its magnitude is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(48 \text{ m/s}^2)^2 + (96 \text{ m/s}^2)^2} = 107.33 \text{ m/s}^2$$

Since the direction of the  $s$  axis is defined by  $\mathbf{u}_s$ , the component of the particle's acceleration along the  $s$  axis can be determined from

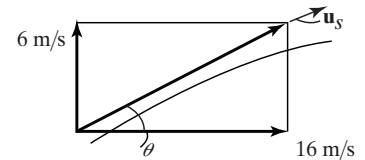
$$\begin{aligned} a_s &= \mathbf{a} \cdot \mathbf{u}_s = [48\mathbf{i} + 96\mathbf{j}] \cdot [0.9363\mathbf{i} + 0.3511\mathbf{j}] \\ &= 78.65 \text{ m/s}^2 = 78.7 \text{ m/s}^2 \end{aligned}$$

**Ans.**

The normal component of the particle's acceleration is

$$a_n = \sqrt{a^2 - a_s^2} = \sqrt{(107.33 \text{ m/s}^2)^2 - (78.65 \text{ m/s}^2)^2} = 73.0 \text{ m/s}^2$$

**Ans.**



**Ans:**

$$\begin{aligned} a_s &= 78.7 \text{ m/s}^2 \\ a_n &= 73.0 \text{ m/s}^2 \end{aligned}$$

**3-59.** Fluid particles have velocity components of  $u = (8y) \text{ m/s}$  and  $v = (6x) \text{ m/s}$ , where  $x$  and  $y$  are in meters. Determine the acceleration of a particle located at point  $(1 \text{ m}, 1 \text{ m})$ . Determine the equation of the streamline, passing through this point.

## SOLUTION

Since the velocity components are independent of time, but a function of position, the flow can be classified as steady nonuniform. For two dimensional flow, ( $w = 0$ ), the Eulerian description is

$$a = \frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ &= 0 + 8y(0) + 6x(8) \\ &= 48x \\ a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + 8y(6) + 6x(0) \\ &= 48y \end{aligned}$$

At point  $x = 1 \text{ m}$  and  $y = 1 \text{ m}$

$$\begin{aligned} a_x &= 48(1) = 48 \text{ m/s}^2 \\ a_y &= 48(1) = 48 \text{ m/s}^2 \end{aligned}$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(48 \text{ m/s}^2)^2 + (48 \text{ m/s}^2)^2} = 67.9 \text{ m/s}^2 \quad \text{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{48 \text{ m/s}^2}{48 \text{ m/s}^2}\right) = 45^\circ \quad \text{Ans.}$$

The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{6x}{8y} \quad (1)$$

$$\int_{1 \text{ m}}^y 8y dy = \int_{1 \text{ m}}^x 6x dx$$

$$4y^2 \Big|_{1 \text{ m}}^y = 3x^2 \Big|_{1 \text{ m}}^x$$

$$4y^2 - 3x^2 = 1 \quad \text{Ans.}$$

**Ans:**

$$a = 67.9 \text{ m/s}^2$$

$$\theta = 45^\circ \quad \swarrow$$

$$4y^2 - 3x^2 = 1$$

**\*3–60.** A fluid has velocity components of  $u = (2y^2)$  m/s and  $v = (8xy)$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the streamline and normal components of acceleration of a particle located at point (1 m, 2 m).

## SOLUTION

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{8xy}{2y^2} = \frac{4x}{y}$$

$$\int_{2 \text{ m}}^y y \, dy = \int_{1 \text{ m}}^x 4x \, dx$$

$$\frac{y^2}{2} \Big|_{2 \text{ m}}^y = 2x^2 \Big|_{1 \text{ m}}^x$$

$$\frac{y^2}{2} - 2 = 2x^2 - 2$$

$$y^2 = 4x^2$$

$$y = 2x$$

(Note that  $x = 1$ ,  $y = 2$  is not a solution  $y = -2x$ .) Two equation from streamline is  $y = 2x$  (A straight line). Thus,  $R \rightarrow \infty$  and since the flow is steady,

$$a_n = \frac{V^2}{R} = 0$$

**Ans.**

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= 0 + 2y^2(0) + (8xy)(4y) = (32xy^2) \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

$$= 0 + 2y^2(8y) + (8xy)(8x)$$

$$= (16y^3 + 64x^2y) \text{ m/s}^2$$

At (1 m, 2 m),

$$a_x = 32(1)(2^2) = 128 \text{ m/s}^2$$

$$a_y = [16(2^3) + 64(1^2)(2)] = 256 \text{ m/s}^2$$

$$a = a_s = \sqrt{a_x^2 + a_y^2} = \sqrt{(128 \text{ m/s}^2)^2 + (256 \text{ m/s}^2)^2}$$

$$a_s = 286 \text{ m/s}^2$$

**Ans.**

**3-61.** A fluid has velocity components of  $u = (2y^2)$  m/s and  $v = (8xy)$  m/s, where  $x$  and  $y$  are in meters. Determine the magnitude of the streamline and normal components of the acceleration of a particle located at point (1 m, 1 m). Find the equation of the streamline passing through this point, and sketch the streamline and normal components of acceleration at this point.

## SOLUTION

$x(\text{m})$	$\sqrt{3}/2$	1.0	2.0	3.0	4.0	5.0
$y(\text{m})$	0	1.0	3.61	5.74	7.81	9.85

Since the velocity component is independent of time and is a function of position, the flow can be classified as steady nonuniform. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{8xy}{2y^2} = \frac{4x}{y}$$

$$\int_{1\text{ m}}^y y \, dy = \int_{1\text{ m}}^x 4x \, dx$$

$$\frac{y^2}{2} \Big|_{1\text{ m}}^y = 2x^2 \Big|_{1\text{ m}}^x$$

$$4x^2 - y^2 = 3 \quad \text{where } x \text{ and } y \text{ are in } m$$

**Ans.**

For two dimensional flow ( $w = 0$ ), the Eulerian description is

$$a = \frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y}$$

Writing the scalar components of this equation along the  $x$  and  $y$  axes,

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= 0 + (2y^2)(0) + (8xy)(4y)$$

$$= (32xy^2) \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

$$= 0 + (2y^2)(8y) + (8xy)(8x)$$

$$= (16y^3 + 64x^2y) \text{ m/s}^2$$

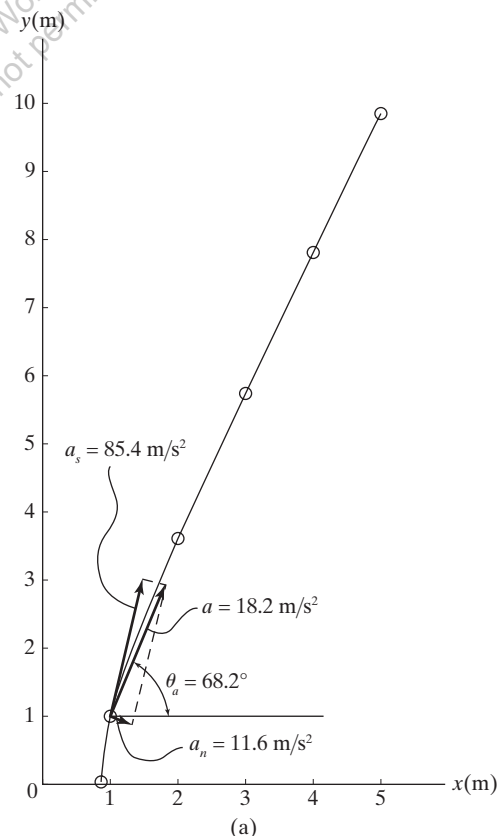
At point  $x = 1$  m and  $y = 1$  m

$$a_x = 32(1)(1^2) = 32 \text{ m/s}^2$$

$$a_y = 16(1^3) + 64(1^2)(1) = 80 \text{ m/s}^2$$

Thus,

$$a = \{32\mathbf{i} + 80\mathbf{j}\} \text{ m/s}^2$$





**3-61. (continued)**

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(32 \text{ m/s}^2)^2 + (80 \text{ m/s}^2)^2} = 86.16 \text{ m/s}^2 = 18.2 \text{ m/s}^2$$

and its direction is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{80 \text{ m/s}^2}{32 \text{ m/s}^2}\right) = 68.2^\circ$$

At point (1 m, 1 m),

$$\tan \theta = \left. \frac{dy}{dx} \right|_{\substack{x=1 \text{ m} \\ y=1 \text{ m}}} = \frac{4(1)}{1} = 4; \theta = 75.96^\circ$$

Thus, the unit vector along the streamline is

$$u = \cos 75.96^\circ \mathbf{i} + \sin 75.96^\circ \mathbf{j} = 0.2425 \mathbf{i} + 0.9701 \mathbf{j}$$

Thus, the streamline component of the acceleration is

$$\begin{aligned} a_s &= \mathbf{a} \cdot \mathbf{u}_s = (32 \mathbf{i} + 80 \mathbf{j}) \cdot (0.2425 \mathbf{i} + 0.9701 \mathbf{j}) \\ &= 85.37 \text{ m/s}^2 = 85.4 \text{ m/s}^2 \end{aligned} \quad \text{Ans.}$$

Then

$$\begin{aligned} a_n &= \sqrt{a^2 - a_s^2} = \sqrt{(86.16 \text{ m/s}^2)^2 - (85.37 \text{ m/s}^2)^2} \\ &= 11.64 \text{ m/s}^2 = 11.6 \text{ m/s}^2 \end{aligned} \quad \text{Ans.}$$

**Ans:**

$$\begin{aligned} 4x^2 - y^2 &= 3 \\ a_s &= 85.4 \text{ m/s}^2 \\ a_n &= 11.6 \text{ m/s}^2 \end{aligned}$$