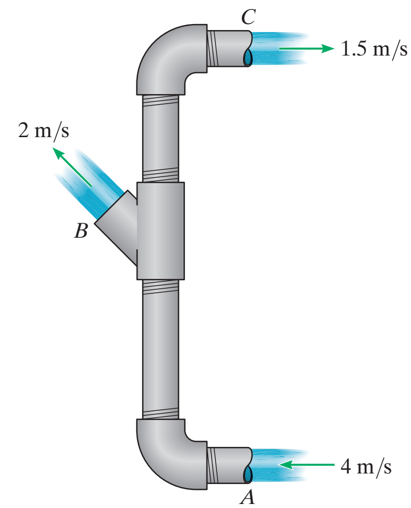
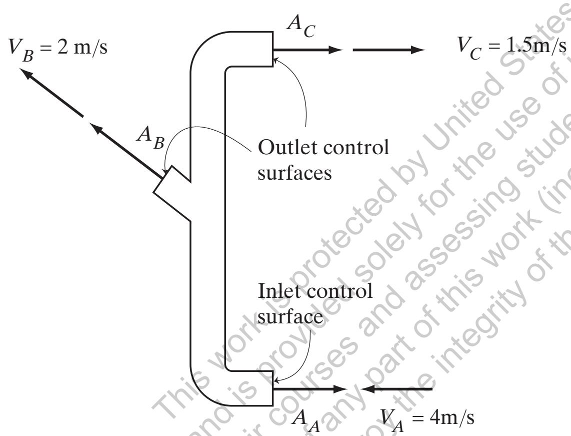
4–1. Water flows steadily through the pipes with the average velocities shown. Outline the control volume that contains the water in the pipe system. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

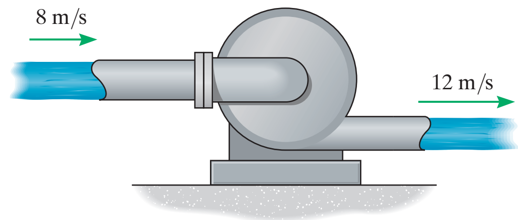


SOLUTION

Since the flow is steady, no local change occurs. However, the water flows in and out of the control volume through the open (inlet and outlet) control surfaces. Thus, convective changes take place.

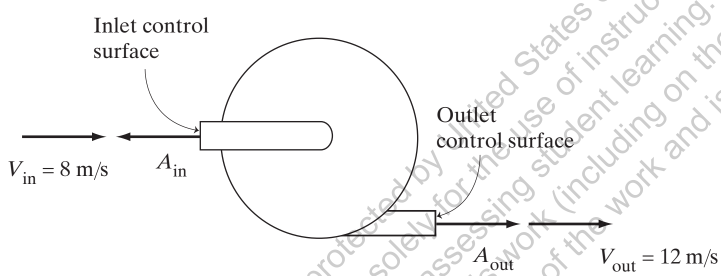


4–2. Water is drawn steadily through the pump. The average velocities are indicated. Select a control volume that contains the water in the pump and extends slightly past it. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

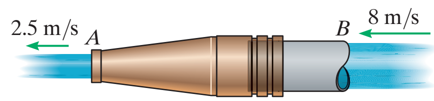


SOLUTION

Since the flow is steady, there is no local change. However, the water flows in and out of the control volume through the open (inlet and outlet) control surfaces. Thus, convective changes take place.

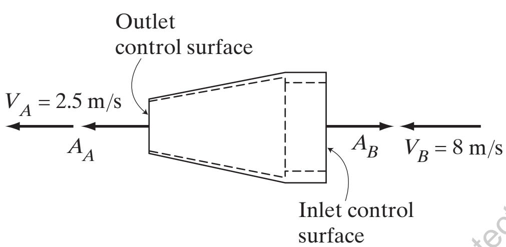


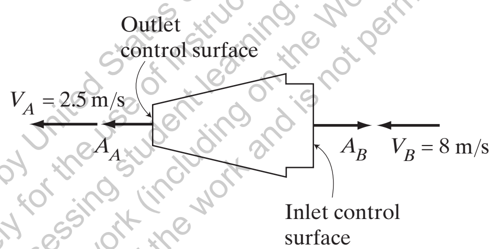
4–3. The average velocities of water flowing steadily through the nozzle are indicated. If the nozzle is glued onto the end of the hose, outline the control volume to be the entire nozzle and the water inside it. Also, select another control volume to be just the water inside the nozzle. In each case, indicate the open control surfaces, and show the positive direction of their areas. Specify the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.



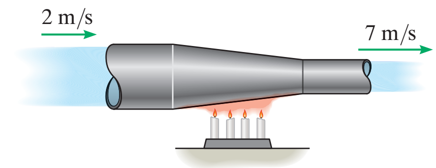
SOLUTION

Since the flow is steady, no local change occurs. However, the water flows in and out of the control volume through the opened (inlet and outlet) control surfaces. Thus, convective changes take place.



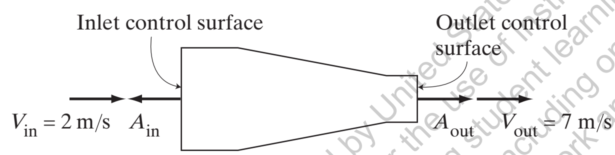


***4–4.** Air flows through the tapered duct, and during this time heat is being added that changes the density of the air within the duct. The average velocities are indicated. Select a control volume that contains the air in the duct. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume the air is incompressible.



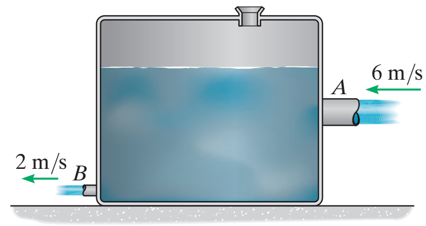
SOLUTION

Since the density of the air within the control volume changes with time due to the increased heating, local changes occur. Also, air flows/in and out of the control volume through the opened (inlet and outlet) control surfaces. This causes a convective change to take place.



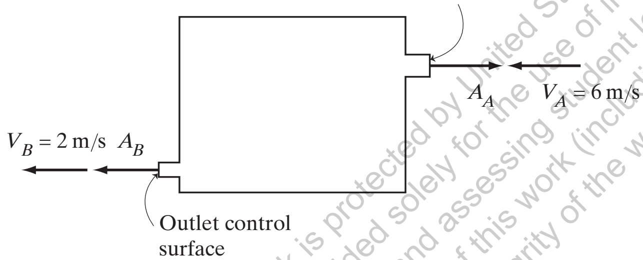
Note: If the heating is constant, then no local changes will occur.

4–5. The tank is being filled with water at A at a rate faster than it is emptied at B; the average velocities are shown. Select a control volume that includes only the water in the tank. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

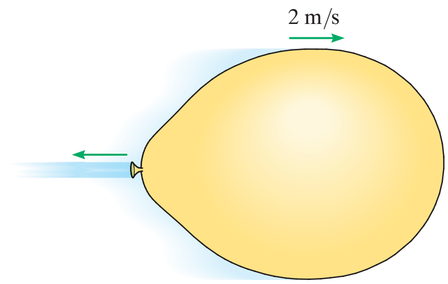


SOLUTION

Since the volume of the control volume changes with time, local changes occur. Also, the water flows in and out of the control volume through the open (inlet and outlet) control surfaces. This causes convective changes to take place.

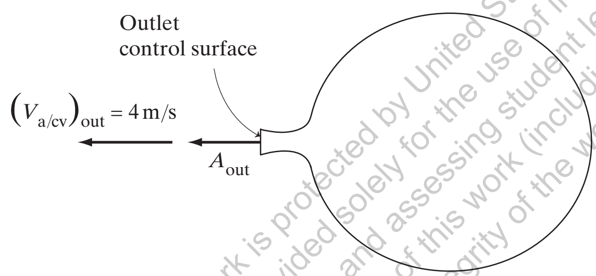


4–6. The toy balloon is filled with air and is released. At the instant shown, the air is escaping at an average velocity of 4 m/s, measured relative to the balloon, while the balloon is accelerating. For an analysis, why is it best to consider the control volume to be moving? Select this control volume so that it contains the air in the balloon. Indicate the open control surface, and show the positive direction of its area. Also, indicate the direction of the velocity through this surface. Identify the local and convective changes that occur. Assume the air to be incompressible.

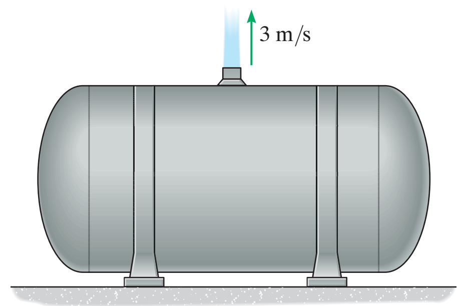


SOLUTION

The control volume is considered moving so that the Reynolds transport theorem can be applied using relative velocities. Since the volume of the control volume (ballon) changes with time, local changes occur. Also, air flows/out from the control volume through the open (outlet) control surface. This causes convective changes to take place.

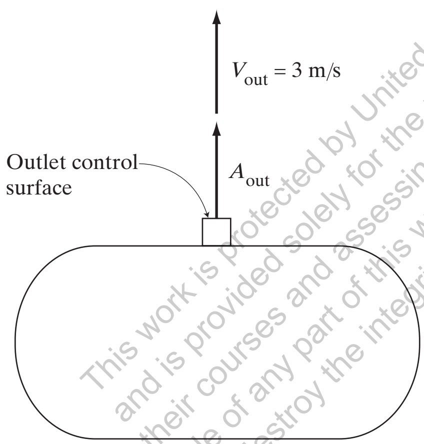


4–7. Compressed air is being released from the tank, and at the instant shown it has a velocity of 3 m/s. Select a control volume that contains the air in the tank. Indicate the open control surface, and show the positive direction of its area. Also, indicate the direction of the velocity through this surface. Identify the local and convective changes that occur. Assume the air to be compressible.

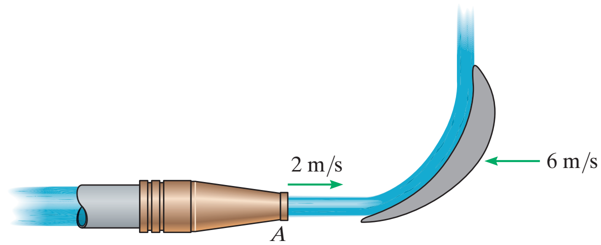


SOLUTION

Since the density of the air changes with time, local changes occur. Also, the air flows out of the control volume through the open (outlet) control surface. This causes convective changes.

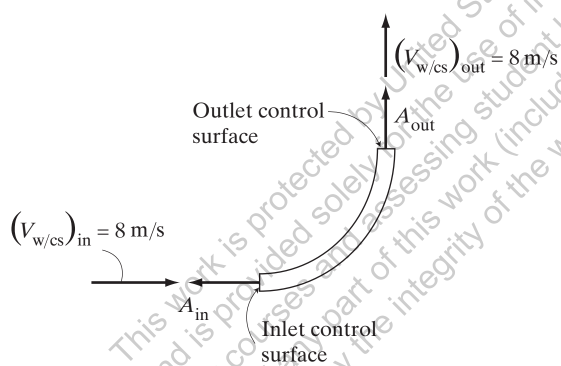


***4–8.** The blade on the turbine is moving to the left at 6 m/s. Water is ejected from the nozzle at A at an average velocity of 2 m/s. For the analysis, why is it best to consider the control volume as moving? Outline this moving control volume that contains the water on the blade. Indicate the open control surfaces, and show the positive direction of their areas through which flow occurs. Also, indicate the magnitudes of the relative velocities and their directions through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

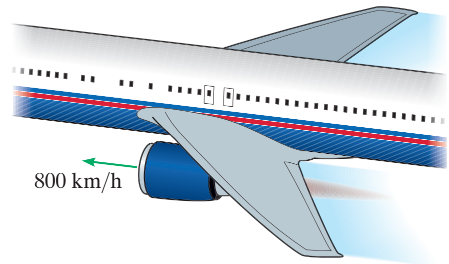


SOLUTION

The control volume is considered moving with the blade so that the flow can be considered steady as measured relative to the control volume. No local changes occurs. Also, the water flows in and out through the open (inlet and outlet) control surfaces. This causes convective changes.

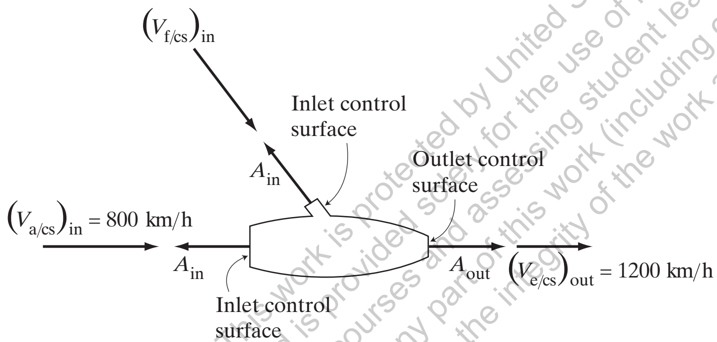


4–9. The jet engine is moving forward with a constant speed of 800 km/h. Fuel from a tank enters the engine and is mixed with the intake air, burned, and exhausted with an average relative velocity of 1200 km/h. Outline the control volume as the jet engine and the air and fuel within it. For an analysis, why is it best to consider this control volume to be moving? Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the magnitudes of the relative velocities and their directions through these surfaces. Identify the local and convective changes that occur. Assume the fuel is incompressible and the air is compressible.

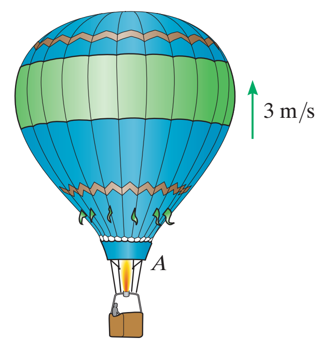


SOLUTION

The control volume is considered moving, so that the Reynolds transport theorem can be applied using relative velocities since the masses are conserved within the control volume the flow is steady, no local changes occur. Also, mass flows in and out from the control volume through the open (inlet and outlet) control surfaces. This causes convective changes to take place.

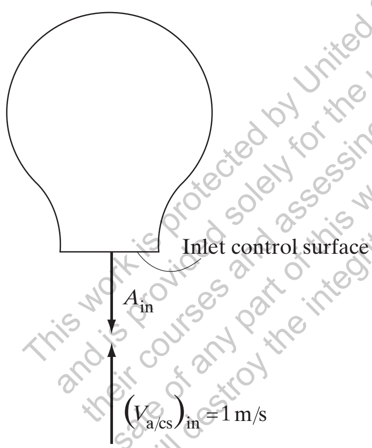


4–10. The balloon is rising at a constant velocity of 3 m/s. Hot air enters from a burner and flows into the balloon at A at an average velocity of 1 m/s, measured relative to the balloon. For an analysis, why is it best to consider the control volume as moving? Outline this moving control volume that contains the air in the balloon. Indicate the open control surface, and show the positive direction of its area. Also, indicate the magnitude of the velocity and its direction through this surface. Identify the local and convective changes that occur. Assume the air to be incompressible.

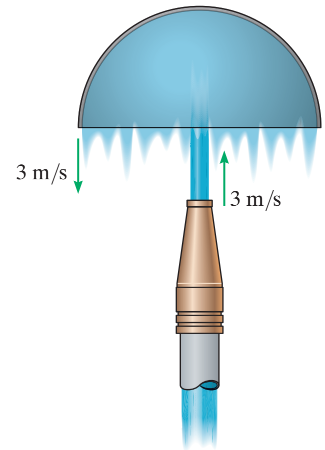


SOLUTION

The control volume is considered moving so that the Reynolds transport theorem can be applied using relative velocities. Since the volume of the control volume (ballon) changes with time, local changes occur. Also, the air flows in the control volume through the opened (inlet) control surface. This causes convective changes to take place.

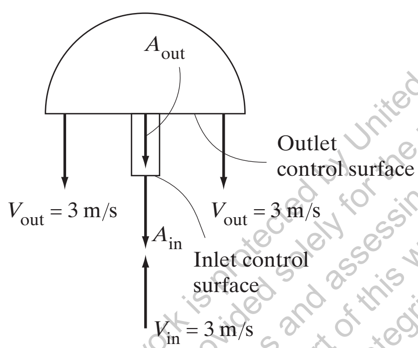


4–11. The hemispherical bowl is suspended in the air by the water stream that flows into and then out of the bowl at the average velocities indicated. Outline a control volume that contains the bowl and the water entering and leaving it. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

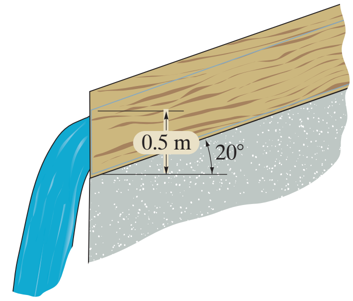


SOLUTION

Since the flow is steady, there is no local change. However, the water flows in and out of the control volume through the open (inlet and outlet) control surface, thus convective changes take place.



***4–12.** Water flows along a rectangular channel having a width of 0.75 m. If the average velocity is 2 m/s, determine the volumetric discharge.



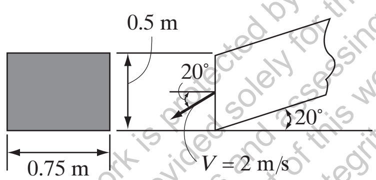
SOLUTION

The vertical cross section of the flow of area $A = (0.75\text{ m})(0.5\text{ m}) = 0.375\text{ m}^2$ is shown shaded in Fig. *a*. The component of velocity perpendicular to this cross section is $V_\perp = (2\text{ m/s})\cos 20° = 1.8794\text{ m/s}$. Thus,

$$Q = \mathbf{V}\cdot\mathbf{A}$$

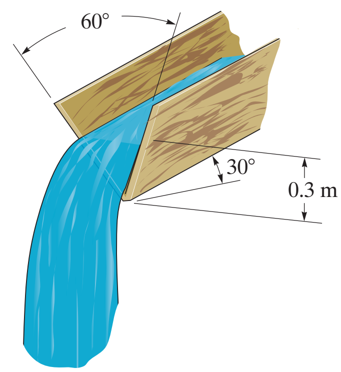
$$= (1.8794\text{ m/s})\left(0.375\text{ m}^2\right)$$

$$= 0.705\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$



(a)

4–13. Water flows along the triangular trough with an average velocity of 5 m/s. Determine the volumetric discharge and the mass flow if the vertical depth of the water is 0.3 m.

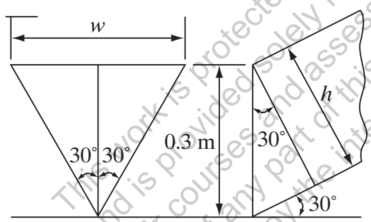


SOLUTION

From the geometry shown in Fig. *a*, the top width w and height h of the cross section perpendicular to the flow are $w = 2[0.3\text{ m}(\tan 30°)] = 0.3464\text{ m}$ and $h = (0.3\text{ m})(\cos 30°) = 0.2598\text{ m}$. Thus, the cross-sectional area of this cross section is $A = \frac{1}{2}(0.3464\text{ m})(0.2598\text{ m}) = 0.045\text{ m}^2$. Thus,

$$Q = \mathbf{V}\cdot\mathbf{A}$$
$$= (5\text{ m/s})(0.045\text{ m}^2)$$
$$= 0.225\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

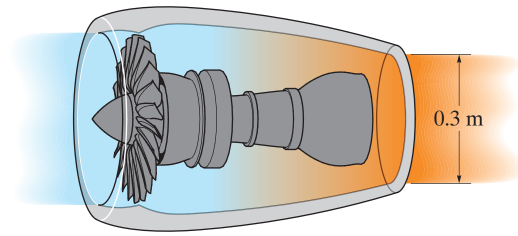
$$\dot{m} = \rho Q = (1000\text{ kg/m}^3)(0.225\text{ m}^3/\text{s})$$
$$= 225\text{ kg/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$Q = 0.225\text{ m}^3/\text{s}$
$\dot{m} = 225\text{ kg/s}$

4–14. Air enters the turbine of a jet engine at a rate of 40 kg/s. If it is discharged with an absolute pressure of 750 kPa and temperature of 120°C, determine its average velocity at the exit. The exit has a diameter of 0.3 m.



SOLUTION

From Appendix A, $R = 286.9\ \text{J/(kg}\cdot\text{K)}$ for air.

$$p = \rho R T$$

$$750(10^3)\ \text{N/m}^2 = \rho\big(286.9\ \text{J/(kg}\cdot\text{K)}\big)(120^\circ + 273)$$

$$\rho = 6.6518\ \text{kg/m}^3$$

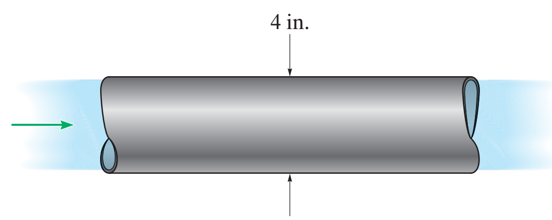
$$\dot{m} = \rho V A$$

$$40\ \text{kg/s} = \big(6.6518\ \text{kg/m}^3\big)V(\pi)(0.15\ \text{m})^2$$

$$V = 85.1\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
8.51 m/s

4–15. Determine the mass flow of CO_2 gas in a 4-in.-diameter duct if it has an average velocity of 20 ft/s. The gas has a temperature of 70°F, and the pressure is 6 psi.



SOLUTION

From Appendix A, $R = 1130\,\dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot\text{R}}$ for CO_2. Here, the absolute pressure is $p = p_{atm} + p_g = 14.7\text{ psi} + 6\text{ psi} = \left(20.7\dfrac{\text{lb}}{\text{in}^2}\right)\left(\dfrac{12\text{ in.}}{1\text{ ft}}\right)^2 = 2980.8\text{ lb/ft}^2$ and $T = 70°\text{F} + 460 = 530°R$

$$p = \rho RT$$

$$(6\text{ psi} + 14.7\text{ psi})\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 = \rho(1130\text{ ft}\cdot\text{lb/slug}\cdot\text{R})(70° + 460)$$

$$\rho = 0.004977\text{ slug/ft}^3$$

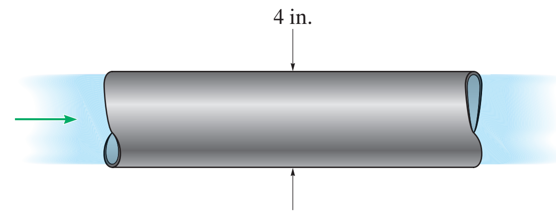
$$\dot{m} = \rho VA$$

$$= (0.004977\text{ slug/ft}^3)(20\text{ ft/s})(\pi)\left(\frac{2}{12}\text{ft}\right)^2$$

$$= 8.69(10^{-3})\text{ slug/s} \qquad \textbf{Ans.}$$

Ans:
$8.69(10^{-3})$ slug/s

***4–16.** Carbon dioxide gas flows through the 4-in.-diameter duct. If it has an average velocity of 10 ft/s and the gage pressure is maintained at 8 psi, plot the variation of mass flow (vertical axis) versus temperature for the temperature range 0°F ≤ T ≤ 100°F. Give values for increments of $\Delta T = 20°$.



SOLUTION

From Appendix A, $R = 1130 \text{ ft}\cdot\text{lb}/(\text{slug}\cdot\text{R})$ for CO_2. Here the absolute pressure is $p = p_{atm} + p_g = 14.7 \text{ psi} + 8 \text{ psi} = \left(22.7\dfrac{\text{lb}}{\text{in}^2}\right)\left(\dfrac{12 \text{ in.}}{1 \text{ ft}}\right)^2 = 3268.8 \text{ lb/ft}^2$ and $T = T_F + 460$.

$$p = \rho R T$$

$$3268.8 \text{ lb/ft}^2 = \rho\big(1130 \text{ ft}\cdot\text{lb}/(\text{slug}\cdot\text{R})\big)(T_F + 460)$$

$$\rho = \left(\frac{2.8927}{T_F + 460}\right) \text{slug/ft}^3$$

The mass flow is

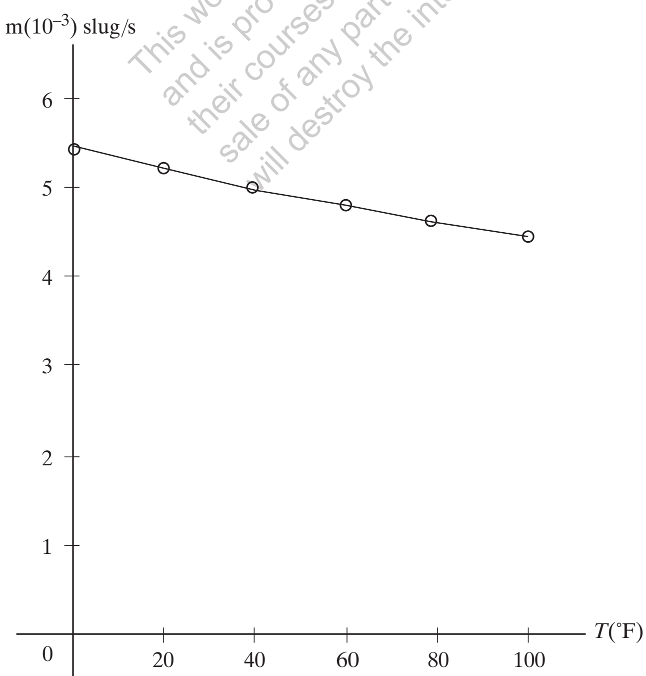
$$\dot{m} = \rho V A$$

$$\dot{m} = \left[\left(\frac{2.8927}{T_F + 460}\right) \text{slug/ft}^3\right](10 \text{ ft/s})\left[\pi\left(\frac{2}{12}\text{ft}\right)^2\right]$$

$$\dot{m} = \left(\frac{2.5244}{T_F + 460}\right) \text{slug/s where } T_F \text{ is in °F.}$$

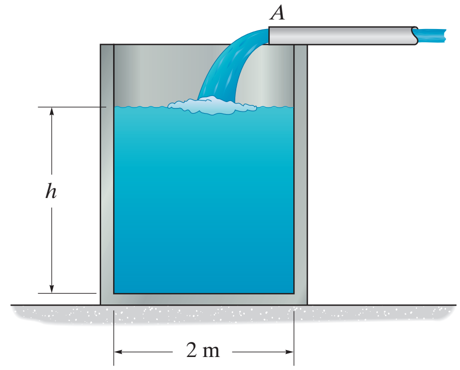
The plot of $\dot{m}$ vs T_F is shown in Fig. *a*.

T(°F)	0	20	40	60	80	100
$\dot{m}\,(10^{-3})$ slug/s	5.49	5.26	5.05	4.85	4.67	4.51



(a)

4–17. Water flowing at a constant rate fills the tank to a height of $h = 3$ m in 5 minutes. If the tank has a width of 1.5 m, determine the average velocity of the flow from the 0.2-m-diameter pipe at A.



SOLUTION

The volume of water in the tank when $t = 5(60)\text{ s} = 300\text{ s}$ is

$$\forall = (3\text{ m})(2\text{ m})(1.5\text{ m}) = 9\text{ m}^3$$

Thus, the discharge through the pipe at A is

$$Q = \frac{\forall}{t} = \frac{9\text{ m}^3}{300\text{ s}} = 0.03\text{ m}^3/\text{s}$$

Then, the average velocity of the water flow through A is

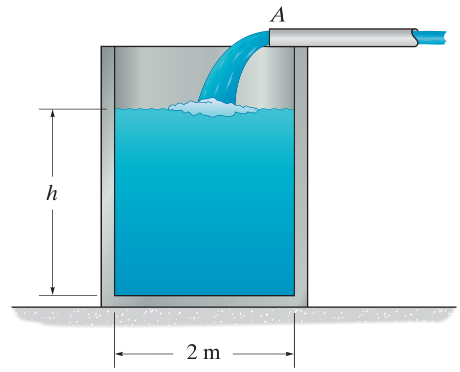
$$Q = VA$$

$$0.03\text{ m}^3/\text{s} = V(\pi)(0.1\text{ m})^2$$

$$V = 0.955\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
0.955 m/s

4–18. Water flows through the pipe at a constant average velocity of 0.5 m/s. Determine the relation between the time needed to fill the tank to a depth of $h = 3$ m and the diameter D of the pipe at A. The tank has a width of 1.5 m. Plot the time in minutes (vertical axis) versus the diameter $0.05 \text{ m} \leq D \leq 0.25 \text{ m}$. Give values for increments of $\Delta D = 0.05$ m.



SOLUTION

The volume of water filled to $h = 3$ m in time t is

$$\forall = (2\text{ m})(1.5\text{ m})(3\text{ m}) = 9\text{ m}^3$$

Thus, the discharge through the pipe is

$$Q = \frac{\forall}{t} = \left(\frac{9}{t}\right)\text{ m}^3/\text{s}$$

Applying

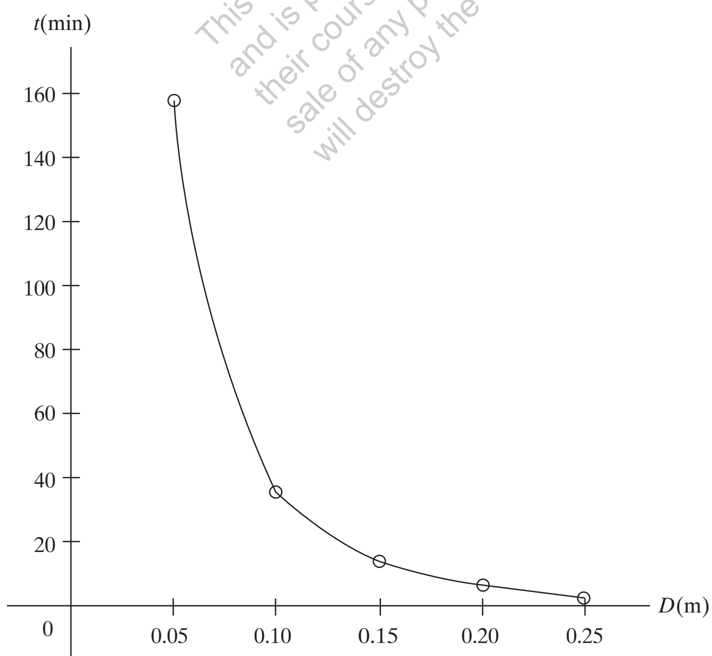
$$Q = VA; \qquad \frac{9}{t} = (0.5\text{ m/s})\left[\frac{\pi}{4}(D^2)\right]$$

$$t = \left[\left(\frac{22.9183}{D^2}\right)\text{s}\right]\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$t = \left(\frac{0.382}{D^2}\right)\text{ min where } d \text{ is in } m \qquad \textbf{Ans.}$$

The plot of t vs D is shown in Fig. a

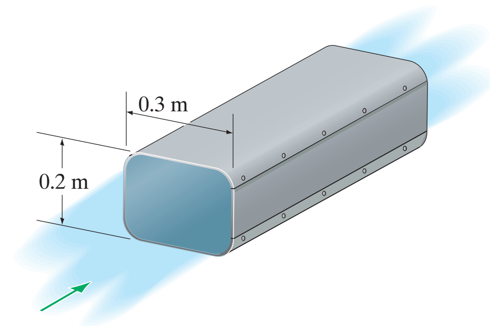
D(m)	0.05	0.10	0.15	0.20	0.25
t(min)	153	38.2	17.0	9.55	6.11



(a)

Ans:
$t = \left(\frac{0.382}{D^2}\right)$ min, where d is in m

4–19. Determine the mass flow of air in the duct if it has an average velocity of 15 m/s. The air has a temperature of 30°C, and the (gage) pressure is 50 kPa.



SOLUTION

From Appendix A, $R = 286.9\ \text{J/kg}\cdot\text{K}$ for air.

$$p = \rho RT$$

$$(50 + 101)(10^3)\,\text{N/m}^2 = \rho(286.9\ \text{J/(kg}\cdot\text{K)}(30° + 273)$$

$$\rho = 1.737\ \text{kg/m}^3$$

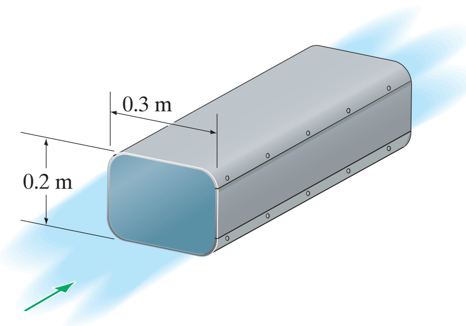
$$\dot{m} = \rho VA$$

$$= (1.737\ \text{kg/m}^3)(15\ \text{m/s})(0.3\ \text{m})(0.2\ \text{m})$$

$$= 1.56\ \text{kg/s} \qquad \textbf{Ans.}$$

Ans:
1.56 kg/s

***4–20.** Air flows through the duct at an average velocity of 20 m/s. If the temperature is maintained at 20°C, plot the variation of the mass flow (vertical axis) versus the (gage) pressure for the range of $0 \le p \le 100$ kPa. Give values for increments of $\Delta p = 20$ kPa. The atmospheric pressure is 101.3 kPa.



SOLUTION

From Appendix A, $R = 286.9\ \text{J/(kg}\cdot\text{K)}$ for air. Here, the absolute pressure is $p = p_g + p_{atm} = (p_g + 101.3)$ kPa

$$p = \rho R T$$

$$(p_g + 101.3)(10^3)\ \text{N/m}^2 = \rho(286.9\ \text{J/(kg}\cdot\text{K)})(20 + 273)\ \text{K}$$

$$\rho = \left[0.01190(p_g + 101.3)\right]\ \text{kg/m}^3$$

The mass flow is

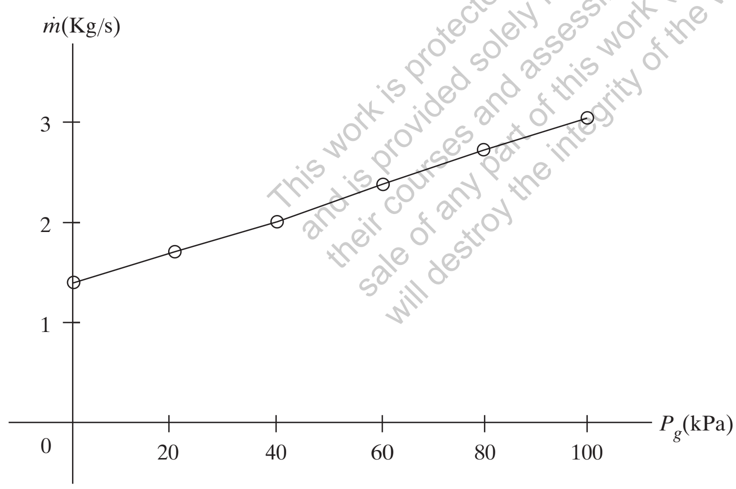
$$\dot{m} = \rho V A$$

$$\dot{m} = \left[0.01190(p_g + 101.3)\text{kg/m}^3\right](20\ \text{m/s})\left[0.3\ \text{m}(0.2\ \text{m})\right]$$

$$\dot{m} = [0.01428(p_g + 101.3)]\text{kg/s where } p_g \text{ is in kPa}$$

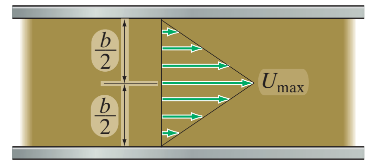
The plot of $\dot{m}$ vs p_g is shown in Fig. *a*.

P_g(kPa)	0	20	40	60	80	100
$\dot{m}$(kg/s)	1.45	1.73	2.02	2.30	2.59	2.87



(a)

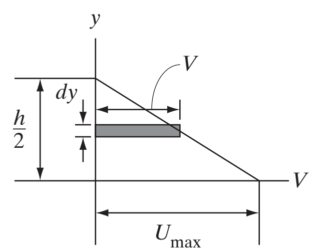
4–21. A fluid flowing between two plates has a velocity profile that is assumed to be linear as shown. Determine the average velocity and volumetric discharge in terms of U_{max}. The plates have a width of w.



SOLUTION

The velocity profile in Fig. *a* can be expressed as

$$\frac{v - 0}{y - \frac{h}{2}} = \frac{U_{max} - 0}{0 - \frac{h}{2}}; \qquad v = U_{max}\left(1 - \frac{2}{h}y\right)$$



(a)

The differential rectangular element of the thickness dy on the cross section will be considered. Thus, $dA = wdy$.

$$Q = \int_A v = dA$$

$$= 2\int_0^{\frac{h}{2}}\left[U_{max}\left(1 - \frac{2}{h}y\right)\right](wdy)$$

$$= 2wU_{max}\int_0^{\frac{h}{2}}\left(1 - \frac{2}{h}y\right)dy$$

$$= 2wU_{max}\left(y - \frac{y^2}{h}\right)\Bigg|_0^{\frac{h}{2}}$$

$$= \frac{wU_{max}h}{2} \qquad \textbf{Ans.}$$

Also,

$$Q = \int_A v \cdot d\mathbf{A} = \text{volume under velocity diagram}$$

$$= \frac{1}{2}(h)(w)(U_{max}) = \frac{wU_{max}h}{2} \qquad \textbf{Ans.}$$

Therefore,

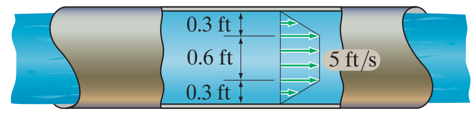
$$V = \frac{Q}{A} = \frac{wU_{max}h}{2(w)(h)} = \frac{U_{max}}{2} \qquad \textbf{Ans.}$$

Ans:

$Q = \frac{wU_{max}h}{2}$

$V = \frac{U_{max}}{2}$

4–22. The velocity profile of a liquid flowing through the pipe is approximated by the truncated conical distribution as shown. Determine the average velocity of the flow. *Hint*: The volume of a cone is $\forall = \frac{1}{3}\pi r^2 h$.



SOLUTION

$$Q = \int_A \mathbf{V} \cdot d\mathbf{A} = \text{the volume enclosed by the velocity profile}$$

Here, this volume is equal to the volume of cone (1) minus that of cone (2) in Fig *a*. From the geometry of cone (1),

$$\frac{V_2}{5\text{ ft/s} + V_2} = \frac{0.3\text{ ft}}{0.6\text{ ft}}; \qquad V_2 = 5\text{ ft/s}$$

Then,

$$V_1 = 5\text{ ft/s} + V_2 = 10\text{ ft/s}$$

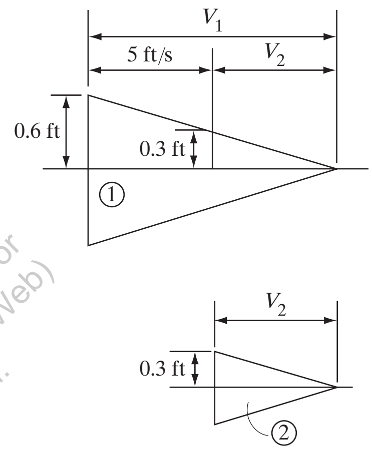
The volume of the cone can be computed by $\forall = \dfrac{1}{3}\pi r^2 h$. Then,

$$Q = \frac{1}{3}\pi(0.6\text{ ft})^2\,(10\text{ ft/s}) - \frac{1}{3}\pi\,(0.3\text{ ft})^2\,(5\text{ ft/s})$$

$$= 1.05\pi\text{ ft}^3\text{/s}$$

The average velocity is

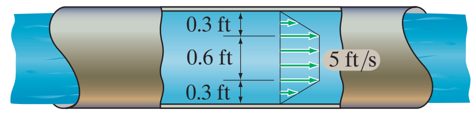
$$V = \frac{Q}{A} = \frac{1.05\pi}{\pi(0.6\text{ ft})^2} = 2.92\text{ ft/s}$$ **Ans.**



(a)

Ans:
2.92 ft/s

4–23. The velocity profile of a liquid flowing through the pipe is approximated by the truncated conical distribution as shown. Determine the mass flow if the liquid has a specific weight of $\gamma = 54.7\ \text{lb/ft}^3$. *Hint*: The volume of a cone is $\forall = \frac{1}{3}\pi r^2 h$.



SOLUTION

$$Q = \int_A \mathbf{V} \cdot d\mathbf{A} = \text{the volume enclosed by the velocity profile}$$

Here, this volume is equal to the volume of cone (1) minus that of code (2) in Fig. *a*. From the geometry of cone (1),

$$\frac{V_2}{5\ \text{ft/s} + V_2} = \frac{0.3\ \text{ft}}{0.6\ \text{ft}}; \qquad V_2 = 5\ \text{ft/s}$$

Then,

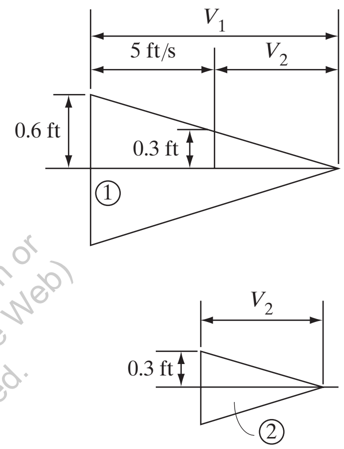
$$V_1 = 5\ \text{ft/s} + V_2 = 10\ \text{ft/s}$$

The volume of the cone can be computed by $\forall = \dfrac{1}{3}\pi r^2 h$. Then,

$$Q = \frac{1}{3}\pi(0.6\ \text{ft})^2(10\ \text{ft/s}) - \frac{1}{3}\pi(0.3\ \text{ft})^2(5\ \text{ft/s})$$
$$= 1.05\pi\ \text{ft}^3/\text{s}$$

Then the mass flow is

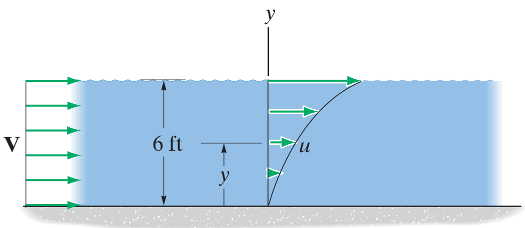
$$\dot{m} = \rho Q = \left(\frac{54.7\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(1.05\pi\ \text{ft}^3/\text{s}) = 5.60\ \text{slug/s} \qquad \textbf{Ans.}$$



(a)

Ans:
5.60 slug/s

***4–24.** Determine the average velocity V of a very viscous fluid that enters the 8-ft-wide rectangular open channel and eventually forms the velocity profile that is approximated by $u = 0.8(1.25y + 0.25y^2)$ ft/s, where y is in feet.



SOLUTION

Here, $dA = b\,dy$. Thus, the discharge is

$$Q = \int_A v\,dA = \int_0^{6\text{ ft}} 0.8(1.25y + 0.25y^2)(8dy)$$

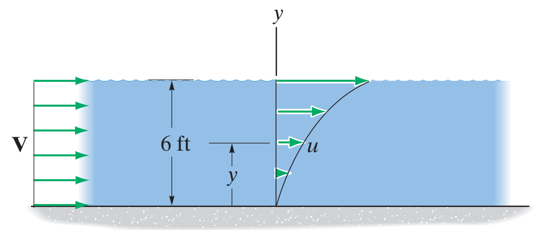
$$= 0.8(8)(0.625y^2 + 0.08333y^3)\Big|_0^{6\text{ ft}}$$

$$= 32.4(8)$$

The average velocity is

$$V = \frac{Q}{A} = \frac{32.4(8)}{6(8)} = 5.40\text{ ft/s}$$ **Ans.**

4–25. Determine the mass flow of a very viscous fluid that enters the 3-ft-wide rectangular open channel and eventually forms the velocity profile that is approximated by $u = 0.8\left(1.25y + 0.25y^2\right)$ ft/s, where y is in ft. Take $\gamma = 40\ \text{lb/ft}^3$.



SOLUTION

Here, $dA = bdy = 3dy$. Thus, the mass flow is

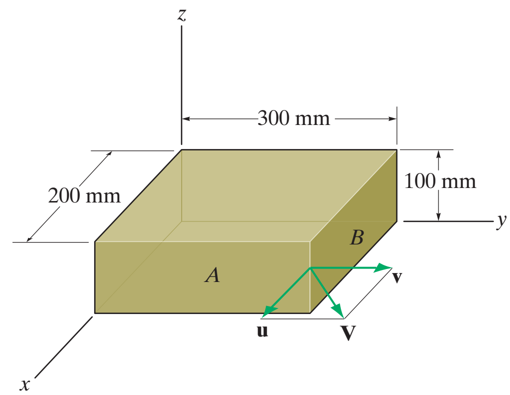
$$\dot{m} = \int_A \rho v dA = \left(\frac{40\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)\int_0^{6\ \text{ft}} 0.8\left(1.25y + 0.25y^2\right)(3dy)$$

$$= 2.9814\left(0.625y^2 + 0.08333y^3\right)\Big|_0^{6\ \text{ft}}$$

$$= 120.75\ \text{slug/s} = 121\ \text{slug/s}$$ **Ans.**

Ans:
121 slug/s

4–26. The velocity field for a flow is defined by $u = (6x)$ m/s and $v = (4y^2)$ m/s, where x and y are in meters. Determine the discharge through the surface at A and at B.



SOLUTION

The velocity perpendicular to and passing through surface A, where $x = 0.2$ m, is

$$V_A = u = (6x)\text{ m/s} = [6(0.2)]\text{ m/s} = 1.2\text{ m/s}$$

The velocity perpendicular to and passing through surface B, where $y = 0.3$ m, is

$$V_B = v = (4y^2)\text{ m/s} = \left[4(0.3^2)\right]\text{ m/s} = 0.36\text{ m/s}$$

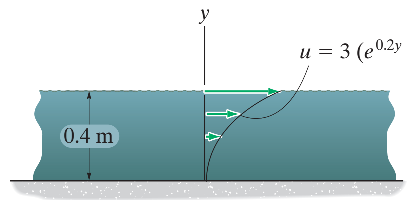
The areas of surfaces A and B are $A_A = (0.3\text{ m})(0.1\text{ m}) = 0.03\text{ m}^2$ and $A_B = (0.2\text{ m})(0.1\text{ m}) = 0.02\text{ m}^2$. Thus,

$$Q_A = V_A A_A = (1.2\text{ m/s})(0.03\text{ m}^2) = 0.036\text{ m}^3/\text{s} \quad \textbf{Ans.}$$

$$Q_B = V_B A_B = (0.36\text{ m/s})(0.02\text{ m}^2) = 0.0072\text{ m}^3/\text{s} \quad \textbf{Ans.}$$

Ans:
$Q_A = 0.036\text{ m}^3/\text{s}$
$Q_B = 0.0072\text{ m}^3/\text{s}$

4–27. The velocity profile in a channel carrying a very viscous liquid is approximated by $u = 3\left(e^{0.2y} - 1\right)$ m/s, where y is in meters. If the channel is 1 m wide, determine the volumetric discharge from the channel.



SOLUTION

The differential rectangular element of thickness dy on the cross section will be considered, Fig. a. Since the channel has a constant width of 1 m, then the area of this element is $dA = (1\text{ m})dy = dy$.

$$Q = \int_A \mathbf{V} \cdot d\mathbf{A}$$

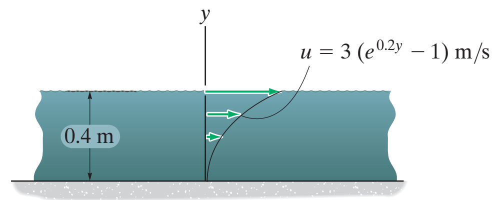
$$= \int_0^{0.4\text{ m}} \left[3\left(e^{0.2y} - 1\right)\right](dy)$$

$$= 3\left(\frac{e^{0.2y}}{0.2} - y\right)\Bigg|_0^{0.4\text{ m}}$$

$$= 0.0493\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$0.0493\text{ m}^3/\text{s}$

***4–28.** The velocity profile in a channel carrying a very viscous liquid is approximated by $u = 3\left(e^{0.2y} - 1\right)$ m/s, where y is in meters. Determine the average velocity of the flow. The channel has a width of 1 m.



SOLUTION

The differential rectangular element of thickness dy on the cross section will be considered, Fig. a. Since the channel has a constant width of 1 m, then the area of this element is $dA = (1\text{ m})dy = dy$.

$$Q = \int_A \mathbf{V} \cdot d\mathbf{A}$$

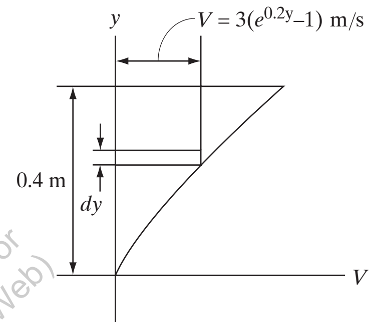
$$= \int_0^{0.4\text{ m}} \left[3\left(e^{0.2y} - 1\right)\right](dy)$$

$$= 3\left(\frac{e^{0.2y}}{0.2} - y\right)\Bigg|_0^{0.4\text{ m}}$$

$$= 0.04931\text{ m}^3/\text{s}$$

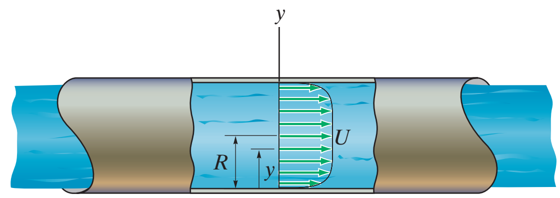
The average velocity is

$$V = \frac{Q}{A} = \frac{0.04931\text{ m}^3/\text{s}}{0.4\text{ m}(1\text{ m})} = 0.123\text{ m/s} = 12.3\text{ mm/s} \qquad \textbf{Ans.}$$

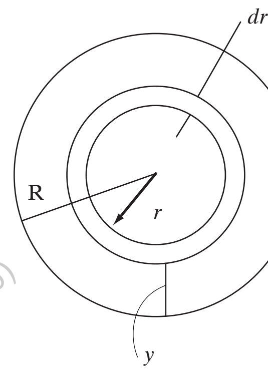


(a)

4–29. The velocity profile for a fluid within the circular pipe for fully developed turbulent flow is modeled using Prandtl's one-seventh power law $u = U\left(y/R\right)^{1/7}$. Determine the average velocity for this case.



SOLUTION



$$\int u dA = \int_0^R U\left(\frac{y}{R}\right)^{\frac{1}{7}} 2\pi r dr, \qquad r = R - y$$

$$= \frac{2\pi U}{R^{\frac{1}{7}}} \int_R^0 y^{\frac{1}{7}}(R - y)(-dy)$$

$$= \frac{2\pi U}{R^{\frac{1}{7}}} \int_0^R \left(R y^{\frac{1}{7}} - y^{\frac{8}{7}}\right) dy$$

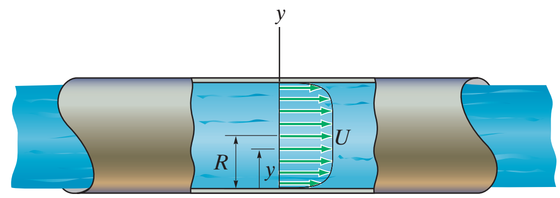
$$= \frac{2\pi U}{R^{\frac{1}{7}}} \left[\frac{7}{8} R y^{\frac{8}{7}} - \frac{7}{15} y^{\frac{15}{7}}\right]_0^R$$

$$= \frac{49}{60}\pi R^2 U$$

$$\bar{u} = \frac{\int u dA}{\int dA} = \frac{\left(\frac{49}{60}\right)\pi R^2 U}{\pi R^2} = \frac{49}{60} U \qquad \textbf{Ans.}$$

Ans:
$\frac{49}{60} U$

4–30. The velocity profile for a fluid within the circular pipe for fully developed turbulent flow is modeled using Prandtl's seventh-power law $u = U\left(y/R\right)^{1/7}$. Determine the mass flow of the fluid if it has a density ρ.



SOLUTION

Referring to Fig. a, $r = R - y$. Thus $dr = -dy$. Also the area of the differential element shown shaded is $dA = 2\pi r dr$. Thus, the mass flow rate is

$$\dot{m} = \int_A \rho v dA = \rho \int_0^R U\left(\frac{y}{R}\right)^{\frac{1}{7}}(2\pi r dr)$$

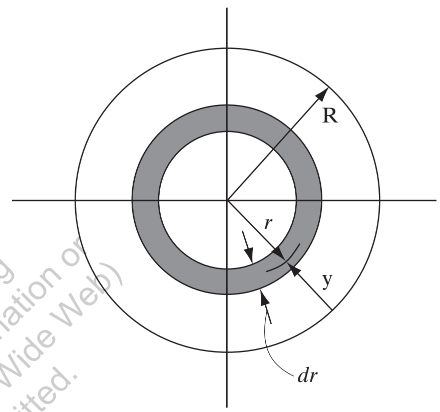
$$= \frac{2\pi\rho U}{R^{\frac{1}{7}}}\int_R^0 y^{\frac{1}{7}}(R - y)(-dy)$$

$$= \frac{2\pi\rho U}{R^{\frac{1}{7}}}\int_0^R \left(Ry^{\frac{1}{4}} - y^{\frac{8}{7}}\right) dy$$

$$= \frac{2\pi\rho U}{R^{\frac{1}{7}}}\left(\frac{7}{8}Ry^{\frac{8}{7}} - \frac{7}{15}y^{\frac{15}{7}}\right)\Bigg|_0^R$$

$$= \frac{2\pi\rho U}{R^{\frac{1}{7}}}\left(\frac{49}{120}R^{\frac{15}{7}}\right)$$

$$= \frac{49\pi}{60}\rho U R^2$$ **Ans.**

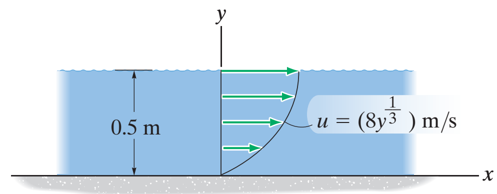


(a)

Ans:

$\dot{m} = \frac{49\pi}{60}\rho U R^2$

4–31. The velocity profile for a liquid in the channel is determined experimentally and found to fit the equation $u = (8y^{1/3})\,\text{m/s}$, where y is in meters. Determine the volumetric discharge if the width of the channel is 0.5 m.



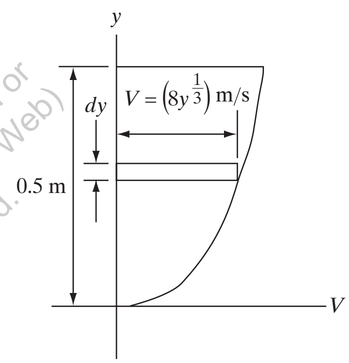
SOLUTION

The differential rectangular element of thickness dy on the cross section will be considered, Fig. *a*. Since the channel has a constant width of 0.5 m, the area of this element is $dA = (0.5\text{ m})dy = 0.5dy$.

$$Q = \int_A \mathbf{V} \cdot d\mathbf{A}$$

$$Q = \int_0^{0.5} \left(8y^{\frac{1}{3}}\right)(0.5\text{ m})dy = 3y^{\frac{4}{3}}\Big|_0^{0.5}$$

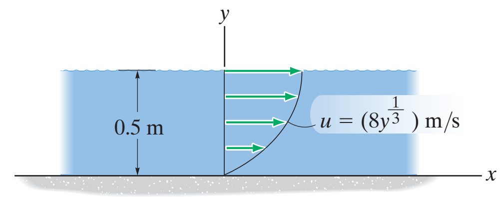
$$Q = 1.19\text{ m}^3/\text{s}$$ **Ans.**



(a)

Ans:
119 m^3/s

***4–32.** The velocity profile for a liquid in the channel is determined experimentally and found to fit the equation $u = (8y^{1/3})$ m/s, where y is in meters. Determine the average velocity of the liquid. The channel has a width of 0.5 m.



SOLUTION

The differential rectangular element of thickness dy on the cross section will be considered, Fig. a. Since the channel has a constant width of 0.5 m, the area of this element is $dA = (0.5\text{ m})\,dy = 0.5\,dy$.

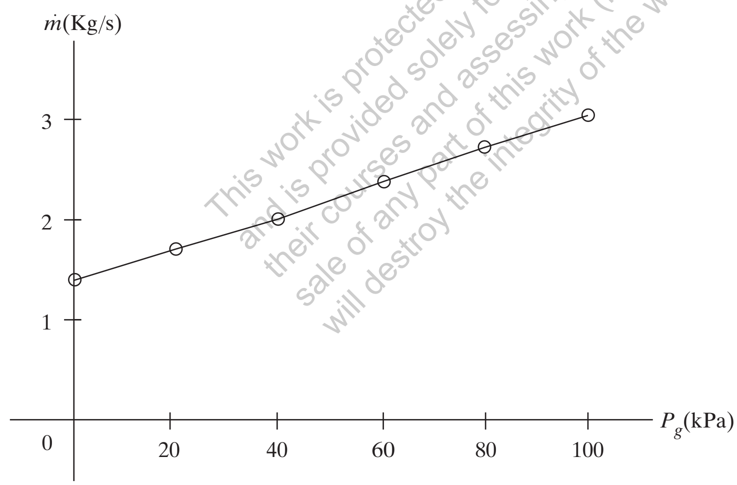
$$Q = \int_A \mathbf{V} \cdot d\mathbf{A}$$

$$Q = \int_0^{0.5\text{ m}} \left(8y^{\frac{1}{3}}\right)(0.5\text{ m})dy = 3y^{\frac{4}{3}}\Big|_0^{0.5} = 1.191\text{ m}^3/\text{s}$$

The average velocity is

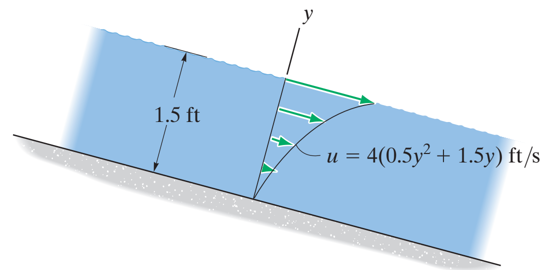
$$V = \frac{Q}{A} = \frac{1.191\text{ m}^3/\text{s}}{(0.5\text{ m})(0.5\text{ m})} = 4.76\text{ m/s} \qquad \textbf{Ans.}$$

P_g(kPa)	0	20	40	60	80	100
$\dot{m}$(kg/s)	1.45	1.73	2.02	2.30	2.59	2.87



(a)

4–33. A very viscous liquid flows down the inclined rectangular channel such that its velocity profile is approximated by $u = 4(0.5y^2 + 1.5y)$ ft/s, where y is in feet. Determine the mass flow if the channel width is 2 ft. Take $\gamma = 60\ \text{lb/ft}^3$.



SOLUTION

The differential rectangular element of thickness dy on the cross section will be considered, Fig. *a*. Since the channel has a constant width of 2 ft, the area of this element is $dA = (2\text{ ft})dy$. Thus,

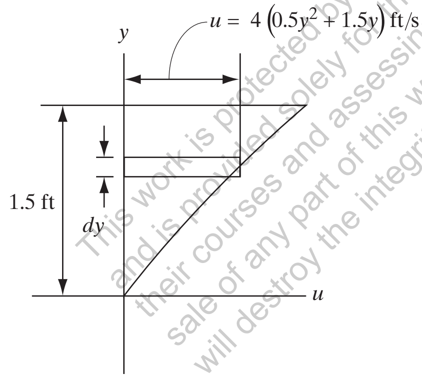
$$\dot{m} = \int_A \rho \mathbf{V} \cdot d\mathbf{A}$$

$$= \left(\frac{60}{32.2}\ \text{slug/ft}^3\right)\int_0^{1.5\text{ ft}} \left[4\left(0.5y^2 + 1.5y\right)\text{ft/s}\right]\left[(2\text{ ft})dy\right]$$

$$= 14.91\int_0^{1.5\text{ ft}} \left(0.5y^2 + 1.5y\right)dy$$

$$= 14.91\left(\frac{0.5y^3}{3} + 0.75y^2\right)\Bigg|_0^{1.5\text{ ft}}$$

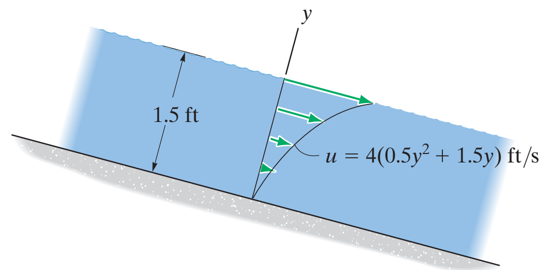
$$= 33.5\ \text{slug/s}$$ **Ans.**



(a)

Ans:
33.5 slug/s

4–34. A very viscous liquid flows down the inclined rectangular channel such that its velocity profile is approximated by $u = 4\left(0.5y^2 + 1.5y\right)$ ft/s, where y is in feet. Determine the average velocity of the liquid if the channel width is 2 ft.



SOLUTION

The differential rectangular element of thickness dy on the cross-section will be considered, Fig. a. Since the channel has a constant width of 2 ft, the area of this element is $dA = 2dy$. Thus,

$$Q = \int_A u\,dA = \int_0^{1.5\text{ ft}} 4\left(0.5y^2 + 1.5y\right)(2dy)$$

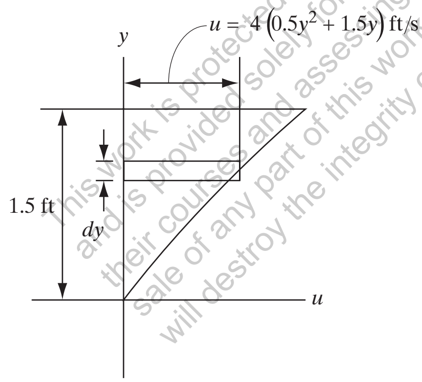
$$= 8\int_0^{1.5\text{ ft}} \left(0.5y^2 + 1.5y\right)dy$$

$$= 8\left(\frac{0.5}{3}y^3 + 0.75y^2\right)\Bigg|_0^{1.5\text{ ft}}$$

$$= 18\text{ ft}^3/\text{s}$$

Thus, the average velocity is

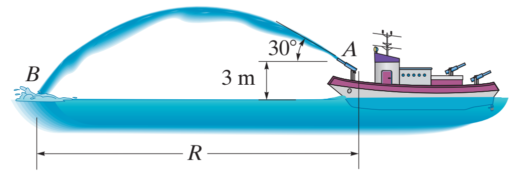
$$V = \frac{Q}{A} = \frac{18}{1.5(2)} = 6.00\text{ ft/s} \qquad \textbf{Ans.}$$



(a)

Ans:
6.00 ft/s

4–35. Determine the volumetric flow through the 50-mm-diameter nozzle of the fire boat if the water stream reaches point B, which is $R = 24$ m from the boat. Assume the boat does not move.



SOLUTION

The xy coordinate system with origin at A is established as shown in Fig. a.

Horizontal Motion.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (V_A)_x t$$

$$24 = 0 + (V_A \cos 30°)t$$

$$t = \frac{27.7128}{V_A} \qquad \textbf{(1)}$$

Vertical Motion.

$$(+\uparrow) \qquad s_y = (s_0)_y + (V_A)_y t + \frac{1}{2} a_c t^2$$

$$-3 = 0 + (V_A \sin 30°)t + \frac{1}{2}(-9.81 \text{ m/s}^2)t^2$$

$$4.905t^2 - 0.5V_A t - 3 = 0 \qquad \textbf{(2)}$$

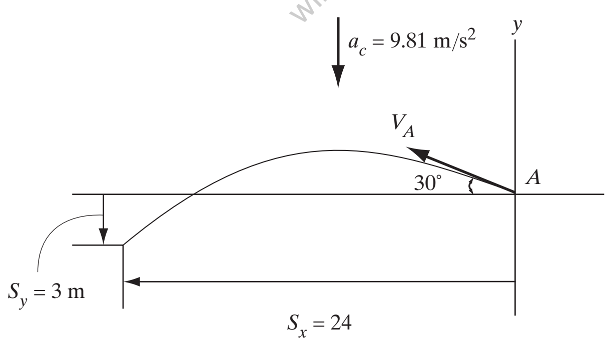
Solving Eqs. (1) and (2),

$$t = 1.854 \text{ s}$$
$$V_A = 14.95 \text{ m/s}$$

Using the result of V_A,

$$Q = V_A A_A = (14.95 \text{ m/s})\left[\frac{\pi}{4}(0.05 \text{ m})^2\right]$$

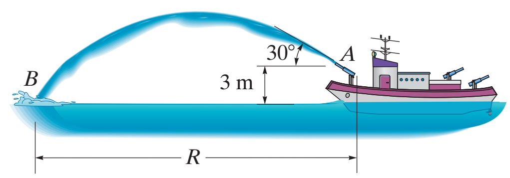
$$= 0.0294 \text{ m}^3/\text{s} \qquad \textbf{Ans.}$$



(a)

Ans:
$0.0294 \text{ m}^3/\text{s}$

***4–36.** Determine the volumetric flow through the 50-mm-diameter nozzle of the fire boat as a function of the distance R of the water stream. Plot this function of flow (vertical axis) versus the distance for $0 \le R \le 25$ m. Give values for increments of $\Delta R = 5$ m. Assume the boat does not move.



SOLUTION

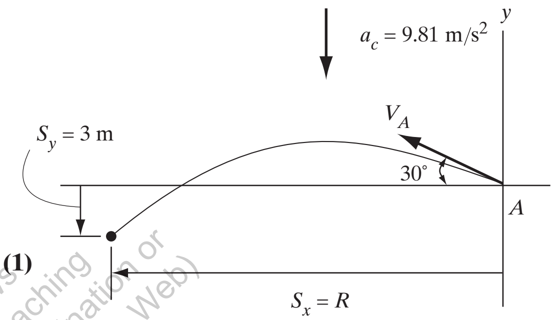
The x-y coordinate system with origin at A is established as shown in Fig. a.

Horizontal Motion

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (V_A)_x t$$

$$R = 0 + (V_A \cos 30°)t$$

$$t = \frac{R}{V_A \cos 30°} = \frac{2R}{\sqrt{3}V_A} \qquad \textbf{(1)}$$



(a)

Vertical Motion

$$(+\uparrow) \qquad s_y = (s_0)_y + (V_A)_y t + \frac{1}{2}a_c t^2$$

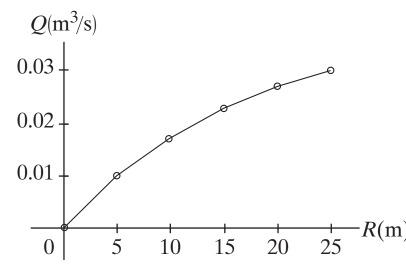
$$-3 \text{ m} = 0 + (V_A \sin 30°)t + \frac{1}{2}(-9.81 \text{ m/s}^2)t^2$$

$$4.905t^2 - 0.5V_A t - 3 = 0 \qquad \textbf{(2)}$$

Substitute Eq (1) into (2)

$$4.905\left(\frac{2R}{\sqrt{3}V_A}\right)^2 - 0.5V_A\left(\frac{2R}{\sqrt{3}V_A}\right) - 3 = 0$$

$$V_A = \left(\frac{6.54R^2}{0.5774R + 3}\right)^{\frac{1}{2}}$$



(b)

Thus, the discharge is

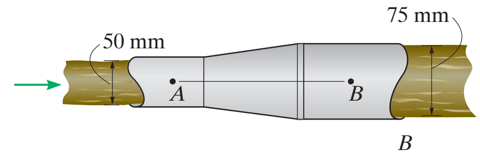
$$Q = VA; \qquad Q = \left(\frac{6.54R^2}{0.5774R + 3}\right)^{\frac{1}{2}}\left[\pi(0.025 \text{ m})^2\right]$$

$$Q = \left[\frac{0.00502R}{(0.5774R + 3)^{\frac{1}{2}}}\right] \text{m}^3/\text{s where } R \text{ is in } m \qquad \textbf{Ans.}$$

The plot of Q vs R is shown in Fig. b

R(m)	0	5	10	15	20	25
Q(m^3/s)	0	0.0103	0.0170	0.0221	0.0263	0.0301

4–37. For a short time, the flow of carbon tetrachloride through the circular pipe transition can be expressed as $Q = (0.8t + 5)(10^{-3})\ \mathrm{m^3/s}$, where t is in seconds. Determine the average velocity and average acceleration of a particle located at A and B when $t = 2$ s.



SOLUTION

When $t = 2$ s,

$$Q = [0.8(2) + 5](10^{-3})\ \mathrm{m^3/s} = 6.6(10^{-3})\ \mathrm{m^3/s}$$

Thus, the velocities at A and B are

$$V_A = \frac{Q}{A_A} = \frac{6.6(10^{-3})\ \mathrm{m^3/s}}{\pi(0.025\ \mathrm{m})^2} = 3.36\ \mathrm{m/s} \qquad \textbf{Ans.}$$

$$V_B = \frac{Q}{A_B} = \frac{6.6(10^{-3})\ \mathrm{m^3/s}}{\pi(0.0375\ \mathrm{m})^2} = 1.49\ \mathrm{m/s} \qquad \textbf{Ans.}$$

Here,

$$V_A = \frac{Q}{A_A} = \frac{(0.8\,t + 5)(10^{-3})\ \mathrm{m^3/s}}{\pi(0.025\ \mathrm{m})^2} = \left[\frac{1.6(0.8\,t + 5)}{\pi}\right]\mathrm{m/s}$$

$$V_B = \frac{Q}{A_B} = \frac{(0.8\,t + 5)(10^{-3})\ \mathrm{m^3/s}}{\pi(0.0375\ \mathrm{m})^2} = \left[\frac{0.7111(0.8\,t + 5)}{\pi}\right]\mathrm{m/s}$$

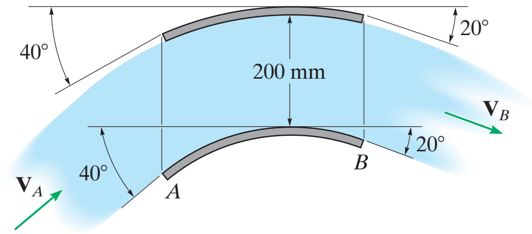
Thus

$$a_A = \frac{dV_A}{dt} = \frac{1.6}{\pi}(0.8) = 0.407\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

$$a_B = \frac{dV_B}{dt} = \frac{0.7111}{\pi}(0.8) = 0.181\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

Ans:
$V_A = 3.36\ \mathrm{m/s}$, $V_B = 1.49\ \mathrm{m/s}$
$a_A = 0.407\ \mathrm{m/s^2}$, $a_B = 0.181\ \mathrm{m/s^2}$

4–38. Air flows through the gap between the vanes at $0.75\ \text{m}^3/\text{s}$. Determine the average velocity of the air passing through the inlet A and the outlet B. The vanes have a width of 400 mm and the vertical distance between them is 200 mm.



SOLUTION

The discharge can be calculated using

$$Q = \int_{cs} \mathbf{V}_a \cdot d\mathbf{A}$$

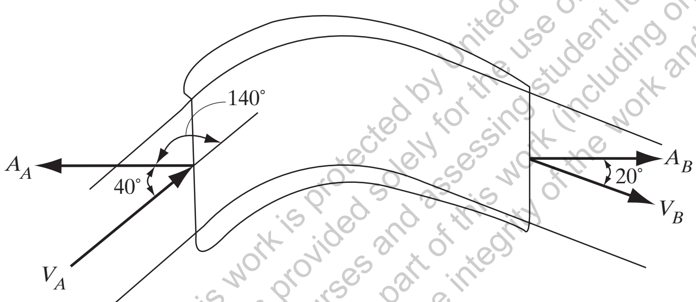
Here, the average velocities will be used. Referring to Fig. a,

$$Q_A = \mathbf{V}_A \cdot \mathbf{A}_A; \qquad -0.75\ \text{m}^3/\text{s} = (V_A \cos 140°)\left[(0.2\ \text{m})(0.4\ \text{m})\right]$$

$$V_A = 12.2\ \text{m/s} \qquad \textbf{Ans.}$$

$$Q_B = \mathbf{V}_B \cdot \mathbf{A}_B; \qquad 0.75\ \text{m}^3/\text{s} = (V_B \cos 20°)\left[(0.2\ \text{m})(0.4\ \text{m})\right]$$

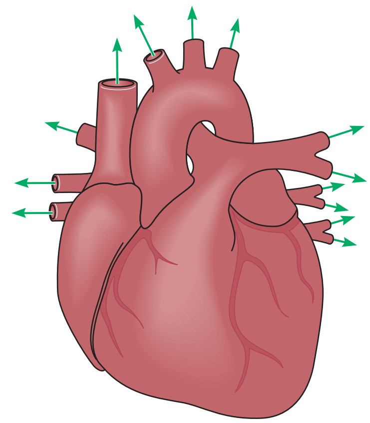
$$V_B = 9.98\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$V_A = 12.2\ \text{m/s}$, $V_B = 9.98\ \text{m/s}$

4–39. The human heart has an average discharge of $0.1\left(10^{-3}\right)\ \mathrm{m^3/s}$, determined from the volume of blood pumped per beat and the rate of beating. Careful measurements have shown that blood cells pass through the capillaries at about 0.5 mm/s. If the average diameter of a capillary is 6 μm, estimate the number of capillaries that must be in the human body.



SOLUTION

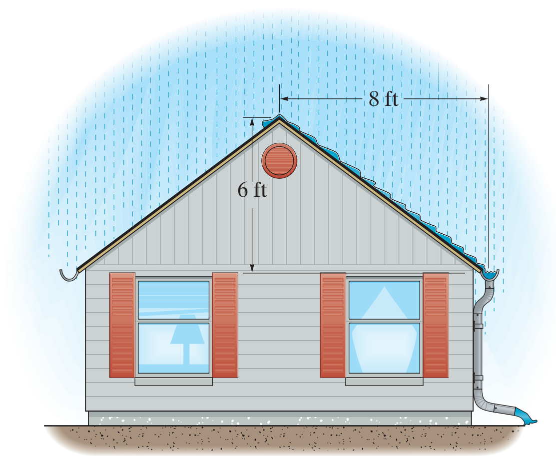
n is the number of the capillaries in the human body. From the discharge of the blood from heart,

$$Q = nAV; \quad 0.1\left(10^{-3}\right)\mathrm{m^3/s} = n\left\{\pi\left[3.0\left(10^{-6}\right)\mathrm{m}\right]^2\right\}\left[0.5\left(10^{-3}\right)\mathrm{m/s}\right]$$

$$n = 7.07\left(10^{9}\right)$$ **Ans.**

Ans:
$7.07\left(10^{9}\right)$

***4–40.** Rain falls vertically onto the roof of the house with an average speed of 12 ft/s. On one side the roof has a width of 15 ft and is sloped as shown. The water accumulates in the gutter and flows out the downspout at $6\ \text{ft}^3/\text{min}$. Determine the amount of falling rainwater in a cubic foot of air. Also, if the average radius of a drop of rain is 0.16 in., determine the number of raindrops in a cubic foot of air. *Hint*: The volume of a drop is $\forall = \frac{4}{3}\pi r^3$.



SOLUTION

The discharge Q_{DP} of the downspout is equal to the discharge of the rain water contained in the air of volume equal to that of the volume shown in Fig. *a*. Here,

$$Q_{DP} = \left(6\ \text{ft}^3/\text{min}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 0.1\ \text{ft}^3/\text{s}$$

And the volume of the air is

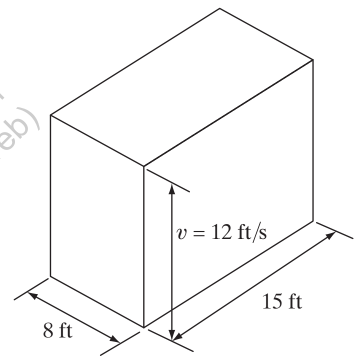
$$Q_a = (8\ \text{ft})(15\ \text{ft})\left(12\ \text{ft/s}\right) = 1440\ \text{ft}^3/\text{s}$$

In another words, $1440\ \text{ft}^3$ of air contains $0.1\ \text{ft}^3$ of rain water. Therefore, for $1\ \text{ft}^3$ of air it contains

$$\forall_w = \left(\frac{0.1\ \text{ft}^3}{1440\ \text{ft}^3}\right)\left(1\ \text{ft}^3\right) = 69.44\left(10^{-6}\right)\text{ft}^3 = 69.4\left(10^{-6}\right)\text{ft}^3 \qquad \textbf{Ans.}$$

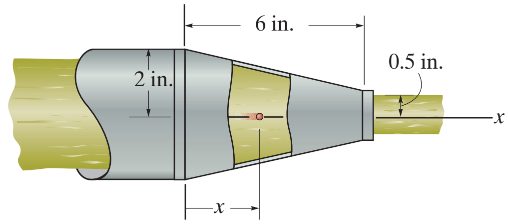
Then the number of rain drops in $1\ \text{ft}^3$ of air is

$$n = \frac{69.44\left(10^{-6}\right)\text{ft}^3}{\frac{4}{3}\pi\left(\frac{0.16}{12}\ \text{ft}\right)^3} = 6.994 = 7 \qquad \textbf{Ans.}$$



(a)

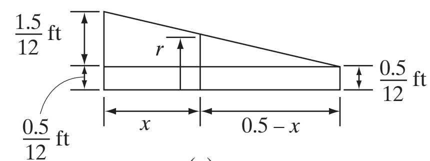
4–41. Acetate flows through the nozzle at $2\ \text{ft}^3/\text{s}$. Determine the time it takes for a particle on the x axis to pass through the nozzle, from $x = 0$ to $x = 6$ in. if $x = 0$ at $t = 0$. Plot the distance-versus-time graph for the particle.



SOLUTION

Since the flow is assumed to be one dimensional and incompressible, its velocity can be determined using

$$u = \frac{Q}{A}$$



(a)

From the geometry shown in Fig. *a*, the radius r of the nozzle's cross-section as a function of x is

$$r = \frac{0.5}{12}\ \text{ft} + \left(\frac{0.5 - x}{0.5}\right)\left(\frac{1.5}{12}\ \text{ft}\right) = \frac{1}{24}(4 - 6x)\ \text{ft}$$

Thus, the cross-sectional area of the nozzle is

$$A = \pi r^2 = \pi\left[\frac{1}{24}(4 - 6x)\right]^2 = \frac{\pi}{576}(4 - 6x)^2\ \text{ft}^2$$

Then

$$u = \frac{Q}{A} = \frac{2\ \text{ft}^3/\text{s}}{\frac{\pi}{576}(4 - 6x)^2\ \text{ft}^2} = \left[\frac{1152}{\pi(4 - 6x)^2}\right]\text{ft/s}$$

Using the definition of velocity and the initial condition of $x = 0$ at $t = 0$,

$$\frac{dx}{dt} = u = \frac{1152}{\pi(4 - 6x)^2}$$

$$\int_0^t dt = \frac{\pi}{1152}\int_0^x (4 - 6x)^2 dx$$

$$t = \frac{\pi}{1152}\int_0^x (36x^2 - 48x + 16)dx$$

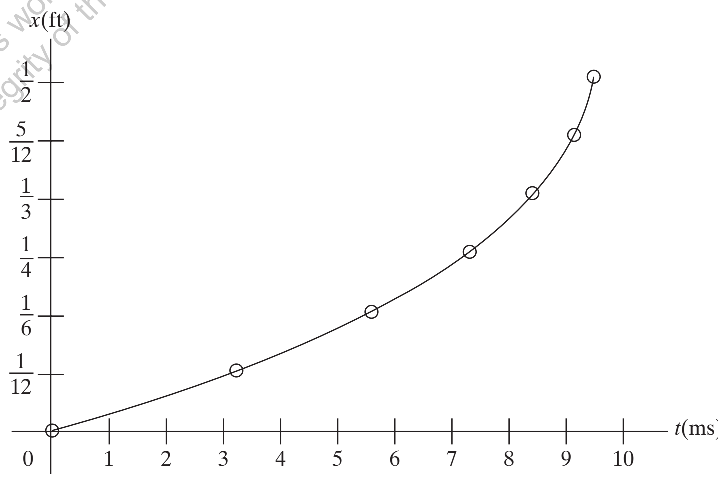
$$t = \frac{\pi}{1152}\left(12x^3 - 24x^2 + 16x\right)$$

$$t = \frac{\pi x}{288}\left(3x^2 - 6x + 4\right) \qquad \textbf{(1)}$$

When $x = \frac{6}{12}\ \text{ft} = 0.5\ \text{ft}$,

$$t = \frac{\pi(0.5)}{288}\left[3\left(0.5^2\right) - 6(0.5) + 4\right] = 9.5448\left(10^{-3}\right)$$

$$= 9.54\ \text{ms} \qquad \textbf{Ans.}$$



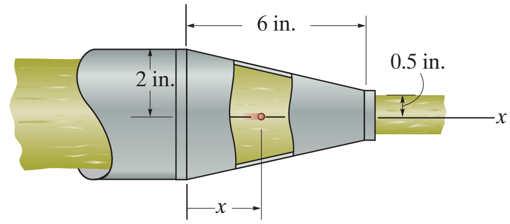
(b)

Using Eq (1), the following tabulation can be computed and the plot of x vs t is shown in Fig. *b*.

x(ft)	0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{12}$	$\frac{1}{2}$
t(ms)	0	3.20	5.61	7.33	8.48	9.18	9.54

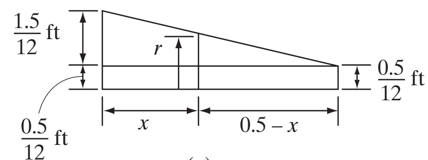
Ans:
9.54 ms

4–42. Acetate flows through the nozzle at $2\ \text{ft}^3/\text{s}$. Determine the velocity and acceleration of a particle on the x axis at $x = 3$ in. When $t = 0, x = 0$.



SOLUTION

Since the flow is assumed to be one dimensional and incompressible, its velocity can be determined using



(a)

$$u = \frac{Q}{A}$$

From the geometry shown in Fig. *a*, the radius r of the nozzle's cross-section as a function of x is

$$r = \frac{0.5}{12}\ \text{ft} + \left(\frac{0.5 - x}{0.5}\right)\left(\frac{1.5}{12}\ \text{ft}\right) = \frac{1}{24}(4 - 6x)\ \text{ft}$$

Thus,

$$A = \pi r^2 = \pi\left[\frac{1}{24}(4 - 6x)\right]^2 = \frac{\pi}{576}(4 - 6x)^2\ \text{ft}^2$$

Then,

$$u = \frac{Q}{A} = \frac{2\ \text{ft}^3/\text{s}}{\dfrac{\pi}{576}(4 - 6x)^2\ \text{ft}^2} = \left[\frac{1152}{\pi(4 - 6x)^2}\right]\ \text{ft/s}$$

Thus, when $x = \left(\dfrac{3}{12}\right)\ \text{ft} = 0.25\ \text{ft}$

$$u = \frac{1152}{\pi\left[4 - 6(0.25)\right]^2} = 58.67\ \text{ft/s} = 58.7\ \text{ft/s} \qquad \textbf{Ans.}$$

The acceleration can be determined using

$$a = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

Since Q is constant $\dfrac{\partial u}{\partial t} = 0$, that is, there is no local change at the print. Since,

$$\frac{\partial u}{\partial x} = \frac{1152}{\pi}\left[(-2)\left[(4 - 6x)^{-3}\right](-6)\right] = \frac{13824}{\pi}\left[\frac{1}{(4 - 6x)^3}\right]$$

Then

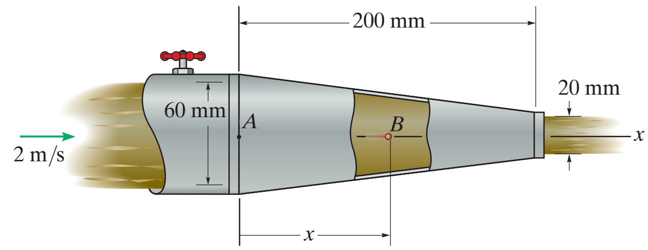
$$a = 0 + u\frac{\partial u}{\partial x}$$
$$= \frac{13824}{\pi}\left[\frac{u}{(4 - 6x)^3}\right]\ \text{ft/s}^2$$

When $x = \dfrac{2}{12}\ \text{ft} = 0.25\ \text{ft}$, $u = 58.67\ \text{ft/s}$. Then

$$a = \frac{13824}{\pi}\left\{\frac{58.67}{\left[4 - 6(0.25)\right]^3}\right\} = 16523\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$u = 58.7\ \text{ft/s}, a = 16\,523\ \text{ft/s}^2$

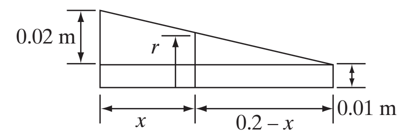
4–43. The tapered pipe transfers ethyl alcohol to a mixing tank such that a particle at A has a velocity of 2 m/s. Determine the velocity and acceleration of a particle at B, where $x = 75$ mm.



SOLUTION

From the geometry shown in Fig. a, the radius r of the pipe as a function of x is

$$r = 0.01\text{ m} + \left(\frac{0.2 - x}{0.2}\right)(0.02\text{ m}) = (0.03 - 0.1x)\text{ m}$$



(a)

Thus, the cross-sectional area of the pipe as a function of x is

$$A = \pi r^2 = \left[\pi(0.03 - 0.1x)^2\right]\text{ m}^2$$

The flow rate is constant which can be determined from

$$Q = u_A A_A = (2\text{ m/s})\left[\pi(0.03\text{ m})^2\right] = 1.8\pi\left(10^{-3}\right)\text{ m}^3/\text{s}$$

Thus, the velocity of the flow as a function of x is

$$u = \frac{Q}{A} = \frac{1.8\pi\left(10^{-3}\right)\text{ m}^3/\text{s}}{\pi(0.03 - 0.1x)^2\text{ m}^2} = \left[\frac{1.8\left(10^{-3}\right)}{(0.03 - 0.1x)^2}\right]\text{m/s}$$

At $x = 0.075$ m,

$$u = \frac{1.8\left(10^{-3}\right)}{\left[0.03 - 0.1(0.075)\right]^2} = 3.556\text{ m/s} = 3.56\text{ m/s} \qquad \textbf{Ans.}$$

The acceleration of the flow can be determined using

$$a = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

Since the flow rate is constant there is no local changes, so that at the point, $\frac{\partial u}{\partial t} = 0$. Here

$$\frac{\partial u}{\partial x} = 1.8\left(10^{-3}\right)(-2)(0.03 - 0.1x)^{-3}(-0.1)$$

$$= \frac{0.36\left(10^{-3}\right)}{(0.03 - 0.1x)^3}$$

Thus

$$a = 0 + u\frac{\partial u}{\partial x}$$

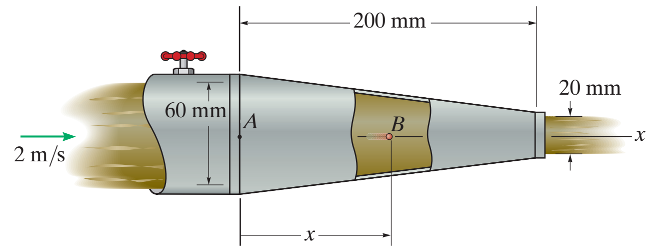
$$= \left[\frac{0.36\left(10^{-3}\right)u}{(0.03 - 0.1x)^3}\right]\text{m/s}^2$$

When $x = 0.075$ m, $u = 3.556$ m/s. Then

$$a = \frac{0.36\left(10^{-3}\right)(3.556)}{\left[0.03 - 0.1(0.075)\right]^3} = 112.37\text{ m/s}^2 = 112\text{ m/s}^2 \qquad \textbf{Ans.}$$

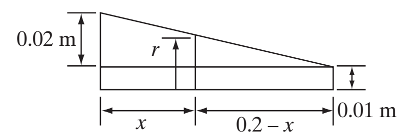
Ans:
$u = 3.56$ m/s, $a = 112$ m/s^2

***4–44.** The tapered pipe transfers ethyl alcohol to a mixing tank such that when a valve is opened, a particle at A has a velocity at A of $2\ \text{m/s}$, which is increasing at $4\ \text{m/s}^2$. Determine the velocity of the same particle when it arrives at B, where $x = 75$ mm.



SOLUTION

From the geometry shown in Fig. a, the radius r of the pipe as a function of x is



(a)

$$r = 0.01\ \text{m} + \left(\frac{0.2 - x}{0.2}\right)(0.02\ \text{m}) = (0.03 - 0.1x)\ \text{m}$$

Thus, the cross-sectional area of the pipe as a function of x is

$$A = \pi r^2 = \left[\pi(0.03 - 0.1x)^2\right]\ \text{m}^2$$

The velocity of a particle passing through the cross-section at A can be determined from

$$\frac{du}{dt} = a; \qquad \int_{2\ \text{m/s}}^{u_A} du = \int_0^t 4\, dt$$

$$u_A - 2 = 4t$$

$$u_A = (4t + 2)\ \text{m/s}$$

Thus, the flow is

$$Q = u_A A_A = \left[(4t + 2)\ \text{m/s}\right]\left[\pi(0.03\ \text{m})^2\right]$$

$$= \pi(0.0036t + 0.0018)\ \text{m}^3/\text{s}$$

Then, the velocity of the flow as a function of t and x is

$$u = \frac{Q}{A} = \frac{\pi(0.0036t + 0.0018)\ \text{m}^3/\text{s}}{\pi(0.03 - 0.1x)^2\ \text{m}^2}$$

$$= \left[\frac{0.0036t + 0.0018}{(0.03 - 0.1x)^2}\right]\ \text{m/s}$$

Using the definition of velocity,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{0.0036t + 0.0018}{(0.03 - 0.1x)^2}$$

$$\int_0^x (0.03 - 0.1x)^2 dx = \int_0^t (0.0036t + 0.0018)dt$$

$$\int_0^x \left(0.01x^2 - 0.006x + 0.0009\right)dx = \int_0^t (0.0036t + 0.0018)dt$$

$$0.003333x^3 - 0.003x^2 + 0.0009x = 0.0018t^2 + 0.0018t$$

***4–44. Continued**

When $x = 0.075$ m,

$$0.003333\left(0.075^3\right) - 0.003\left(0.075^2\right) + 0.0009(0.075) = 0.0018t^2 + 0.0018t$$

$$t^2 + t - 0.02890625 = 0$$

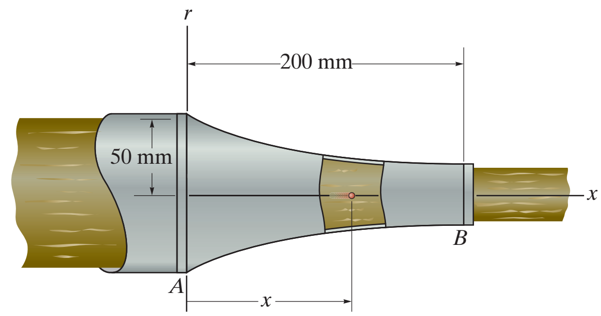
$$t = 0.02812 \text{ s}$$

When $x = 0.075$ m, $t = 0.02812$ s,

$$u = \frac{0.0036(0.02812) + 0.0018}{\left[0.03 - 0.1(0.075)\right]^2}$$

$$= 3.7555 \text{ m/s} = 3.76 \text{ m/s} \qquad \textbf{Ans.}$$

4–45. The radius of the circular duct varies as $r = \left(0.05e^{-3x}\right)$ m, where x is in meters. The flow of a fluid at A is $Q = 0.004\ \text{m}^3/\text{s}$ at $t = 0$, and it is increasing at $dQ/dt = 0.002\ \text{m}^3/\text{s}^2$. If a fluid particle is originally located at $x = 0$ when $t = 0$, determine the time for this particle to arrive at $x = 100$ mm.



SOLUTION

The discharge as a function of time t is

$$Q = 0.004\ \text{m}^3/\text{s} + \left(0.002\ \text{m}^3/\text{s}^2\right)t$$
$$= (0.004 + 0.002t)\ \text{m}^3/\text{s}$$

The cross-sectional Area of the duct as a function of x is

$$A = \pi r^2 = \pi\left(0.05e^{-3x}\right)^2 = \left(0.0025\pi e^{-6x}\right)\ \text{m}^2$$

Thus, the velocity of the flow is

$$u = \frac{Q}{A} = \frac{(0.004 + 0.002t)\ \text{m}^3/\text{s}}{\left(0.0025\pi e^{-6x}\right)\ \text{m}^2}$$
$$= \frac{1.6}{\pi}e^{6x} + \frac{0.8t}{\pi}e^{6x}$$
$$= \frac{4}{5\pi}e^{6x}(t + 2)$$

Using the definition of velocity,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{4}{5\pi}e^{6x}(t + 2)$$
$$\int_0^x \frac{dx}{e^{6x}} = \frac{4}{5\pi}\int_0^t (t + 2)dt$$
$$-\left(\frac{1}{6}e^{-6x}\right)\Bigg|_0^x = \frac{4}{5\pi}\left(\frac{t^2}{2} + 2t\right)\Bigg|_0^t$$
$$\frac{1}{6}\left(1 - e^{-6x}\right) = \frac{2}{5\pi}\left(t^2 + 4t\right)$$

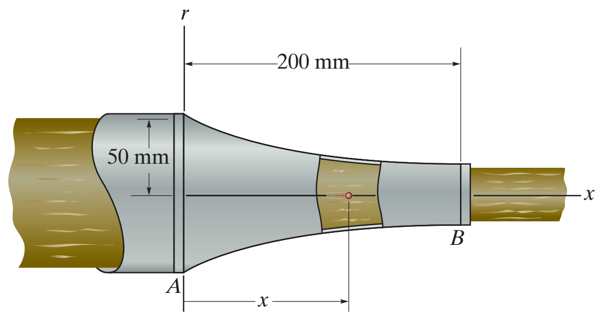
When $x = 0.1$ m,

$$\frac{1}{6}\left[1 - e^{-6(0.1)}\right] = \frac{2}{5\pi}\left(t^2 + 4t\right)$$
$$t^2 + 4t - 0.5906 = 0$$
$$t = \frac{-4 \pm \sqrt{4^2 - 4(1)(-0.5906)}}{2(1)} > 0$$

$t = 0.143$ s **Ans.**

Ans:
0.413 s

4–46. The radius of the circular duct varies as $r = \left(0.05e^{-3x}\right)$ m, where x is in meters. If the flow of the fluid at A is $Q = 0.004\ \text{m}^3/\text{s}$ at $t = 0$, and it is increasing at $dQ/dt = 0.002\ \text{m}^3/\text{s}^2$, determine the time for this particle to arrive at $x = 200$ mm.



SOLUTION

The discharge as a function of time t is

$$Q = 0.004\ \text{m}^3/\text{s} + \left(0.002\ \text{m}^3/\text{s}^2\right)t$$
$$= (0.004 + 0.002t)\ \text{m}^3/\text{s}$$

The cross-sectional area of the duct as a function of x is

$$A = \pi r^2 = \pi\left(0.05e^{-3x}\right)^2 = \left(0.0025\pi\, e^{-6x}\right)\ \text{m}^2$$

Thus, the velocity of the flow is

$$u = \frac{Q}{A} = \frac{(0.004 + 0.002t)\ \text{m}^3/\text{s}}{\left(0.0025\pi e^{-6x}\right)\ \text{m}^2}$$
$$= \frac{1.6}{\pi}e^{6x} + \frac{0.8t}{\pi}e^{6x}$$
$$= \frac{4}{5\pi}e^{6x}(t + 2)$$

Using the definition of velocity,

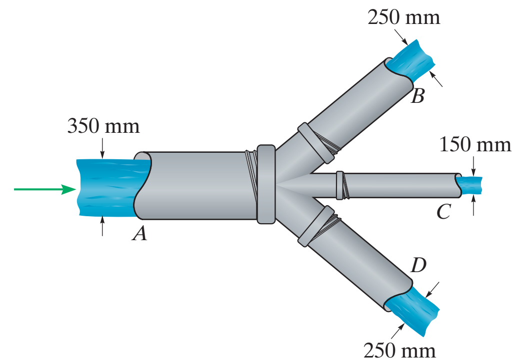
$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{4}{5\pi}e^{6x}(t + 2)$$
$$\int_0^x \frac{dx}{e^{6x}} = \frac{4}{5\pi}\int_0^t (t + 2)dt$$
$$-\frac{1}{6}\left(e^{-6x}\right)\Big|_0^x = \frac{4}{5\pi}\left(\frac{t^2}{2} + 2t\right)\Big|_0^t$$
$$\frac{1}{6}\left(1 - e^{-6x}\right) = \frac{2}{5\pi}\left(t^2 + 4t\right)$$

At point E, $x = 0.2$ m.

$$\frac{1}{6}\left[1 - e^{-6(0.2)}\right] = \frac{2}{5\pi}\left(t^2 + 4t\right)$$
$$t^2 + 4t - 0.9147 = 0$$
$$t = \frac{-4 \pm \sqrt{4^2 - 4(1)(-0.9147)}}{2(1)} > 0$$
$$t = 0.217\ \text{s}$$ **Ans.**

Ans:
0.217 s

4–47. Water flows through the pipe at A at 300 kg/s, and then out the double wye with an average velocity of 3 m/s through B and an average velocity of 2 m/s through C. Determine the average velocity at which it flows through D.



SOLUTION

$$\dot{m} = \rho V_A A_A$$

$$300 \text{ kg/s} = \left(1000 \text{ kg/m}^3\right)(V_A)\left[\pi(0.175 \text{ m})^2\right]$$

$$V_A = 3.118 \text{ m/s}$$

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation. Since the fluid is water which has a constant density, then

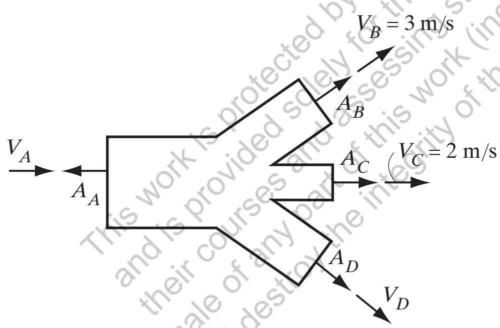
$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$-V_A A_A + V_B A_B + V_C A_C + V_D A_D = 0$$

$$-(3.118 \text{ m/s})\left[\pi(0.175 \text{ m})^2\right] + (3 \text{ m/s})\left[\pi(0.125 \text{ m})^2\right] + (2 \text{ m/s})\left[\pi(0.075 \text{ m})^2\right]$$

$$+ V_D\left[\pi(0.125 \text{ m})^2\right] = 0$$

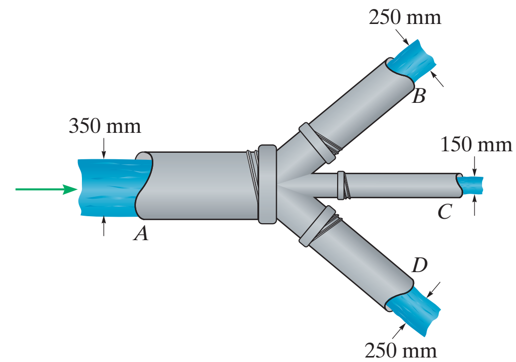
$$V_D = 2.39 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
2.39 m/s

*4–48. If water flows at 150 kg/s through the double wye at B, at 50 kg/s through C, and at 150 kg/s through D, determine the average velocity of flow through the pipe at A.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation. Since $\dot{m} = \rho V \cdot A$, then

$$\frac{\partial}{dt}\int_{cv} \rho d V\!\!\!/ + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \dot{m}_A + \dot{m}_B + \dot{m}_C + \dot{m}_D = 0$$

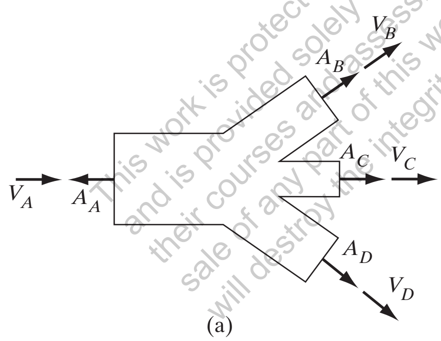
$$-\dot{m}_A + 150 \text{ kg/s} + 50 \text{ kg/s} + 150 \text{ kg/s} = 0$$

$$\dot{m}_A = 350 \text{ kg/s}$$

$$\dot{m}_A = \rho V_A A_A$$

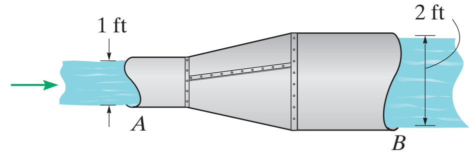
$$350 \text{ kg/s} = \left(1000 \text{ kg/m}^3\right)(V_A)\left[\pi(0.175 \text{ m})^2\right]$$

$$V_A = 3.64 \text{ m/s}$$ **Ans.**



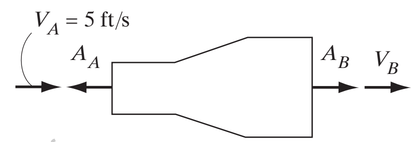
(a)

4–49. Air having a specific weight of 0.0795 lb/ft^3 flows into the duct at A with an average velocity of 5 ft/s. If its density at B is $0.00206 \text{ slug/ft}^3$, determine its average velocity at B.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.



(a)

Continuity Equation. The density of the air at A and B is different. The density at A is $\rho_A = \dfrac{\gamma_A}{g} = \dfrac{0.0795 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} = 0.002469 \text{ slug/ft}^3$. Then,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

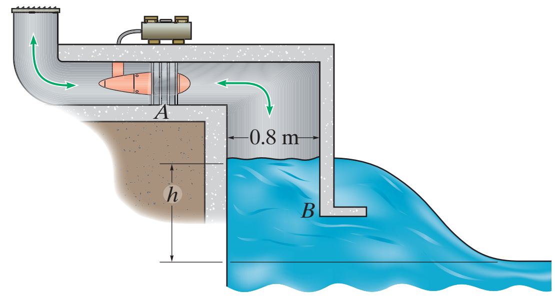
$$0 - \rho_A V_A A_A + \rho_B V_B A_B = 0$$

$$-(0.002469 \text{ slug/ft}^3)(5 \text{ ft/s})\left[\pi(0.5 \text{ ft})^2\right] + (0.00206 \text{ slug/ft}^3)(V_B)\left[\pi(1 \text{ ft})^2\right] = 0$$

$$V_B = 1.50 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
1.50 ft/s

4–50. An oscillating water column (OWC), or gully generator, is a device for producing energy created by ocean waves. As noted, a wave will push water up into the air chamber, forcing the air to pass through a turbine, producing energy. As the wave falls back, the air is drawn into the chamber, reversing the rotational direction of the turbine, but still creating more energy. Assuming a wave will reach an average height of $h = 0.5$ m in the 0.8-m-diameter chamber at B, and it falls back at an average speed of 1.5 m/s determine the speed of the air as it moves through the turbine at A, which has a net area of 0.26 m^2. The air temperature at A is $T_A = 20°\text{C}$, and at B it is $T_B = 10°\text{C}$.



SOLUTION

Here, the control volume contains the air in the air chamber from A to B. Thus, it is a fixed control Volume,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

Since the flow is steady and the volume of the control volume does not change with time, there are no local changes. Thus,

$$0 - \rho_A V_A A_A + \rho_B V_B A_B = 0$$

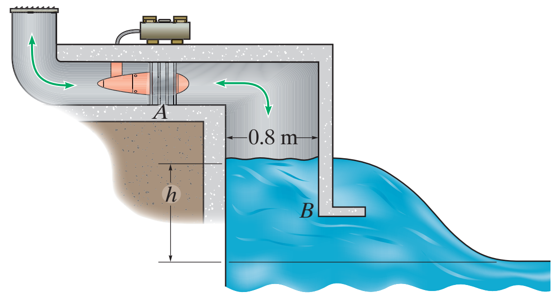
From Appendix A, at $T_A = 20°\text{C}$, $\rho_A = 1.202 \text{ kg/m}^3$ and at $T_B = 10°\text{C}$, $\rho_B = 1.247 \text{ kg/m}^3$. Thus,

$$-\left(1.202 \text{ kg/m}^3\right)(V_A)\left(0.26 \text{ m}^2\right) + \left(1.247 \text{ kg/m}^3\right)\left(1.5 \text{ m/s}\right)\left[\pi(0.4 \text{ m})^2\right] = 0$$

$$V_A = 3.01 \text{ m/s}$$ **Ans.**

Ans:
3.01 m/s

4–51. An oscillating water column (OWC), or gully generator, is a device for producing energy created by ocean waves. As noted, a wave will push water up into the air chamber, forcing the air to pass through a turbine, producing energy. As the wave falls back, the air is drawn into the chamber, reversing the rotational direction of the turbine, but still creating more energy. Determine the speed of the air as it moves through the turbine at A, which has a net open area of $0.26\ \text{m}^2$, if the speed of the water in the 0.8-m-diameter chamber is 5 m/s. The air temperature at A is $T_A = 20°\text{C}$, and at B it is $T_B = 10°\text{C}$.



SOLUTION

Here, the control volume contains the air in the air chamber from A to B. Thus, it is a fixed control volume.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

Since the flow is steady and the volume of the control volume does not change with time, there are no local changes. Thus

$$0 + \rho_A V_A A_A + \rho_B(-V_B A_B) = 0$$

From Appendix A, at $T_A = 20°\text{C}$, $\rho_A = 1.202\ \text{kg/m}^3$ and at $T_B = 10°\text{C}$, $\rho_B = 1.247\ \text{kg/m}^3$. Thus,

$$(1.202\ \text{kg/m}^3)(V_A)(0.26\ \text{m}^2) + (1.247\ \text{kg/m}^3)\left\{-V_B\left[\pi(0.4\ \text{m})^2\right]\right\} = 0$$

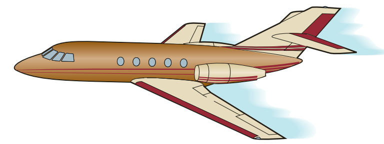
$$V_A = 2.0057\, V_B$$

Here, $V_B = 5\ \text{m/s}$, so that

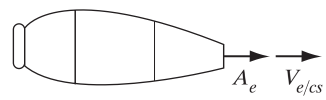
$$V_A = 2.0057(5\ \text{m/s}) = 10.0\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
10.0 m/s

***4–52.** A jet engine draws in air at 25 kg/s and jet fuel at 0.2 kg/s. If the density of the expelled air–fuel mixture is 1.356 kg/m^3, determine the average velocity of the exhaust relative to the plane. The exhaust nozzle has a diameter of 0.4 m.



SOLUTION



(a)

Control Volume. If the control volume moves with the plane, the flow is steady if viewed from the plane. No local changes occur within this control volume.

Continuity Equation. Since $\dot{m} = \rho \mathbf{V}_{f/\text{cs}} \cdot \mathbf{A}$, then

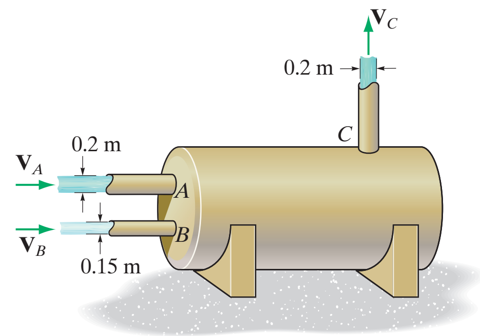
$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{V}_{f/\text{cs}} \cdot d\mathbf{A} = 0$$

$$0 - \dot{m}_a - \dot{m}_f + \rho V_{e/\text{cs}} A_e = 0$$

$$-25\ \text{kg/s} - 0.2\ \text{kg/s} + \left(1.356\ \text{kg/m}^3\right)\left(V_{e/\text{cs}}\right)\left[\pi(0.2\ \text{m})^2\right] = 0$$

$$V_{e/\text{cs}} = 148\ \text{m/s}$$ **Ans.**

4–53. Carbon dioxide flows into the tank at A at $V_A = 4\text{ m/s}$, and nitrogen flows in at B at $V_B = 3\text{ m/s}$. Both enter at a gage pressure of 300 kPa and a temperature of 250°C. Determine the steady mass flow of the mixed gas at C.



SOLUTION

From Appendix A, the values of the gas constants for CO_2 and nitrogen are $R_{CO_2} = 188.9\text{ J/kg}\cdot\text{K}$ and $R_N = 296.8\text{ J/kg}\cdot\text{K}$.

$$p = \rho RT$$

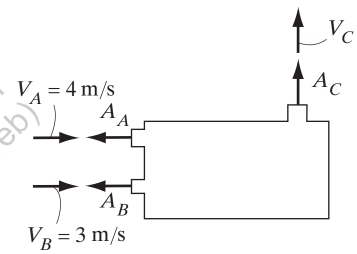
$$(300 + 101.3)(10^3) = \rho_{CO_2}(188.9\text{ J/kg}\cdot\text{K})(250° + 273)\text{ K}$$

$$\rho_{CO_2} = 4.062\text{ kg/m}^3$$

and

$$(300 + 101.3)(10^3) = \rho_N(296.8\text{ J/kg}\cdot\text{K})(250° + 273)\text{ K}$$

$$\rho_N = 2.585\text{ kg/m}^3$$



(a)

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation. Since the densities of the fluids through the open control surfaces are different but of constant value, then

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

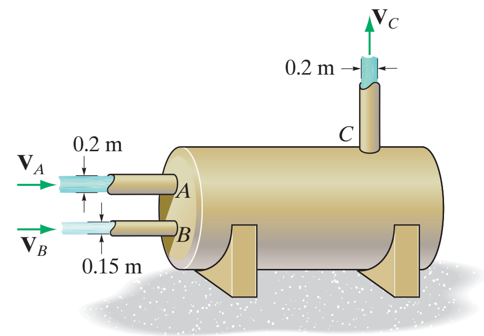
$$0 - \rho_{CO_2}V_A A_A - \rho_N V_B A_B + \dot{m}_m = 0$$

$$-(4.062\text{ kg/m}^3)(4\text{ m/s})\left[\pi(0.1\text{ m})^2\right] - (2.585\text{ kg/m}^3)(3\text{ m/s})\left[\pi(0.075\text{ m})^2\right] + \dot{m}_m = 0$$

$$\dot{m}_m = 0.647\text{ kg/s}$$ **Ans.**

Ans:
0.647 kg/s

4–54. Carbon dioxide flows into the tank at A at $V_A = 10\text{ m/s}$, and nitrogen flows in at B with a velocity of $V_B = 6\text{ m/s}$. Both enter at a pressure of 300 kPa and a temperature of 250°C. Determine the average velocity of the mixed gas leaving the tank at a steady rate at C. The mixture has a density of $\rho = 1.546\text{ kg/m}^3$.



SOLUTION

From Appendix A, the values of the gas constants for CO_2 and nitrogen are $R_{CO_2} = 188.9\text{ J/kg}\cdot\text{K}$ and $R_N = 296.8\text{ J/kg}\cdot\text{K}$.

$$p = \rho RT$$

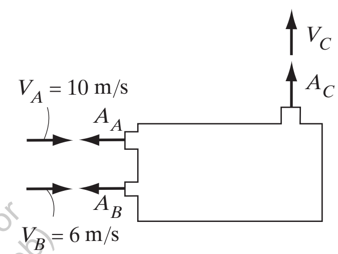
$$(300 + 101.3)(10^3) = \rho_{CO_2}(188.9\text{ J/kg}\cdot\text{K})(250 + 273)\text{ K}$$

$$\rho_{CO_2} = 4.062\text{ kg/m}^3$$

and

$$(300 + 101.3)(10^3) = \rho_N(296.8\text{ J/kg}\cdot\text{K})(250 + 273)\text{ K}$$

$$\rho_N = 2.585\text{ kg/m}^3$$



(a)

Control Volume. The fixed control volume is shown in Fig. a. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - \rho_{CO_2}V_A A_A - \rho_N V_B A_B + \rho_m V_C A_C = 0$$

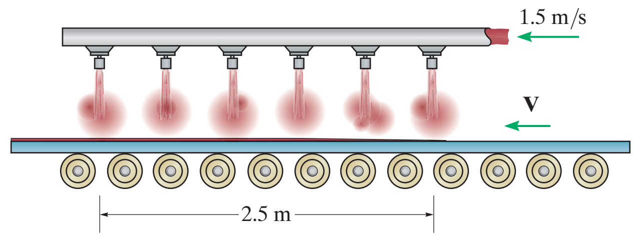
$$-(4.062\text{ kg/m}^3)(10\text{ m/s})\left[\pi(0.1\text{ m})^2\right] - (2.585\text{ kg/m}^3)(6\text{ m/s})\left[\pi(0.075\text{ m})^2\right]$$

$$+ (1.546\text{ kg/m}^3)(V_C)\left[\pi(0.1\text{ m})^2\right] = 0$$

$$V_C = 31.9\text{ m/s}$$ **Ans.**

Ans:
31.9 m/s

4–55. The flat strip is sprayed with paint using the six nozzles, each having a diameter of 2 mm. They are attached to the 20-mm-diameter pipe. The strip is 50 mm wide, and the paint is to be 1 mm thick. If the average speed of the paint through the pipe is 1.5 m/s, determine the required speed V of the strip as it passes under the nozzles.



SOLUTION

Since the flow is steady, there is no local change. Also, ρ for the point is constant and the average velocities will be used. Thus, the continuity equation reduces to

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_p A_p + 6 V_{no} A_{no} = 0$$

However, $Q_{no} = V_{no} A_{no}$. Then

$$-V_p A_p + 6 Q_{no} = 0$$

$$Q_{no} = \frac{V_p A_p}{6} = \frac{(1.5 \text{ m/s})\left[\pi (0.01 \text{ m})^2\right]}{6} = 25(10^{-6})\pi \text{ m}^3/\text{s}$$

Each nozzle has to cover 0.5 m of length and put $\frac{1\left(10^{-3}\right) \text{ m}}{6} = 0.1667\left(10^{-3}\right) \text{ m}$ thickness of paint on the strip. Thus, the volume of paint required is

$$\mathcal{V}_p = (0.5 \text{ m})(0.05 \text{ m})\left[0.1667\left(10^{-3}\right) \text{ m}\right] = 4.1667\left(10^{-6}\right) \text{m}^3$$

Thus,

$$\mathcal{V}_p = Q_{no} t; \qquad 4.1667\left(10^{-6}\right) \text{m}^3 = \left[25\left(10^{-6}\right)\pi \text{ m}^3/\text{s}\right] t$$

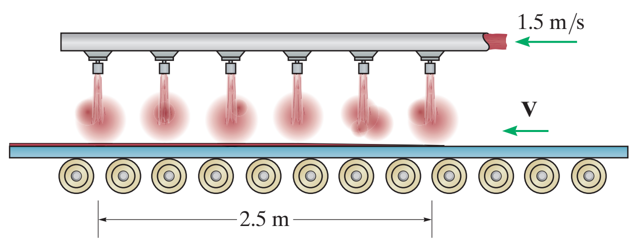
$$t = 0.05305 \text{ s}$$

Then the required speed of the strip is

$$V = \frac{0.5 \text{ m}}{0.05305 \text{ s}} = 9.42 \text{ m/s}$$ **Ans.**

Ans:
9.42 m/s

***4–56.** The flat strip is sprayed with paint using the six nozzles, which are attached to the 20-mm-diameter pipe. The strip is 50 mm wide and the paint is to be 1 mm thick. If the average speed of the point through the pipe is 1.5 m/s, determine the required speed V of the strip as it passes under the nozzles as a function of the diameter of the pipe. Plot this function of speed (vertical axis) versus diameter for $10 \text{ mm} \le D \le 30 \text{ mm}$. Give values for increments of $\Delta D = 5$ mm.



SOLUTION

Since the flow is steady, there is no local change. Also, ρ for the paint is constant and the average velocities will be used. Thus,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_p A_p + 6V_{no}A_{no} = 0$$

However, $Q_{no} = V_{no}A_{no}$. Then

$$Q_{no} = \frac{V_p A_p}{6} = \frac{(1.5 \text{ m/s})\left[\frac{\pi}{4}\left(\frac{D}{1000}\right)^2\right]}{6} = \left[62.5\left(10^{-9}\right)\pi D^2\right] \text{m}^3/\text{s}$$

Each nozzle has to cover 0.5 m of length and put $\dfrac{1\left(10^{-3}\right) \text{ m}}{6} = 0.1667\left(10^{-3}\right)$ m thickness of paint on the strip. Thus, the volume of paint required is

$$\forall_p = (0.5 \text{ m})(0.05 \text{ m})\left[0.1667\left(10^{-3}\right) \text{ m}\right] = 4.1667\left(10^{-6}\right) \text{m}^3$$

Thus,

$$\forall_p = Q_{no}\, t; \quad 4.1667\left(10^{-6}\right) \text{m}^3 = \left[62.5\left(10^{-9}\right)\pi D^2 \text{ m}^3/\text{s}\right] t$$

$$t = \left(\frac{21.22}{D^2}\right) \text{s}$$

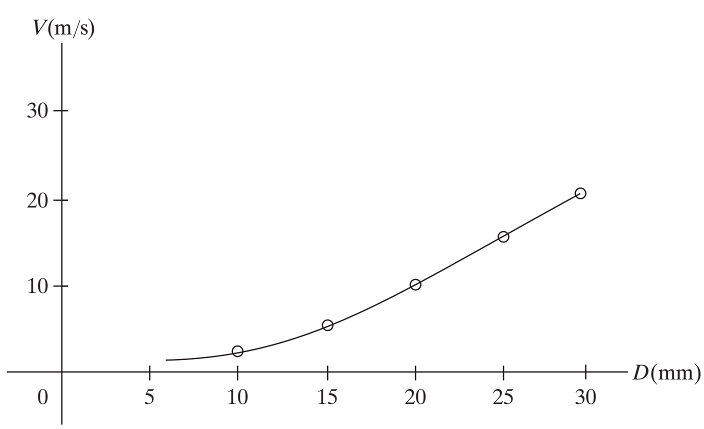
Then the required speed of the strip is

$$V = \frac{0.5 \text{ m}}{\left(21.22/D^2\right) \text{s}}$$

$$V = \left(0.0236\, D^2\right) \text{m/s where } D \text{ is in mm} \qquad \textbf{Ans.}$$

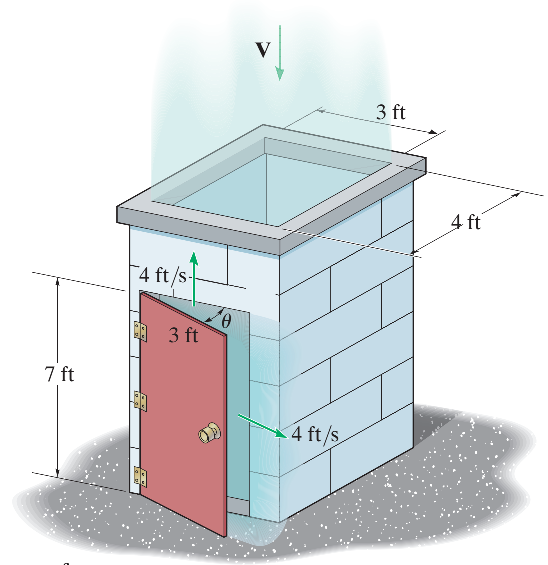
The plot of V vs D is shown in Fig. *a*.

D(mm)	10	15	20	25	30
V(m/s)	2.36	5.30	9.42	14.7	21.2



(a)

4–57. Pressurized air in a building well flows out through the partially opened door with an average velocity of 4 ft/s. Determine the average velocity of the air as it flows down from the top of the building well. Assume the door is 3 ft wide and $\theta = 30°$.



SOLUTION

The control volume contains the air in the building well and in the circular sector of the door opening. We use a circular sector because the velocity must be *normal* to the area through which it flows. It can be considered fixed. Also, since in this case the air is assumed to be incompressible, the flow is steady, thus there is no local change. Here, the density of the air remains constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + (V_{out})_1(A_{out})_1 + (V_{out})_2(A_{out})_2 = 0 \qquad \textbf{(1)}$$

Here, the entrance open control surface is the cross-section of the building well above the door.

$$A_{in} = (4\text{ ft})(3\text{ ft}) = 12\text{ ft}^2$$

The exit open control surfaces are the top and side of the door opening.

$$(A_{out})_1 = \frac{1}{2}r^2\theta = \frac{1}{2}(3\text{ ft})^2\left(\frac{30°}{180°}\pi\text{ rad}\right) = 0.75\pi\text{ ft}^2$$

$$(A_{out})_2 = r\,\theta h = (3\text{ ft})\left(\frac{30°}{180°}\pi\text{ rad}\right)(7\text{ ft}) = 3.5\pi\text{ ft}^2$$

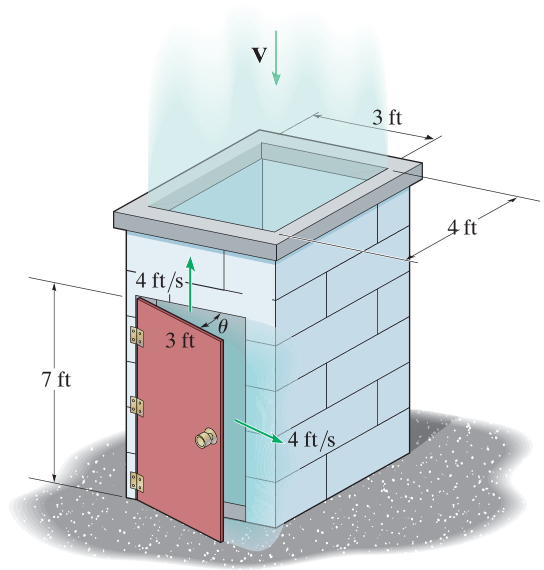
Substituting these values into Eq. (1),

$$-V_{in}(12\text{ ft}^2) + (4\text{ ft/s})(0.75\pi\text{ ft}^2) + (4\text{ ft/s})(3.5\pi\text{ ft}^2) = 0$$

$$V_{in} = 4.45\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
4.45 ft/s

4–58. Pressurized air in a building well flows out through the partially opened door with an average velocity of 4 ft/s. Determine the average velocity of the air as it flows down from the top of the building well as a function of the door opening θ. Plot this function of velocity (vertical axis) versus θ for $0° \le \theta \le 50°$. Give values for increments of $\Delta\theta = 10°$.



SOLUTION

The control volume contains the air in the building well and in the circular sector of the door opening. We use a circular sector because the velocity must be *normal* to the area through which it flows. It can be considered fixed. Also, since in this case the air is assumed to be incompressible, the flow is steady, thus there is no local change. Here, the density of the air remains constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + (V_{out})_1(A_{out})_1 + (V_{out})_2(A_{out})_2 = 0 \qquad \textbf{(1)}$$

Here, the entrance open control surface is the cross-section of the building well above the door.

$$A_{in} = (4\text{ ft})(3\text{ ft}) = 12\text{ ft}^2$$

The exit open control surfaces are the top and side of the door opening.

$$(A_{out})_1 = \frac{1}{2}r^2\theta = \frac{1}{2}(3\text{ ft})^2\left(\frac{\theta}{180°}\pi\text{ rad}\right) = (0.025\pi\theta)\text{ ft}^2$$

$$(A_{out})_2 = r\theta h = (3\text{ ft})\left(\frac{\theta}{180°}\pi\text{ rad}\right)(7\text{ ft}) = (0.1167\pi\theta)\text{ ft}^2$$

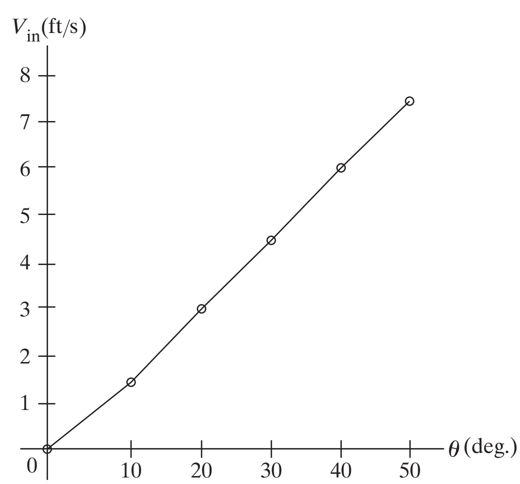
Substituting these values into Eq. (1),

$$-V_{in}(12\text{ ft}^2) + (4\text{ ft/s})(0.025\pi\theta\text{ ft}^2) + (4\text{ ft/s})(0.1167\pi\theta\text{ ft}^2) = 0$$

$$V_{in} = (0.0472\pi\theta)\text{ ft/s} \text{ where } \theta \text{ is in degrees.} \qquad \textbf{Ans.}$$

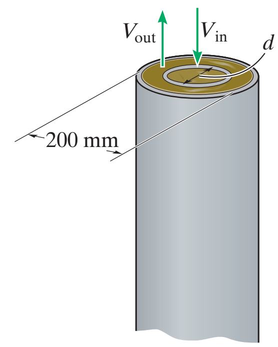
The plot of V_{in} vs θ is shown in Fig. *a*.

θ(deg.)	0	10	20	30	40	50
V_{in}(ft/s)	0	1.48	2.97	4.45	5.93	7.42



Ans:
$V_{in} = (0.0472\pi\theta)$ ft/s, where θ is in degrees

4–59. Drilling fluid is pumped down through the center pipe of a well and then rises up within the annulus. Determine the diameter d of the inner pipe so that the average velocity of the fluid remains the same in both regions. Also, what is this average velocity if the discharge is 0.02 m^3/s? Neglect the thickness of the pipes.



SOLUTION

The control volume considered is the volume of drilling fluid in pipe which is fixed. Here, the flow is steady, thus there are no local changes. Also, the density of the fluid is constant and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + V_{out}A_{out} = 0 \qquad \textbf{(1)}$$

Here, it is required that $V_{in} = V_{out}$. Also, $A_{in} = \frac{\pi}{4}d^2$ and $A_{out} = \frac{\pi}{4}(0.2\text{ m})^2 - \frac{\pi}{4}d^2$.

Then

$$-V\left(\frac{\pi}{4}d^2\right) + V\left[\frac{\pi}{4}(0.2\text{ m})^2 - \frac{\pi}{4}d^2\right] = 0$$

$$2d^2 = 0.04$$

$$d = 0.1414\text{ m} = 141\text{ mm} \qquad \textbf{Ans.}$$

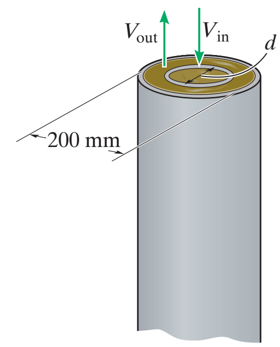
Considering the flow in the center pipe,

$$Q = VA; \quad 0.02\text{ m}^3/\text{s} = V\left[\frac{\pi}{4}(0.1414\text{ m})^2\right]$$

$$V = 1.27\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$d = 141\text{ mm}$
$V = 1.27\text{ m/s}$

***4–60.** Drilling fluid is pumped down through the center pipe of a well and then rises up within the annulus. Determine the velocity of the fluid forced out of the well as a function of the diameter d of the inner pipe, if the velocity of the fluid forced into the well is maintained at $V_{in} = 2\text{ m/s}$. Neglect the thickness of the pipes. Plot this velocity (vertical axis) versus the diameter for $50\text{ mm} \le d \le 150\text{ mm}$. Give values for increments of $\Delta d = 25\text{ mm}$.



SOLUTION

The control volume is the volume of the drilling fluid in the pipe which is fixed. Here, the flow is steady thus there is no local change. Also the density of the fluid is constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + V_{out}A_{out} = 0$$

Here,
$$A_{in} = \frac{\pi}{4}\left(\frac{d}{1000}\right)^2 = 0.25\left(10^{-6}\right)\pi\, d^2 \text{ and}$$

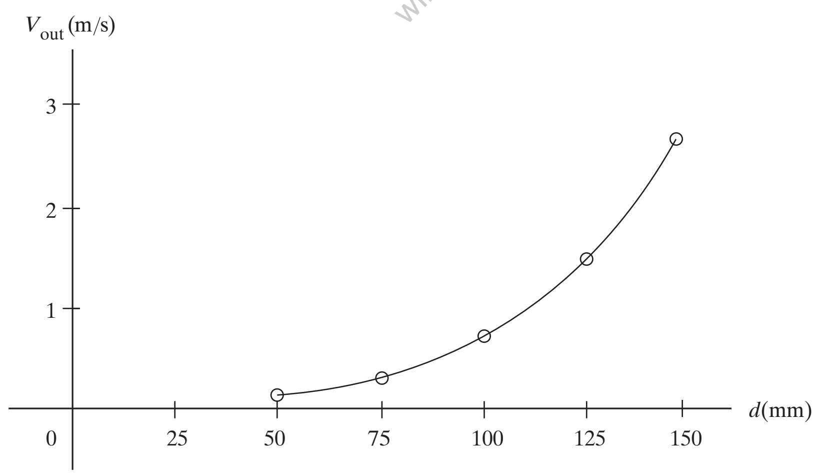
$$A_{out} = \frac{\pi}{4}\left[(0.2\text{ m})^2 - \left(\frac{d}{1000}\right)^2\right] = \frac{\pi}{4}\left[0.04 - \left(10^{-6}\right)d^2\right]$$

$$-(2\text{ m/s})\left[0.25(10^{-6})\pi\, d^2\right] + V_{out}\left\{\frac{\pi}{4}\left[0.04 - (10^{-6})d^2\right]\right\} = 0$$

$$V_{out} = \left[\frac{2\left(10^{-6}\right)d^2}{0.04 - \left(10^{-6}\right)d^2}\right]\text{ m/s where } d \text{ is in mm} \qquad \textbf{Ans.}$$

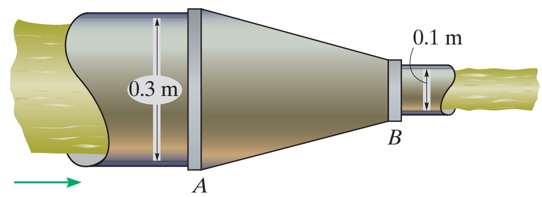
The plot of V_{out} vs d is shown in Fig. *a*.

d(mm)	50	75	100	125	150
V_{out}(m/s)	0.133	0.327	0.667	1.28	2.57



(a)

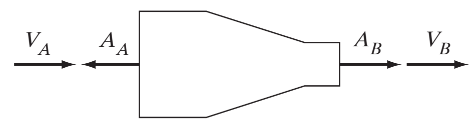
4–61. The unsteady flow of glycerin through the reducer is such that at A its velocity is $V_A = (0.8\,t^2)$ m/s, where t is in seconds. Determine its average velocity at B, and its average acceleration at A, when $t = 2$ s. The pipes have the diameters shown.



SOLUTION

When $t = 2$ s, the velocity of the flow at A is

$$V_A = 0.8(2)^2 = 3.20\text{ m/s}$$



(a)

Control Volume. The fixed control volume is shown in Fig. *a*. Since the volume of the control volume does not change over time, no local changes occur within this control volume.

Continuity Equation. Since the water has a constant density, then

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(3.20\text{ m/s})\left[\pi\left(\frac{0.3\text{ m}}{2}\right)^2\right] + V_B\left[\pi\left(\frac{0.1\text{ m}}{2}\right)^2\right] = 0$$

$$V_B = 28.8\text{ m/s}$$ **Ans.**

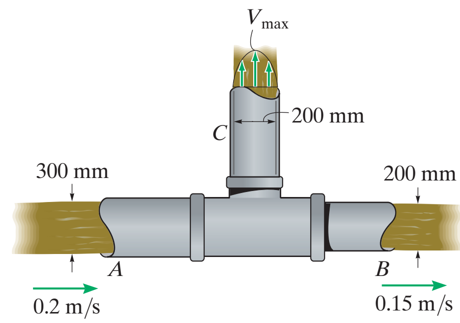
With $u = V_A$ and $v = w = 0$, we have

$$a_A = \frac{\partial V_A}{\partial t}$$

$$= 1.6t\big|_{t=2\text{ s}} = 3.20\text{ m/s}^2$$ **Ans.**

Ans:
$V_B = 28.8$ m/s
$a_A = 3.20$ m/s^2

4–62. Oil flows into the pipe at A with an average velocity of 0.2 m/s and through B with an average velocity of 0.15 m/s. Determine the maximum velocity V_{max} of the oil as it emerges from C if the velocity distribution is parabolic, defined by $v_C = V_{max}(1 - 100r^2)$, where r is in meters measured from the centerline of the pipe.



SOLUTION

The control volume considered is fixed as it contain the oil in the pipe. Also, the flow is steady and so no local changes occur. Here, the density of the oil is constant. Then

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

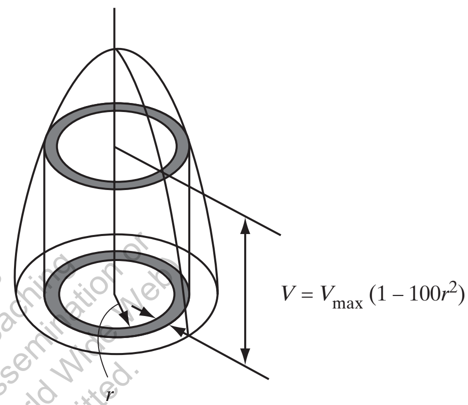
$$0 - V_A A_A + V_B A_B + \int_A V_c dA = 0$$

$$-(0.2 \text{ m/s})\left[\pi(0.15 \text{ m})^2\right] + (0.15 \text{ m/s})\left[\pi(0.1 \text{ m})^2\right]$$

$$+ \int_0^{0.1 \text{ m}} V_{max}(1 - 100r^2)(2\pi r dr) = 0$$

$$-3(10^{-3})\pi \text{m}^3 + 5(10^{-3})\pi V_{max} = 0$$

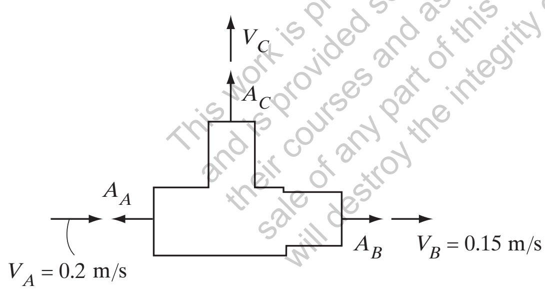
$$V_{max} = 0.6 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Note: The integral in the above equation is equal to the volume under the velocity profile, while in this case is a paraboloid.

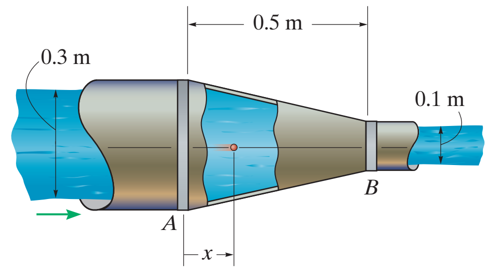
$$\int_A V_c dA = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(0.1 \text{ m})^2(V_{max}) = 5(10^{-3})\pi V_{max}.$$



(b)

Ans:
0.6 m/s

4–63. The unsteady flow of linseed oil is such that at A it has a velocity of $V_A = (0.7t + 4)$ m/s, where t is in seconds. Determine the acceleration of a fluid particle located at $x = 0.2$ m when $t = 1$ s. *Hint*: Determine $V = V(x, t)$, then use Eq. 3–4.

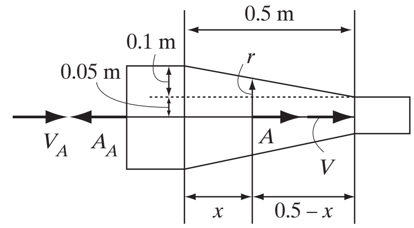


SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the volume does not change over time, no local changes occur within this control volume.

Continuity Equation. Referring to the geometry shown in Fig. *a*, the radius of the pipe at an arbitrary distance x is

$$\frac{r - 0.05}{0.1} = \frac{0.5 - x}{0.5}; \quad r = (0.15 - 0.2x) \text{ m}$$



(a)

Then,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + VA = 0$$

$$-(0.7t + 4)\left[\pi(0.15 \text{ m})^2\right] + V\left[\pi(0.15 - 0.2x)^2\right] = 0$$

$$V = \frac{0.0225(0.7t + 4)}{(0.15 - 0.2x)^2}$$

For A differential control volume at $x = 0.2$ m, with $u = V$ and $v = w = 0$,

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

$$= \frac{0.0225(0.7)}{(0.15 - 0.2x)^2} + \left(\frac{0.0225(0.7t + 4)}{(0.15 - 0.2x)^2}\right)\left(\frac{0.0225\left[(0.15 - 0.2x)^2\right](0) - (0.7t + 4)(2)(0.15 - 0.2x)(-0.2)\right]}{(0.15 - 0.2x)^4}\right)$$

$$= \frac{0.01575}{(0.15 - 0.2x)^2} + \left[\frac{0.0225(0.7t + 4)}{(0.15 - 0.2x)^2}\right]\left[\frac{0.009(0.7t + 4)}{(0.15 - 0.2x)^3}\right]$$

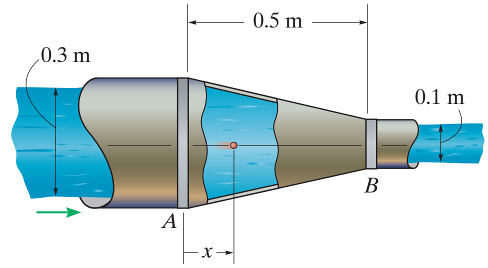
For $t = 1$ s, $x = 0.2$ m,

$$a = 1.3017 + 8.7397(31.781)$$

$$= 279 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
279 m/s^2

***4–64.** The unsteady flow of linseed oil is such that at A it has a velocity of $V_A = (0.4t^2)$ m/s, where t is in seconds. Determine the acceleration of a fluid particle located at $x = 0.25$ m when $t = 2$ s *Hint*: Determine $V = V(x, t)$, then use Eq. 3–4.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the volumes does not change over time, no local changes occur within this control volume.

Continuity Equation. Referring to the geometry shown in Fig. *a*, the radius of the pipe at an arbitrary distance x is

$$\frac{r - 0.05}{0.1} = \frac{0.5 - x}{0.5}; \quad r = (0.15 - 0.2x)\text{ m}$$

Then,

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + VA = 0$$

$$-(0.4t^2)\left[\pi(0.15\text{ m})^2\right] + V\left[\pi(0.15 - 0.2x)^2\right] = 0$$

$$V = \frac{0.009t^2}{(0.15 - 0.2x)^2}$$

For A differential control volume at $x = 0.25$ m, with $u = V$ and $v = w = 0$,

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

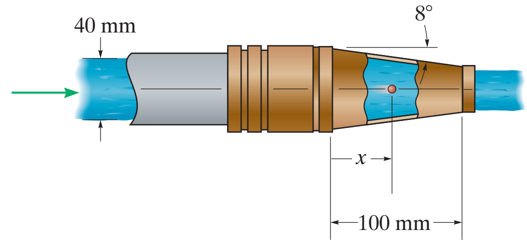
$$= \frac{0.018t}{(0.15 - 0.2x)^2} + \left[\frac{0.009t}{(0.15 - 0.2x)^2}\right]\left[\frac{(0.15 - 0.2x)^2(0) - (0.009t^2)(2)(0.15 - 0.2x)(-0.2)}{(0.15 - 0.2x)^4}\right]$$

$$= \frac{0.018t}{(0.15 - 0.2x)^2} + \left[\frac{0.009t^2}{(0.15 - 0.2x)^2}\right]\left[\frac{0.0036t^2}{(0.15 - 0.2x)^3}\right]$$

For $t = 2$ s, $x = 0.25$ m,

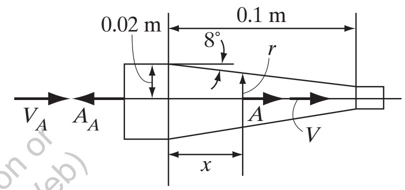
$$a = 3.6 + 3.6(14.4)$$
$$= 55.4\text{ m/s}^2$$ **Ans.**

4–65. Water flows through the nozzle at a rate of $0.2\ \text{m}^3/\text{s}$. Determine the velocity V of a particle as it moves along the centerline as a function of x.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.



(a)

Continuity Equation. From the geometry shown in Fig. *a*,

$$r = 0.02\ \text{m} - x \tan 8° = (0.02 - 0.1405x)\ \text{m}$$

Realizing that $Q_A = V_A A_A = 0.2\ \text{m}^3/\text{s}$,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - Q_A + VA = 0$$

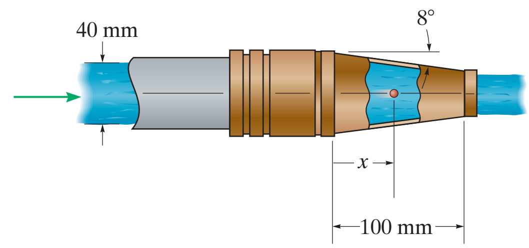
$$-0.2\ \text{m}^3/\text{s} + V\left[\pi(0.02 - 0.1405x)^2\right] = 0$$

$$V = \frac{0.0637}{(0.02 - 0.141x)^2} \qquad \textbf{Ans.}$$

Ans:

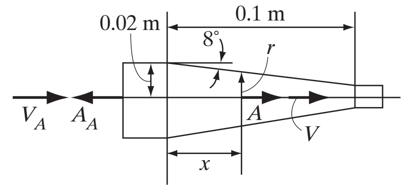
$$V = \frac{0.0637}{(0.02 - 0.141x)^2}$$

4–66. Water flows through the nozzle at a rate of $0.2\ \text{m}^3/\text{s}$. Determine the acceleration of a particle as it moves along the centerline as a function of x.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.



(a)

Continuity Equation. From the geometry shown in Fig. *a*,

$$r = 0.02\ \text{m} - x\tan 8^\circ = (0.02 - 0.1405x)\ \text{m}$$

Realizing that $Q_A = V_A A_A = 0.2\ \text{m}^3/\text{s}$,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - Q_A + VA = 0$$

$$-0.2\ \text{m}^3/\text{s} + V\left[\pi(0.02 - 0.1405x)^2\right] = 0$$

$$V = \frac{0.06366}{(0.02 - 0.1405x)^2}$$

Since the flow is one dimensional, the acceleration can be determined using

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

Here, $\dfrac{\partial V}{\partial t} = 0$ and

$$\frac{\partial V}{\partial x} = \frac{(0.02 - 0.1405x)^2(0) - 0.06366(2)(0.02 - 0.1405x)(-0.1405)}{(0.02 - 0.1405x)^4}$$

$$= \frac{0.01789}{(0.02 - 0.1405x)^3}$$

Thus,

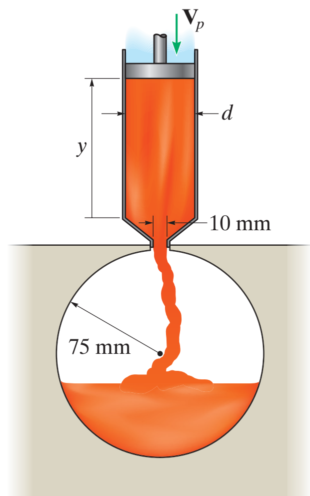
$$a = 0 + \left[\frac{0.06366}{(0.02 - 0.1405x)^2}\right]\left[\frac{0.01789}{(0.02 - 0.1405x)^3}\right]$$

$$= \left[\frac{1.14(10^{-3})}{(0.02 - 0.141x)^5}\right]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$$\left[\frac{1.14\left(10^{-3}\right)}{(0.02 - 0.141x)^5}\right]\ \text{m/s}^2$$

4–67. The cylindrical plunger traveling at $V_p = \left(0.004t^{1/2}\right)$ m/s, where t is in seconds, injects a liquid plastic into the mold to make a solid ball. If $d = 50$ mm, determine the amount of time needed to do this if the volume of the ball is $\forall = \frac{4}{3}\pi r^3$.



SOLUTION

The control volume considered is the volume of the liquid plastic contained in the plunger. Its volume changes with time, Fig. *a*. The volume $\forall_0$ of the lower portion of the control volume is constant.

$$\frac{\partial}{\partial t}\int_{cv} \rho_p d\forall + \int_{cs} \rho_p \mathbf{V}_{p/cs} \cdot d\mathbf{A} = 0$$

Since ρ_p is constant, it can be factored out of the integrals. Also, the average velocity will be used. Thus, the above equation becomes

$$\rho_p \frac{d\forall}{dt} + \rho_p V_A A_A = 0$$

Since, $Q_A = V_A A_A$,

$$\frac{d\forall}{dt} + Q_A = 0 \qquad \textbf{(1)}$$

The volume of the control volume is

$$\forall = \pi(0.025 \text{ m})^2 y + \forall_0 = 0.625\left(10^{-3}\right)\pi y + \forall_0$$

$$\frac{d\forall}{dt} = 0.625(10^{-3})\pi \frac{dy}{dt}$$

However, $\frac{dy}{dt} = -V_p = \left(-0.004t^{\frac{1}{2}}\right)$m/s. The negative sign indicates that $\mathbf{V}_p$ is directed in the opposite sense to positive y.

$$\frac{d\forall}{dt} = 0.625\left(10^{-3}\right)\pi\left(-0.004t^{\frac{1}{2}}\right) = \left[-2.5\left(10^{-6}\right)\pi t^{\frac{1}{2}}\right] \text{ m}^3/\text{s}$$

The negative sign indicates that the volume is decreasing.

$$-2.5\left(10^{-6}\right)\pi t^{\frac{1}{2}} + Q_A = 0$$

$$Q_A = \left(2.5\left(10^{-6}\right)\pi t^{\frac{1}{2}}\right) \text{ m}^3/\text{s}$$

The volume of the ball is

$$\forall_s = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.075 \text{ m})^3 = 0.5625\left(10^{-3}\right)\pi \text{ m}^3$$

The time required to fill up the mold is given by

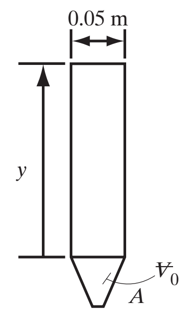
$$\int Q_A dt = \forall_s$$

$$\int_0^T 2.5\left(10^{-6}\right)\pi t^{\frac{1}{2}} dt = 0.5625\left(10^{-3}\right)\pi$$

$$\int_0^T t^{\frac{1}{2}}\, dt = 225$$

$$\frac{2}{3}T^{\frac{3}{2}} = 225$$

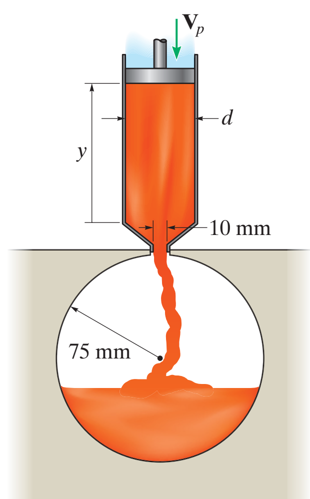
$$T = (337.5)^{\frac{2}{3}} = 48.5 \text{ s} \qquad \textbf{Ans.}$$



(a)

Ans:
48.5 s

***4–68.** The cylindrical plunger traveling at $V_p = \left(0.004\,t^{\frac{1}{2}}\right)$ m/s, where t is in seconds, injects a liquid plastic into the mold to make a solid ball. Determine the time needed to fill the mold as a function of the plunger diameter d. Plot the time needed to fill the mold (vertical axis) versus the diameter of the plunger for 10 mm $\le d \le$ 50 mm. Give values for increments of $\Delta d = 10$ mm. The volume of the ball is $\mathcal{V} = \frac{4}{3}\pi r^3$.



SOLUTION

The control volume is the volume of the liquid plastic contained in the plunger for which its volume changes with time, Fig. *a*. The volume $\mathcal{V}_0$ of the lower portion of the control volume is constant.

$$\frac{\partial}{\partial t}\int_{cv} \rho_p d\mathcal{V} + \int_{cs} \rho_p \mathbf{V}_{p/cs}\cdot d\mathbf{A} = 0$$

Since ρ_p is constant, it can be factored out of the integral. Also, the average velocity will be used. Thus, the above equation becomes

$$\rho_p \frac{\partial \mathcal{V}}{\partial t} + \rho_p V_A A_A = 0$$

Since $Q_A = V_A A_A$,

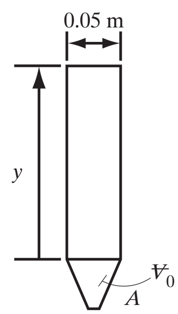
The volume of the control volume is

$$\mathcal{V} = \left(\frac{\pi}{4}d^2\right)y + V_0$$

$$\frac{\partial \mathcal{V}}{\partial t} = \frac{\pi}{4}d^2\frac{\partial y}{\partial t}$$

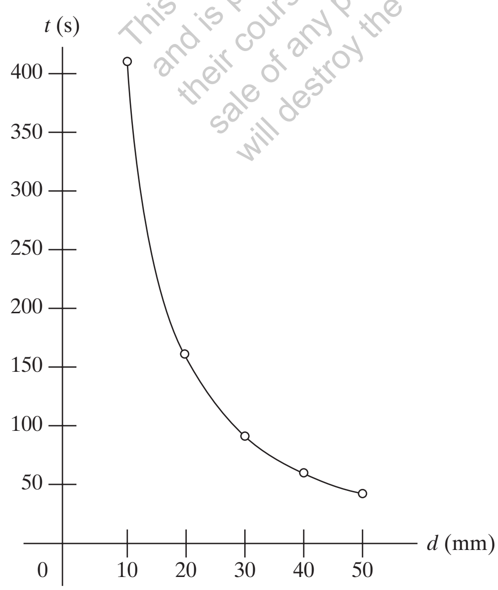
However, $\frac{\partial y}{\partial t} = -V_p = \left(-0.004\,t^{\frac{1}{2}}\right)$ m/s. The negative sign indicates that V_p is directed in the opposite sense to that of positive y.

$$\frac{\partial \mathcal{V}}{\partial t} = \frac{\pi}{4}d^2\left(-0.004t^{\frac{1}{2}}\right) = \left(-0.001\pi d^2 t^{\frac{1}{2}}\right)\ \text{m}^3/\text{s}$$



(a)

d(mm)	10	20	30	40	50
t(s)	414	164	88.9	60.6	45.0



(b)

***4–68. Continued**

The negative sign indicates that the volume is decreasing. Substituting into Eq (1),

$$= -0.001\pi d^2 t^{\frac{1}{2}} + Q_A = 0$$

$$Q_A = \left(0.001\pi d^2 t^{\frac{1}{2}}\right)\text{m}^3/\text{s}$$

The volume of the sphere (mold) is

$$\mathcal{V}_s = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.075\text{ m})^3 = 0.5625\left(10^{-3}\right)\pi\text{ m}^3$$

The time to fill up the sphere is

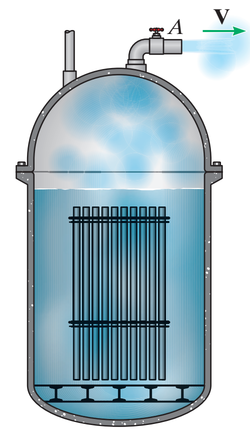
$$dt = \frac{V_s}{Q_A}; \qquad dt = \frac{0.5625\left(10^{-3}\right)\pi\text{ m}^3}{0.001\pi d^2 t^{\frac{1}{2}}}$$

$$\int_o^t t^{\frac{1}{2}}\,dt = 0.5625 d^{-2}$$

$$\frac{2}{3}t^{\frac{3}{2}} = 0.5625 d^{-2}$$

$$t = 0.8929 d^{-\frac{4}{3}}$$ **Ans.**

4–69. The pressure vessel of a nuclear reactor is filled with boiling water having a density of $\rho_w = 850\ \text{kg/m}^3$. Its volume is 185 m^3. Due to failure of a pump, needed for cooling, the pressure release valve A is opened and emits steam having a density of $\rho_s = 35\ \text{kg/m}^3$ and an average speed of $V = 400\ \text{m/s}$. If it passes through the 40-mm-diameter pipe, determine the time needed for all the water to escape. Assume that the temperature of the water and the velocity at A remain constant.



SOLUTION

The steam has a steady flow and the density of the water in the pressure vessel is constant since the temperature is assumed to be constant. Here, the control volume is changing since it contains the water in the vessel.

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\forall + \int_{cs} \rho_s \mathbf{V} \cdot d\mathbf{A} = 0$$

Since ρ_w and ρ_s are constant, they can be factored out from the integrals. Also, the average velocity of the steam will be used. Then $\int_{cs} \mathbf{V} \cdot d\mathbf{A} = V_s A$.

$$\rho_w \frac{\partial}{\partial t}\int_{cv} d\forall + \rho_s V_s A = 0$$

$$\rho_w \frac{\partial \forall}{\partial t} + \rho_s V_s A = 0$$

$$\frac{\partial \forall}{\partial t} = -\frac{\rho_s V_s A}{\rho_w} = -\frac{(35\ \text{kg/m}^3)(400\ \text{m/s})\left[\pi(0.02\ \text{m})^2\right]}{850\ \text{kg/m}^3}$$

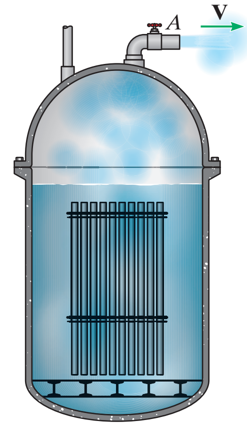
$$= -0.02070\ \text{m}^3/\text{s}$$

The negative sign indicates that the volume of water is decreasing. Thus, the time needed for all the water to escape is

$$t = \frac{\forall}{\partial \forall/\partial t} = \frac{185\ \text{m}^3}{0.02070\ \text{m}^3/\text{s}} = (8938\ \text{s})\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = 2.48\ \text{hr} \qquad \textbf{Ans.}$$

Ans:
2.48 hr

4–70. The pressure vessel of a nuclear reactor is filled with boiling water having a density of $\rho_w = 850\ \text{kg/m}^3$. Its volume is 185 m^3. Due to failure of a pump, needed for cooling, the pressure release valve is opened and emits steam having a density of $\rho_s = 35\ \text{kg/m}^3$. If the steam passes through the 40-mm-diameter pipe, determine the average speed through the pipe as a function of the time needed for all the water to escape. Plot the speed (vertical axis) versus the time for $0 \le t \le 3$ h. Give values for increments of $\Delta t = 0.5$ h. Assume that the temperature of the water remains constant.



SOLUTION

The steam has a steady flow and the densities of the water in the pressure vessel and the steam are constant since the temperature is assumed to be constant. Here the control volume is changing since it contains the water in the vessel.

$$\frac{\partial}{\partial t}\int_{cv} \rho_w dV\!\!\!\!- + \int_{cs} \rho_s \mathbf{V}_s \cdot d\mathbf{A} = 0$$

Since ρ_w and ρ_s are constants, they can be factored out from the integrals. Also the average velocity of the steam will be used. Then

$$\int_{cs} \mathbf{V}_s \cdot d\mathbf{A} = V_s A.$$

$$\rho_w \frac{\partial}{\partial t}\int_{cv} dV\!\!\!\!- + \rho_s V_s A = 0$$

$$\frac{\partial V\!\!\!\!-}{\partial t} = -\frac{\rho_s V_s A}{\rho_w} = -\frac{(35\ \text{kg/m}^3)(V_s)\left[\pi(0.02\ \text{m})^2\right]}{850\ \text{kg/m}^3}$$

$$\frac{\partial V\!\!\!\!-}{\partial t} = \left[-51.74(10^{-6})\,V_s\right]\ \text{m}^3/\text{s}$$

The negative sign indicates that the volume of water is decreasing. Thus, the time needed for all the water to escape is

$$t = \frac{V\!\!\!\!-}{\partial V\!\!\!\!-/\partial t} = \frac{185\ \text{m}^3}{\left[51.74(10^{-6})\,V_s\right]\ \text{m}^3/\text{s}}$$

$$t = \left\{\left[\frac{3.5753(10^6)}{V_s}\right]\text{s}\right\}\left(\frac{1\ \text{hr}}{3600}\right)$$

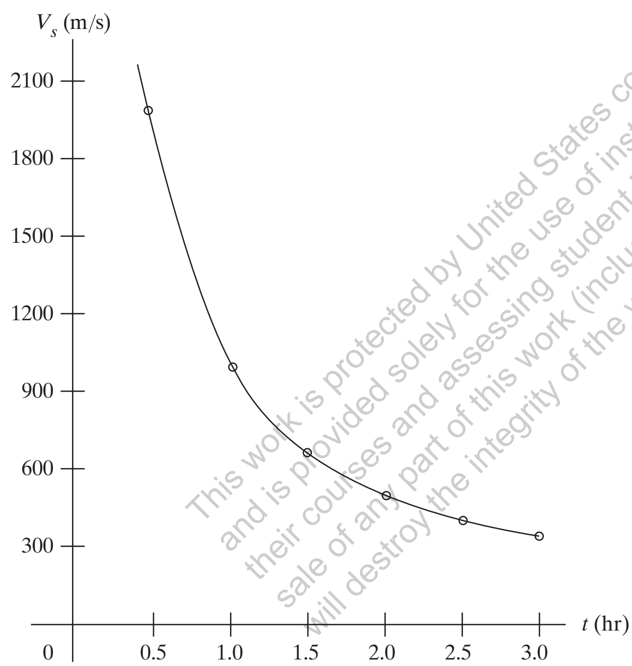
4–70. Continued

$$t = \frac{993.14}{V_s}$$

$$V_s = \left(\frac{993}{t}\right) \text{ m/s where } t \text{ is in hrs}$$ **Ans.**

The plot of V_s vs t is shown in Fig. *a*

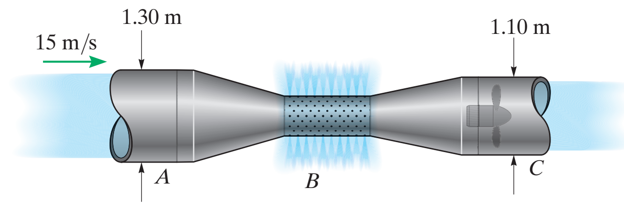
t(hr)	0	0.5	1.0	1.5	2.0	2.5	3.0
V_s(m/s)	∞	1986	993	662	497	397	331



(a)

Ans:
$V_s = \left(\frac{993}{t}\right)$ m/s, where t is in hrs

4–71. The wind tunnel is designed so that the lower pressure outside the testing region draws air out in order to reduce the boundary layer or frictional effects along the wall within the testing tube. Within region B there are 2000 holes, each 3 mm in diameter. If the pressure is adjusted so that the average velocity of the air through each hole is 40 m/s, determine the average velocity of the air exiting the tunnel at C. Assume the air is incompressible.



SOLUTION

The control volume is fixed since it contains the air in the tunnel. Since the flow is steady, no local changes take place. Also, the density of air is constant (incompressible) and the average velocities will be used.
Thus,

$$\frac{\partial}{\partial t}\int_{cv} \rho d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A}$$

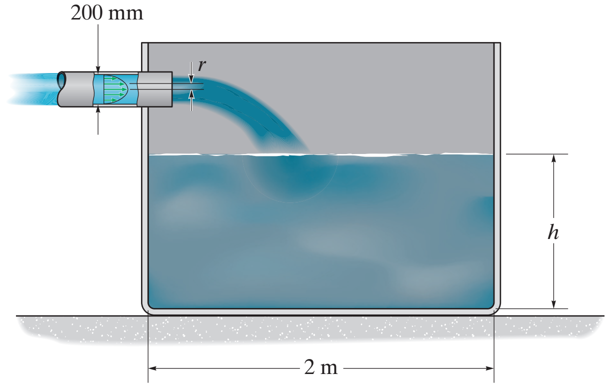
$$0 - V_A A_A + 2000\, V_B A_B + V_C A_C = 0$$

$$-(15\text{ m/s})\left[\pi(0.65\text{ m})^2\right] + 2000(40\text{ m/s})\left[\pi(0.0015\text{ m})^2\right] + V_C\left[\pi(0.55\text{ m})^2\right] = 0$$

$$V_C = 20.4\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
20.4 m/s

***4–72.** Water flows through the pipe such that it has a parabolic velocity profile $V = 3(1 - 100r^2)$ m/s, where r is in meters. Determine the time needed to fill the tank to a depth of $h = 1.5$ m if $h = 0$ when $t = 0$. The width of the tank is 3 m.



SOLUTION

The control volume is the volume of the water in the tank. Thus, its volume changes with time

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\forall + \int_{cs} \rho_w \mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_w is constant (incompressible), it can be factor out of the integrals.

$$\rho_w \frac{\partial \forall}{\partial t} + \rho_w \int_{cs} \mathbf{V}\cdot d\mathbf{A} = 0$$

$$\frac{\partial \forall}{\partial t} + Q_{out} - Q_{in} = 0 \qquad \textbf{(1)}$$

Here, $Q_{out} = 0$ and

$$Q_{in} = \int_A v dA = \int_0^{0.1\text{m}} 3(1 - 100r^2)(2\pi r dr) = (0.015\pi)\ \text{m}^3/\text{s}$$

The integral $\int_A v dA$ can also be determined by computing the volume under the velocity profile, which in this case is a paraboloid.

$$\int_A v dA = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(0.1\ \text{m})^2(3\ \text{m/s}) = (0.015\pi)\ \text{m}^3/\text{s}$$

Also, the volume of the control volume at a particular instant is

$$\forall = (2\text{m})(3\text{m})(h) = 6h$$

Thus,

$$\frac{d\forall}{dt} = 6\frac{dh}{dt}$$

Substituting these results into Eq (1)

$$6\frac{dh}{dt} - 0.015\pi = 0$$

$$\frac{dh}{dt} = 0.0025\pi$$

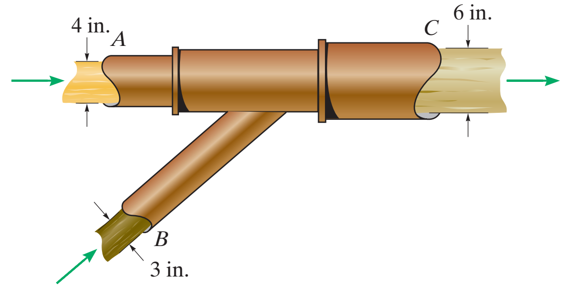
$$\int_0^{1.5\ \text{m}} dh = 0.0025\pi \int_0^t dt$$

$$1.5 = 0.0025\pi t$$

$$t = (190.99\ \text{s})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 3.18\ \text{min} \qquad \textbf{Ans.}$$

4–73. Ethyl alcohol flows through pipe A with an average velocity of 4 ft/s, and oil flows through pipe B at 2 ft/s. Determine the average density at which the mixture flows through the pipe at C. Assume uniform mixing of the fluids occurs within a 200 in^3 volume of the pipe assembly. Take $\rho_{ea} = 1.53\ \text{slug/ft}^3$ and $\rho_o = 1.70\ \text{slug/ft}^3$.



SOLUTION

The fluids are assumed to be incompressible, and so their volumes remain constant. Also the volume within the pipe is constant. Therefore

$$-V_A A_A - V_B A_B + V_C A_C = 0$$

$$-(4\ \text{ft/s})\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right] - (2\ \text{ft/s})\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right] + V_C\left[\pi\left(\frac{3}{12}\ \text{ft}\right)^2\right] = 0$$

$$V_C = 2.278\ \text{ft/s}$$

Applying the conservation of mass for steady flow.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \rho_{ea} V_A A_A - \rho_o V_B A_B + \rho_C V_C A_C = 0$$

Thus

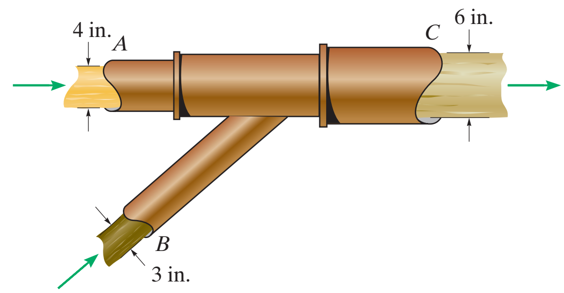
$$\rho_C = \frac{\rho_{ea} V_A A_A + \rho_o V_B A_B}{V_c A_c}$$

$$\rho_C = \frac{(1.53\ \text{slug/ft}^3)(4\ \text{ft/s})\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right] + (1.70\ \text{slug/ft}^3)(2\ \text{ft/s})\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right]}{(2.278\ \text{ft/s})\left(\pi\left(\frac{3}{12}\ \text{ft}\right)^2\right)}$$

$$\rho_C = 1.57\ \text{slug/ft}^3$$ **Ans.**

Ans:
$1.57\ \text{slug/ft}^3$

4–74. Ethyl alcohol flows through pipe A at 0.05 ft^3/s, and oil flows through pipe B at 0.03 ft^3/s. Determine the average density of the two fluids as the mixture flows through the pipe at C. Assume uniform mixing of the fluids occurs within a 200 in^3 volume of the pipe assembly. Take $\rho_{ea} = 1.53\ \text{slug/ft}^3$ and $\rho_o = 1.70\ \text{slug/ft}^3$.



SOLUTION

The fluids are assumed to be incompressible, so their volumes remain constant. Also, the volume within the pipe is constant. Therefore

$$-Q_A - Q_B + Q_C = 0$$

$$-0.05\ \text{ft}^3/\text{s} - 0.03\ \text{ft}^3/\text{s} + Q_C = 0$$

$$Q_C = 0.08\ \text{ft}^3/\text{s}$$

Applying the conservation of mass for steady flow

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

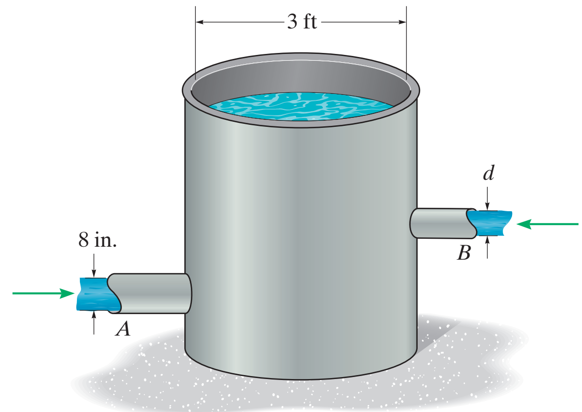
$$0 - \rho_{ea} Q_A - \rho_o Q_B + \rho_m Q_C = 0$$

$$0 - \left(1.53\ \text{slug/ft}^3\right)\left(0.05\ \text{ft}^3/\text{s}\right) - \left(1.70\ \text{slug/ft}^3\right)\left(0.03\ \text{ft}^3/\text{s}\right) + \rho_C\left(0.08\ \text{ft}^3/\text{s}\right) = 0$$

$$\rho_C = 1.59\ \text{slug/ft}^3 \qquad \textbf{Ans.}$$

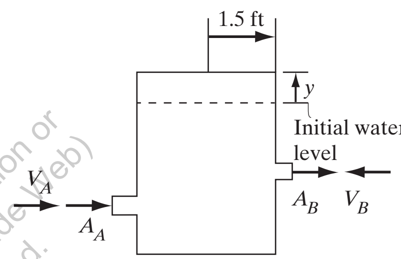
Ans:
$1.59\ \text{slug/ft}^3$

4–75. Water flows into the tank through two pipes. At A the flow is 400 gal/h, and at B it is 200 gal/h when $d = 6$ in. Determine the rate at which the level of water is rising in the tank. There are 7.48 gal/ft^3.



SOLUTION

Control volume. The deformable control volume shown in Fig. a will be considered. If the initial control volume is $\forall_0$, then its volume at any given instant is



(a)

$$\forall = \forall_0 + \pi(1.5\text{ ft})^2 y = \left[\forall_0 + 2.25\pi y\right]\text{ ft}^3$$

Continuity Equation. Realizing that $Q = \int_{cs} \mathbf{V}\cdot d\mathbf{A}$ and

$$Q_A = \left(400\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.01485\text{ ft}^3/\text{s}$$

$$Q_B = \left(200\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.007427\text{ ft}^3/\text{s}$$

Since the density of water is constant,

$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A}\right] = 0$$

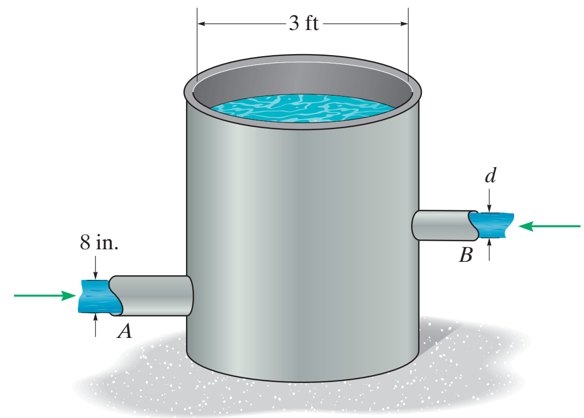
$$\frac{\partial}{\partial t}\left(\forall_0 + 2.25\pi y\right) - 0.01485\text{ ft}^3/\text{s} - 0.007427\text{ ft}^3/\text{s} = 0$$

$$2.25\pi\frac{\partial y}{\partial t} = 0.02228$$

$$\frac{\partial y}{\partial t} = 3.15\left(10^{-3}\right)\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$3.15\left(10^{-3}\right)$ ft/s

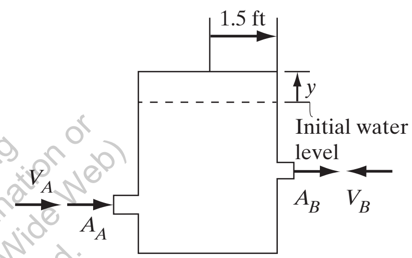
***4–76.** Water flows into the tank through two pipes. At A the flow is 400 gal/h. Determine the rate at which the level of water is rising in the tank as a function of the discharge of the inlet pipe B. Plot this rate (vertical axis) versus the discharge for $0 \le Q_B \le 300$ gal/h. Give values for increments of $\Delta Q_B = 50$ gal/h. There are 7.48 gal/ft^3.



SOLUTION

The deformable control volume shown in Fig. *a* will be considered. If the initial volume of this control volume is $\forall_0$, then its volume at any given instant is

$$\forall = \forall_0 + \pi(1.5\text{ ft})^2 y = (\forall_0 + 2.25\pi y)\text{ ft}^3$$



(a)

The discharges at A and B are

$$Q_A = \left(400\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.01485\text{ ft}^3/\text{s}$$

$$\left(Q_B\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \left[37.14(10^{-6})Q_B\right]\text{ ft}^3/\text{s}$$

Since the density of water is constant and $Q = \int_{cs} \mathbf{V}\cdot d\mathbf{A}$,

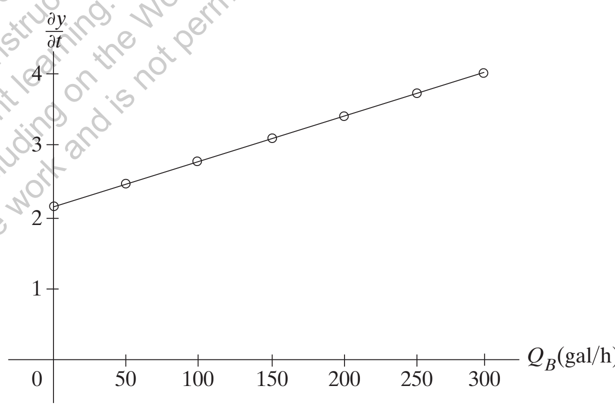
$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho\left(\frac{\partial\forall}{\partial t} - Q_A - Q_B\right) = 0$$

$$\frac{\partial}{\partial t}(\forall_0 + 2.25\pi y) = Q_A + Q_B$$

$$2.25\pi\frac{\partial y}{\partial t} = 0.01485 + 37.14(10^{-6})Q_B$$

$$\frac{\partial y}{\partial t} = \left[2.10(10^{-3}) + 5.25(10^{-6})Q_B\right]\text{ ft/s where } Q_B \text{ is in gal/h}$$ **Ans.**

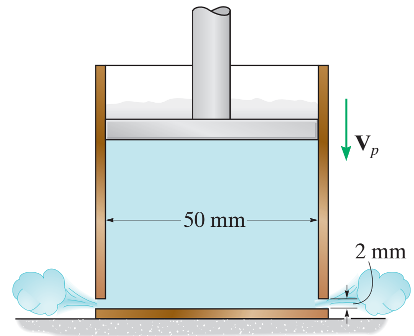


(b)

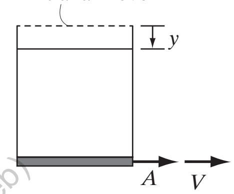
The plot of $\frac{\partial y}{\partial t}$ vs Q_B is shown in Fig. *b*.

Q_B(gal/h)	0	50	100	150	200	250	300
$\frac{\partial y}{\partial t}(10^{-3}\text{ft/s})$	2.10	2.36	2.63	2.89	3.15	3.41	3.68

4–77. The piston is traveling downwards at $V_p = 3\ \text{m/s}$, and as it does, air escapes radially outward through the entire bottom of the cylinder. Determine the average speed of the escaping air. Assume the air is incompressible.



SOLUTION



(a)

Control Volume. The deformable control volume shown in Fig. *a* will be considered. If the initial control volume is $\forall_0$, then its volume at any given instant is

$$\forall = \forall_0 - \pi(0.025\ \text{m})^2 y = \left[\forall_0 - 0.625(10^{-3})\pi y\right]\text{m}^3$$

Continuity Equation. Since the air is assumed to be incompressible, its density is constant.

$$\rho\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs} \mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\left[\forall_0 - 0.625(10^{-3})\pi y\right] + V\left[2\pi(0.025\ \text{m})(0.002\ \text{m})\right] = 0$$

$$-0.625(10^{-3})\pi\frac{\partial y}{\partial t} + 0.1(10^{-3})\pi V = 0$$

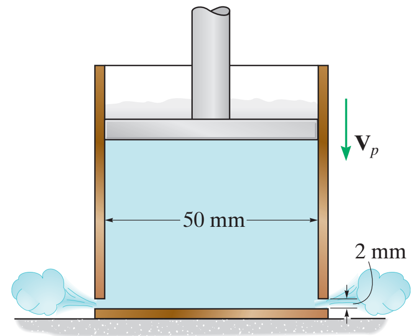
$$V = 6.25\frac{dy}{dt}$$

However, $\dfrac{dy}{dt} = 3\ \text{m/s}$. Then

$$V = 6.25(3\ \text{m/s}) = 18.8\ \text{m/s}$$ **Ans.**

Ans:
$18.8\ \text{m/s}$

4–78. The piston is travelling downwards with a velocity V_p, and as it does, air escapes radially outward through the entire bottom of the cylinder. Determine the average velocity of the air at the bottom as a function of V_p. Plot this average velocity of the escaping air (vertical axis) versus the velocity of the piston for $0 \le V_p \le 5$ m/s. Give values for increments of $\Delta V_p = 1$ m/s. Assume the air is incompressible.



SOLUTION

The deformable control volume shown in Fig. *a* will be considered. If the initial control volume is $\forall_0$, then its volume at any given instant is

$$\forall = \forall_0 - \pi(0.025\text{ m})^2 y = \left[\forall_0 - 0.625\left(10^{-3}\right)\pi y\right]\text{ m}^3$$

Since the air is assumed to be incompressible, its density is constant.

$$\rho\left[\frac{\partial}{\partial t}\int_{\text{cv}} d\forall + \int_{\text{cs}} \mathbf{V}\cdot d\mathbf{A}\right] = 0$$

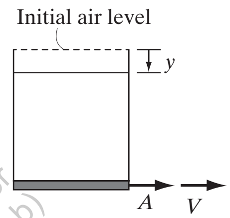
$$\frac{\partial}{\partial t}\left[\forall_0 - 0.625\left(10^{-3}\right)\pi y\right] + V\left[2\pi(0.025\text{ m})(0.002\text{ m})\right] = 0$$

$$-0.625\left(10^{-3}\right)\pi\frac{\partial y}{\partial t} + 0.1\left(10^{-3}\right)\pi V = 0$$

$$V = 6.25\frac{\partial y}{\partial t}$$

However, $\dfrac{\partial y}{\partial t} = V_p$. Then

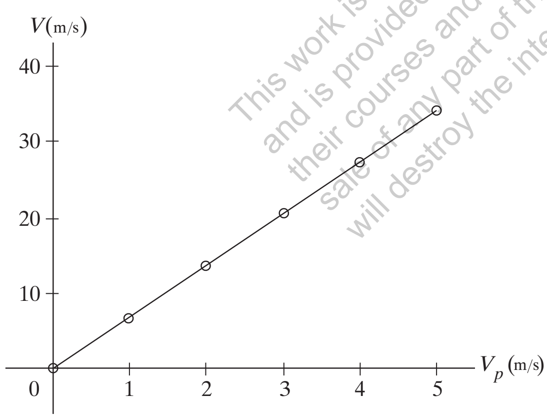
$$V = (6.25\,V_p)\text{ m/s}$$ **Ans.**



(a)

The plot of V vs V_p is shown in Fig. *b*

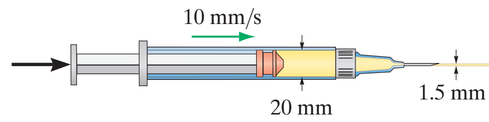
V_p(m/s)	0	1	2	3	4	5
V(m/s)	0	6.25	12.5	18.75	25.0	31.25



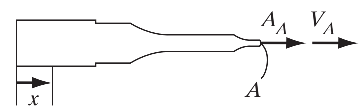
(b)

Ans:
$V = (6.25V_p)$ m/s

4–79. The cylindrical syringe is actuated by applying a force on the plunger. If this causes the plunger to move forward at 10 mm/s, determine the average velocity of the fluid passing out of the needle.



SOLUTION



(a)

Control Volume. The deformable control volume is shown in Fig. *a*. If the volume of the control volume is initially $\mathcal{V}_0$ then at any instant its volume is

$$\mathcal{V} = \mathcal{V}_0 - \frac{\pi}{4}(0.02\text{ m})^2 x = \left[\mathcal{V}_0 - 0.1\left(10^{-3}\right)\pi x\right]\text{ m}^3$$

Continuity Equation. With the fluid assumed to be incompressible, ρ is constant. since $\mathbf{V}_A$ and $\mathbf{A}_A$ are in the same sense, $Q_A = V_A A_A = V_A\left[\frac{\pi}{4}(0.0015\text{ m})^2\right] = 0.5625\left(10^{-6}\right)\pi V_A$.

$$\frac{\partial}{\partial t}\int_{cv}\rho d\mathcal{V} + \int_{cs} p\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho_w\left[\frac{\partial}{\partial t}(\mathcal{V}) + V_A A_A\right] = 0$$

$$\frac{\partial}{\partial t}\left[\mathcal{V}_0 - 0.1\left(10^{-3}\right)\pi x\right] + 0.5625\left(10^{-6}\right)\pi V_A = 0$$

$$-0.1\left(10^{-3}\right)\pi\frac{\partial x}{\partial t} + 0.5625\left(10^{-6}\right)\pi V_A = 0$$

However,

$$\frac{\partial x}{\partial t} = 10\text{ mm/s} = 0.01\text{ m/s}$$

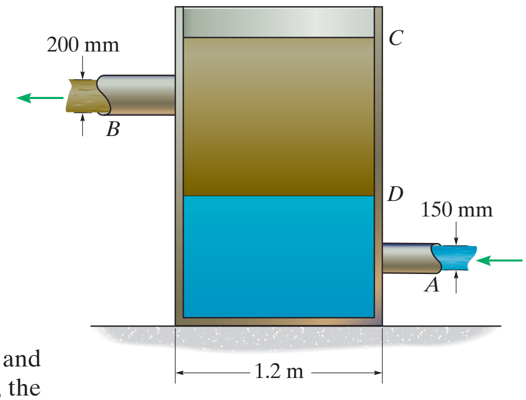
Then

$$\left[0.1\left(10^{-3}\right)\pi\right](0.01) + 0.5625\left(10^{-6}\right)\pi V_A = 0$$

$$V_A = 1.78\text{ m/s}$$ **Ans.**

Ans:
1.78 m/s

***4–80.** Water enters the cylindrical tank at A with an average velocity of 2 m/s, and oil exits the tank at B with an average velocity of 1.5 m/s. Determine the rates at which the top level C and interface level D are moving. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

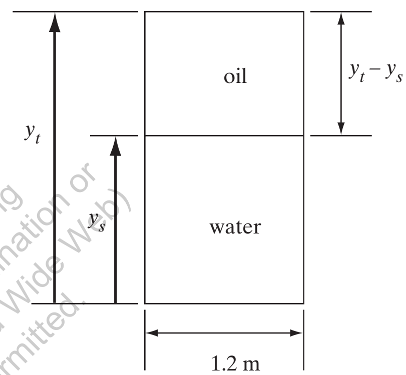
We will consider two control volumes separately namely one contains water and the other contains oil in the tank their volume changes with time, Fig. *a*. Here, the densities of water and oil are constant and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv}\rho dV + \int \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

For water,

$$\rho_w \frac{\partial V\!\!\!/_w}{\partial t} - \rho_w V_A A_A = 0$$

$$\frac{\partial V\!\!\!/_w}{\partial t} - V_A A_A = 0 \qquad \textbf{(1)}$$



(a)

Here,

$$V\!\!\!/_w = \left[\pi(0.6\ \text{m})^2\right]y_s = 0.36\ \pi y_s$$

$$\frac{\partial V\!\!\!/_w}{\partial t} = 0.36\pi\frac{\partial y_s}{\partial t}$$

Substitute this result into Eq (1)

$$0.36\pi\frac{\partial y_s}{\partial t} - (2\ \text{m/s})\left[\pi(0.075\ \text{m})^2\right] = 0$$

$$V_p = \frac{\partial y_s}{\partial t} = 0.0312\ \text{m/s} \qquad \textbf{Ans.}$$

Positive sign indicates that the separation level is rising.

For the oil,

$$\rho_o\frac{\partial V\!\!\!/_0}{\partial t} + \rho_o V_B A_B = 0$$

$$\frac{\partial V\!\!\!/_0}{\partial t} + V_B A_B = 0 \qquad \textbf{(2)}$$

Here

$$V\!\!\!/_0 = \left[\pi(0.6\ \text{m})^2\right](y_t - y_s)$$

$$= 0.36\pi(y_t - y_s)$$

$$\frac{\partial V\!\!\!/_0}{\partial t} = 0.36\pi\left(\frac{\partial y_t}{\partial t} - \frac{\partial y_s}{\partial t}\right)$$

$$= 0.36\ \pi\left(\frac{\partial y_t}{\partial t} - 0.03125\ \text{m/s}\right)$$

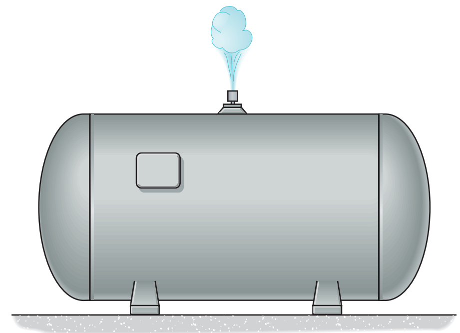
Substituting this result into Eq. (2),

$$0.36\ \pi\left(\frac{\partial y_t}{\partial t} - 0.03125\right) + (1.5\ \text{m/s})\left[\pi(0.1\ \text{m})^2\right] = 0$$

$$V_C = \frac{\partial y_t}{\partial t} = -0.0104\ \text{m/s} \qquad \textbf{Ans.}$$

The negative sign indicates that the top level descends.

4–81. The tank contains air at a temperature of 20°C and absolute pressure of 500 kPa. Using a valve, the air escapes with an average speed of 120 m/s through a 15-mm-diameter nozzle. If the volume of the tank is 1.25 m^3, determine the rate of change in the density of the air within the tank at this instant. Is the flow steady or unsteady?



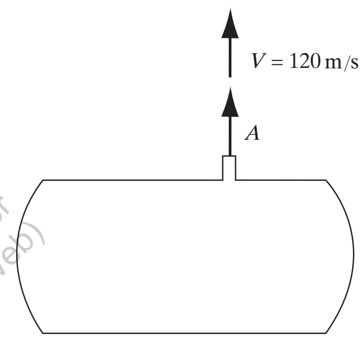
SOLUTION

From Appendix A, the gas constant for air is $R = 286.9\ \mathrm{J/(kg\cdot K)}$

$$p = \rho R T$$

$$500\left(10^3\right)\ \mathrm{N/m^2} = \rho\left(286.9\ \mathrm{J/(kg\cdot K)}\right)(20°\mathrm{C} + 273)$$

$$\rho = 5.948\ \mathrm{kg/m^3}$$



(a)

Control Volume. The control volume is shown in Fig. *a*. The control volume does not change, but the density of the air changes and therefore results in local changes.

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cv} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$\frac{\partial \rho}{\partial t}(\mathcal{V}) + \rho V A = 0$$

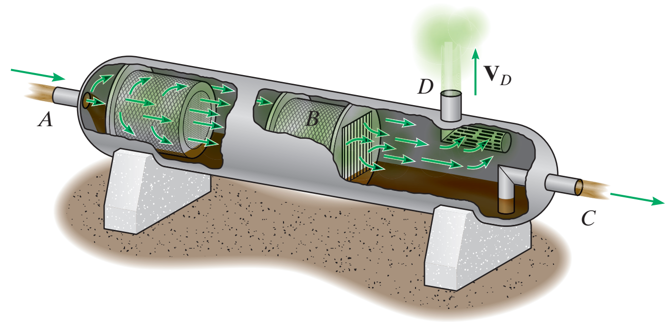
$$\frac{\partial \rho}{\partial t}\left(1.25\ \mathrm{m^3}\right) + \left(5.948\ \mathrm{kg/m^3}\right)\left(120\mathrm{m/s}\right)\left[\pi(0.0075\ \mathrm{m})^2\right] = 0$$

$$\frac{\partial \rho}{\partial t} = -0.101\ \mathrm{kg/\left(m^3\cdot s\right)}$$ **Ans.**

The negative sign indicates that the density of the air is decreasing. Flow is unsteady, since the pressure within the tank is decreasing and this affects the flow.

Ans:
$-0.101\ \mathrm{kg/\left(m^3\cdot s\right)}$, unsteady

4–82. The natural gas (methane) and crude oil mixture enters the separator at A at $6\ \text{ft}^3/\text{s}$ and passes through the mist extractor at B. Crude oil flows out at $800\ \text{gal/min}$ through the pipe at C, and natural gas leaves the 2-in-diameter pipe at D at $V_D = 300\ \text{ft/s}$. Determine the specific weight of the mixture that enters the separator at A. The process takes place at a constant temperature of 68°F. Take $\rho_o = 1.71\ \text{slug/ft}^3$, $\rho_{me} = 1.29\left(10^{-3}\right)\ \text{slug/ft}^3$. Note $1\ \text{ft}^3 = 7.48\ \text{gal}$.



SOLUTION

The control volume is fixed which is the volume of the crude oil and natural gas contained in the tank. Here, the flow is steady. Thus, no local changes take place. Also, the densities of the gas oil mixture, gas and oil separation are constant, and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\mathcal{V} + \int_{cs} \rho\, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \rho_{\text{mix}} V_A A_A + \rho_{co} V_C A_C + \rho_{me} V_D A_D = 0 \qquad \textbf{(1)}$$

From Appendix A, $\rho_{co} = 1.71\ \text{slug/ft}^3$ and $\rho_m = 1.29\left(10^{-3}\right)\ \text{slug/ft}^3$ when $T = 68°\text{F}$. Also, $Q_C = V_C A_C = \left(800\,\dfrac{\text{gal}}{\text{min}}\right)\left(\dfrac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 1.783\ \text{ft}^3/\text{s}$ and

$Q_A = V_A A_A = 6\ \text{ft}^3/\text{s}$. Substituting these results into Eq. (1),

$$-\rho_{\text{mix}}(6\ \text{ft}^3/\text{s}) + \left(1.71\ \text{slug/ft}^3\right)\left(1.783\ \text{ft}^3/\text{s}\right)$$

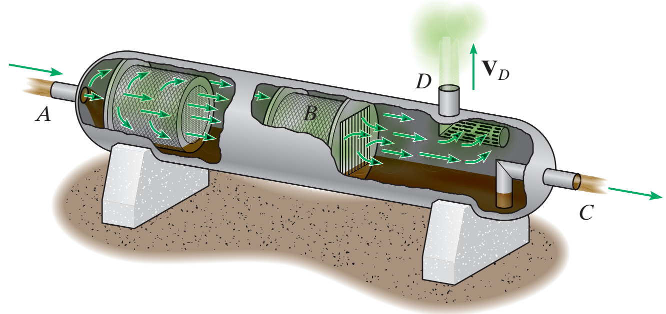
$$+ \left[1.29\left(10^{-3}\right)\text{slug/ft}^3\right]\left(300\ \text{ft/s}\right)\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] = 0$$

$$\rho_{\text{mix}} = 0.5094\ \text{slug/ft}^3$$

$$\gamma_{\text{mix}} = \rho_{\text{mix}}\, g = \left(0.5094\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right) = 16.4\ \text{lb/ft}^3 \qquad \textbf{Ans.}$$

Ans:
$16.4\ \text{lb/ft}^3$

4–83. The natural gas (methane) and crude oil mixture having a density of 0.51 slug/ft^3 enters the separator at A at 6 ft^3/s, and crude oil flows out through the pipe at C at 800 gal/min. Determine the average velocity of the natural gas that leaves the 2-in.-diameter pipe at D. The process takes place at a constant temperature of 68°F. Take $\rho_o = 1.71\ \text{slug/ft}^3$, $\rho_{me} = 1.29\left(10^{-3}\right)\ \text{slug/ft}^3$. Note $1\ \text{ft}^3 = 7.48$ gal.



SOLUTION

The control volume is fixed which is the volume of the mixture of crude oil and natural gas contained in the tank. Here, the flow is steady. Thus, no local changes take place. Also the densities of the oil gas mixture, gas and oil separation, are constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - \rho_{mix}V_A A_A + \rho_{CO}V_C A_C + \rho_{me}V_D A_D = 0 \qquad \textbf{(1)}$$

From Appendix A, $\rho_{CO} = 1.71\ \text{slug/ft}^3$ and $\rho_{me} = 1.29\left(10^{-3}\right)\ \text{slug/ft}^3$ at $T = 68°\text{F}$

Also, $Q_C = \left(800\,\dfrac{\text{gal}}{\text{min}}\right)\left(\dfrac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 1.783\ \text{ft}^3/\text{s}$ and $Q_A = V_A A_A = 6\ \text{ft}^3/\text{s}$.

Substituting these results into Eq 1,

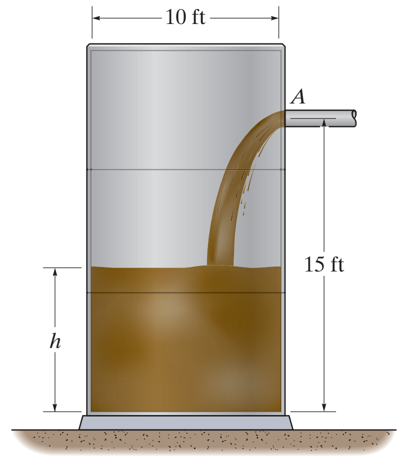
$$-\left(0.51\ \text{slug/ft}^3\right)\left(6\ \text{ft}^3/\text{s}\right) + \left(1.71\ \text{slug/ft}^3\right)\left(1.783\ \text{ft}^3/\text{s}\right)$$

$$+ \left[1.29\left(10^{-3}\right)\text{slug/ft}^3\right]\left(V_D\right)\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] = 0$$

$$V_D = 422\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
422 ft/s

***4–84.** The cylindrical storage tank is being filled using a pipe having a diameter of 3 in. Determine the rate at which the level in the tank is rising if the flow into the tank at A is 40 gal/min . Note 1 $ft^3 = 7.48$ gal.



SOLUTION

The control volume is the volume of oil contained in the tank, which changes with time. Here, the density of the oil is constant and the average velocity will be used

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho\, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\rho \frac{\partial \mathcal{V}}{\partial t} - \rho V_A A_A = 0$$

Since $Q_A = V_A A_A = \left(40 \dfrac{\text{gal}}{\text{min}}\right)\left(\dfrac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 0.08913\text{ ft}^3/\text{s}$. Then

$$\frac{\partial \mathcal{V}}{\partial t} = 0.08913 \qquad \textbf{(1)}$$

Here, the volume of the control volume at a particular instant is

$$\mathcal{V} = \pi r^2 h = \pi(5\text{ ft})^2 h = 25\pi h$$

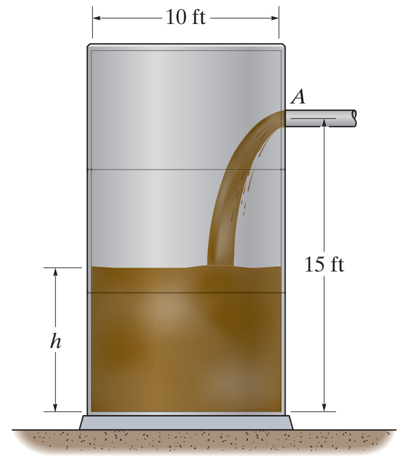
$$\frac{\partial \mathcal{V}}{\partial t} = 25\pi \frac{\partial h}{\partial t}$$

Substituting this result into Eq (1)

$$25\pi \frac{\partial h}{\partial t} = 0.08913$$

$$\frac{\partial h}{\partial t} = 1.13\left(10^{-3}\right)\text{ ft/s} \qquad \textbf{Ans.}$$

4–85. The cylindrical storage tank is being filled using a pipe having a diameter of D. Determine the rate at which the level is rising as a function of D if the velocity of the flow into the tank is 6 ft/s. Plot this rate (vertical axis) versus the diameter for $0 \le D \le 6$ in. Give values for increments of $\Delta D = 1$ in.



SOLUTION

The control volume is the volume of oil contained in the tank of which its volume changes with time. Here, the density of the oil is constant and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\rho\left(\frac{\partial \mathcal{V}}{\partial t} - V_A A_A\right) = 0$$

$$\frac{\partial \mathcal{V}}{\partial t} = V_A A_A = (6\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{D}{12}\right)^2\right]$$

$$\frac{\partial \mathcal{V}}{\partial t} = 0.03272\, D^2 \qquad \textbf{(1)}$$

Here, the volume of the control volume at a particular instant is

$$\mathcal{V} = \pi r^2 h = \pi(5\text{ ft})^2 h = 25\pi h$$

$$\frac{\partial \mathcal{V}}{\partial t} = 25\pi \frac{\partial h}{\partial t}$$

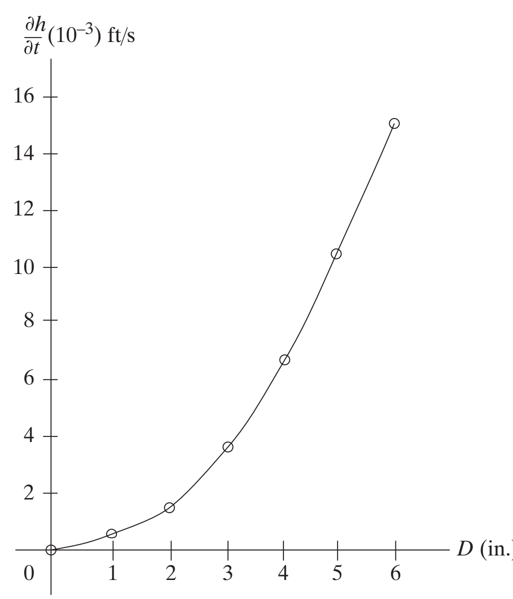
Substituting this result into Eq (1),

$$25\pi \frac{\partial h}{\partial t} = 0.03272\, D^2$$

$$\frac{\partial h}{\partial t} = \left[0.417\left(10^{-3}\right)D^2\right] \text{ ft/s where } D \text{ is in inches.} \qquad \textbf{Ans.}$$

The plot of $\frac{\partial h}{\partial t}$ vs D is shown in Fig. *a*.

D(in.)	0	1	2	3	4	5	6
$\frac{\partial h}{\partial t}(10^{-3})$ft/s	0	0.417	1.67	3.75	6.67	10.4	15.0

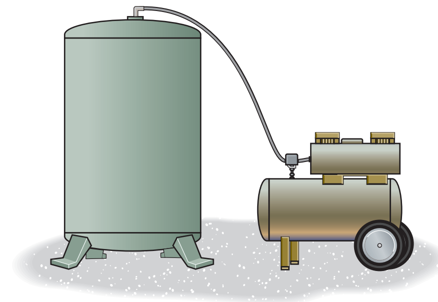


(a)

Ans:

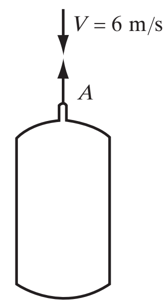
$$\frac{\partial h}{\partial t} = \left[0.417\left(10^{-3}\right) D^2\right] \text{ ft/s, where } D \text{ is in in.}$$

4–86. Air is pumped into the tank using a hose having an inside diameter of 6 mm. If the air enters the tank with an average speed of 6 m/s and has a density of $1.25\ \mathrm{kg/m^3}$, determine the initial rate of change in the density of the air within the tank. The tank has a volume of $0.04\ \mathrm{m^3}$.



SOLUTION

Control Volume. The control volume is shown in Fig. *a*. The control volume does not change but the density of the air changes and therefore results in local changes.



(a)

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv}\rho\, d\mathcal{V} + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

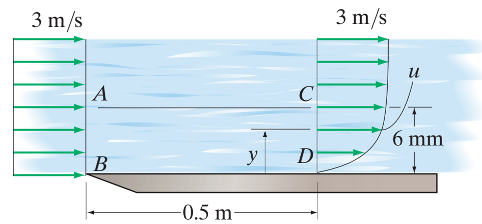
$$\frac{\partial \rho}{\partial t}(\mathcal{V}) - \rho V A = 0$$

$$\frac{\partial \rho}{\partial t}\left(0.04\ \mathrm{m^3}\right) - \left(1.25\ \mathrm{kg/m^3}\right)\left(6\ \mathrm{m/s}\right)\left[\pi(0.003\ \mathrm{m})^2\right] = 0$$

$$\frac{\partial \rho}{\partial t} = 0.00530\ \mathrm{kg/\left(m^3 \cdot s\right)} \qquad \textbf{Ans.}$$

Ans:
$0.00530\ \mathrm{kg/\left(m^3 \cdot s\right)}$

4–87. As air flows over the plate, frictional effects on its surface tend to form a boundary layer in which the velocity profile changes from that of being uniform to one that is parabolic, defined by $u = \left[1000y - 83.33\left(10^3\right)y^2\right]$ m/s, where y is in meters, $0 \le y < 6$ mm. If the plate is 0.2 m wide and this change in velocity occurs within the distance of 0.5 m, determine the mass flow through the sections AB and CD. Since these results will not be the same, how do you account for the mass flow difference? Take $\rho = 1.226\ \mathrm{kg/m^3}$.

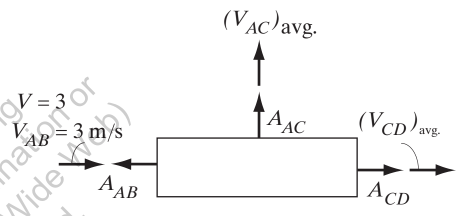


SOLUTION

Mass Flow Rate. For section AB, since ρ is constant and the velocity has a constant magnitude,

$$\dot{m}_{AB} = \rho V_{AB} A_{AB}$$

$$= \left(1.226\ \mathrm{kg/m^3}\right)(3\ \mathrm{m/s})\left[0.006\ \mathrm{m}(0.2\ \mathrm{m})\right] = 0.00441\ \mathrm{kg/s} = 4.41\ \mathrm{g/s} \qquad \textbf{Ans.}$$



(a)

For section CD, since the velocity is a function of y, a differential element of thickness dy, which has an area $dA = b\,dy = (0.2\ \mathrm{m})dy$, is chosen. Thus,

$$\dot{m}_{CD} = \rho \int u\,dA$$

$$= \left(1.226\ \mathrm{kg/m^3}\right)\left(\int_0^{0.006\ \mathrm{m}} \left[1000y - 83.33\left(10^3\right)y^2\right]\ \mathrm{m/s}\right)(0.2\ \mathrm{m})dy$$

$$= 0.2452\left[500y^2 - 27.78\left(10^3\right)y^3\right]\Big|_0^{0.006\ \mathrm{m}}$$

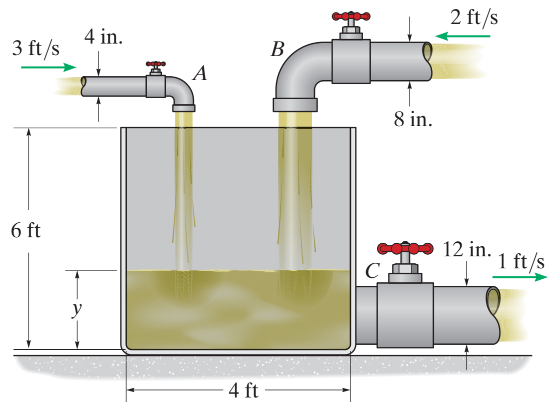
$$= 0.00294\ \mathrm{kg/s} = 2.94\ \mathrm{g/s} \qquad \textbf{Ans.}$$

To satisfy continuity the difference between $\dot{m}_{AB}$ and $\dot{m}_{CD}$ requires that mass flows through the control surface AC as indicated on the control volume in Fig. a.

What this means is that the streamlines in fact cannot be horizontal, as the figure implies. The fluid velocity must have a vertical component in addition to the horizontal one.

Ans:
2.94 g/s

***4–88.** Kerosene flows into the rectangular tank through pipes A and B, at 3 ft/s and 2 ft/s, respectively. It exits at C at a constant rate of 1 ft/s. Determine the rate at which the surface of the kerosene is rising. The base of the tank is 6 ft by 4 ft.



SOLUTION

The control volume is the volume of the kerosene in the tank. Thus its volume changes with time.

$$\frac{\partial}{\partial t}\int_{cv} \rho_{ke}\, d\forall + \int_{cs} \rho_{ke}\mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_{ke} is constant (incompressible), it can be factored out of the integral.

$$\rho_{ke}\frac{\partial \forall}{\partial t} + \rho_{ke}\int_{cs} \mathbf{V}\cdot d\mathbf{A} = 0$$

Here, we will use the average velocities.

$$\frac{d\forall}{dt} - V_A A_A - V_B A_B + V_C A_C = 0$$

$$\frac{d\forall}{dt} - (3\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{4}{12}\text{ ft}\right)^2\right] - (2\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{8}{12}\text{ ft}\right)^2\right] + (1\text{ ft/s})\left[\frac{\pi}{4}(1\text{ ft})^2\right] = 0$$

$$\frac{d\forall}{dt} = 0.1745\text{ ft}^3/\text{s} \qquad \textbf{(1)}$$

The volume of the control volume at a particular instant is

$$\forall = (6\text{ ft})(4\text{ ft})y = (24y)\text{ ft}^3$$

Thus

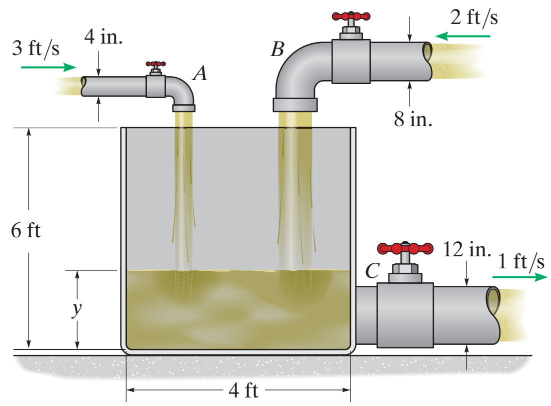
$$\frac{\partial \forall}{\partial t} = 24\frac{\partial y}{\partial t}$$

Substituting this result into Eq (1),

$$24\frac{\partial y}{\partial t} = 0.1745$$

$$\frac{\partial y}{\partial t} = 0.00727\text{ ft/s} \qquad \textbf{Ans.}$$

4–89. Kerosene flows into the 4-ft-diameter cylindrical tank through pipes A and B, at 3 ft/s and 2 ft/s, respectively. It exists at C at a constant rate of 1 ft/s. Determine the time required to fill the tank if $y = 0$ when $t = 0$.



SOLUTION

The control volume is the volume of the kerosene in the tank. Thus, its volume changes with time.

$$\frac{\partial}{\partial t}\int_{cv} \rho_{ke}\, d\forall + \int_{cs} \rho_{ke}\mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_{ke} is constant (incompressible), it can be factored out of the integral

$$\rho_{ke}\frac{\partial \forall}{\partial t} + \rho_{ke}\int_{cs}\mathbf{V}\cdot d\mathbf{A} = 0$$

Here, we will use the average velocities

$$\frac{d\forall}{dt} - V_A A_A - V_B A_B + V_C A_C = 0$$

$$\frac{d\forall}{dt} - (3\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{4}{12}\text{ ft}\right)^2\right] - (2\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{8}{12}\text{ ft}\right)^2\right] + (1\text{ ft/s})\left[\frac{\pi}{4}(1\text{ ft})^2\right] = 0$$

$$\frac{d\forall}{dt} = 0.1745\text{ ft}^3/\text{s} \qquad \textbf{(1)}$$

The volume of the control volume at a particular instant is

$$\forall = \frac{\pi}{4}(4\text{ ft})^2 y = (4\pi y)\text{ ft}^3$$

Thus,

$$\frac{d\forall}{dt} = 4\pi\frac{dy}{dt}$$

Substituting this result into Eq (1),

$$4\pi\frac{dy}{dt} = 0.1745$$

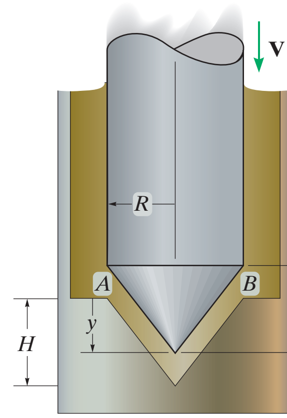
$$\int_0^{6\text{ ft}} dy = 0.01389\int_0^t dt$$

$$6 = 0.01389t$$

$$t = (432.5)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 7.20\text{ min.} \qquad \textbf{Ans.}$$

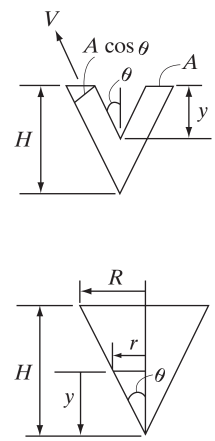
Ans:
7.20 min

4–90. The conical shaft is forced into the conical seat at a constant speed of V_0. Determine the average velocity of the liquid as it is ejected from the horizontal section AB as a function of y. *Hint*: The volume of a cone is $\forall = \frac{1}{3}\pi r^2 h$.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered.



(a)

$$\frac{r}{R} = \frac{y}{H};\ r = \frac{R}{H}y$$

Then, the volume of the control volume at any instant is

$$\forall = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{R}{H}y\right)^2 y = \frac{\pi R^2}{3H^2}\left(H^3 - y^3\right)$$

Continuity Equation. Since the density of the liquid is constant,

$$\rho\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs} \mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall + V(A\cos\theta) = 0$$

$$\frac{\partial}{\partial t}\left[\frac{\pi R^2}{3H^2}\left(H^3 - y^3\right)\right] + V\left[\pi\left[R^2 - \left(\frac{R}{H}y\right)^2\right]\cos\theta\right] = 0$$

$$\frac{\pi R^2}{3H^2}\left(-3y^2\right)\frac{dy}{dt} + V\left[\frac{\pi R^2}{H^2}\left(H^2 - y^2\right)\right]\cos\theta = 0$$

$$V = \left(\frac{y^2}{H^2 - y^2}\right)\frac{dy}{dt}(\sec\theta)$$

However, $\frac{dy}{dt} = V_0$ and $\sec\theta = \frac{\sqrt{H^2 + R^2}}{H}$. Then,

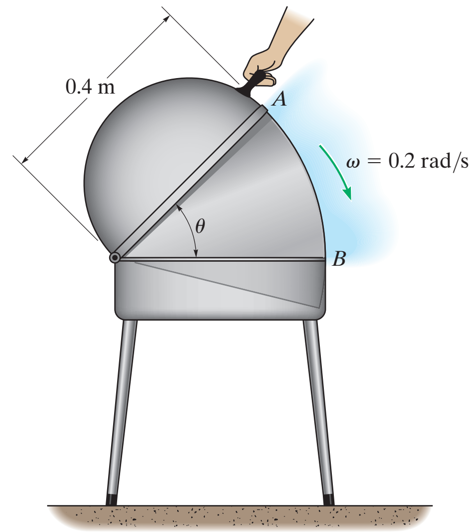
$$V = \left(\frac{y^2}{H^2 - y^2}\right)V_0\frac{\sqrt{H^2 + R^2}}{H}$$

$$V = V_0\frac{y^2\sqrt{H^2 + R^2}}{H\left(H^2 - y^2\right)}$$ **Ans.**

Ans:

$$V = V_0\frac{y^2\sqrt{H^2 + R^2}}{H\left(H^2 - y^2\right)}$$

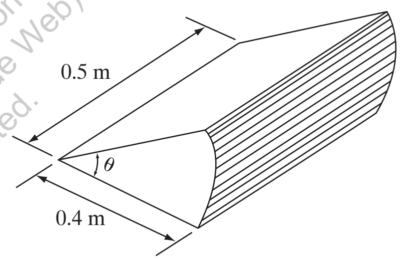
4–91. The 0.5-m-wide lid on the barbecue grill is being closed at a constant angular velocity of $\omega = 0.2$ rad/s, starting at $\theta = 90°$. In the process, the air between A and B will be pushed out in the *radial direction* since the sides of the grill are covered. Determine the average velocity of the air that emerges from the front of the grill at the instant $\theta = 45°$ rad. Assume that the air is incompressible.



SOLUTION

The flow is considered one dimensional since its velocity is directed in the radial direction only. The control volume is shown in Fig. *a*, and its volume changes with time. At a particular instant it is

$$\forall = \frac{1}{2}r^2\theta b = \frac{1}{2}(0.4\text{ m})^2\theta(0.5\text{ m}) = 0.04\theta\text{ m}^3$$



(a)

Thus,

$$\frac{d\forall}{dt} = 0.04\frac{d\theta}{dt}$$

However,

$$\frac{d\theta}{dt} = \omega = -0.2\text{ rad/s}.$$

Then

$$\frac{d\forall}{dt} = -0.008\text{ m}^3/\text{s}$$

Notice that negative sign indicates that the volume is decreasing with time. The opened control surface is shown shaded in Fig. *a*. Its area is

$$A = r\theta b = (0.4\text{ m})\theta(0.5\text{ m}) = 0.2\theta$$

Thus,

$$\frac{\partial}{\partial t}\int_{\text{cv}}\rho_a\, d\forall + \int_{\text{cs}}\rho_a\mathbf{V}\cdot d\mathbf{A} = 0$$

Since here air is assumed to be incompressible, ρ_a is constant. Also, the average velocity of the air is directed radially outward. Thus, it always acts perpendicular to the opened control surface. Hence, the above equation becomes

$$\rho_a\frac{\partial\forall}{\partial t} + \rho_a V_a A = 0$$

$$\frac{\partial\forall}{\partial t} = -V_a A$$

$$V_a = -\frac{\partial\forall/\partial t}{A} = -\frac{-0.008}{0.2\theta} = \frac{0.04}{\theta}\text{ m/s}$$

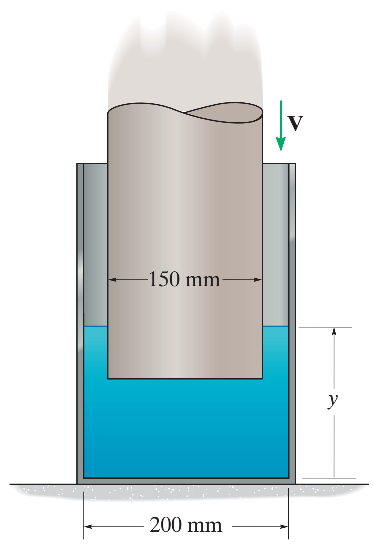
When

$$\theta = 45° = \frac{\pi}{4}\text{ rad},$$

$$V_A = \frac{0.04}{\pi/4} = 0.0509\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
0.0509 m/s

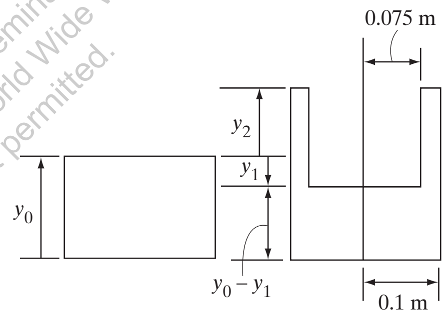
***4–92.** The cylinder is pushed down into the tube at a rate of $V = 5$ m/s. Determine the average velocity of the liquid as it rises in the tube.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered. If the initial water level in the tube is y_0, Fig. *b*, then the control volume at any instant is

$$\begin{aligned}\forall &= \pi(0.1\text{ m})^2(y_0 - y_1) + \pi(0.1\text{ m})^2(y_1 + y_2) - \pi(0.075\text{ m})^2(y_1 + y_2)\\ &= \pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)\end{aligned}$$



(b) (a)

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv}\rho_w d\forall + \int_{cs}\rho_w \mathbf{V}_{f/cs}\cdot d\mathbf{A} = 0$$

Since no water enters or leaves the control volume at any instant, $\int_{cs}\rho_w \mathbf{V}_{f/cs}\cdot d\mathbf{A} = 0$.

Then,

$$\frac{\partial}{\partial t}\int_{cv}\rho_w d\forall = 0$$

$$\rho_w \frac{\partial}{\partial t}\forall = 0$$

$$\frac{\partial}{\partial t}\left[\pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)\right] = 0$$

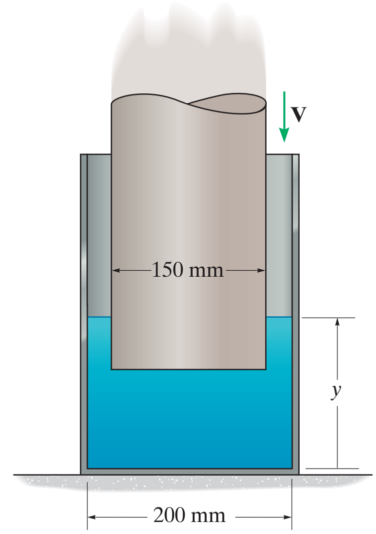
$$-0.005625\frac{\partial y_1}{\partial t} + 0.004375\frac{\partial y_2}{\partial t} = 0$$

$$\frac{\partial y_2}{\partial t} = 1.2857\frac{\partial y_1}{\partial t}$$

However, $\dfrac{\partial y_1}{\partial t} = V_r = 5$ m/s. Then

$$\frac{\partial y_2}{\partial t} = 1.2857(5\text{ m/s}) = 6.43\text{ m/s}$$ **Ans.**

4–93. Determine the speed V at which the cylinder must be pushed down into the tube so that the liquid in the tube rises with an average velocity of 4 m/s.



SOLUTION

Control Volume. The deformable control volume shown in Fig. a will be considered. If the initial water level in the tube is y_0, Fig. b, then the control volume at any instant is

$$\forall = \pi(0.1\text{ m})^2(y_0 - y_1) + \pi(0.1\text{ m})^2(y_1 + y_2) - \pi(0.075\text{ m})^2(y_1 + y_2)$$

$$= \pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)$$

Continuity Equation. $$\frac{\partial}{\partial t}\int_{cv}\rho_w d\forall + \int_{cs}\rho_w \mathbf{V}_{f/cs}\cdot d\mathbf{A} = 0$$

Since no water enters or leaves the control volume at any instant, $\int_{cs}\rho_w \mathbf{V}_{f/cs}\cdot d\mathbf{A} = 0$.

Then,

$$\frac{\partial}{\partial t}\int_{cv}\rho_w d\forall = 0$$

$$\rho_w \frac{\partial}{\partial t}\forall = 0$$

$$\frac{\partial}{\partial t}\left[\pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)\right] = 0$$

$$-0.005625\frac{\partial y_1}{\partial t} + 0.004375\frac{\partial y_2}{\partial t} = 0$$

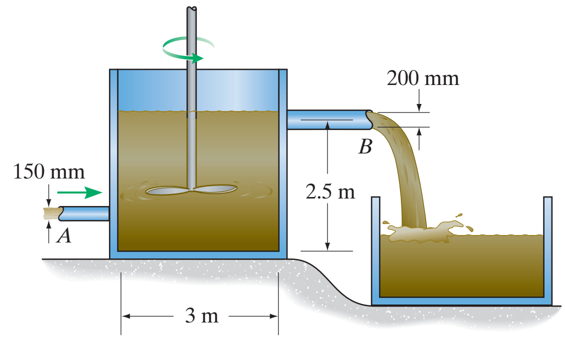
$$\frac{\partial y_1}{\partial t} = 0.7778\frac{\partial y_2}{\partial t}$$

However, $\frac{\partial y_1}{\partial t} = V_r$ and $\frac{\partial y_2}{\partial t} = 4\text{ m/s}$. Then

$$V_r = 0.7778(4\text{ m/s}) = 3.11\text{ m/s}$$ **Ans.**

Ans:
3.11 m/s

4–94. The tank originally contains oil. If kerosene having a mass flow of 0.2 kg/s enters the tank at A and mixes with the oil, determine the rate of change of the density of the mixture in the tank if 0.28 kg/s of the mixture exits the tank at the overflow B. The tank is 3 m wide.



SOLUTION

Control Volume. The control volume is shown in Fig. *a*. The control volume does not change but the density of the mixture changes and therefore results in local changes.

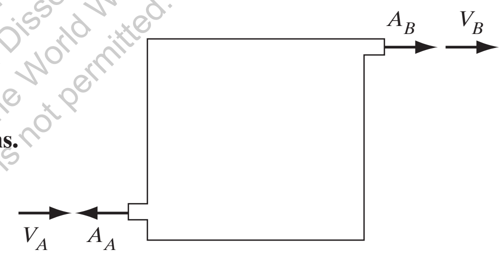
Continuity Equation. Since the liquids are incompressible and the volume is constant, we have

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cv} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\frac{\partial \rho}{\partial t}(3\text{ m})(3\text{ m})(2.5\text{ m}) - 0.2\text{ kg/s} + 0.28\text{ kg/s} = 0$$

$$\frac{\partial \rho}{\partial t} = -0.00356\text{ kg/}\left(\text{m}^3 \cdot \text{s}\right) \qquad \textbf{Ans.}$$

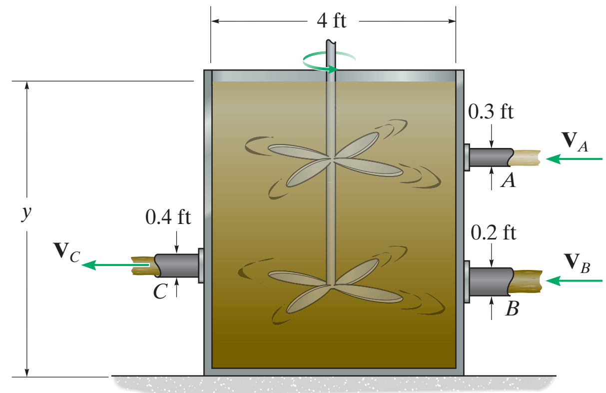
The negative sign indicates that the density of the mixture is decreasing.



(a)

Ans:
$-0.00356\text{ kg/}(\text{m}^3 \cdot \text{s})$

4–95. Benzene flows through the pipe at A with an average velocity of 4 ft/s, and kerosene flows through the pipe at B with an average velocity of 6 ft/s. Determine the required average velocity V_C of the mixture from the tank at C so that the level of the mixture within the tank remains constant at $y = 3$ ft. The tank has a width of 3 ft. What is the density of the mixture leaving the tank at C? Take $\rho_b = 1.70\ \text{slug/ft}^3$ and $\rho_{ke} = 1.59\ \text{slug/ft}^3$.



SOLUTION

The density of the mixture can be determined from

$$\rho_m = \frac{\rho_b Q_A + \rho_{ke} Q_B}{Q_A + Q_B}$$

Here,

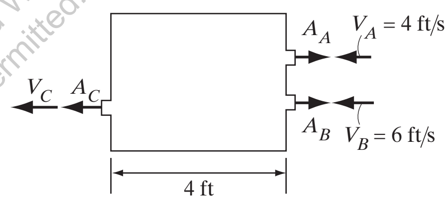
$$Q_A = V_A A_A = (4\ \text{ft/s})\left[\pi(0.15\ \text{ft})^2\right] = 0.09\pi\ \text{ft}^3/\text{s}$$

$$Q_B = V_B A_B = (6\ \text{ft/s})\left[\pi(0.1\ \text{ft})^2\right] = 0.06\pi\ \text{ft}^3/\text{s}$$

Then

$$\rho_m = \frac{(1.70\ \text{slug/ft}^3)(0.09\pi\ \text{ft}^3/\text{s}) + (1.59\ \text{slug/ft}^3)(0.06\pi\ \text{ft}^3/\text{s})}{0.09\pi\ \text{ft}^3/\text{s} + 0.06\pi\ \text{ft}^3/\text{s}}$$

$$= 1.656\ \text{slug/ft}^3 \qquad \textbf{Ans.}$$



Here, the control volume is fixed since it contains the mixture of which the volume does not change. The flow is steady thus there are no local changes. Here, we will use the average velocities,

$$\frac{\partial}{dt}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \rho_b V_A A_A - \rho_{ke} V_B A_B + \rho_m V_c A_c = 0$$

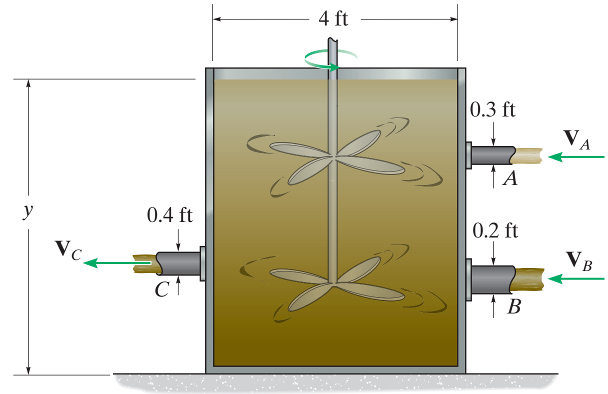
$$-(-1.70\ \text{slug/ft}^3)(0.09\ \pi\text{ft}^3/\text{s}) - (1.59\ \text{slug/ft}^3)(0.06\pi\ \text{ft}^3/\text{s})$$

$$+ (1.656\ \text{slug/ft}^3)(V_C)\left[\pi(0.2\ \text{ft})^2\right] = 0$$

$$V_C = 3.75\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
$\rho_m = 1.656\ \text{slug/ft}^3$
$V_C = 3.75\ \text{ft/s}$

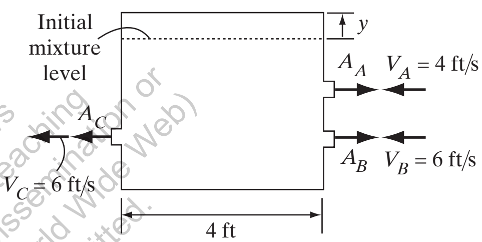
***4–96.** Benzene flows through the pipe at A with an average velocity of 4 ft/s, and kerosene flows through the pipe at B with an average velocity of 6 ft/s. If the average velocity of the mixture leaving the tank at C is $V_C = 5$ ft/s, determine the rate at which the level in the tank is changing. The tank has a width of 3 ft. Is the level rising or falling? What is the density of the mixture leaving the tank at C? Take $\rho_b = 1.70\ \text{slug/ft}^3$ and $\rho_{ke} = 1.59\ \text{slug/ft}^3$.



SOLUTION

The density of the mixture can be determined from

$$\rho_m = \frac{\rho_b Q_A + \rho_{ke} Q_B}{Q_A + Q_B}$$



(a)

Here,

$$Q_A = V_A A_A = (4\ \text{ft/s})\left[\pi(0.15\ \text{ft})^2\right] = 0.09\pi\ \text{ft}^3/\text{s}$$

$$Q_B = V_B A_B = (6\ \text{ft/s})\left[\pi(0.1\ \text{ft})^2\right] = 0.06\pi\ \text{ft}^3/\text{s}$$

$$Q_C = V_C A_C = (5\ \text{ft/s})\left[\pi(0.2\ \text{ft})^2\right] = 0.2\pi\ \text{ft}^3/\text{s}$$

Then

$$\rho_m = \frac{(1.70\ \text{slug/ft}^3)(0.09\pi\ \text{ft}^3/\text{s}) + (1.59\ \text{slug/ft}^3)(0.06\pi\ \text{ft}^3/\text{s})}{0.09\pi\ \text{ft}^3/\text{s} + 0.06\pi\ \text{ft}^3/\text{s}}$$

$$= 1.656\ \text{slug/ft}^3 \qquad \textbf{Ans.}$$

Here, the volume of the control volume changes with time since it contains the mixture in the tank. Its volume is

$$\forall = (4\ \text{ft})(3\ \text{ft})y = 12y$$

$$\frac{\partial \forall}{\partial t} = 12\frac{\partial y}{\partial t}$$

Here, the densities of the liquids are constant and the average velocity will be used.

$$\rho_m \frac{\partial \forall}{\partial t} - \rho_b V_A A_A - \rho_a V_B A_B + \rho_m V_C A_C = 0$$

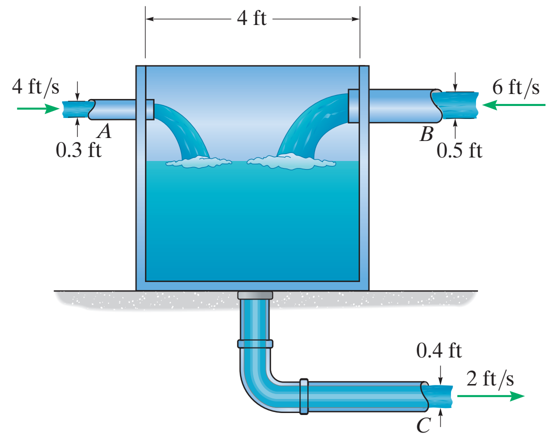
$$(1.656\ \text{slug/ft}^3)\left(12\frac{\partial y}{\partial t}\right) - (1.70\ \text{slug/ft}^3)(0.09\pi\ \text{ft}^3/\text{s})$$

$$-\ (1.59\ \text{slug/ft}^3)(0.06\pi\ \text{ft}^3/\text{s}) + (1.656\ \text{slug/ft}^3)(0.2\pi\ \text{ft}^3/\text{s}) = 0$$

$$\frac{\partial y}{\partial t} = -0.0131\ \text{ft/s} \qquad \textbf{Ans.}$$

The negative sign indicates the level of the mixture is falling.

4–97. The three pipes are connected to the water tank. If the average velocities of water flowing through the pipes are $V_A = 4$ ft/s, $V_B = 6$ ft/s, and $V_C = 2$ ft/s, determine the rate at which the water level in the tank changes. The tank has a width of 3 ft.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered.

$$\forall = (4\text{ ft})(3\text{ ft})y = (12y)\text{ ft}^3$$

Continuity Equation. Since water has a constant density,

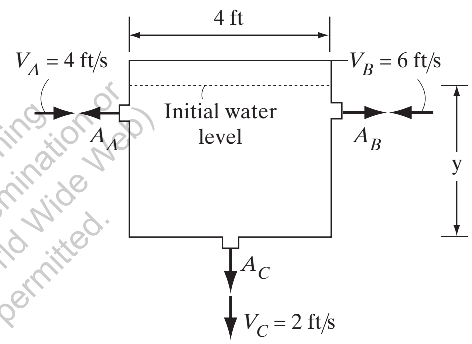
$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs} \mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall - V_A A_A - V_B A_B + V_C A_C = 0$$

$$\frac{\partial}{\partial t}(12y) - (4\text{ ft/s})\left[\pi(0.15\text{ ft})^2\right] - (6\text{ft/s})\left[\pi(0.25\text{ ft})^2\right] + (2\text{ ft/s})\left[\pi(0.2\text{ ft})^2\right] = 0$$

$$12\frac{\partial y}{\partial t} = 1.2095$$

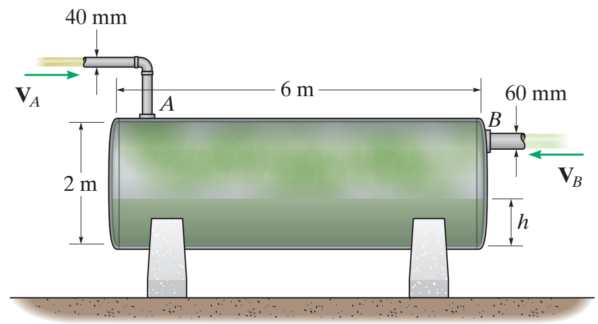
$$\frac{\partial y}{\partial t} = 0.101\text{ ft/s} \qquad \textbf{Ans.}$$



(a)

Ans:
0.101 ft/s

4–98. The 2-m-diameter cylindrical emulsion tank is being filled at A with cyclohexanol at an average rate of $V_A = 4$ m/s and at B with thiophene at an average rate of $V_B = 2$ m/s. Determine the rate at which the depth increases as a function of depth h.



SOLUTION

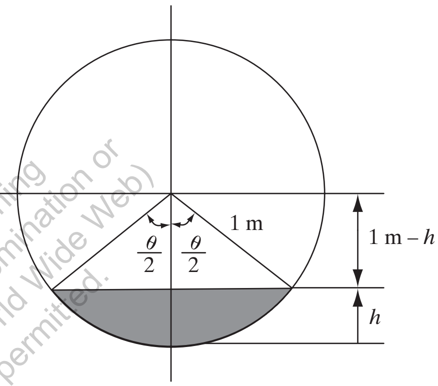
The control volume considered is the volume of liquid mixture contained in the sector of the tank (shown shaded in Fig. *a*) which changes with time. The volume of this control volume at a particular instant is

$$\forall = \left\{\frac{1}{2}(1\text{ m})^2\theta - \frac{1}{2}\left[2(1\text{ m})\sin\frac{\theta}{2}(1\text{ m})\cos\frac{\theta}{2}\right]\right\}(6\text{ m}) = 3(\theta - \sin\theta)$$

$$\frac{\partial \forall}{\partial t} = 3\left(\frac{\partial\theta}{\partial t} - \cos\theta\frac{\partial\theta}{\partial t}\right) = 3(1 - \cos\theta)\frac{\partial\theta}{\partial t}$$

However, $\cos\theta = 2\cos^2\dfrac{\theta}{2} - 1$ and $\cos\dfrac{\theta}{2} = \dfrac{1-h}{1} = 1 - h$. Thus $\cos\theta = 2(1-h)^2 - 1$. Then

$$\frac{\partial \forall}{\partial t} = 3\left\{1 - \left[2(1-h)^2 - 1\right]\right\}\frac{d\theta}{dt} = 6\left(2h - h^2\right)\frac{d\theta}{dt} \qquad \textbf{(1)}$$



(a)

Here,

$$\cos\frac{\theta}{2} = 1 - h$$

$$\left(-\frac{1}{2}\sin\frac{\theta}{2}\right)\frac{d\theta}{dt} = -\frac{dh}{dt}$$

$$\frac{d\theta}{dt} = \frac{2}{\sin\frac{\theta}{2}}\frac{dh}{dt}$$

However, $\sin\dfrac{\theta}{2} = \dfrac{\sqrt{1^2 - (1-h)^2}}{1} = \sqrt{2h - h^2}$. Thus,

$$\frac{d\theta}{dt} = \frac{2}{\sqrt{2h - h^2}}\frac{dh}{dt}$$

Substituting this result into Eq. (1),

$$\frac{d\forall}{dt} = 6\left(2h - h^2\right)\left(\frac{2}{\sqrt{2h - h^2}}\frac{dh}{dt}\right) = 12\sqrt{2h - h^2}\frac{dh}{dt} \qquad \textbf{(2)}$$

Since the liquids are assumed incompressible, there volume remain the same. Thus,

$$\left(\frac{d\forall}{dt}\right) = V_A A_A + V_B A_B = (4\text{ m/s})(\pi)(0.02\text{ m})^2 + (2\text{ m/s})(\pi)(0.03\text{ m})^2$$

$$= 0.010681\text{ m}^3/\text{s}$$

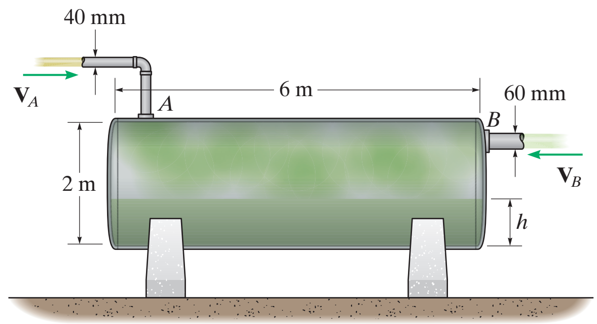
From Eq. (2),

$$0.010681 = 12\sqrt{2h - h^2}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{0.890\left(10^{-3}\right)}{\sqrt{2h - h^2}}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$$\frac{0.890\left(10^{-3}\right)}{\sqrt{2h - h^2}}\text{ m/s}$$

4–99. The 2-m-diameter cylindrical emulsion tank is being filled at A with cyclohexanol at an average rate of $V_A = 4$ m/s and at B with thiophene at an average rate of $V_B = 2$ m/s. Determine the rate at which the depth of the mixture is increasing when $h = 1$ m. Also, what is the average density of the mixture? Take $\rho_{cy} = 779$ kg/m^3, and $\rho_t = 1051$ kg/m^3.



SOLUTION

See the solution to part 4–98. When $h = 1$ m,

$$\frac{dh}{dt} = \frac{0.890\left(10^{-3}\right)}{\sqrt{2(1) - 1^2}} = 0.890\left(10^{-3}\right)\text{ m/s} \qquad \textbf{Ans.}$$

The average density of the mixture is

$$\rho_{\text{avg}} = \frac{\rho_{cy}(Q_A) + \rho_t(Q_B)}{Q_A + Q_B}$$

$$= \frac{779\text{ kg/m}^3(\pi)(0.02\text{ m})^2(4\text{ m/s}) + 1051\text{ kg/m}^3(\pi)(0.03\text{ m})^2(2\text{ m/s})}{\pi(0.02\text{ m})^2(4\text{ m/s}) + \pi(0.03\text{ m})^2(2\text{ m/s})}$$

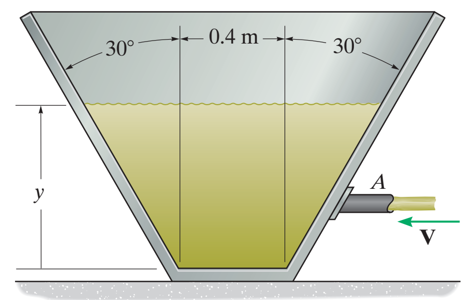
$$= 923\text{ kg/m}^3 \qquad \textbf{Ans.}$$

Ans:

$\frac{dh}{dt} = 0.890\left(10^{-3}\right)$ m/s

$\rho_{\text{avg}} = 923$ kg/m^3

***4–100.** Hexylene glycol is flowing into the trapezoidal container at a constant rate of 600 kg/min. Determine the rate at which the level is rising when $y = 0.5$ m. The container has a constant width of 0.5 m. $\rho_{hg} = 924\ \text{kg/m}^3$.



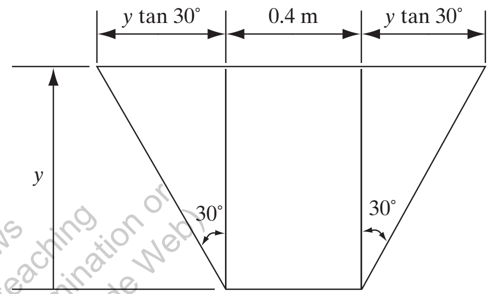
SOLUTION

The volume of the control volume considered changes with time since it contains the hexylene glycol in the tank, Fig. *a*. Its volume is

$$\forall = \frac{1}{2}(0.4\text{ m} + 2y\tan 30° + 0.4\text{ m})(y)(0.5\text{ m})$$

$$= \left(0.2y + 0.5\tan 30° y^2\right)\text{ m}^3$$

$$\frac{\partial \forall}{\partial t} = 0.2\frac{dy}{dt} + \tan 30° y\frac{\partial y}{\partial t} = \left(0.2 + \tan 30° y\right)\frac{\partial y}{\partial t}$$



(a)

Thus,

$$\frac{\partial \forall}{\partial t}\int_{cv}\rho_{hg}\,d\forall + \int_{cs}\rho_{hg}\mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_{hg} is constant, it can be factored out from the integrals. Also, the average velocity will be used. Thus, this equation reduces to

$$\rho_{hg}\frac{\partial \forall}{\partial t} - \rho_{hg}V_A A_A = 0$$

Here $\rho_{hg}V_A A_A = \dot{m}_{hg} = \left(600\dfrac{\text{kg}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 10\text{ kg/s}$. Then

$$\left(924\text{ kg/m}^3\right)(0.2 + \tan 30° y)\frac{\partial y}{\partial t} - 10\text{ kg/s} = 0$$

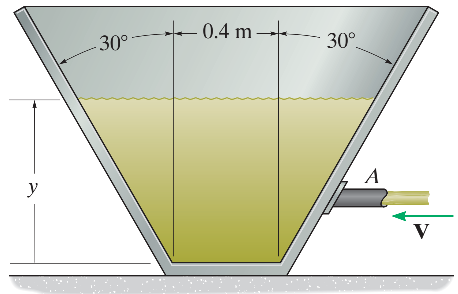
$$\frac{dy}{dt} = \left[\frac{5}{462\left(0.2 + \tan 30° y\right)}\right]\text{m/s.}$$

When

$$y = 0.5\text{ m}$$

$$\frac{dy}{dt} = \frac{5}{462\left[0.2 + \tan 30°(0.5)\right]} = 0.0221\text{ m/s} \qquad \textbf{Ans.}$$

4–101. Hexylene glycol is flowing into the container at a constant rate of 600 kg/min. Determine the rate at which the level is rising when $y = 0.5$ m. The container is in the form of a conical frustum. *Hint*: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. $\rho_{hg} = 924\ \text{kg/m}^3$.



SOLUTION

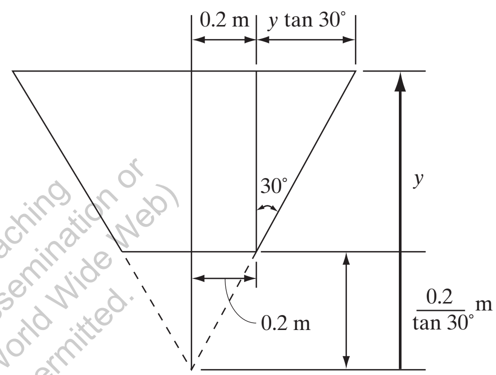
Since the control volume contains hexylene glycol in the tank, Fig. *a*, its volume is

$$V = \frac{1}{3}\pi(0.2 + y\tan 30°)^2 + \left(y + \frac{0.2}{\tan 30°}\text{ m}\right) - \frac{1}{3}\pi(0.2\text{ m})^2\left(\frac{0.2}{\tan 30°}\text{ m}\right)$$

$$= \frac{1}{3}\pi\left(\frac{1}{3}y^3 + 0.2\sqrt{3}y^2 + 0.12y\right)$$

$$\frac{\partial V}{\partial t} = \frac{1}{3}\pi\left(y^2\frac{\partial y}{\partial t} + 0.4\sqrt{3}y\frac{\partial y}{\partial t} + 0.12\frac{\partial y}{\partial t}\right)$$

$$= \frac{1}{3}\pi\left(y^2 + 0.4\sqrt{3}y + 0.12\right)\frac{\partial y}{\partial t}$$



(a)

Thus,

$$\frac{\partial}{\partial t}\int_{cv}\rho_{hg}\,dV + \int_{cs}\rho_{hg}\mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_{hg} is constant, it can be factored out from the integrals. Also, the average velocity will be used. Thus, the equation reduces to

$$\rho_{hg}\frac{\partial V}{\partial t} - \rho_{hg}V_A A_A = 0$$

Here, $\rho_{hg}V_A A_A = \dot{m}_{hg} = \left(600\frac{\text{kg}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10\text{ kg/s}$. Then

$$\left(924\text{ kg/m}^3\right)\left[\frac{1}{3}\pi\left(y^2 + 0.4\sqrt{3}y + 0.12\right)\frac{\partial y}{\partial t}\right] - 10\text{ kg/s} = 0$$

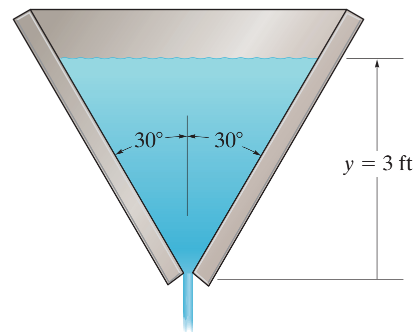
$$\frac{\partial y}{\partial t} = \left[\frac{15}{462\pi\left(y^2 + 0.4\sqrt{3}y + 0.12\right)}\right]\text{ m/s}$$

When $y = 0.5$ m

$$\frac{\partial y}{\partial t} = \frac{15}{462\pi\left[0.5^2 + 0.4\sqrt{3}(0.5) + 0.12\right]} = 0.0144\text{ m/s}$$ **Ans.**

Ans:
0.0144 m/s

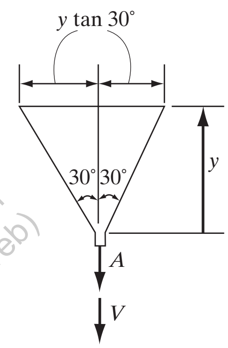
4–102. Water in the triangular trough is at a depth of $y = 3$ ft. If the drain is opened at the bottom, and water flows out at a rate of $V = (8.02y^{1/2})$ ft/s, where y is in feet, determine the time needed to fully drain the trough. The trough has a width of 2 ft. The slit at the bottom has a cross-sectional area of 24 in^2.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a*. will be considered. Its volume at any instant is

$$\forall = 2\left[\frac{1}{2}(y\tan 30°)y\right](2\text{ ft}) = (1.1547y^2)\text{ ft}^3$$



(a)

Continuity Equation. Since water has a constant density.

$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall + VA = 0$$

$$\frac{\partial}{\partial t}(1.1547y^2) + (8.02y^{\frac{1}{2}})\left(\frac{24}{144}\text{ ft}^2\right) = 0$$

$$2.3094y\frac{\partial y}{\partial t} = -1.3367y^{\frac{1}{2}}$$

$$\frac{\partial y}{\partial t} = -0.5788y^{-\frac{1}{2}}$$

Integrating,

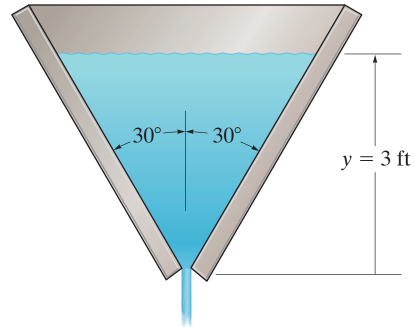
$$\int_0^t dt = \int_{3\text{ ft}}^0 -1.7277y^{\frac{1}{2}}dy$$

$$t = -1.7277\left(\frac{2}{3}y^{\frac{3}{2}}\right)\Bigg|_{3\text{ ft}}^0$$

$$t = 5.99\text{ s}$$ **Ans.**

Ans:
5.99 s

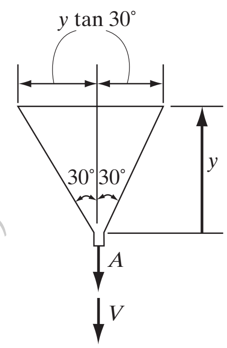
4–103. Water in the triangular trough is at a depth of $y = 3$ ft. If the drain is opened at the bottom, and water flows out at a rate of $V = \left(8.02y^{1/2}\right)$ ft/s, where y is in feet, determine the time needed for the water to reach a depth of $y = 2$ ft. The trough has a width of 2 ft. The slit at the bottom has a cross-sectional area of 24 in^2.



SOLUTION

Control volume. The deformable control volume shown in Fig. *a* will be considered. Its volume at any instant is

$$\forall = 2\left[\frac{1}{2}(y \tan 30°)y\right](2\text{ ft}) = \left(1.1547y^2\right)\text{ ft}^3$$



(a)

Continuity Equation. Since water has a constant density.

$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs} \mathbf{V}_{f/cs} \cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall + VA = 0$$

$$\frac{\partial}{\partial t}\left(1.1547y^2\right) + \left(8.02y^{\frac{1}{2}}\right)\left(\frac{24}{144}\text{ ft}^2\right) = 0$$

$$2.3094y\frac{\partial y}{\partial t} = -1.3367y^{\frac{1}{2}}$$

$$\frac{\partial y}{\partial t} = -0.5788y^{-\frac{1}{2}}$$

Integrating,

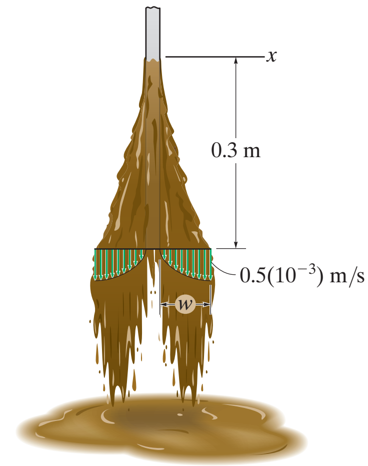
$$\int_0^t dt = \int_{3\text{ ft}}^{2\text{ ft}} -1.7277y^{\frac{1}{2}}dy$$

$$t = -1.7277\left(\frac{2}{3}y^{\frac{3}{2}}\right)\Big|_{3\text{ ft}}^{2\text{ ft}}$$

$$t = 2.73\text{ s}$$ **Ans.**

Ans:
2.73 s

***4–104.** As part of a manufacturing process, a 0.1-m-wide plate is dipped into hot tar and then lifted out, causing the tar to run down and then off the sides of the plate as shown. The thickness w of the tar at the bottom of the plate decreases with time t, but it still is assumed to maintain a linear variation along the plate as shown. If the velocity profile at the bottom of the plate is approximately parabolic, such that $u = \left[0.5(10^{-3})(x/w)^{1/2}\right]$ m/s, where x and w are in meters, determine w as a function of time. Initially, when $t = 0$, $w = 0.02$ m.



SOLUTION

The flow is considered one dimensional since its velocity is directed downward. The control volume is shown in Fig. *a* and its volume changes with time. At a particular instant it is

$$\forall = \frac{1}{2}w(0.3\text{ m})(0.1\text{ m}) = (0.015w)\text{ m}^3$$

Thus,

$$\frac{\partial \forall}{\partial t} = 0.015\frac{\partial w}{\partial t}$$

The opened control surface is shown shaded in Fig. *a*. The differential area element is $dA = bdx$.

$$\frac{\partial}{\partial t}\int_{cv}\rho_t d\forall + \int_{cs}\rho_t \mathbf{V}\cdot d\mathbf{A} = 0$$

Since the tar is assumed to be incompressible, ρ_t is constant. Also, the velocity of the tar is always directed perpendicular to the opened surface. Hence the above equation reduces to

$$\rho_t\frac{\partial \forall}{\partial t} + \rho_t\int_{cs}udA = 0$$

$$\frac{\partial \forall}{\partial t} = -\int_{cs}udA$$

The negative sign indicates that $\forall$ is decreasing with time. Then

$$0.015\frac{dw}{dt} = -\int_0^w 0.5(10^{-3})\left(\frac{x}{w}\right)^{\frac{1}{2}}(0.1dx)$$

$$0.015\frac{dw}{dt} = -\frac{0.5(10^{-3})(0.1)}{w^{\frac{1}{2}}}\left(\frac{2}{3}x^{\frac{3}{2}}\right)\Big|_0^w$$

$$0.015\frac{dw}{dt} = -3.333(10^{-5})w$$

Or, the integral $\int_{cs}udA$ is equal to the volume of the parabolic block under the velocity profile, ie, $\int_{cs}udA = \frac{2}{3}\left[0.5(10^{-3})\right]w(0.1) = 0.0333(10^{-3})w$

$$\frac{dw}{dt} = -2.222(10^{-3})w$$

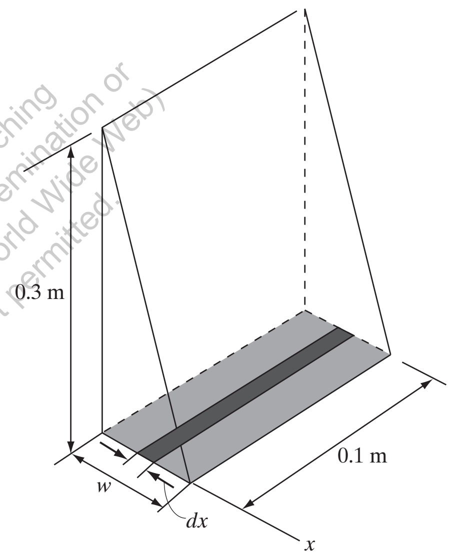
$$-\int_{0.02\text{ m}}^{w}\frac{dw}{2.222(10^{-3})w} = \int_0^t dt$$

$$-450\ln\frac{w}{0.02} = t$$

$$\ln\frac{w}{0.02} = -\frac{t}{450}$$

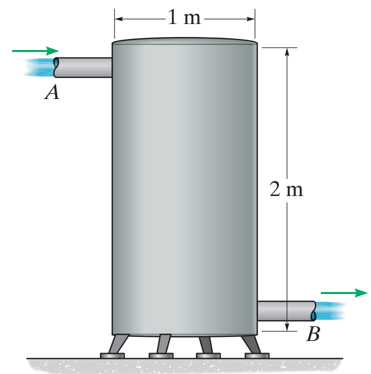
$$\frac{w}{0.02} = e^{-t/450}$$

$$w = \left(0.02e^{-t/450}\right)\text{ m} \qquad \textbf{Ans.}$$



(a)

4–105. The cylindrical tank in a food-processing plant is filled with a concentrated sugar solution having an initial density of $\rho_s = 1400\ \text{kg/m}^3$. Water is piped into the tank at A at $0.03\ \text{m}^3/\text{s}$ and mixes with the sugar solution. If an equal flow of the diluted solution exits at B, determine the amount of water that must be added to the tank so that the density of the sugar solution is reduced by 10% of its original value.



SOLUTION

The control volume considered here is the volume of the tank. It is a fixed control volume since its volume does not change throughout the mixing.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{V}_{ds} \cdot d\mathbf{A} = 0$$

$$\mathcal{V}\frac{\partial \rho}{\partial t} + \rho Q - \rho_w Q = 0$$

$$\mathcal{V}\frac{\partial \rho}{\partial t} = Q(\rho_w - \rho)$$

$$\int_{\rho_s}^{\rho} \frac{\partial \rho}{\rho_w - \rho} = \frac{Q}{\mathcal{V}}\int_0^t \partial t$$

$$-\ln(\rho_w - \rho)\Big|_{\rho_s}^{\rho} = \frac{Q}{\mathcal{V}}t$$

$$-\ln\left(\frac{\rho_w - \rho}{\rho_w - \rho_s}\right) = \frac{Q}{\mathcal{V}}t$$

$$t = \frac{\mathcal{V}}{Q}\ln\left(\frac{\rho_w - \rho_s}{\rho_w - \rho}\right)$$

Here, $\mathcal{V} = \pi(0.5\ \text{m})^2(0.2\ \text{m}) = 0.5\pi\ \text{m}^3$, and it is required that $\rho = 0.9\rho_s = 0.9\left(1400\ \text{kg/m}^3\right) = 1260\ \text{kg/m}^3$. Then

$$t = \left(\frac{0.5\pi\ \text{m}^3}{0.03\ \text{m}^3/\text{s}}\right)\ln\left(\frac{1000\ \text{kg/m}^3 - 1400\ \text{kg/m}^3}{1000\ \text{kg/m}^3 - 1260\ \text{kg/m}^3}\right)$$

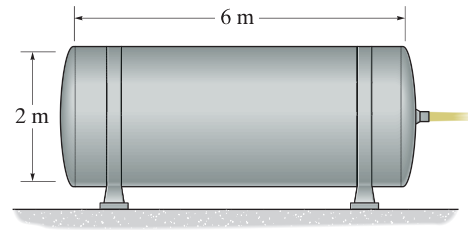
$$= 22.556$$

The amount of water to be added is

$$\mathcal{V}_w = Qt = \left(0.03\ \text{m}^3/\text{s}\right)(22.556) = 0.677\ \text{m}^3 \qquad \textbf{Ans.}$$

Ans:
$0.677\ \text{m}^3$

4–106. The cylindrical pressure vessel contains methane at an initial absolute pressure of 2 MPa. If the nozzle is opened, the mass flow depends upon the absolute pressure and is $\dot{m} = 3.5\left(10^{-6}\right)p$ kg/s, where p is in pascals. Assuming the temperature remains constant at 20°C, determine the time required for the pressure to drop to 1.5 MPa.



SOLUTION

From Appendix A, the gas constant for Methane is $R = 518.3\ \text{J}/(\text{kg}\cdot\text{K})$. Using the ideal gas law with $T = 20°\text{C} + 273 = 293\ \text{K}$ which is constant throughout,

$$p = \rho RT; \qquad p = \rho\left(518.3\ \text{J}/(\text{kg}\cdot\text{k})\right)(293\ \text{k})$$

$$p = 151861.9\rho$$

$$\rho = 6.5849\left(10^{-6}\right)p \tag{1}$$

The control volume considered is the volume of tank which contains Methane. Since the tank is fully filled at all times, the control volume can be classified as fixed.

$$\frac{d}{dt}\int_{cv}\rho\, dV + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

Here $\int_{cv} dV = V$ (fixed control volume) and $\dot{m} = \int_{cv}\rho\mathbf{V}\cdot\mathbf{A}$. Then

$$V\frac{d\rho}{dt} + \dot{m} = 0 \tag{2}$$

Since, $V = \pi(1\ \text{m})^2(6\ \text{m}) = 6\pi\ \text{m}^3$ and $\dot{m} = \left[3.5\left(10^{-6}\right)p\right]$ kg/s, then Eq (1),

$$\frac{d\rho}{dt} = 6.5849\left(10^{-6}\right)\frac{dp}{dt}$$

Substitute these results into Eq. (2),

$$6\pi\left[6.5849\left(10^{-6}\right)\frac{dp}{dt}\right] + 3.5\left(10^{-6}\right)p = 0$$

$$\frac{dp}{dt} = -0.02820p$$

$$\int_{p_0}^{p}\frac{dp}{p} = -0.02820\int_0^t dt$$

$$\ln\left(\frac{p}{p_0}\right) = -0.02820t$$

$$t = -35.46\ln\left(\frac{p}{p_0}\right)$$

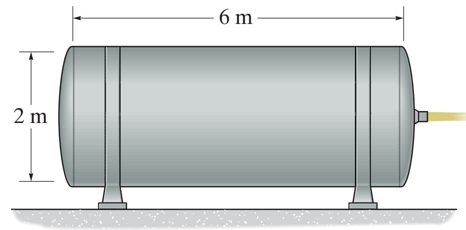
Here $p_0 = 2$ Mpa. Thus when $p = 1.5$ Mpa,

$$t = -35.46\ln\left(\frac{1.5\ \text{MPa}}{2\ \text{MPa}}\right)$$

$$= 10.2\ \text{s}$$ **Ans.**

Ans:
10.2 s

4–107. The cylindrical pressure vessel contains methane at an initial absolute pressure of 2 MPa. If the nozzle is opened, the mass flow depends upon the absolute pressure and is $\dot{m} = 3.5(10^{-6})p$ kg/s, where P is in pascals. Assuming the temperature remains constant at 20°C, determine the pressure in the tank as a function of time. Plot this pressure (vertical axis) versus the time for $0 \le t \le 15$ s. Give values for increments of $\Delta t = 3$ s.



SOLUTION

From Appendix A, the gas constant for Methane is $R = 518.3$ J/kg·K using the ideal gas law with $T = 20°\text{C} + 273 = 293$ K, which is constant throughout,

$$p = \rho RT; \qquad p = \rho(518.3\ \text{J/kg}\cdot\text{k})(293\ \text{k})$$

$$p = 151861.9\rho$$

$$\rho = 6.5849(10^{-6})p \tag{1}$$

The control volume considered is the volume of tank which contains Methane. Since the tank is fully filled at all times, the control volume can be classified as fixed.

$$\frac{d}{dt}\int_{cv}\rho d\forall + \int_{cs}\rho \mathbf{V}_{ds}\cdot d\mathbf{A} = 0$$

Here $\int_{cv} d\forall = \forall$ (fixed control volume) and $\dot{m} = \int_{cv}\rho\mathbf{V}\cdot d\mathbf{A}$. Then

$$\forall\frac{d\rho}{dt} + \dot{m} = 0 \tag{2}$$

Since, $\forall = \pi(1\ \text{m})^2(6\ \text{m}) = 6\pi\ \text{m}^3$ and $\dot{m} = \left[3.5(10^{-6})p\right]$ kg/s, then from Eq (1),

$$\frac{d\rho}{dt} = 6.5849(10^{-6})\frac{dp}{dt}$$

Substitute these results into Eq (2),

$$6\pi\left[6.5849(10^{-6})\frac{\partial p}{\partial t}\right] + 3.5(10^{-6})p = 0$$

$$\frac{dp}{dt} = -0.02820p$$

$$\int_{p_0}^{p}\frac{dp}{p} = -0.02820\int_0^t dt$$

$$\ln\left(\frac{p}{p_0}\right) = -0.02820t$$

$$\frac{p}{p_0} = e^{-0.02820t}$$

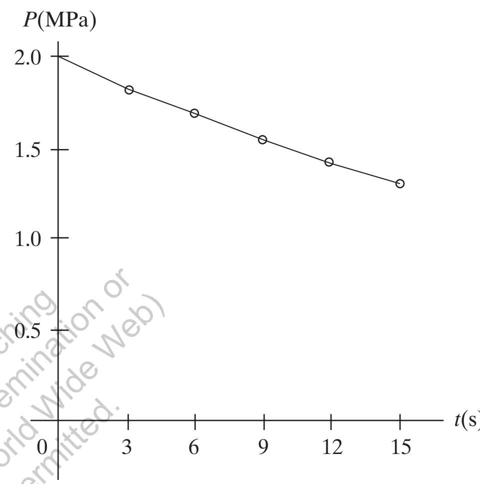
$$p = p_0e^{-0.02820t}$$

Here $p_0 = 2$MPa, then

$$p = (2e^{-0.02820t})\text{MPa, where } t \text{ is in seconds}$$ **Ans.**

The plot of p vs t is shown in Fig. a

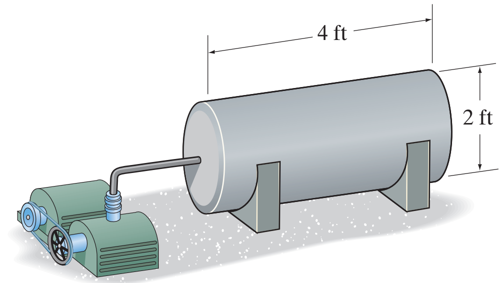
t(s)	0	3	6	9	12	15
p(MPa)	2.0	1.84	1.69	1.55	1.43	1.31



(a)

Ans:
$p = (2\,e^{-0.0282t})$ MPa, where t is in seconds

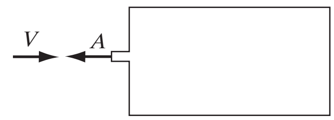
***4–108.** As nitrogen is pumped into the closed cylindrical tank, the mass flow through the tube is $\dot{m} = \left(0.8\rho^{-1/2}\right)$ slug/s. Determine the density of the nitrogen within the tank when $t = 5$ s from the time the pump is turned on. Assume that initially there is 0.5 slug of nitrogen in the tank.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. This control volume has a constant volume of

$$\mathcal{V} = \pi(1\text{ ft})^2(4\text{ ft}) = 4\pi\text{ ft}^3$$



(a)

The density of the nitrogen within the control volume changes with time and therefore contributes to local changes.

Continuity Equation. Realizing that $\dot{m} = \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A}$,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\frac{\partial \rho}{\partial t}\mathcal{V} - \dot{m} = 0$$

$$4\pi\frac{\partial \rho}{\partial t} - 0.8\rho^{\frac{-1}{2}} = 0$$

$$\frac{\partial \rho}{\partial t} = \frac{0.2}{\pi}\rho^{\frac{-1}{2}}$$

Integrating,

$$\int_0^t dt = \int_{\rho_o}^{\rho} 5\pi\rho^{\frac{1}{2}}\, d\rho$$

$$t = 5\pi\left(\frac{2}{3}\rho^{\frac{3}{2}}\right)\Bigg|_{\rho_o}^{\rho}$$

$$t = \frac{10\pi}{3}\left(\rho^{\frac{3}{2}} - \rho_o^{\frac{3}{2}}\right)$$

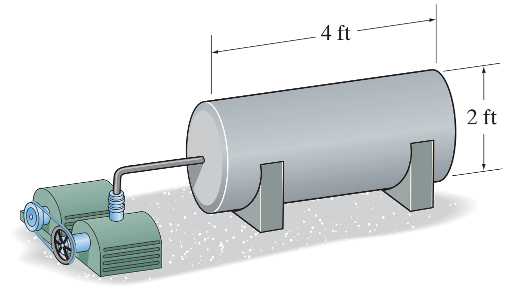
$$\rho = \left(\frac{3t}{10\pi} + \rho_o^{\frac{3}{2}}\right)^{\frac{2}{3}} \text{ slug/ft}^3$$

Here, $\rho_o = \dfrac{0.5\text{ slug}}{4\pi\text{ ft}^3} = \dfrac{0.125}{\pi}$ slug/ft^3. Then, when $t = 5$ s,

$$\rho = \left[\frac{3(5)}{10\pi} + \left(\frac{0.125}{\pi}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}} \text{ slug/ft}^3$$

$$= 0.618 \text{ slug/ft}^3$$ **Ans.**

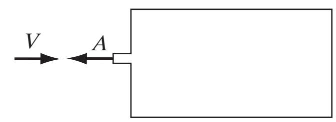
4–109. As nitrogen is pumped into the closed cylindrical tank, the mass flow through the tube is $m = (0.8\rho^{-1/2})$ slug/s. Determine the density of the nitrogen within the tank when $t = 10$ s from the time the pump is turned on. Assume that initially there is 0.5 slug of nitrogen in the tank.



SOLUTION

Control Volume. The fixed volume is shown in Fig. a. This control volume has a constant volume of

$$\forall = \pi(1\text{ ft})^2(4\text{ ft}) = 4\pi\text{ ft}^3$$



(a)

The density of the nitrogen within the control volume changes with time and therefore contributes to local changes.

Continuity Equation. Realizing that $\dot{m} = \int_{cs} \rho\mathbf{V}\cdot d\mathbf{A}$,

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\frac{\partial\rho}{\partial t}\forall - \dot{m} = 0$$

$$4\pi\frac{\partial\rho}{\partial t} - 0.8\rho^{\frac{-1}{2}} = 0$$

$$\frac{\partial\rho}{\partial t} = \frac{0.2}{\pi}\rho^{\frac{-1}{2}}$$

Integrating,

$$\int_0^t dt = \int_{\rho_o}^{\rho} 5\pi\rho^{\frac{1}{2}}d\rho$$

$$t = 5\pi\left(\frac{2}{3}\rho^{\frac{3}{2}}\right)\Bigg|_{\rho_o}^{\rho}$$

$$t = \frac{10\pi}{3}\left(\rho^{\frac{3}{2}} - \rho_o^{\frac{3}{2}}\right)$$

$$\rho = \left(\frac{3t}{10\pi} + \rho_o^{\frac{3}{2}}\right)^{\frac{2}{3}}\text{ slug/ft}^3$$

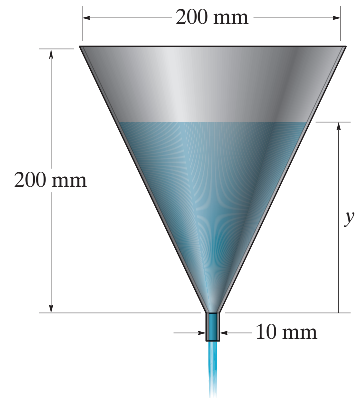
Here, $\rho_o = \dfrac{0.5\text{ slug}}{4\pi\text{ ft}^3} = \dfrac{0.125}{\pi}\text{ slug/ft}^3$. Then, when $t = 10$ s,

$$\rho = \left[\frac{3(10)}{10\pi} + \left(\frac{0.125}{\pi}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}}\text{ slug/ft}^3$$

$$= 0.975\text{ slug/ft}^3 \qquad \textbf{Ans.}$$

Ans:
0.975 slug/ft^3

4–110. Water flows out of the stem of the funnel at an average speed of $V = \left(3e^{-0.05t}\right)$ m/s, where t is in seconds. Determine the average speed at which the water level is falling at the instant $y = 100$ mm. At $t = 0$, $y = 200$ mm.



SOLUTION

The control volume is the volume of the water in the funnel. This volume changes with time.

$$\frac{d}{dt}\int_{cv} \rho_w d\forall + \int_{cs} \rho_w \mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_w is constant (in compressible), it can be factored out from the integrals

$$\rho_w \frac{d\forall}{dt} + \rho_w \int_{cs} \mathbf{V}\cdot d\mathbf{A} = 0$$

Here, the average velocity will be used. Then

$$\frac{d\forall}{dt} + VA = 0$$

$$\frac{d\forall}{dt} + \left(3e^{-0.05t}\right)\left[\frac{\pi}{4}(0.01\text{ m})^2\right]$$

$$\frac{d\forall}{dt} + 75\left(10^{-6}\right)\pi e^{-0.05t} = 0 \qquad \textbf{(1)}$$

The volume of the control volume at a particular instant is

$$\forall = \frac{1}{3}\pi \tau^2 y$$

From the geometry shown in Fig. a,

$$\frac{r}{y} = \frac{0.1\text{ m}}{0.2\text{ m}}; \quad r = \frac{1}{2}y$$

Then

$$\forall = \frac{1}{3}\pi\left(\frac{1}{2}y\right)^2 y = \left(\frac{1}{12}\pi y^3\right)\text{ m}^3$$

$$\frac{d\forall}{dt} = \frac{1}{12}\pi\left(3y^2\frac{dy}{dt}\right)$$

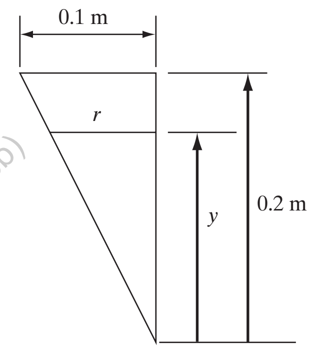
$$\frac{d\forall}{dt} = \frac{1}{4}\pi y^2\frac{dy}{dt}$$

Substitute into Eq. (1),

$$\frac{1}{4}\pi y^2\frac{dy}{dt} + 75\left(10^{-6}\right)\pi e^{-0.05t} = 0$$

$$y^2\frac{dy}{dt} = -0.3\left(10^{-3}\right)e^{-0.05t} \qquad \textbf{(2)}$$

$$\frac{dy}{dt} = -\frac{0.3\left(10^{-3}\right)e^{-0.05t}}{y^2} \qquad \textbf{(3)}$$



(a)

4–110. Continued

Separating the variables of Eq. (2)

$$\int_{0.2\text{ m}}^{0.1\text{ m}} y^2 dy = -0.3\left(10^{-3}\right)\int_0^t e^{-0.05t} dt$$

$$\left.\frac{y^3}{3}\right|_{0.2\text{ m}}^{0.1\text{ m}} = -0.3\left(10^{-3}\right)\left.\left(\frac{e^{-0.05t}}{-0.05}\right)\right|_0^t$$

$$-2.3333\left(10^{-3}\right) = 6\left(10^{-3}\right)e^{-0.05t}\Big|_0^t$$

$$-2.3333\left(10^{-3}\right) = 6\left(10^{-3}\right)\left(e^{-0.05t} - 1\right)$$

$$e^{-0.05t} = 0.6111$$

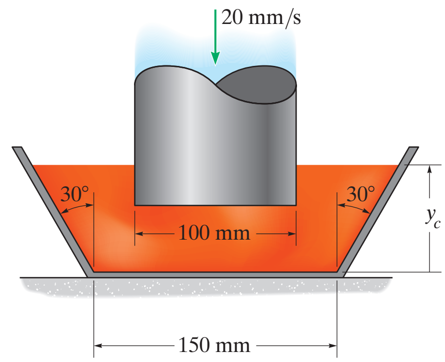
Substitute this value and $y = 0.1$ m into Eq. (3),

$$\frac{dy}{dt} = -\frac{0.3\left(10^{-3}\right)(0.6111)}{0.1^2} = -0.0183 \text{ m/s} \qquad \textbf{Ans.}$$

The negative sign indicates that y is decreasing, ie., the water level is falling.

Ans:
-0.0183 m/s

4–111. A part is manufactured by placing molten plastic into the trapezoidal container and then moving the cylindrical die down into it at a constant speed of 20 mm/s. Determine the average speed at which the plastic rises in the form as a function of y_c. The container has a width of 150 mm.



SOLUTION

The control volume segments shown shaded in Fig. *a* can be consider fixed at a particular instant. At this instant, no local changes occur since the molten plastic is incompressible. Also, its density is constant. If we use average velocities. Also then

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 + V_B A_B - Q_A = 0 \qquad \textbf{(1)}$$

Here, $A_B = (2y_c \tan 30° + 0.15\text{ m})(0.15\text{ m}) - \pi(0.05\text{ m})^2$

$$= (0.3 \tan 30° y_c + 0.01465)\text{ m}^2$$

$$V_B = \frac{\partial y_c}{\partial t}$$

The volume of the die submerged in the plastic is

$$\forall_d = \pi(0.05\text{ m})^2 y_d = (0.0025\pi y_d)\text{ m}^3$$

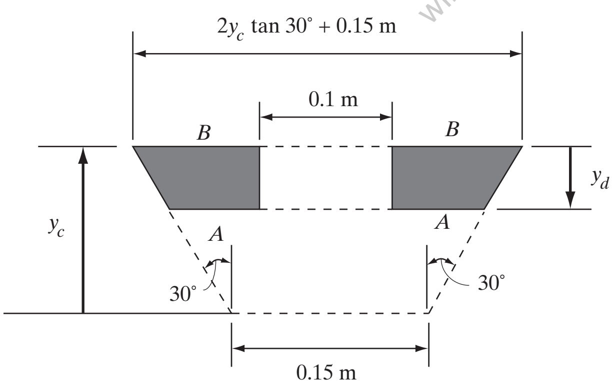
Realizing that $\dfrac{dy_d}{dt} = V_d = 0.02$ m/s, then

$$Q_A = \frac{d\forall_2}{dt} = 0.0025\pi\frac{dy_2}{dt} = (0.0025\pi)(0.02\text{ m/s}) = 50(10^{-6})\pi\text{ m}^3/\text{s}$$

Substitute these results into Eq (1),

$$\frac{dy_o}{dt}(0.3 \tan 30° y_c + 0.01465) - 50(10^{-6})\pi = 0$$

$$\frac{dy_c}{dt} = \left(\frac{0.157(10^{-3})}{0.173y_c + 0.0146}\right)\text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:

$$\frac{dy_c}{dt} = \left(\frac{0.157\ (10^{-3})}{0.173y_c + 0.0146}\right)\text{ m/s}$$