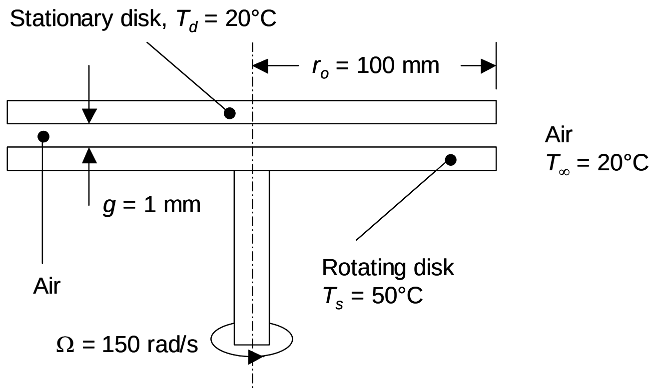
PROBLEM 6.17

KNOWN: Dimensions and temperatures of rotating and stationary disks, air gap spacing between disks, rotational speed. Correlation for the local Nusselt number.

FIND: Value of the average Nusselt number, total heat flux from the disk's top surface, total power requirement. Comment on the nature of the flow between the disks.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible viscous dissipation.

PROPERTIES: *Table A-4*, air ($\overline{T}$ = (50°C + 20°C)/2 = 35°C ≈ 308K): $\nu = 16.69\times10^{-6}$ m²/s, k = 0.0269 W/m·K.

ANALYSIS: From the problem statement, $Nu_r = \dfrac{h(r)r}{k} = 70\left(1+e^{-140G}\right)Re_{r_o}^{-0.456}Re_r^{0.478}$.

Since $Re_r = \Omega r^2/\nu$, the local heat transfer coefficient is

$$h(r) = k\left[70\left(1+e^{-140G}\right)\left(\frac{\Omega r_o^2}{\nu}\right)^{-0.456}\left(\frac{\Omega}{\nu}\right)^{0.478}\right]r^{-0.044}$$

The average heat transfer coefficient may be evaluated from

$$\overline{h} = \frac{1}{A_s}\int_{A_s} h(r)dA_s = \frac{2\pi k}{\pi r_o^2}\left[70\left(1+e^{-140G}\right)\left(\frac{\Omega r_o^2}{\nu}\right)^{-0.456}\left(\frac{\Omega}{\nu}\right)^{0.478}\right]\int_0^{r_o} r\times r^{-0.044}dr$$

or

$$\overline{h} = \frac{1.022k}{r_o^2}\left[70\left(1+e^{-140G}\right)\left(\frac{\Omega r_o^2}{\nu}\right)^{-0.456}\left(\frac{\Omega}{\nu}\right)^{0.478}\right]r_o^{1.956}$$

Continued…

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Substituting values,

$$\bar{h} = \frac{1.022 \times 0.0269\,\text{W/m}\cdot\text{K}}{(0.100\text{m})^2}$$

$$\times\left[70\left(1+e^{-140\times 0.01}\right)\left(\frac{150\text{rad/s}\times\left(0.100\text{m}\right)^2}{16.69\times10^{-6}\text{m}^2\text{/s}}\right)^{-0.456}\left(\frac{150\text{rad/s}}{16.69\times10^{-6}\text{m}^2\text{/s}}\right)^{0.478}\right]\left(0.100\text{m}\right)^{1.956}$$

or

$$\bar{h} = 30.8\ \text{W/m}^2\cdot\text{K} \qquad <$$

The average Nusselt number is

$$\overline{Nu}_D = \bar{h}D/k = 30.8\text{W/m}^2\cdot\text{K}\times0.200\text{m}/0.0269\text{W/m}\cdot\text{K} = 229 \qquad <$$

The heat flux from the top surface of the disk is

$$q'' = \bar{h}(T_s - T_d) = 30.8\text{W/m}^2\cdot\text{K}\times(50-20)°\text{C} = 924\text{W/m}^2 \qquad <$$

Therefore, the total electric power requirement is

$$P = q''A_s = q''\pi r_o^2 = 924\text{W/m}^2\times\pi\times(0.100\text{m})^2 = 29\text{W} \qquad <$$

Note that if only conduction were occurring between the two disks, the heat flux would be

$$q'' = (k/g)(T_s - T_d) = (0.0269\text{W/m}\cdot\text{K}/0.001\text{m})\times(50-20)°\text{C} = 807\text{W/m}^2.$$

The conduction heat flux is slightly less than the calculated quantity with rotation, suggesting that advection in the cross-gap direction is small, and that heat transfer between the disks is dominated by conduction. The laminar flow between the disks is characterized by very small velocities in the cross-gap direction.

COMMENTS: (1) The slight increase in heat transfer rate is due to edge effects where air can exit or enter the space between the disks, and enhance heat transfer between the disks by mixing. (2) The Reynolds number is $Re_{r_o} = \Omega r_o^2/\nu = 150\text{rad/s}\times\left(0.100\text{m}\right)^2/16.69\times10^{-6}\text{m}^2/\text{s} = 90{,}000$. Transition to turbulent flow begins at a Reynolds number of approximately 180,000 for this configuration. (3) See Pelle and Harmand, "Heat Transfer Measurements in an Opened Rotor-Stator System Air-Gap," *Experimental Thermal and Fluid Science*, 'Vol. 31, pp. 165 – 180, 2007, for more information.