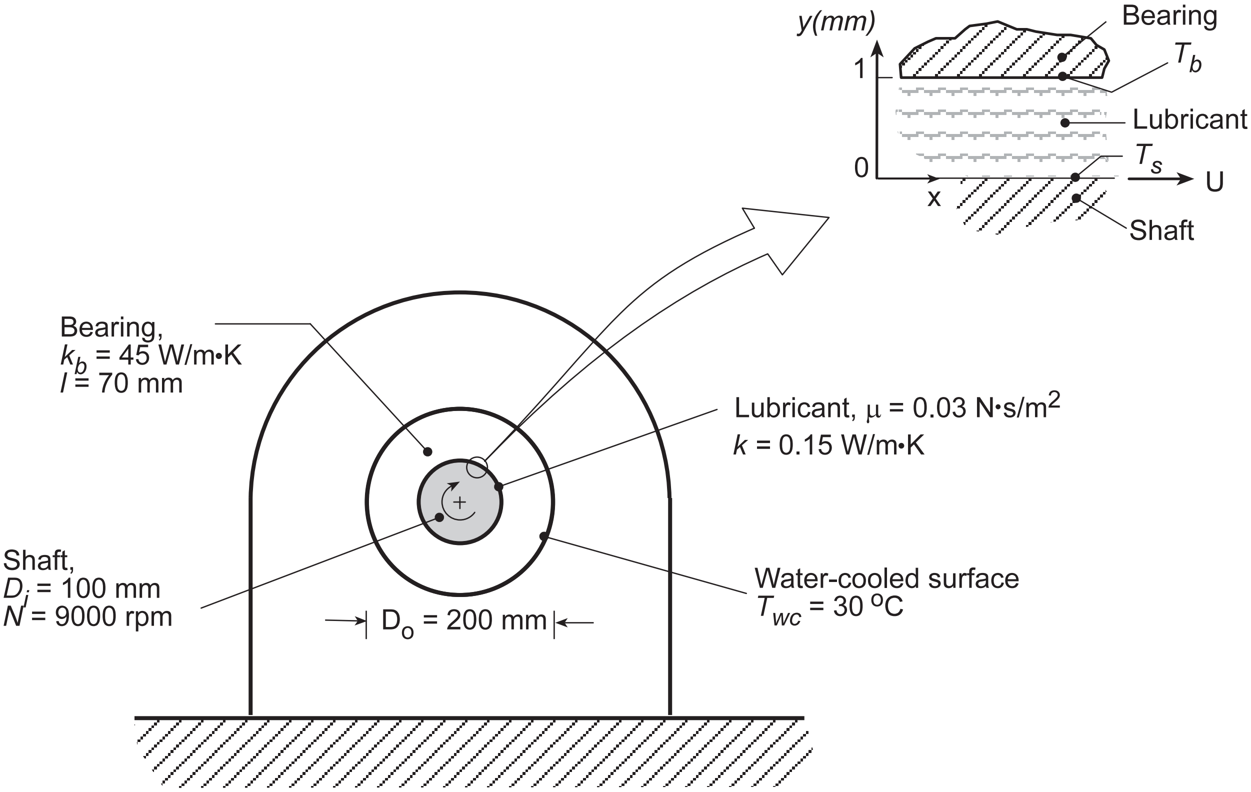
PROBLEM 6S.8

KNOWN: Shaft of diameter 100 mm rotating at 9000 rpm in a journal bearing of 70 mm length. Uniform gap of 1 mm separates the shaft and bearing filled with lubricant. Outer surface of bearing is water-cooled and maintained at T_{wc} = 30°C.

FIND: (a) Viscous dissipation in the lubricant, $\mu\Phi(W/m^3)$, (b) Heat transfer rate from the lubricant, assuming no heat lost through the shaft, and (c) Temperatures of the bearing and shaft, T_b and T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed Couette flow, (3) Incompressible fluid with constant properties, and (4) Negligible heat lost through the shaft.

ANALYSIS: (a) The viscous dissipation, μΦ, Eq. 6S.20, for Couette flow from Example 6S.1, is

$$\mu\Phi = \mu\left(\frac{du}{dy}\right)^2 = \mu\left(\frac{U}{L}\right)^2 = 0.03\,\text{N}\cdot\text{s/m}^2\left(\frac{47.1\,\text{m/s}}{0.001\,\text{m}}\right)^2 = 6.656\times10^7\ \text{W/m}^3 \qquad <$$

where the velocity distribution is linear and the tangential velocity of the shaft is

$$U = \pi DN = \pi(0.100\,\text{m})\times9000\,\text{rpm}\times(\text{min}/60\text{s}) = 47.1\,\text{m/s}\,.$$

(b) The heat transfer rate from the lubricant volume ∀ through the bearing is

$$q = \mu\Phi\cdot\forall = \mu\Phi(\pi D\cdot L\cdot\ell) = 6.65\times10^7\ \text{W/m}^3(\pi\times0.100\,\text{m}\times0.001\,\text{m}\times0.070\,\text{m}) = 1462\,\text{W} \qquad <$$

where ℓ = 70 mm is the length of the bearing normal to the page.

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PROBLEM 6S.8 (Cont.)

(c) From Fourier's law, the heat rate through the bearing material of inner and outer diameters, D_i and D_o, and thermal conductivity k_b is, from Eq. (3.32),

$$q_r = \frac{2\pi \ell k_b \left(T_b - T_{wc}\right)}{\ln\left(D_o / D_i\right)}$$

$$T_b = T_{wc} + \frac{q_r \ln\left(D_o / D_i\right)}{2\pi \ell k_b}$$

$$T_b = 30^\circ\mathrm{C} + \frac{1462\,\mathrm{W} \ln\left(200/100\right)}{2\pi \times 0.070\,\mathrm{m} \times 45\,\mathrm{W/m \cdot K}} = 81.2^\circ\mathrm{C}$$ <

To determine the temperature of the shaft, $T(0) = T_s$, first the temperature distribution must be found beginning with the general solution, Example 6S.1,

$$T(y) = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + C_3 y + C_4$$

The boundary conditions are, at y = 0, the surface is adiabatic

$$\left.\frac{dT}{dy}\right)_{y=0} = 0 \qquad\qquad C_3 = 0$$

and at y = L, the temperature is that of the bearing, T_b

$$T(L) = T_b = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 L^2 + 0 + C_4 \qquad\qquad C_4 = T_b + \frac{\mu}{2k} U^2$$

Hence, the temperature distribution is

$$T(y) = T_b + \frac{\mu}{2k} U^2 \left(1 - \frac{y^2}{L^2}\right)$$

and the temperature at the shaft, y = 0, is

$$T_s = T(0) = T_b + \frac{\mu}{2k} U^2 = 81.3^\circ\mathrm{C} + \frac{0.03\,\mathrm{N \cdot s/m^2}}{2 \times 0.15\,\mathrm{W/m \cdot K}}\left(47.1\,\mathrm{m/s}\right)^2 = 303^\circ\mathrm{C}$$ <