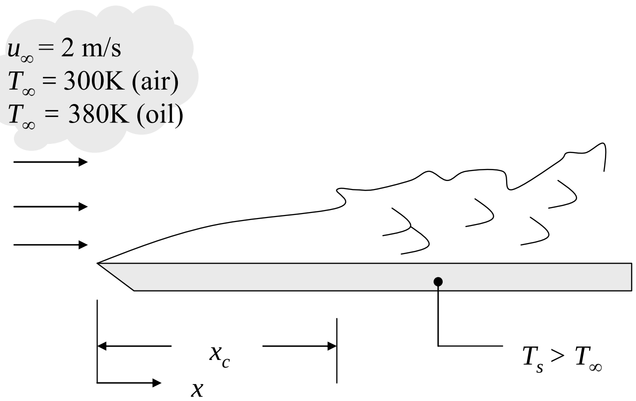
PROBLEM 6.38

KNOWN: Flow over a flat plate. Velocity and temperature of two fluids. Variation of boundary layer thickness with *x* for laminar flow.

FIND: (a) Location where transition to turbulence occurs for each fluid, (b) Plot of velocity boundary layer thickness for $0 \le x \le x_c$ for each fluid, (c) Plot of thermal boundary layer thickness over the same range. Which fluid has the largest local temperature gradient at the surface, Nusselt number, and heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible flow, (3) Transition occurs at a critical Reynolds number of 5×10^5.

PROPERTIES: *Table A.4*, Air (T = 300 K): $\nu = 15.89 \times 10^{-6}$ m²/s, k = 0.0263 W/m·K, Pr = 0.707. *Table A.5*, Engine Oil (T = 380 K): $\nu = 16.9 \times 10^{-6}$ m²/s, k = 0.136 W/m·K, Pr = 233.

ANALYSIS: (a) Transition occurs at $Re_{x,c} = u_\infty x/\nu = 5 \times 10^5$. Therefore, for air

$$x_c = 5\times10^5 \frac{\nu}{u_\infty} = 5\times10^5 \frac{15.89\times10^{-6}\ \text{m}^2/\text{s}}{2\ \text{m/s}} = 4.0\ \text{m}$$ <

The value for engine oil is 4.2 m. <

(b) The velocity boundary layer thickness is given by $\dfrac{\delta}{x} = \dfrac{5}{\sqrt{Re_x}}$. Thus, for air

$$\delta = \frac{5x}{\sqrt{Re_x}} = 5\sqrt{x\nu/u_\infty} = 5\sqrt{\frac{15.89\times10^{-6}\ \text{m}^2/\text{s}\times x}{2\ \text{m/s}}} = 0.0141x^{1/2}$$ <

where x and δ are expressed in meters. The corresponding result for engine oil is $\delta = 0.0145x^{1/2}$. <

These results are plotted below.

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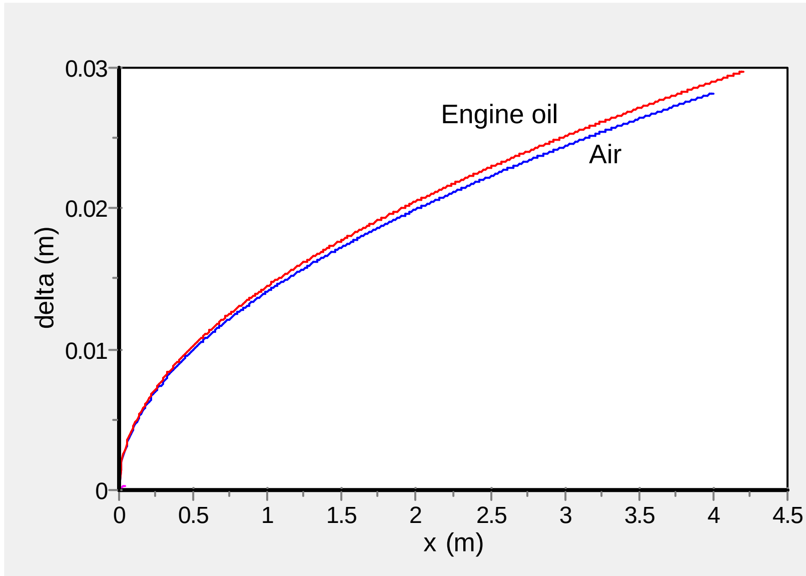
PROBLEM 6.38 (Cont.)



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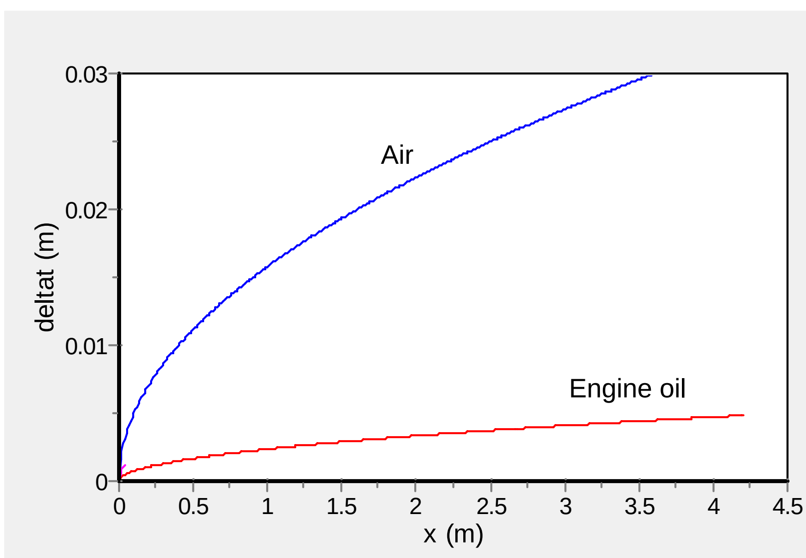
(c) From Eq. 6.34 with $n = 1/3$, $\delta_t = \delta{\cdot}Pr^{-1/3}$. Thus, for air

$$\delta_t = \delta Pr^{-1/3} = 0.0141x^{1/2} \times (0.707)^{-1/3} = 0.0158x^{1/2}$$ <

Similarly for engine oil $\delta_t = 0.00586x^{1/2}$. <

The results for the two fluids are shown below.



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The two fluids are subjected to the same temperature difference between the surface and the free stream. Since the thermal boundary layer thickness is the distance over which the temperature varies from the surface temperature to the free stream temperature, the fluid with the smaller value of δ_t must have a larger temperature gradient, $-\partial T/\partial y|_{y=0}$.

Therefore, engine oil has the larger temperature gradient at the surface. <

The local Nusselt number is given by $Nu = hx/k$, where h is defined in Equation 6.5. Therefore,

$$Nu = \frac{-\partial T/\partial y|_{y=0}}{T_s - T_\infty} x$$

At a given x location, since $T_s - T_\infty$ is the same for both fluids, the fluid with the larger temperature gradient has the larger local Nusselt number.

Engine oil has the larger local Nusselt number. <

The heat transfer coefficient is given by Equation 6.5:

$$h = \frac{-k_f \partial T/\partial y|_{y=0}}{T_s - T_\infty}$$

Since engine oil has a larger temperature gradient *and* a larger thermal conductivity, it is associated with a larger heat transfer coefficient. <

COMMENTS: (1) Since the kinematic viscosity of the two fluids is nearly the same, their local Reynolds numbers, transition locations, and velocity boundary layer thicknesses are comparable. (2) The much higher Prandtl number of the engine oil results in a much thinner thermal boundary layer and consequently a larger temperature gradient at the surface and higher heat transfer coefficient.