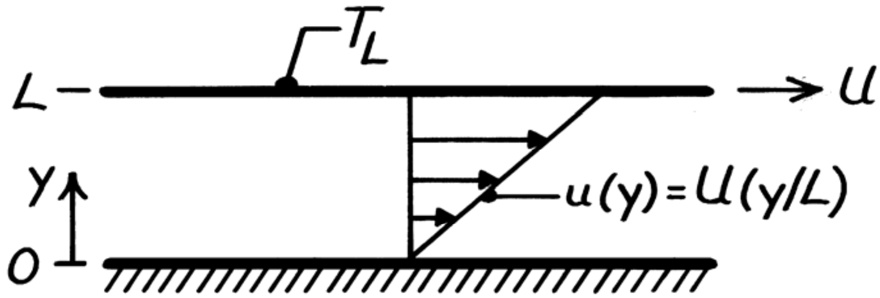
PROBLEM 6S.6

KNOWN: Couette flow with moving plate isothermal and stationary plate insulated.

FIND: Temperature of stationary plate and heat flux at the moving plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Couette flow.

ANALYSIS: The energy equation is given by

$$0 = k\left[\frac{\partial^2 T}{\partial y^2}\right] + \mu\left[\frac{\partial u}{\partial y}\right]^2$$

Integrating twice find the general form of the temperature distribution,

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{k}\left[\frac{U}{L}\right]^2 \qquad \frac{\partial T}{\partial y} = -\frac{\mu}{k}\left[\frac{U}{L}\right]^2 y + C_1$$

$$T(y) = -\frac{\mu}{2k}\left[\frac{U}{L}\right]^2 y^2 + C_1 y + C_2.$$

Consider the boundary conditions to evaluate the constants,

$$\partial T/\partial y\Big|_{y=0} = 0 \quad \rightarrow \quad C_1 = 0 \quad \text{and} \quad T(L) = T_L \quad \rightarrow \quad C_2 = T_L + \frac{\mu}{2k}U^2.$$

Hence, the temperature distribution is

$$T(y) = T_L + \left[\frac{\mu U^2}{2k}\right]\left[1 - \left[\frac{y}{L}\right]^2\right].$$

The temperature of the lower plate (y = 0) is

$$T(0) = T_L + \left[\frac{\mu U^2}{2k}\right]. \qquad <$$

The heat flux to the upper plate (y = L) is

$$q''(L) = -k\frac{\partial T}{\partial y}\Big|_{y=L} = \frac{\mu U^2}{L}. \qquad <$$

COMMENTS: The heat flux at the top surface may also be obtained by integrating the viscous dissipation over the fluid layer height. For a control volume about a unit area of the fluid layer,

$$\dot{E}''_g = \dot{E}''_{out} \qquad \int_0^L \mu\left[\frac{\partial u}{\partial y}\right]^2 dy = q''(L) \qquad q''(L) = \frac{\mu U^2}{L}.$$