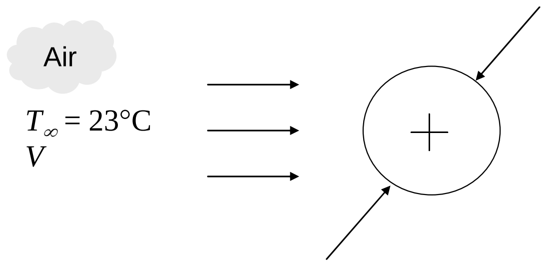
PROBLEM 6.26

KNOWN: Critical Reynolds number for a cylinder in cross flow. Critical Mach number.

FIND: Critical cylinder diameter below which, if the flow of air at atmospheric pressure and temperature is turbulent, compressibility effects may be important.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions. (2) Air behaves as an ideal gas.

PROPERTIES: Table A.4, air (T = 300 K): $\mathcal{M}$ = 28.97 kg/kmol, c_p = 1.007 kJ/kg·K, μ = 184.6 × 10^{-7} N·s/m^2.

ANALYSIS: The density of an ideal gas may be found from the equation of state,

$$\rho = \frac{p}{RT}$$

and the speed of sound for an ideal gas is $a = \sqrt{\gamma RT}$.

From the definition of the Mach number, the air velocity may be expressed as

$$V = Ma \cdot a$$

Substituting the preceding equations into the definition of the Reynolds number yields

$$Re_D = \frac{VD}{\nu} = \frac{VD\rho}{\mu} = \frac{Ma\sqrt{\gamma RT}\,Dp}{\mu RT} D = \frac{Ma}{\mu}\sqrt{\frac{\gamma}{RT}}Dp$$

Letting $Re = Re_c$ and $Ma = Ma_c$, the preceding equation can be rearranged to write an expression for the critical cylinder diameter,

$$D_c = \frac{Re_c}{Ma_c}\sqrt{\frac{RT}{\gamma}}\frac{\mu}{p}$$

Before evaluating the critical cylinder diameter, we note that the gas constant for air is

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PROBLEM 6.26 (Cont.)

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8315 \text{ J/kmol}\cdot\text{K}}{28.97 \text{ kg/kmol}} = 287 \text{ J/kg}\cdot\text{K}$$

and the specific heat at constant volume, c_v, is

$$c_v = c_p - R = 1007 \text{ J/kg}\cdot\text{K} - 287 \text{ J/kg}\cdot\text{K} = 720 \text{ J/kg}\cdot\text{K}$$

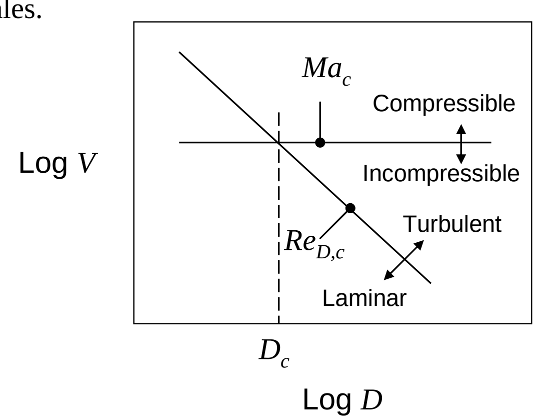
Therefore, the ratio of specific heats for air is

$$\gamma = \frac{c_p}{c_v} = \frac{1007 \text{ J/kg}\cdot\text{K}}{720 \text{ J/kg}\cdot\text{K}} = 1.399$$

For the conditions of the problem, the critical cylinder diameter is

$$D_c = \frac{Re_c}{Ma_c}\sqrt{\frac{RT}{\gamma}}\frac{\mu}{p} = \frac{2\times10^5}{0.3}\sqrt{\frac{287 \text{ J/kg}\cdot\text{K}\times300 \text{ K}}{1.399}}\times\frac{184.6\times10^{-7}\text{ N}\cdot\text{s/m}^2}{1.0133\times10^5 \text{ N/m}^2} = 0.030 \text{ m} = 30 \text{ mm} \quad <$$

COMMENTS: (1) The expression for the critical Reynolds number ($Re_{D,c} = 2\times10^5$) is plotted using *log-log* scales in the figure below. Laminar flow occurs to the left of the sloped line, while turbulent flow occurs to the right of the sloped line. The velocity associated with the critical Mach number is identified by the horizontal line that separates regions of incompressible flow (below the horizontal line) and compressible flow (above the horizontal line). From this plot, it is evident that below the critical cylinder diameter, D_c, if the flow is turbulent, compressibility effects may be important. If the flow is laminar, the flow may or may not be compressible. In general, turbulent flow is difficult to achieve in situations involving small length scales.



(2) The value of the critical Reynolds number is geometry-dependent. Care must be taken to apply the correct value of the critical Reynolds number in any calculation involving convection heat transfer.