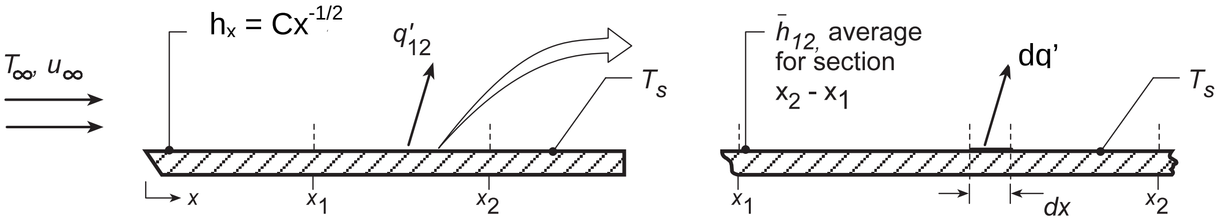
PROBLEM 6.13

KNOWN: Variation of local convection coefficient with distance x from a heated plate with a uniform temperature T_s.

FIND: (a) An expression for the average coefficient $\bar{h}_{12}$ for the section of length $(x_2 - x_1)$ in terms of C, x_1 and x_2, and (b) An expression for $\bar{h}_{12}$ in terms of x_1 and x_2, and the average coefficients $\bar{h}_1$ and $\bar{h}_2$, corresponding to lengths x_1 and x_2, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow over a plate with uniform surface temperature, T_s, and (2) Spatial variation of local coefficient is of the form $h_x = Cx^{-1/2}$, where C is a constant.

ANALYSIS: (a) The heat transfer rate per unit width from a longitudinal section, $x_2 - x_1$, can be expressed as

$$q'_{12} = \bar{h}_{12}(x_2 - x_1)(T_s - T_\infty) \tag{1}$$

where $\bar{h}_{12}$ is the average coefficient for the section of length $(x_2 - x_1)$. The heat rate can also be written in terms of the local coefficient, Eq. (6.7), as

$$q'_{12} = \int_{x_1}^{x_2} h_x dx (T_s - T_\infty) = (T_s - T_\infty)\int_{x_1}^{x_2} h_x dx \tag{2}$$

Combining Eq. (1) and (2),

$$\bar{h}_{12} = \frac{1}{(x_2 - x_1)}\int_{x_1}^{x_2} h_x dx \tag{3}$$

and substituting for the form of the local coefficient, $h_x = Cx^{-1/2}$, find that

$$\bar{h}_{12} = \frac{1}{(x_2 - x_1)}\int_{x_1}^{x_2} Cx^{-1/2}dx = \frac{C}{x_2 - x_1}\left[\frac{x^{1/2}}{1/2}\right]_{x_1}^{x_2} = 2C\frac{x_2^{1/2} - x_1^{1/2}}{x_2 - x_1} \tag{4}$$ <

(b) The heat rate, given as Eq. (1), can also be expressed as

$$q'_{12} = \bar{h}_2 x_2 (T_s - T_\infty) - \bar{h}_1 x_1 (T_s - T_\infty) \tag{5}$$

which is the difference between the heat rate for the plate over the section $(0 - x_2)$ and over the section $(0 - x_1)$. Combining Eqs. (1) and (5), find,

$$\bar{h}_{12} = \frac{\bar{h}_2 x_2 - \bar{h}_1 x_1}{x_2 - x_1} \tag{6}$$ <

COMMENTS: (1) Note that

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{1}{x}\int_0^x Cx^{-1/2}dx = 2Cx^{-1/2} \tag{7}$$

or $\bar{h}_x = 2h_x$. Substituting Eq. (7) into Eq. (6), see that the result is the same as Eq. (4).