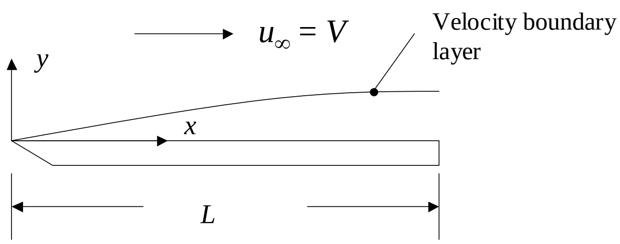
PROBLEM 6.24

KNOWN: Nondimensional form of the *x*-direction velocity boundary layer equation and boundary conditions, expressions for x^*, y^*, u^* and v^*. Laminar, incompressible flow.

FIND: Expressions for (a) the velocity boundary conditions and (b) *x*-momentum equation in dimensional form.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: (a) From Equation 6.23, the boundary conditions in nondimensional form are

$$u^*(x^*,0) = 0 \qquad (1)$$
$$v^*(x^*,0) = 0 \qquad (2)$$
$$u^*(x^*,\infty) = \frac{u_\infty(x^*)}{V} \qquad (3)$$

Substituting $u^* = \frac{u}{V}$ from Equation 6.19 and $x^* = \frac{x}{L}$ from Equation 6.18 into Equation (1) yields $\frac{u}{V}\left(x/L,0\right) = 0$. After multiplying both sides of the resultant equation by *V*, we have $u\left(x/L, y=0\right) = 0$.

Hence, the *x*-component of the fluid velocity at any *x* location along the surface is zero. <

Substituting $v^* = v/V$ from Equation 6.19 and $x^* = \frac{x}{L}$ from Equation 6.18 into Equation (2) yields $\frac{v}{V}\left(x/L,0\right) = 0$. After multiplying both sides of the equation by *V*, we have $v\left(x/L, y=0\right) = 0$.

Hence, the *y*-component of the fluid velocity at any *x* location along the surface is zero. <

The preceding two results are the familiar *zero velocity boundary conditions* that exist at an impenetrable, stationary surface.

Substituting $u^* = \frac{u}{V}$ from Equation 6.19 and $x^* = \frac{x}{L}$ from Equation 6.18 into Equation (3) yields $\frac{u}{V}\left(x/L,\infty\right) = \frac{u_\infty\left(x/L\right)}{V} = \frac{V}{V} = 1$. After multiplying both sides of the resultant equation by *V*, we have $u\left(x/L, y \to \infty\right) = V = u_\infty$. Hence, the *x*-component of the velocity at any *x* location outside of the boundary layer is equal to the free stream value. <

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PROBLEM 6.24 (Cont.)

(b) Note that $\dfrac{\partial u^*}{\partial x^*} = \dfrac{\partial(u/V)}{\partial(x/L)} = \dfrac{L}{V}\dfrac{\partial u}{\partial x}$. Likewise, $\dfrac{\partial u^*}{\partial y^*} = \dfrac{\partial(u/V)}{\partial(y/L)} = \dfrac{L}{V}\dfrac{\partial u}{\partial y}$ and $\dfrac{\partial^2 u^*}{\partial y^{*2}} = \dfrac{\partial}{\partial y^*}\left(\dfrac{\partial u^*}{\partial y^*}\right) = \dfrac{\partial}{\partial(y/L)}\left(\dfrac{L}{V}\dfrac{\partial u}{\partial y}\right) = \dfrac{L^2}{V}\dfrac{\partial^2 u}{\partial y^2}$. Also, from the definition of p^*, we note that $-\dfrac{\partial p^*}{\partial x^*} = -\dfrac{\partial(p_\infty/\rho V^2)}{\partial(x/L)} = -\dfrac{L}{\rho V^2}\dfrac{\partial p_\infty}{\partial x} = 0$. Substituting the preceding expressions, along with the definition of the Reynolds number, $Re_L = \rho VL/\mu = VL/\nu$ into Equation 6.21 yields

$\dfrac{u}{V}\dfrac{L}{V}\dfrac{\partial u}{\partial x} + \dfrac{v}{V}\dfrac{L}{V}\dfrac{\partial u}{\partial y} = 0 + \dfrac{\nu}{VL}\dfrac{L^2}{V}\dfrac{\partial^2 u}{\partial y^2}$. Multiplying both sides of the preceding equation by $\dfrac{V^2}{L}$ gives

$u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \nu\dfrac{\partial^2 u}{\partial y^2}$ which is identical to Equation 6.16 for the case where $\dfrac{dp_\infty}{dx} = 0$. <

COMMENTS: (1) Equations 6.21 and 6.23 are nondimensional forms of Equations 6.16 and the no-slip boundary conditions. When converted to their nondimensional forms, the equations explicitly illustrate the importance of the Reynolds number in describing the velocity boundary layer. (2) For a flat plate subject to parallel flow, the Reynolds number is usually expressed as $Re_L = u_\infty L/\nu$, since $u_\infty = V$.