PROBLEM 6.22

KNOWN: Pressure dependence of the dynamic viscosity, thermal conductivity and specific heat.

FIND: (a) Variation of the kinematic viscosity and thermal diffusivity with pressure for an incompressible liquid and an ideal gas, (b) Value of the thermal diffusivity of air at 350 K for pressures of 1, 5 and 10 atm, (c) Location where transition occurs for air flow over a flat plate with $T_\infty = 350$ K, p = 1, 5 and 10 atm, and $u_\infty = 2$ m/s.

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Transition at $Re_{x,c} = 5 \times 10^5$, (4) Ideal gas behavior.

PROPERTIES: Table A.4, air (350 K): $\mu = 208.2 \times 10^{-7}$ N·s/m², k = 0.030 W/m·K, $c_p = 1009$ J/kg·K, $\rho = 0.995$ kg/m³.

ANALYSIS:

(a) For an ideal gas

$$p = \rho RT \text{ or } \rho = p/RT \tag{1}$$

while for an incompressible liquid, ρ = constant (2)

The kinematic viscosity is $\nu = \mu/\rho$ (3)

Therefore, for an ideal gas

$$\nu = \mu RT/p \quad \text{or} \quad \nu \propto p^{-1} \tag{4}$$ <

and for an incompressible liquid

$$\nu = \mu/\rho \quad \text{or } \nu \text{ is independent of pressure.}$$ <

The thermal diffusivity is

$$\alpha = k/\rho c$$

Therefore, for an ideal gas,

$$\alpha = kRT/pc \quad \text{or} \quad \alpha \propto p^{-1} \tag{6}$$ <

For an incompressible liquid $\alpha = k/\rho c$ or α is independent of pressure <

(b) For T = 350 K, p = 1 atm, the thermal diffusivity of air is

$$\alpha = \frac{0.030 \text{ W/m}\cdot\text{K}}{0.995 \text{ kg/m}^3 \times 1009 \text{ J/kg}\cdot\text{K}} = 29.9 \times 10^{-6} \text{ m}^2\text{/s}$$ <

Using Equation 6, at p = 5 atm,

Continued…

PROBLEM 6.22 (Cont.)

$$\alpha = 29.9 \times 10^{-6}\ \text{m}^2/\text{s}/5 = 5.98 \times 10^{-6}\ \text{m}^2/\text{s}$$ $<$

At p = 10 atm,

$$\alpha = 29.9 \times 10^{-6}\ \text{m}^2/\text{s}/10 = 2.99 \times 10^{-6}\ \text{m}^2/\text{s}$$ $<$

(c) For transition over a flat plate,

$$Re_{x,c} = \frac{x_c u_\infty}{\nu} = 5 \times 10^5$$

Therefore

$$x_c = 5 \times 10^5 (\nu/u_\infty)$$

For $T_\infty = 350$ K, p = 1 atm,

$$\nu = \mu/\rho = 208.2 \times 10^{-7}\ \text{N} \cdot \text{s/m}^2 \big/ 0.995\ \text{kg/m}^3 = 20.92 \times 10^{-6}\ \text{m}^2/\text{s}$$

Using Equation 4, at p = 5 atm

$$\nu = 20.92 \times 10^{-6}\ \text{m}^2/\text{s} \big/ 5 = 4.18 \times 10^{-6}\ \text{m}^2/\text{s}$$

At p = 10 atm,

$$\nu = 20.92 \times 10^{-6}\ \text{m}^2/\text{s} \big/ 10 = 2.09 \times 10^{-6}\ \text{m}^2/\text{s}$$

Therefore, at p = 1 atm

$$x_c = 5 \times 10^5 \times 20.92 \times 10^{-6}\ \text{m}^2/\text{s}/(2\text{m/s}) = 5.23\ \text{m}$$ $<$

At p = 5 atm,

$$x_c = 5 \times 10^5 \times 4.18 \times 10^{-6}\ \text{m}^2/\text{s}/(2\text{m/s}) = 1.05\ \text{m}$$ $<$

At p = 10 atm

$$x_c = 5 \times 10^5 \times 2.09 \times 10^{-6}\ \text{m}^2/\text{s}/(2\text{m/s}) = 0.523\ \text{m}$$ $<$

COMMENT: Note the strong dependence of the transition length upon the pressure for the gas (the transition length is independent of pressure for the incompressible liquid).