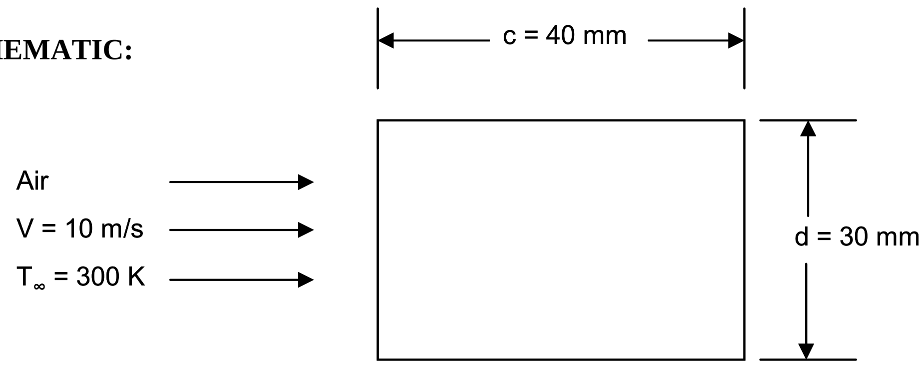
PROBLEM 6.10

KNOWN: Expression for face-averaged Nusselt numbers on a cylinder of rectangular cross section. Dimensions of the cylinder.

FIND: Average heat transfer coefficient over the entire cylinder. Plausible explanation for variations in the face-averaged heat transfer coefficients.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

PROPERTIES: Table A.4, air (300 K): k = 0.0263 W/m·K, $\nu = 1.589 \times 10^{-5}$ m^2/s, Pr = 0.707.

ANALYSIS:

For the square cylinder, c/d = 40 mm/30 mm = 1.33

$$Re_d = \frac{Vd}{\nu} = \frac{10\text{ m/s} \times 30 \times 10^{-3}\text{ m}}{1.589 \times 10^{-5}\text{ m}^2/\text{s}} = 18{,}880$$

Therefore, for the front face C = 0.674, m = ½. For the sides, C = 0.107, m = 2/3 while for the back C = 0.153, m = 2/3.

Front face:

$$Nu_{d,f} = 0.674 \times 18{,}880^{1/2} \times 0.707^{1/3} = 82.44$$

$$\overline{h}_f = \frac{kNu_d}{d} = \frac{0.0263\text{ W/m}\cdot\text{K} \times 82.44}{30 \times 10^{-3}\text{ m}} = 72.27\text{ W/m}^2\cdot\text{K}$$

Side faces:

$$Nu_{d,s} = 0.107 \times 18{,}880^{2/3} \times 0.707^{1/3} = 67.36$$

$$\overline{h}_s = \frac{kNu_{d,s}}{d} = \frac{0.0263\text{ W/m}\cdot\text{K} \times 67.36}{30 \times 10^{-3}\text{ m}} = 59.05\text{ W/m}^2\cdot\text{K}$$

Back face:

$$Nu_{d,b} = 0.153 \times 18{,}880^{2/3} \times 0.707^{1/3} = 96.43$$

$$\overline{h}_b = \frac{kNu_{d,b}}{d} = \frac{0.0263\text{ W/m}\cdot\text{K} \times 96.43}{30 \times 10^{-3}\text{ m}} = 84.54\text{ W/m}^2\cdot\text{K}$$

Continued…

PROBLEM 6.10 (Cont.)

For the entire square cylinder of unit length,

$$\bar{h} = \frac{\bar{h}_f A_f + 2\bar{h}_s A_s + \bar{h}_b A_b}{A_f + 2A_s + A_b}$$

$$\bar{h} = \frac{\left(\begin{array}{l} 72.27\ \text{W/m}^2 \cdot \text{K} \times 30 \times 10^{-3}\ \text{m} + 2 \times 59.05\ \text{W/m}^2 \cdot \text{K} \times 40 \times 10^{-3}\ \text{m} \\ + 84.54\ \text{W/m}^2 \cdot \text{K} \times 30 \times 10^{-3}\ \text{m} \end{array} \right)}{(2 \times 30 \times 10^{-3}\text{m} + 2 \times 40 \times 1^{-3}\text{m})}$$

$$\bar{h} = 67.35\ \text{W/m}^2 \cdot \text{K}$$ <

The face-averaged heat transfer coefficients are largest on the back face and smallest on the side faces. Plausible explanations for the variations of the face-averaged heat transfer coefficients are complex fluid flow patterns including vortex shedding on the back face and development of relatively thick boundary layers along the sides.

COMMENT: See S.Y. Yoo, J.H. Park, C.H. Chung and M.K. Chung, Journal of Heat Transfer, Vol. 125, pp. 1163-1169, 2003 for details.