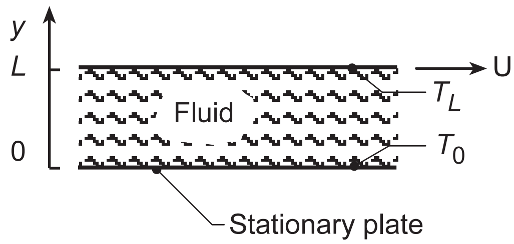
PROBLEM 6S.9

KNOWN: Couette flow with heat transfer.

FIND: (a) Dimensionless form of temperature distribution, (b) Conditions for which top plate is adiabatic, (c) Expression for heat transfer to lower plate when top plate is adiabatic.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) incompressible fluid with constant properties, (3) Negligible body forces, (4) Couette flow.

ANALYSIS: (a) From Example 6.4, the temperature distribution is

$$T = T_0 + \frac{\mu}{2k}U^2\left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + \left(T_L - T_0\right)\frac{y}{L}$$

$$\frac{T - T_0}{T_L - T_0} = \frac{\mu U^2}{2k\left(T_L - T_0\right)}\left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + \frac{y}{L}$$

or, with

$$\theta \equiv \left(T - T_0\right)/T_L - T_0\,, \qquad \eta \equiv y/L\,,$$

$$\Pr \equiv c_p\mu/k\,, \qquad Ec \equiv U^2/c_p\left(T_L - T_0\right)$$

$$\theta = \frac{\Pr\cdot Ec}{2}\left(\eta - \eta^2\right) + \eta = \eta\left[1 + \frac{1}{2}\Pr\cdot Ec\left(1 - \eta\right)\right] \qquad (1)$$ <

(b) For there to be zero heat transfer at the top plate, $(dT/dy)_{y=L} = 0$. Hence, $(d\theta/d\eta)_{\eta=1} = 0$.

$$\left.\frac{d\theta}{d\eta}\right)_{\eta=1} = \frac{\Pr\cdot Ec}{2}\left(1 - 2\eta\right)\Big|_{\eta=1} + 1 = -\frac{\Pr\cdot Ec}{2} + 1 = 0$$

There is no heat transfer at the top plate if,

$$Ec\cdot\Pr = 2. \qquad (2)$$ <

(c) The heat transfer rate to the lower plate (per unit area) is

$$q_0'' = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{\left(T_L - T_0\right)}{L}\left.\frac{d\theta}{d\eta}\right|_{\eta=0}$$

$$q_o'' = -k\frac{T_L - T_0}{L}\left[\frac{\Pr\cdot Ec}{2}\left(1 - 2\eta\right)\Big|_{\eta=0} + 1\right]$$

$$q_0'' = -k\frac{T_L - T_0}{L}\left(\frac{\Pr\cdot Ec}{2} + 1\right) = -2k\left(T_L - T_0\right)/L$$ <

Continued...

PROBLEM 6S.9 (Cont.)

(d) Using Eq. (1), the dimensionless temperature distribution is plotted as a function of dimensionless distance, $\eta = y/L$. When Pr·Ec = 0, there is no dissipation and the temperature distribution is linear, so that heat transfer is by conduction only. As Pr ·Ec increases, viscous dissipation becomes more important. When Pr·Ec = 2, heat transfer to the upper plate is zero. When Pr·Ec > 2, the heat rate is out of the oil film at both surfaces.