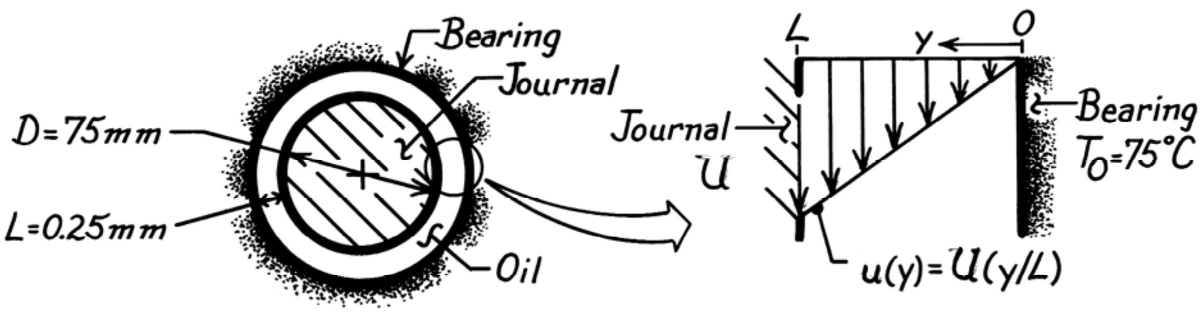
PROBLEM 6S.3

KNOWN: Diameter, clearance, rotational speed and fluid properties of a lightly loaded journal bearing. Temperature of bearing.

FIND: (a) Temperature distribution in the fluid, (b) Rate of heat transfer from bearing and operating power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Couette flow.

PROPERTIES: Oil (Given): $\rho = 800\ \text{kg/m}^3$, $\nu = 10^{-5}\ \text{m}^2/\text{s}$, $k = 0.13$ W/m·K; $\mu = \rho\nu = 8 \times 10^{-3}$ kg/s·m.

ANALYSIS: (a) For Couette flow, the velocity distribution is linear, u(y) = U(y/L), and the energy equation and general form of the temperature distribution are

$$k\frac{d^2T}{dy^2} = -\mu\left[\frac{du}{dy}\right]^2 = -\mu\left[\frac{U}{L}\right]^2 \qquad T = -\frac{\mu}{2k}\left[\frac{U}{L}\right]^2 y^2 + \frac{C_1}{k}y + C_2.$$

Considering the boundary conditions $dT/dy)_{y=L} = 0$ and $T(0) = T_0$, find $C_2 = T_0$ and $C_1 = \mu U^2/L$. Hence,

$$T = T_0 + \left(\mu U^2\right)/k\left[\left(y/L\right) - 1/2\left(y/L\right)^2\right]. \qquad <$$

(b) Applying Fourier's law at y = 0, the rate of heat transfer per unit length to the bearing is

$$q' = -k(\pi\ D)\frac{dT}{dy}\bigg]_{y=0} = -(\pi\ D)\frac{\mu U^2}{L} = -\left(\pi \times 75 \times 10^{-3}\text{m}\right)\frac{8\times 10^{-3}\text{kg/s}\cdot\text{m}\ (14.14\ \text{m/s})^2}{0.25\times 10^{-3}\text{m}} = -1507.5\ \text{W/m}$$

where the velocity is determined as

$$U = (D/2)\omega = 0.0375\text{m} \times 3600\ \text{rev/min}\ \left(2\pi\ \text{rad/rev}\right)/\left(60\ \text{s/min}\right) = 14.14\ \text{m/s}.$$

The journal power requirement is

$$P' = F'_{(y=L)}U = \tau_{s(y=L)} \cdot \pi\ D \cdot U$$

$$P' = 452.5\text{kg/s}^2 \cdot \text{m}\ \left(\pi \times 75 \times 10^{-3}\text{m}\right)14.14\text{m/s} = 1507.5\text{kg}\cdot\text{m/s}^3 = 1507.5\text{W/m} \qquad <$$

where the shear stress at y = L is

$$\tau_{s(y=L)} = \mu\left(\partial\ u/\partial\ y\right)_{y=L} = \mu\frac{U}{L} = 8\times 10^{-3}\text{kg/s}\cdot\text{m}\left[\frac{14.14\ \text{m/s}}{0.25\times 10^{-3}\text{m}}\right] = 452.5\ \text{kg/s}^2\cdot\text{m}.$$

COMMENTS: Note that $q' = P'$, which is consistent with the energy conservation requirement.