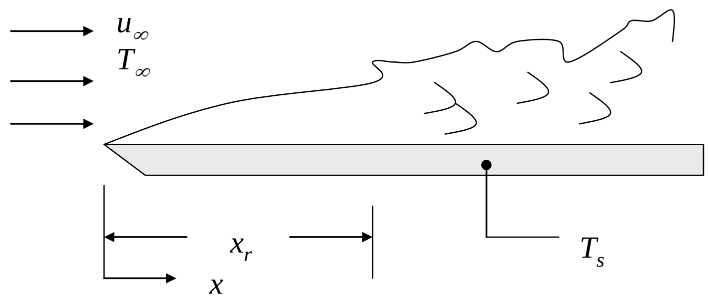
PROBLEM 6.23

KNOWN: Velocity of water flowing over a flat plate. Length of plate. Variation of local convection coefficient with x. Water temperature.

FIND: Average convection coefficient for roughness applied over the range $0 \le x_r \le L$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Transition occurs at a critical Reynolds number of 5×10^5 for the smooth plate, (4) Incompressible flow.

PROPERTIES: *Table A.6*, Water (T = 300 K): $\rho = v_f^{-1} = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2.

ANALYSIS: For roughness applied over the range $0 \le x_r \le x_c = 0.43$ m, transition occurs at x_r. From Eq. 6.10,

$$\overline{h} = \frac{1}{L}\int_0^L h dx = \frac{1}{L}\left[\int_0^{x_r} h_{\text{lam}} dx + \int_{x_r}^{L} h_{\text{turb}} dx\right] = \frac{1}{L}\left[\frac{C_{\text{lam}}}{0.5} x^{0.5}\Big|_0^{x_r} + \frac{C_{\text{turb}}}{0.8} x^{0.8}\Big|_{x_r}^{L}\right]$$

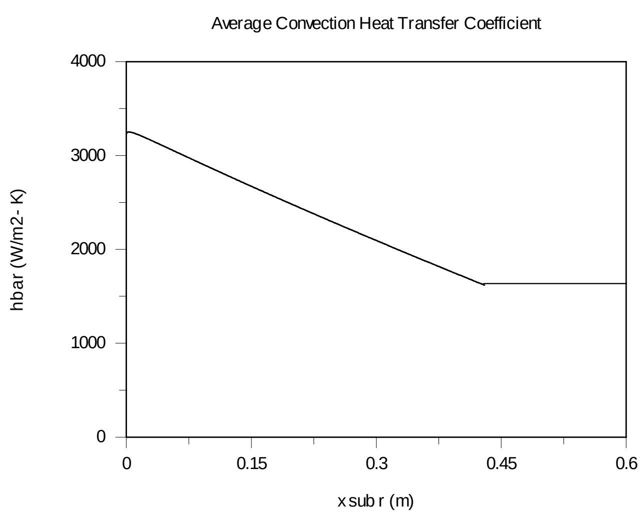
$$= \frac{1}{L}\left[\frac{C_{\text{lam}}}{0.5} x_r^{0.5} + \frac{C_{\text{turb}}}{0.8}\left(L^{0.8} - x_r^{0.8}\right)\right] = \frac{1}{0.6\text{ m}}\left[\frac{395\text{ W/m}^{1.5}\cdot\text{K}}{0.5} x_r^{0.5} + \frac{2330\text{ W/m}^{1.8}\cdot\text{K}}{0.8}\left((0.6\text{ m})^{0.8} - x_r^{0.8}\right)\right]$$

$$= \frac{1}{0.6\text{ m}}\left[\frac{395\text{ W/m}^{1.5}\cdot\text{K}}{0.5} x_r^{0.5} + \frac{2330\text{ W/m}^{1.8}\cdot\text{K}}{0.8}\left((0.6\text{ m})^{0.8} - x_r^{0.8}\right)\right]$$

$$= 1317\text{ W/m}^{2.5}\cdot\text{K} \times x_r^{0.5} + 3226\text{ W/m}^2\cdot\text{K} - 4854\text{ W/m}^{2.8}\cdot\text{K} \times x_r^{0.8}$$

Roughness applied over the range $x_r > 0.43$ has no affect on the transition since the transition occurs at $x_c = 0.43$ m for the smooth plate. That is, $\overline{h} = 1620\text{ W/m}^2\cdot\text{K}$.

This result is plotted below for $0 \le x_r \le 0.6$ m.



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PROBLEM 6.23 (Cont.)

The maximum value of $\overline{h}$ exists when transition occurs very close to the leading edge of the plate, at $x_r = 0.003$ m. It does not occur exactly at the leading edge because the laminar heat transfer coefficient equation yields a slightly higher value than the turbulent heat transfer coefficient equation very near $x = 0$. <

The minimum value of $\overline{h}$ exists when $x_r > x_c = 0.43$ m. <

COMMENTS: (1) Turbulent heat transfer coefficients are usually (but not always) larger than laminar heat transfer coefficients. Therefore, tripping the transition to turbulence at or near the leading edge results in enhanced heat transfer. (2) The conclusion that the laminar heat transfer coefficient is slightly higher than the turbulent heat transfer coefficient very near $x = 0$ may not be accurate. Turbulent heat transfer coefficient measurements are usually not performed very close to the leading edge since, in most cases, turbulence develops further downstream. (3) Adding roughness at x locations downstream of where the transition to turbulence would normally occur has no influence on the transition or the average heat transfer rate.