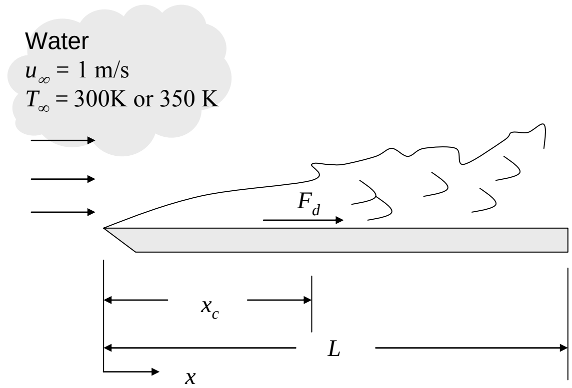
PROBLEM 6.47

KNOWN: Velocity of water flowing over a flat plate. Length and width of plate. Variation of local convection coefficient with x for $T = 300$ K and $T = 350$ K. Locations of turbulence transition.

FIND: Drag force for both water temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Transition occurs at a critical Reynolds number of 5×10^5, (3) Incompressible flow.

PROPERTIES: *Table A.6*, Water ($T = 300$ K): $\mu = 855 \times 10^{-6}$ N·s/m^2, $k = 0.613$ W/m·K. Water ($T = 350$ K): $\mu = 365 \times 10^{-6}$ N·s/m^2, $k = 0.668$ W/m·K.

ANALYSIS: According to the Reynolds analogy, Eq. 6.35

$$C_f \frac{Re_L}{2} = Nu$$

This relationship holds for the local values of C_f and Nu. The local shear stress can be expressed as

$$\tau_s = C_f \frac{\rho u_\infty^2}{2} = \frac{Nu}{Re_L} \rho u_\infty^2 = \frac{hx}{k} \frac{\rho u_\infty \nu}{L} = \frac{\mu u_\infty}{kL} hx = Bhx$$

where $B = \mu u_\infty / kL$. Therefore, $\tau_{s,\text{lam}} = BC_{\text{lam}} x^{0.5}$ and $\tau_{s,\text{turb}} = BC_{\text{turb}} x^{0.8}$. Now the drag force can be found:

$$F_d = \int_0^L \tau_s dx \cdot W = \left[\int_0^{x_c} BC_{\text{lam}} x^{0.5} dx + \int_{x_c}^L BC_{\text{turb}} x^{0.8} dx\right] W = BW\left[C_{\text{lam}} \frac{x_c^{1.5}}{1.5} + C_{\text{turb}}\left(\frac{L^{1.8}}{1.8} - \frac{x_c^{1.8}}{1.8}\right)\right]$$

At $T = 300$ K,

$$F_d = \frac{855 \times 10^{-6} \text{ N} \cdot \text{s/m}^2 \times 1 \text{ m/s}}{0.613 \text{ W/m} \cdot \text{K} \times 0.6 \text{ m}} \times 1 \text{ m}$$
$$\times \left[395 \text{ W/m}^{1.5} \cdot \text{K} \times \frac{(0.43 \text{ m})^{1.5}}{1.5} + 2330 \text{ W/m}^{1.8} \cdot \text{K}\left(\frac{(0.6 \text{ m})^{1.8}}{1.8} - \frac{(0.43 \text{ m})^{1.8}}{1.8}\right)\right]$$

$$F_d = 0.714 \text{ N}$$ <

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Similarly, at $T = 350$ K with $C_{\text{lam}} = 477$ W/m$^{1.5}$·K, $C_{\text{turb}} = 3600$ W/m$^{1.8}$·K and $x_c = 0.19$ m,

$$F_d = 0.659\ \text{N} \qquad <$$

COMMENTS: (1) Even though transition to turbulence occurs earlier for the $T = 350$ K case, the net effect of the much smaller viscosity is a reduction in the drag force. (2) It would be incorrect to apply Reynolds' analogy, Eq. 6.35, directly to the average values of C_f and *Nu* because of the presence of x in the definition of the Nusselt number. Applying Eq. 6.35 directly to the average values would result in the incorrect values $F_d = 1.36$ and 1.22 N for the 300 K and 350 K cases, respectively.