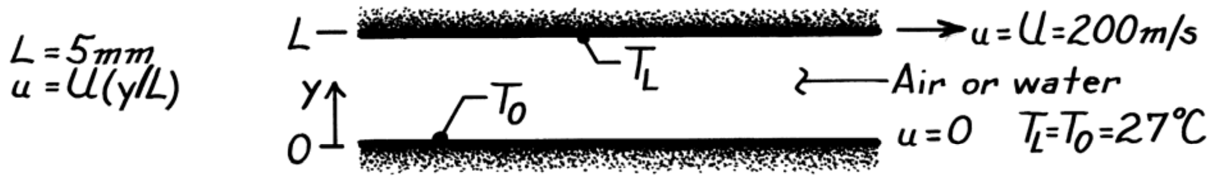
PROBLEM 6S.4

KNOWN: Conditions associated with the Couette flow of air or water.

FIND: (a) Force and power requirements per unit surface area, (b) Viscous dissipation, (c) Maximum fluid temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Fully-developed Couette flow, (2) Incompressible fluid with constant properties.

PROPERTIES: *Table A-4*, Air (300K): $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 26.3 × 10^{-3} W/m·K; *Table A-6*, Water (300K): $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K.

ANALYSIS: (a) The force per unit area is associated with the shear stress. Hence, with the linear velocity profile for Couette flow, $\tau = \mu(\mathrm{du/dy}) = \mu(\mathrm{U/L})$.

Air: $$\tau_{\mathrm{air}} = 184.6\times10^{-7}\ \mathrm{N\cdot s/m^2}\times\frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}} = 0.738\ \mathrm{N/m^2}$$ <

Water: $$\tau_{\mathrm{water}} = 855\times10^{-6}\ \mathrm{N\cdot s/m^2}\times\frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}} = 34.2\ \mathrm{N/m^2}.$$

With the required power given by P/A = $\tau \cdot \mathrm{U}$,

Air: $$(\mathrm{P/A})_{\mathrm{air}} = \left(0.738\ \mathrm{N/m^2}\right)200\ \mathrm{m/s} = 147.6\ \mathrm{W/m^2}$$ <

Water: $$(\mathrm{P/A})_{\mathrm{water}} = \left(34.2\ \mathrm{N/m^2}\right)200\ \mathrm{m/s} = 6840\ \mathrm{W/m^2}.$$

(b) The viscous dissipation is $\mu\Phi = \mu(\mathrm{du/dy})^2 = \mu(\mathrm{U/L})^2$. Hence,

Air: $$(\mu\Phi)_{\mathrm{air}} = 184.6\times10^{-7}\ \frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\left[\frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}}\right]^2 = 2.95\times10^4\ \mathrm{W/m^3}$$ <

Water: $$(\mu\Phi)_{\mathrm{water}} = 855\times10^{-6}\ \frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\left[\frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}}\right]^2 = 1.37\times10^6\ \mathrm{W/m^3}.$$

(c) From the solution to Part 4 of Example 6S.1, the location of the maximum temperature corresponds to y_{max} = L/2. Hence, $T_{max} = T_0 + \mu U^2/8k$ and

Air: $$(T_{max})_{\mathrm{air}} = 27^\circ\mathrm{C} + \frac{184.6\times10^{-7}\ \mathrm{N\cdot s/m^2}\,(200\ \mathrm{m/s})^2}{8\times0.0263\ \mathrm{W/m\cdot K}} = 30.5^\circ\mathrm{C}$$ <

Water: $$(T_{max})_{\mathrm{water}} = 27^\circ\mathrm{C} + \frac{855\times10^{-6}\ \mathrm{N\cdot s/m^2}\,(200\ \mathrm{m/s})^2}{8\times0.613\ \mathrm{W/m\cdot K}} = 34.0^\circ\mathrm{C}.$$

COMMENTS: (1) The viscous dissipation associated with the entire fluid layer, $\mu\Phi(\mathrm{LA})$, must equal the power, P. (2) Although $(\mu\Phi)_{\mathrm{water}} >> (\mu\Phi)_{\mathrm{air}}$, $k_{\mathrm{water}} >> k_{\mathrm{air}}$. Hence, $T_{\mathrm{max,water}} \approx T_{\mathrm{max,air}}$.