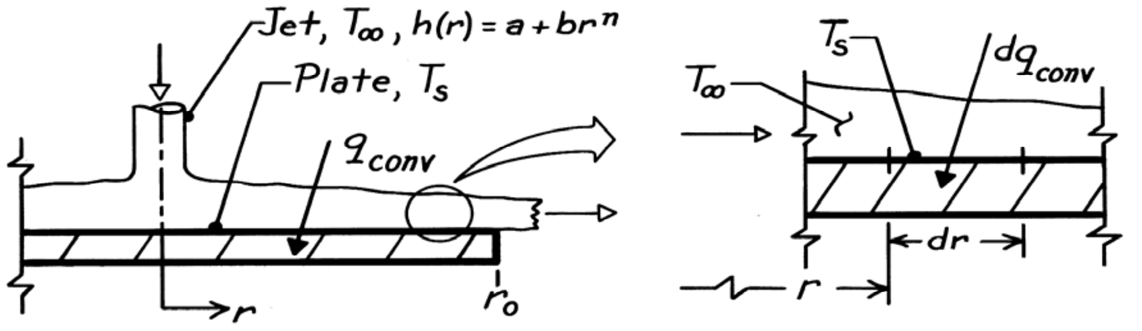
PROBLEM 6.9

KNOWN: Expression for the local heat transfer coefficient of a circular, hot gas jet at T_∞ directed normal to a circular plate at T_s of radius r_o.

FIND: Heat transfer rate to the plate by convection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Flow is axisymmetric about the plate, (3) For h(r), a and b are constants and $n \neq -2$.

ANALYSIS: The convective heat transfer rate to the plate follows from Newton's law of cooling

$$q_{conv} = \int_A dq_{conv} = \int_A h(r) \cdot dA \cdot (T_\infty - T_s).$$

The local heat transfer coefficient is known to have the form,

$$h(r) = a + br^n$$

and the differential area on the plate surface is

$$dA = 2\pi r\, dr.$$

Hence, the heat rate is

$$q_{conv} = \int_0^{r_o} \left(a + br^n\right) \cdot 2\pi\ r\ dr \cdot (T_\infty - T_s)$$

$$q_{conv} = 2\pi(T_\infty - T_s)\left[\frac{a}{2}r^2 + \frac{b}{n+2}r^{n+2}\right]_0^{r_o}$$

$$q_{conv} = 2\pi\left[\frac{a}{2}r_o^2 + \frac{b}{n+2}r_o^{n+2}\right]\ (T_\infty - T_s). \qquad <$$

COMMENTS: Note the importance of the requirement, $n \neq -2$. Typically, the radius of the jet is much smaller than that of the plate.