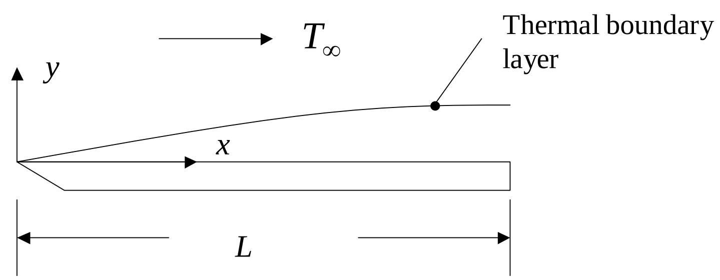
PROBLEM 6.25

KNOWN: Nondimensional form of the thermal boundary layer equation and boundary conditions, expressions for x^*, y^*, u^*, v^* and T^*. Laminar, incompressible flow with negligible viscous dissipation.

FIND: Expressions for (a) the thermal boundary conditions and (b) energy equation in dimensional form.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: (a) From Equation 6.24, the thermal boundary conditions in nondimensional form are

$$T^*(x^*,0) = 0 \qquad (1)$$
$$T^*(x^*,\infty) = 1 \qquad (2)$$

Substituting $T^* = \dfrac{T - T_s}{T_\infty - T_s}$ from Equation 6.20 and $x^* = \dfrac{x}{L}$ from Equation 6.18 into Equation (1) yields $\dfrac{T(x/L,0) - T_s}{T_\infty - T_s} = 0$. After multiplying both sides of the resultant equation by $T_\infty - T_s$, we have $T(x/L, y=0) = T_s$. Hence, the fluid temperature at any x location along the surface is T_s. <

Substituting $T^* = \dfrac{T - T_s}{T_\infty - T_s}$ from Equation 6.20 and $x^* = \dfrac{x}{L}$ from Equation 6.18 into Equation (2) yields $\dfrac{T(x/L, y\to\infty) - T_s}{T_\infty - T_s} = 1$. After multiplying both sides of the equation by $T_\infty - T_s$, we have $T(x/L, y\to\infty) = T_\infty$. Hence, the fluid temperature at any x location outside of the boundary layer is equal to the free stream value. <

(b) Note that $\dfrac{\partial T^*}{\partial x^*} = \dfrac{\partial\left(\dfrac{T - T_s}{T_\infty - T_s}\right)}{\partial(x/L)} = \dfrac{L}{(T_\infty - T_s)}\dfrac{\partial T}{\partial x}$. Likewise, $\dfrac{\partial T^*}{\partial y^*} = \dfrac{\partial\left(\dfrac{T - T_s}{T_\infty - T_s}\right)}{\partial(y/L)} = \dfrac{L}{(T_\infty - T_s)}\dfrac{\partial T}{\partial y}$.

Also, note that $\dfrac{\partial^2 T^*}{\partial y^{*2}} = \dfrac{\partial}{\partial y^*}\left(\dfrac{\partial T^*}{\partial y^*}\right) = \dfrac{L}{(T_\infty - T_s)}\dfrac{\partial}{\partial(y/L)}\dfrac{\partial T}{\partial y} = \dfrac{L^2}{(T_\infty - T_s)}\dfrac{\partial^2 T}{\partial y^2}$. Substituting the preceding expressions, along with $x^* = x/L$, $y^* = y/L$, $Re_L = \dfrac{VL}{\nu}$ and $Pr = \dfrac{\nu}{\alpha}$ into Equation 6.22

Continued…

PROBLEM 6.25 (Cont.)

yields $\frac{u}{V}\frac{L}{\left(T_\infty - T_s\right)}\frac{\partial T}{\partial x} + \frac{v}{V}\frac{L}{\left(T_\infty - T_s\right)}\frac{\partial T}{\partial y} = \frac{\nu}{VL}\frac{\alpha}{\nu}\frac{L^2}{\left(T_\infty - T_s\right)}\frac{\partial^2 T}{\partial y^2}$. Multiplying both sides by $\frac{V\left(T_\infty - T_s\right)}{L}$ gives $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$ which is identical to Equation 6.17 when viscous dissipation is negligible. $<$

COMMENTS: (1) Equations 6.22 and 6.24 are nondimensional forms of Equations 6.17 and the boundary conditions. When converted to their nondimensional forms, the resulting equations explicitly illustrate the importance of the Reynolds and Prandtl numbers in describing the thermal boundary layer. (2) For a flat plate subject to parallel flow, the Reynolds number is usually expressed as $Re_L = \rho u_\infty L/\mu$, or $u_\infty L/\nu$ since $u_\infty = V$.