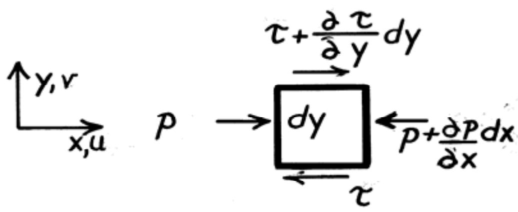
PROBLEM 6S.1

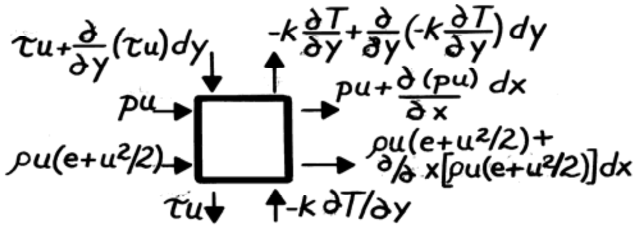
KNOWN: Two-dimensional flow conditions for which v = 0 and T = T(y).

FIND: (a) Verify that u = u(y), (b) Derive the x-momentum equation, (c) Derive the energy equation.

SCHEMATIC:



Pressure & shear forces



Energy fluxes

ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Negligible body forces, (4) v = 0, (5) T = T(y) or $\partial T/\partial x = 0$, (6) Thermal energy generation occurs only by viscous dissipation.

ANALYSIS: (a) From the mass continuity equation, it follows from the prescribed conditions that $\partial u/\partial x = 0$. Hence u = u(y).

(b) From Newton's second law of motion, $\Sigma F_x =$ (Rate of increase of fluid momentum)$_x$,

$$\left[p-\left[p+\frac{\partial p}{\partial x}dx\right]\right] dy\cdot 1+\left[-\tau+\left[\tau+\frac{\partial \tau}{\partial y}dy\right]\right] dx\cdot 1=\left\{(\rho u)u+\frac{\partial}{\partial x}[(\rho u)u]dx\right\}dy\cdot 1-(\rho u)u\,dy\cdot 1$$

Hence, with $\tau=\mu(\partial u/\partial y)$, it follows that

$$-\frac{\partial p}{\partial x}+\frac{\partial \tau}{\partial y}=\frac{\partial}{\partial x}\left[(\rho u)u\right]=0 \qquad \frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}.$$ <

(c) From the conservation of energy requirement and the prescribed conditions, it follows that $\dot{E}_{in}-\dot{E}_{out}=0$, or

$$\left[pu+\rho u\left(e+u^2/2\right)\right] dy\cdot 1+\left[-k\frac{\partial T}{\partial y}+\tau u+\frac{\partial (\tau u)}{\partial y}dy\right] dx\cdot 1$$

$$-\left\{pu+\frac{\partial}{\partial x}(pu)dx+\rho u\left(e+u^2/2\right)+\frac{\partial}{\partial x}\left[\rho u\left(e+u^2/2\right)\right] dx\right\} dy\cdot 1-\left[\tau u-k\frac{\partial T}{\partial y}+\frac{\partial}{\partial y}\left[-k\frac{\partial T}{\partial y}\right] dy\right] dx\cdot 1=0$$

or,
$$\frac{\partial(\tau u)}{\partial y}-\frac{\partial}{\partial x}(pu)-\frac{\partial}{\partial x}\left[\rho u\left(e+u^2/2\right)\right]+\frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]=0$$

$$\tau\frac{\partial u}{\partial y}+u\frac{\partial \tau}{\partial y}-u\frac{\partial p}{\partial x}+k\frac{\partial^2 T}{\partial y^2}=0.$$

Noting that the second and third terms cancel from the momentum equation,

$$\mu\left[\frac{\partial u}{\partial y}\right]^2+k\left[\frac{\partial^2 T}{\partial y^2}\right]=0.$$ <