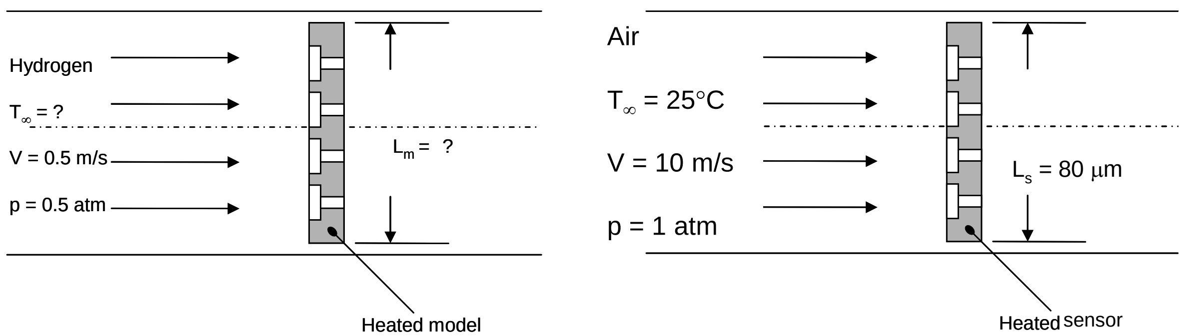
PROBLEM 6.44

KNOWN: Characteristic length of a microscale chemical detector, free stream velocity and temperature, hydrogen wind tunnel pressure and free stream velocity.

FIND: Model length scale and hydrogen temperature needed for similarity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible microscale or nanoscale effects, (4) Ideal gas behavior.

PROPERTIES: Table A.4, air (T = 25 °C): $Pr_s = 0.707$, $\nu_s = 15.71 \times 10^{-6}$ m^2/s, hydrogen (250 K) $Pr = 0.707$, $\nu = 81.4 \times 10^{-6}$ m^2/s.

ANALYSIS: For similarity we require $Re_m = Re_s$ and $Pr_m = Pr_s$. For the sensor,

$$Re_s = \frac{V_s L_s}{\nu_s} = \frac{10\text{ m/s} \times 80 \times 10^{-6}\text{ m}}{1.571 \times 10^{-5}\text{ m}^2/\text{s}} = 50.93$$

$$Pr_s = 0.707$$

For the model, $Pr_m = Pr_s = 0.707$.

From Table A.4, we note $Pr_s = 0.707$, $\nu = 81.4 \times 10^{-6}$ m^2/s at $T_\infty = 250$ K and p = 1 atm. <

The value of the Prandtl number is independent of pressure for an ideal gas. The kinematic viscosity is pressure-dependent. Hence,

$$\nu(\text{at } 0.5\text{ atm}) = \frac{\mu}{\rho(\text{at } 0.5\text{ atm})} = \frac{\mu}{\rho(\text{at } 1.0\text{ atm})} \times \frac{\rho(\text{at } 1.0\text{ atm})}{\rho(\text{at } 0.5\text{ atm})}$$

For an ideal gas,

$$\nu(\text{at } 0.5\text{ atm}) = \nu(\text{at } 1.0\text{ atm}) \times \frac{1.0\text{ atm}}{0.5\text{ atm}} = 2\nu(\text{at } 1.0\text{ atm})$$

Therefore,

$$\nu_m = 81.4 \times 10^{-6}\text{ m}^2/\text{s} \times 2 = 163 \times 10^{-6}\text{ m}^2/\text{s}$$

For similarity,

Continued…

PROBLEM 6.44 (Cont.)

$$Re_m = Re_s = 50.93 = \frac{V_m L_m}{\nu_m} = \frac{0.5 \text{ m/s} \times L_m}{163 \times 10^{-6} \text{ m}^2\text{/s}}$$

or $\quad L_m = 16.6 \times 10^{-3}$ m = 16.6 mm <

COMMENTS: (1) From Section 2.2.1, we know that the mean free path of air at room conditions is approximately 80 nm. Since L_s is three orders of magnitude greater than the mean free path, the air may be treated as a continuum. (2) Hydrogen can leak from enclosures easily. By keeping the wind tunnel pressure below atmospheric, we avoid possible leakage of flammable hydrogen into the lab. Also, if leaks occur, air must enter the wind tunnel. It is much easier to seal against air leaks than hydrogen leaks. (3) $Pr_m = 0.707$ at 100 K also. However, the operation of the hydrogen wind tunnel at such a low temperature would be much more difficult than at 250 K.