PROBLEM 6.31

KNOWN: Form of the Nusselt number correlation for forced convection and fluid properties. Properties of xenon and He-Xe mixture. Temperature and pressure. Expression for specific heat for monatomic gases.

FIND: Figures of merit for air, pure helium, pure xenon, and He-Xe mixture containing 0.75 mole fraction of helium.

PROPERTIES: *Table A-4*, Air (300 K): k = 0.0263 W/m·K, ν = 15.89 × 10^{-6} m^2/s, Pr = 0.707. *Table A-4,* Helium (300 K): k = 0.152 W/m·K, ν = 122 × 10^{-6} m^2/s, Pr = 0.680. Pure xenon (given): k = 0.006 W/m·K, μ = 24.14 × 10^{-6} $N\cdot s/m^2$. He-Xe mixture (given): k = 0.0713 W/m·K, μ = 25.95 × 10^{-6} $N\cdot s/m^2$.

ANALYSIS: With $Nu_L \sim Re_L^m \, Pr^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{VL}{\nu}\right)^m Pr^n \sim \frac{V^m}{L^{1-m}}\left(\frac{k\,Pr^n}{\nu^m}\right)$$

The figure of merit is therefore

$$F_F = \frac{k\,Pr^n}{\nu^m} \qquad (1)$$

For xenon and the He-Xe mixture, we must find the density and specific heat. Proceeding for pure xenon:

$$\rho = \frac{P\mathcal{M}}{\Re T} = \frac{1\text{ atm} \times 131.29\text{ kg/kmol}}{8.205 \times 10^{-2}\text{ m}^3 \cdot \text{atm/kmol} \cdot \text{K} \times 300\text{ K}} = 5.33\text{ kg/m}^3$$

$$c_p = \frac{5}{2}\frac{\Re}{\mathcal{M}} = \frac{5}{2}\frac{8.315 \times 10^3\text{ J/kmol}\cdot\text{K}}{131.29\text{ kg/kmol}} = 158\text{ J/kg}$$

Thus $\nu = \mu/\rho$ = 24.14 × 10^{-6} $N\cdot s/m^2$/5.33 kg/m^3 = 4.53 × 10^{-6} m^2/s and Pr = $\mu c_p/k$ = 24.14 × 10^{-6} $N\cdot s/m^2$ × 158 J/kg/0.006 W/m·K = 0.636.

For the He-Xe mixture, the molecular weight of the mixture can be found from

$$\mathcal{M}_{mix} = 0.75\text{ kmol He/kmol} \times 4.0\text{ kg/kmol He} + 0.25\text{ kmol Xe/kmol} \times 131.29\text{ kg/kmol Xe}$$
$$= 35.82\text{ kg/kmol}$$

from which we can calculate ρ = 1.46 kg/m^3, c_p = 580 J/kg·K, $\nu = \mu/\rho$ = 25.95 × 10^{-6} $N\cdot s/m^2$/1.46 kg/m^3 = 1.78 × 10^{-5} m^2/s, and Pr = $\mu c_p/k$ = 25.95 × 10^{-6} $N\cdot s/m^2$ × 580 J/kg/0.0713 W/m·K = 0.211.

Finally, for the four fluids, with m = 0.85 and n = 0.33, we can calculate the figure of merit from Equation (1):

	F_F ($W\cdot s^{0.85}/m^{2.7}\cdot K$)
Air	281
Helium	284
Xenon	180
He-Xe	465

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COMMENTS: The effectiveness of the He-Xe mixture is much higher than that of pure He, pure Xe, or air. By blending He and Xe, the high thermal conductivity of helium and the high density of xenon are both exploited in a manner that leads to a high figure of merit.