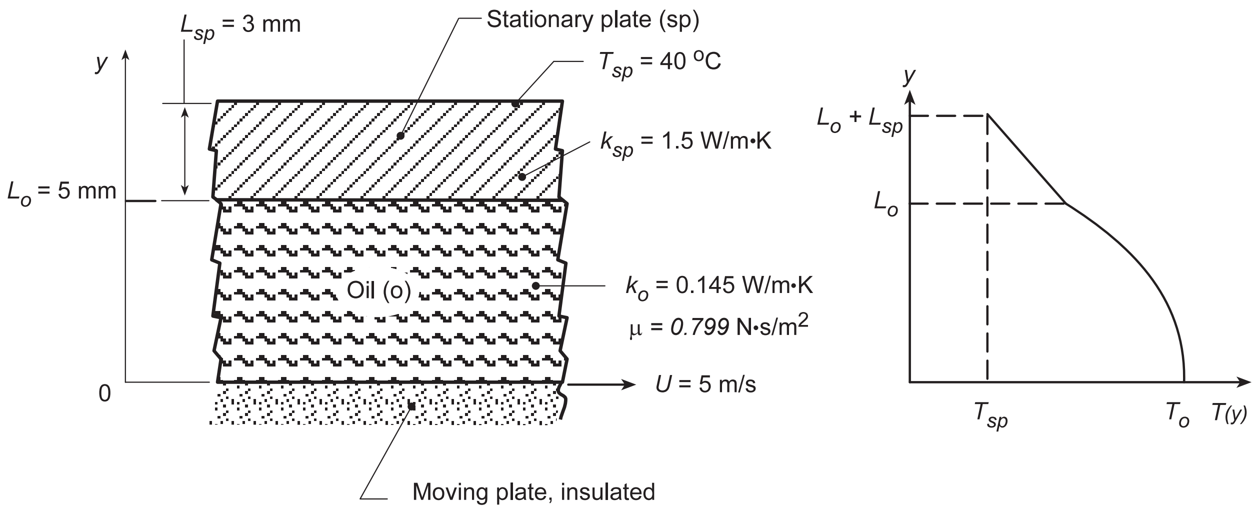
PROBLEM 6S.7

KNOWN: Couette flow with heat transfer. Lower (insulated) plate moves with speed U and upper plate is stationary with prescribed thermal conductivity and thickness. Outer surface of upper plate maintained at constant temperature, T_{sp} = 40°C.

FIND: (a) On T-y coordinates, sketch the temperature distribution in the oil and the stationary plate, and (b) An expression for the temperature at the lower surface of the oil film, T(0) = T_o, in terms of the plate speed U, the stationary plate parameters (T_{sp}, k_{sp}, L_{sp}) and the oil parameters (μ, k_o, L_o). Determine this temperature for the prescribed conditions.

SCHEMATIC:



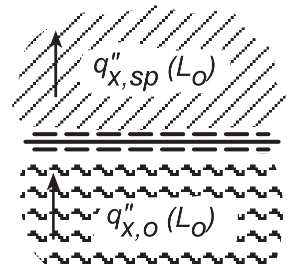
ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed Couette flow and (3) Incompressible fluid with constant properties.

ANALYSIS: (a) The temperature distribution is shown above with these key features: linear in plate, parabolic in oil film, discontinuity in slope at plate-oil interface, and zero gradient at lower plate surface.

(b) From Example 6S.1, the general solution to the conservation equations for the temperature distribution in the oil film is

$$T_o(y) = -Ay^2 + C_3 y + C_4 \qquad \text{where} \qquad A = \frac{\mu}{2k_o}\left(\frac{U}{L_o}\right)^2$$

and the boundary conditions are,



At y = 0, insulated boundary $\left.\frac{dT_o}{dy}\right)_{y=0} = 0;\quad C_3 = 0$

At y = L_o, heat fluxes in oil and plate are equal, $q''_o(L_o) = q''_{sp}(L_o)$

Continued...

PROBLEM 6S.7 (Cont.)

$$-k_o \left.\frac{dT_o}{dy}\right)_{y=L_o} = \frac{T_o(L_o) - T_{sp}}{R_{sp}} \qquad \begin{cases} \left.\dfrac{dT_o}{dy}\right)_{y=L_o} = -2AL_o \\ R_{sp} = L_{sp}/k_{sp} \qquad\qquad T_o(L_o) = -AL_o^2 + C_4 \end{cases}$$

$$C_4 = T_{sp} + AL_o^2\left[1 + 2\frac{k_o}{L_o}\frac{L_{sp}}{k_{sp}}\right]$$

Hence, the temperature distribution at the lower surface is

$$T_o(0) = -A \cdot 0 + C_4$$

$$T_o(0) = T_{sp} + \frac{\mu}{2k_o}U^2\left[1 + 2\frac{k_o}{L_o}\frac{L_{sp}}{k_{sp}}\right] \qquad <$$

Substituting numerical values, find

$$T_o(0) = 40^\circ C + \frac{0.799\,N \cdot s/m^2}{2 \times 0.145\,W/m \cdot K}(5\,m/s)^2\left[1 + 2\frac{0.145}{5} \times \frac{3}{1.5}\right] = 116.9^\circ C \qquad <$$

COMMENTS: (1) Give a physical explanation about why the maximum temperature occurs at the lower surface.

(2) Sketch the temperature distribution if the upper plate moved with a speed U while the lower plate is stationary and all other conditions remain the same.