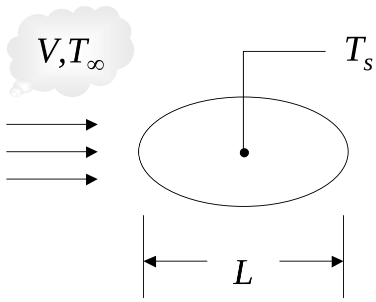
# PROBLEM 6.34

**KNOWN:** Base fluid (water) and nanofluid properties. Fixed surface and ambient temperatures, fixed characteristic velocity. Fixed geometry. Form of Nusselt number correlation.

**FIND:** (a) Prandtl numbers of the base fluid and nanofluid. (b) Ratio of Reynolds numbers of the two fluids and ratio of Nusselt numbers necessary to provide the same convection heat transfer coefficients. (c) Whether the base fluid can provide greater convection heat transfer rates than the nanofluid.

**SCHEMATIC**:




**ASSUMPTIONS**: (1) Steady-state conditions, (2) Constant properties, (3) Negligible viscous dissipation.

**PROPERTIES:** Table A.6 ($T$ = 300 K): Water; $k_{\text{bf}}$ = 0.613 W/m·K, $\rho_{\text{bf}}$ = 997 kg/m$^3$, $c_{p,\text{bf}}$ = 4.179 kJ/kg·K, $\mu_{\text{bf}}$ = 855 × 10$^{-6}$ N·s/m$^2$. Example 2.2: Nanofluid, $k_{\text{nf}}$ = 0.705 W/m·K, $\rho_{\text{nf}}$ = 1146 kg/m$^3$, $c_{p,\text{nf}}$ = 3.587 kJ/kg·K, $\mu_{\text{nf}}$ = 962 × 10$^{-6}$ N·s/m$^2$.

**ANALYSIS:** (a) The Prandtl numbers for the water and nanofluid are:

$$Pr_{\text{bf}} = \frac{\mu_{\text{bf}} c_{p,\text{bf}}}{k_{\text{bf}}} = \frac{855 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 \times 4179\ \text{J/kg}\cdot\text{K}}{0.613\ \text{W/m}\cdot\text{K}} = 5.83 \qquad <$$

$$Pr_{\text{nf}} = \frac{\mu_{\text{nf}} c_{p,\text{nf}}}{k_{\text{nf}}} = \frac{962 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 \times 3587\ \text{J/kg}\cdot\text{K}}{0.705\ \text{W/m}\cdot\text{K}} = 4.89 \qquad <$$

(b) For a given velocity and characteristic length,

$$\frac{Re_{\text{nf}}}{Re_{\text{bf}}} = \frac{\rho_{\text{nf}}\mu_{\text{bf}}}{\rho_{\text{bf}}\mu_{\text{nf}}} = \frac{1146\ \text{kg/m}^3 \times 855 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2}{997\ \text{kg/m}^3 \times 962 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 1.022 \qquad <$$

For the same convection heat transfer coefficients,

$$\frac{\bar{h}_{\text{nf}}}{\bar{h}_{\text{bf}}} = 1 = \frac{\overline{Nu}_{\text{nf}} k_{\text{nf}} L_{\text{bf}}}{\overline{Nu}_{\text{bf}} k_{\text{bf}} L_{\text{nf}}} \quad \text{or} \quad \frac{\overline{Nu}_{\text{nf}}}{\overline{Nu}_{\text{bf}}} = \frac{k_{\text{bf}}}{k_{\text{nf}}} = \frac{0.613\ \text{W/m}\cdot\text{K}}{0.705\ \text{W/m}\cdot\text{K}} = 0.870 \qquad <$$

(c) For the base fluid to have a greater convection heat transfer rate would require $\bar{h}_{bf} > \bar{h}_{nf}$, or $\overline{Nu}_{bf} > \frac{1}{0.870}\overline{Nu}_{nf}$. Using the relationship provided in the problem statement,

$C\,\text{Re}_{bf}^{m}\,\text{Pr}_{bf}^{1/3} > \frac{C}{0.870}\text{Re}_{nf}^{n}\,\text{Pr}_{nf}^{1/3}$ which may be rearranged to yield

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$$\left(\frac{\mathrm{Re}_{nf}}{\mathrm{Re}_{bf}}\right)^{m} < 0.870\left(\frac{\mathrm{Pr}_{bf}}{\mathrm{Pr}_{nf}}\right)^{1/3} = 0.870\left(\frac{5.83}{4.92}\right)^{1/3} = 0.921$$

Since ($Re_{\mathrm{nf}}/Re_{\mathrm{bf}}$) > 1 and $m > 1$, the equation cannot be satisfied. Therefore we conclude that the base fluid would not provide greater convection heat transfer rates than the nanofluid as long as $m$ is positive. $<$

**COMMENTS:** (1) The conclusion regarding the relative efficacy of the nanofluid to the base fluid is valid only for situations involving unconstrained boundary layers. (2) For internal flow situations, such as will be discussed in Chapter 8, one cannot draw a general conclusion that the nanofluid would outperform the base fluid. In fact, in common instances, the base fluid would outperform the nanofluid.