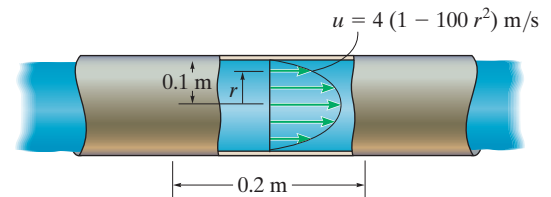
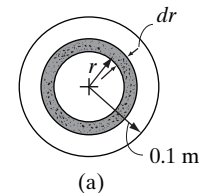


6-1. Determine the linear momentum of a mass of fluid in a 0.2-m length of pipe if the velocity profile for the fluid is a paraboloid as shown. Compare this result with the linear momentum of the fluid using the average velocity of flow. Take $\rho = 800 \text{ kg/m}^3$.



SOLUTION

The shell differential element that has a thickness dr and length 0.2 m shown shaded in Fig. *a* has a volume of $dV = (2\pi r dr)(0.2 \text{ m}) = 0.4\pi r dr$. Thus, the mass of this element is $dm = \rho dV = (800 \text{ kg/m}^3)(0.4\pi r dr) = 320\pi r dr$. The linear momentum of the fluid is



$$\begin{aligned} L &= \int_m v dm \\ &= \int_0^{0.1 \text{ m}} 4(1 - 100r^2)(320\pi r dr) \\ &= 1280\pi \int_0^{0.1 \text{ m}} (r - 100r^3) dr \\ &= 1280\pi \left(\frac{r^2}{2} - 25r^4 \right) \Big|_0^{0.1 \text{ m}} \\ &= 10.05 \text{ kg} \cdot \text{m/s} = 10.1 \text{ kg} \cdot \text{m/s} \end{aligned}$$

Ans.

The ring differential element shown shaded in Fig. *a* has an area of $dA = 2\pi r dr$. Therefore

$$\begin{aligned} V_{\text{avg}} &= \frac{\int v dA}{A} \\ &= \frac{\int_0^{0.1 \text{ m}} 4(1 - 100r^2)(2\pi r dr)}{\pi(0.1 \text{ m})^2} \\ &= \frac{8\pi \int_0^{0.1 \text{ m}} (r - 100r^3) dr}{\pi(0.1 \text{ m})^2} \\ &= \frac{8\pi \left(\frac{r^2}{2} - 25r^4 \right) \Big|_0^{0.1 \text{ m}}}{\pi(0.1 \text{ m})^2} \\ &= 2 \text{ m/s} \end{aligned}$$

The mass of the fluid is $m\rho V = (800 \text{ kg/m}^3)[\pi(0.1 \text{ m})^2](0.2 \text{ m}) = 1.6\pi \text{ kg}$. Thus,

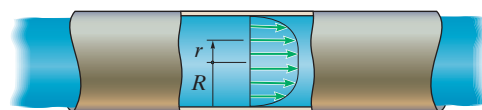
$$\begin{aligned} L &= mV_{\text{avg}} = \rho V V_{\text{avg}} = (1.6\pi \text{ kg})(2 \text{ m/s}) \\ &= 10.05 \text{ kg} \cdot \text{m/s} = 10.1 \text{ kg} \cdot \text{m/s} \end{aligned}$$

Ans.

Ans:

$L = 10.1 \text{ kg} \cdot \text{m/s}$ by either method.

6-2. Flow through the circular pipe is turbulent, and the velocity profile can be modeled using Prandtl's one-seventh power law, $v = V_{\max} (1 - r/R)^{1/7}$. If ρ is the density, show that the momentum of the fluid per unit time passing through the pipe is $(49/72)\pi R^2 \rho V_{\max}^2$. Then show that $V_{\max} = (60/49)V$, where V is the average velocity of the flow. Also, show that the momentum per unit time is $(50/49)\pi R^2 \rho V^2$.



SOLUTION

The amount of mass per unit time passing through a differential ring element of area dA (shown shaded in Fig. *a*) on the cross-section is

$$d\dot{m} = \rho V dA$$

Then the momentum per unit time passing through this element is

$$d\dot{L} = (d\dot{m})V = (\rho V dA)V = \rho V^2 dA$$

Thus, for the entire cross-section,

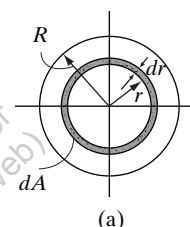
$$\dot{L} = \int_A d\dot{L} = \int_A \rho V^2 dA$$

Here $dA = 2\pi r dr$. Then

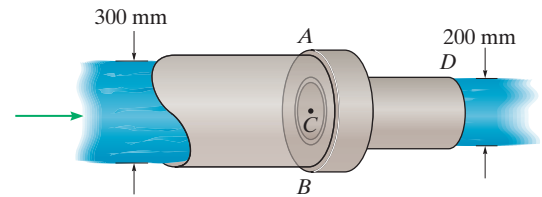
$$\begin{aligned} \dot{L} &= \int_0^R \rho \left[V_{\max} \left(1 - \frac{r}{R} \right)^{1/7} \right]^2 (2\pi r dr) \\ &= 2\pi \rho V_{\max}^2 \int_0^R r \left(1 - \frac{r}{R} \right)^{2/7} dr \end{aligned}$$

Let $u = 1 - \frac{r}{R}$, then $r = R(1 - u)$ and $dr = -Rdu$. Also, the integration limits are $r = 0, u = 1$ and $r = R, u = 0$. Thus,

$$\begin{aligned} \dot{L} &= 2\pi \rho V_{\max}^2 \int_1^0 R(1 - u) \left(u^{2/7} \right) (-Rdu) \\ &= 2\pi R^2 \rho V_{\max}^2 \int_1^0 \left(u^{9/7} - u^{2/7} \right) du \\ &= 2\pi R^2 \rho V_{\max}^2 \left(\frac{7}{16} u^{16/7} - \frac{7}{9} u^{9/7} \right) \Big|_1^0 = \frac{49}{72} \pi R^2 \rho V_{\max}^2 \quad \text{(Q.E.D.)} \end{aligned}$$



6-3. Oil flows at $0.05 \text{ m}^3/\text{s}$ through the transition. If the pressure at the transition C is 8 kPa , determine the resultant horizontal shear force acting along the seam AB that holds the cap to the larger pipe. Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_C A_C; \quad 0.05 \text{ m}^3/\text{s} = V_C [\pi (0.15 \text{ m})^2]$$

$$V_C = 0.7074 \text{ m/s}$$

$$Q = V_D A_D; \quad 0.05 \text{ m}^3/\text{s} = V_D [\pi (0.1 \text{ m})^2]$$

$$V_D = 1.592 \text{ m/s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. a.

Since D is open to the atmosphere, $p_D = 0$.

Linear Momentum. Since the flow is steady and incompressible,

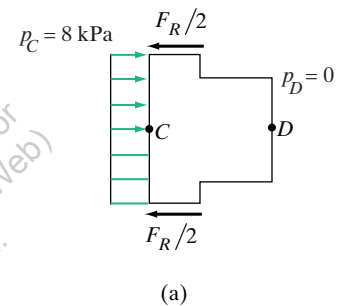
$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\rightarrow \Sigma F = \rho Q (V_D - V_C);$$

$$[8(10^3) \text{ N/m}^2] [\pi (0.15 \text{ m})^2] - F_R = (900 \text{ kg/m}^3) (0.05 \text{ m}^3/\text{s}) (1.592 \text{ m/s} - 0.7074 \text{ m/s})$$

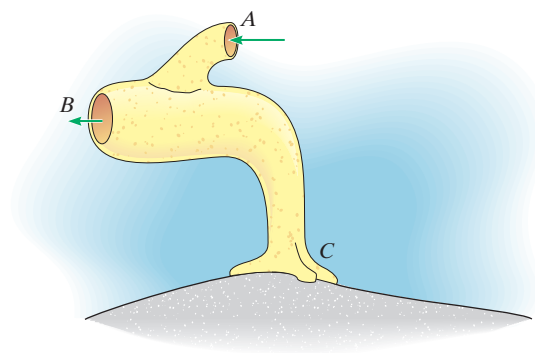
$$F = 526 \text{ N}$$

Ans.



Ans:
526 N

***6-4.** A small marine ascidian called a styela fixes itself on the sea floor and then allows moving water to pass through it in order to feed. If the opening at A has a diameter of 2 mm, and at the exit B the diameter is 1.5 mm, determine the horizontal force needed to keep this organism attached to the rock at C when the water is moving at 0.2 m/s into the opening at A . Take $\rho = 1050 \text{ kg/m}^3$.



SOLUTION

The flow is steady and the sea water can be considered as an ideal fluid (incompressible and inviscid) such that average velocities can be used and $\rho_{\text{sw}} = 1050 \text{ kg/m}^3$. The control volume considered contains the sea water in “styela”, Fig. a . Since the depth of points A and B are almost the same, the pressure forces acting on opened control surfaces A and B can be considered the same and are assumed to cancel each other. Continuity requires

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\text{cv}} \rho_{\text{sw}} dV + \int_{\text{cs}} \rho_{\text{sw}} \mathbf{V} \cdot d\mathbf{A} &= 0 \\ 0 - V_A A_A + V_B A_B &= 0 \\ -(0.2 \text{ m/s}) [\pi(0.001 \text{ m})^2] + V_B [\pi(0.00075 \text{ m})^2] &= 0 \\ V_B &= 0.3556 \text{ m/s} \end{aligned}$$

Applying the linear momentum equation,

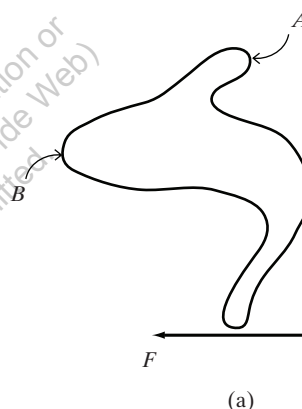
$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho_{\text{sw}} dV + \int_{\text{cs}} \mathbf{V} \rho_{\text{sw}} \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along x axis by referring to the FBD of the control volume, Fig. a

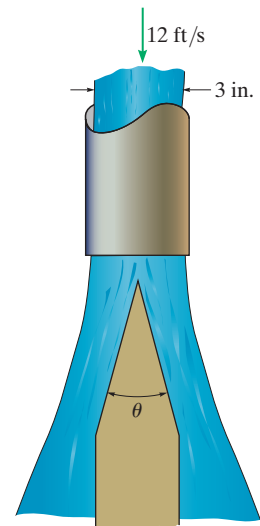
$$\begin{aligned} (\pm) \Sigma F_x &= 0 + (-V_A) \rho_{\text{sw}} (-V_A A_A) + (-V_B) \rho_{\text{sw}} (V_B A_B) \\ -F &= (-0.2 \text{ m/s})(1050 \text{ kg/m}^3) \{ -(0.2 \text{ m/s}) [\pi(0.001 \text{ m})^2] \} \\ &\quad + (-0.3556 \text{ m/s})(1050 \text{ kg/m}^3) \{ (0.3556 \text{ m/s}) [\pi(0.00075 \text{ m})^2] \} \\ F &= 0.103(10^{-3}) \text{ N} = 0.103 \text{ mN} \end{aligned}$$

Ans.

Note: The direction of F implies that if the styela were detached from the rock, it would drift upstream. In reality, it would drift downstream due to forces on its closed surface, which were not considered.



6-5. Water exits the 3-in.-diameter pipe at a velocity of 12 ft/s and is split by the wedge diffuser. Determine the force the flow exerts on the diffuser. Take $\theta = 30^\circ$.



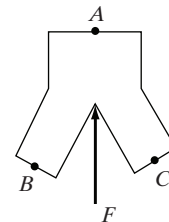
SOLUTION

We consider steady flow of an ideal fluid.

$$\begin{aligned} Q_A &= V_A A_A \\ &= (12 \text{ ft/s}) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] \\ &= 0.5890 \text{ ft}^3/\text{s} \end{aligned}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since this is free flow, $p_A = p_B = p_C = 0$.

Linear Momentum. Since the change in elevation is negligible and the pressure at *A*, *B*, and *C* is zero gauge, $V_A = V_B = V_C = 12 \text{ ft/s}$ (Bernoulli equation). The flow is steady and incompressible.



(a)

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\begin{aligned} +\uparrow \Sigma F_y &= \rho Q_B (V_B)_y + \rho Q_C (V_C)_y - \rho Q_A (V_A)_y \\ F &= \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) [Q_B (-12 \cos 15^\circ \text{ ft/s}) + Q_C (-12 \cos 15^\circ \text{ ft/s}) - (0.5890 \text{ ft}^3/\text{s}) (-12 \text{ ft/s})] \\ &= 1.9379 [7.0686 - 12 \cos 15^\circ (Q_B + Q_C)] \end{aligned}$$

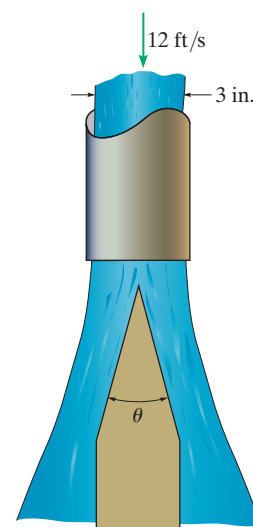
However, $Q_B + Q_C = Q_A = 0.5891 \text{ ft}^3/\text{s}$. Then,

$$\begin{aligned} F &= 1.9379 [7.0686 - 12 \cos 15^\circ (0.5890)] \\ &= 0.4668 \text{ lb} = 0.467 \text{ lb} \end{aligned}$$

Ans.

Ans:
0.467 lb

6–6. Water exits the 3-in.-diameter pipe at a velocity of 12 ft/s, and is split by the wedge diffuser. Determine the force the flow exerts on the diffuser as a function of the diffuser angle θ . Plot this force (vertical axis) versus θ for $0 \leq \theta \leq 30^\circ$. Give values for increments of $\Delta\theta = 5^\circ$.



SOLUTION

The discharge is

$$Q_A = V_A A_A = (12 \text{ ft/s}) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] = 0.1875\pi \text{ ft}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Since this is a free flow, $p_A = p_B = p_C = 0$. Also, since the change in elevation is negligible, $V_A = V_B = V_C = 12 \text{ ft/s}$. The flow is steady and incompressible. Thus

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

The vertical component of this equation gives

$$\begin{aligned} + \uparrow \Sigma F_y &= 0 + [-(V_A)_y] \rho (-V_A A_A) + [-(V_B)_y] \rho (V_B A_B) + [-(V_C)_y] \rho (V_C A_C) \\ F &= \left(\frac{624 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) [(-12 \text{ ft/s})(-0.1875\pi \text{ ft}^3/\text{s}) + (-12 \cos \theta/2 \text{ ft/s})Q_B + (-12 \cos \theta/2 \text{ ft/s})Q_C] \\ F &= 23.25[0.1875\pi - (Q_B + Q_C) \cos \theta/2] \end{aligned}$$

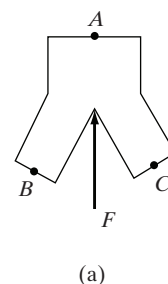
However, continuity requires that $Q_A = Q_B + Q_C$. Then

$$F = 23.25(0.1875\pi - 0.1875\pi \cos \theta/2)$$

$$F = [13.7(1 - \cos \theta/2)] \text{ lb}$$

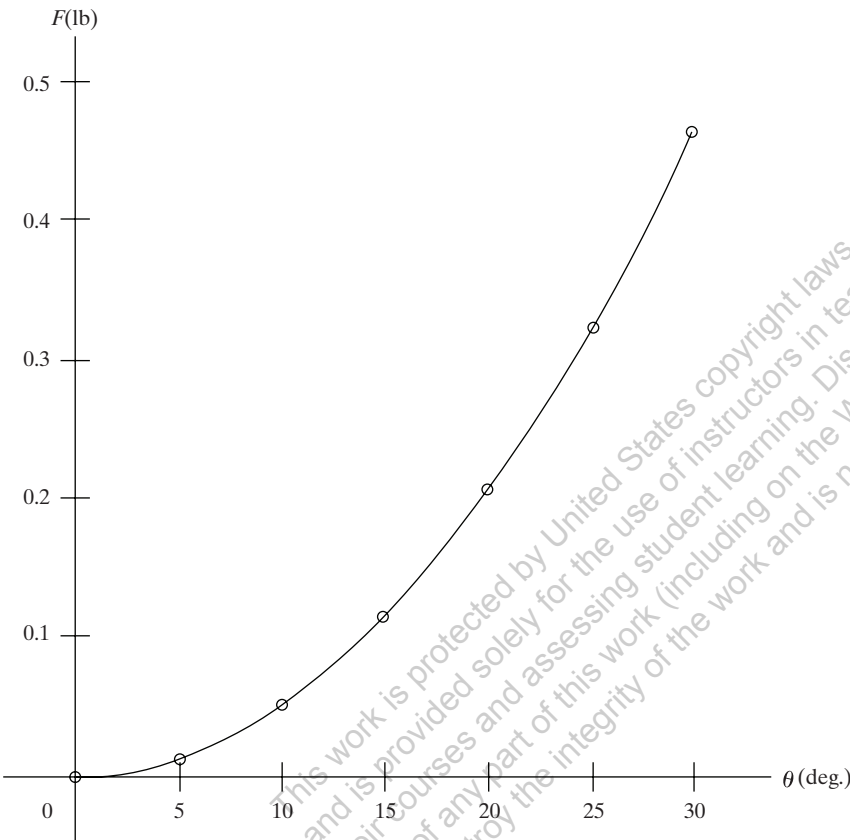
Ans.

The plot of F vs θ is shown in Fig. *b*.



6-6. Continued

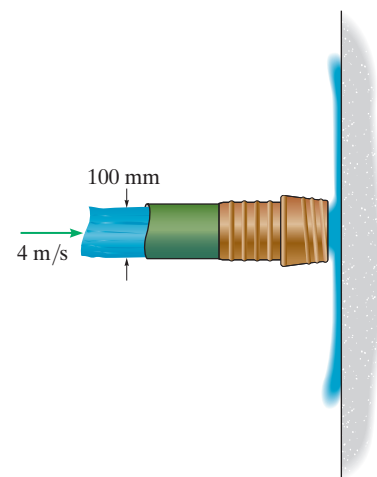
$\theta(\text{deg.})$	0	5	10	15	20	25	30
$F(\text{lb})$	0	0.0130	0.0521	0.117	0.208	0.325	0.467



(b)

Ans:
 $F = [13.7(1 - \cos \theta/2)] \text{ lb}$

6–7. Water flows through the hose with a velocity of 4 m/s. Determine the force that the water exerts on the wall. Assume the water does not splash back off the wall.



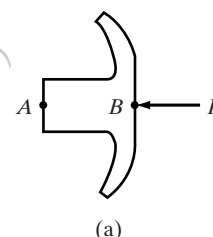
SOLUTION

We consider steady flow of an ideal fluid.

$$Q = VA = (4 \text{ m/s})[\pi(0.05 \text{ m})^2] = 0.03142 \text{ m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since the flow is free, $p_A = p_B = 0$.

Linear Momentum. The horizontal component of flow velocity is zero when the water jet hits the wall, $(V_{\text{out}})_x = 0$. Since the flow is steady and incompressible,



$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho \mathbf{V} dV + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\pm \Sigma F_x = 0 + (V_A)(\rho)(-Q_A)$$

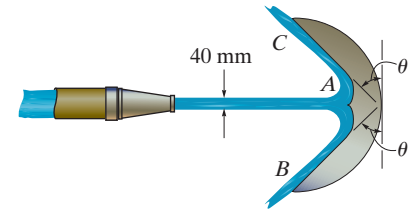
$$-F = (4 \text{ m/s})(1000 \text{ kg/m}^3)(-0.03142 \text{ m}^3/\text{s})$$

$$F = 126 \text{ N}$$

Ans.

Ans:
126 N

***6–8.** The nozzle has a diameter of 40 mm. If it discharges water with a velocity of 20 m/s against the fixed blade, determine the horizontal force exerted by the water on the blade. The blade divides the water evenly at an angle of $\theta = 45^\circ$.



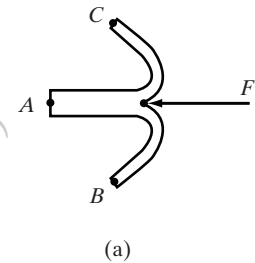
SOLUTION

We consider steady flow of an ideal fluid.

$$Q_A = V_A A_A = (20 \text{ m/s}) [\pi (0.02 \text{ m})^2] = 0.02513 \text{ m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since this is a free flow, $p_A = p_B = p_C$.

Linear Momentum. Since the change in elevation is negligible and the pressure at *A*, *B*, and *C* is zero gauge, $V_A = V_B = V_C = 20 \text{ m/s}$ (Bernoulli equation). Since the flow is steady and incompressible,



$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

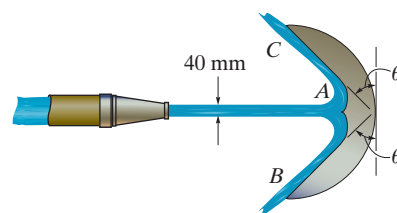
$$\begin{aligned} \Sigma F_x &= (-V_B)_x \rho Q_B - (V_C)_x \rho Q_C + (V_A)_x \rho (-Q_A) \\ -F &= (1000 \text{ kg/m}^3) [Q_B (-20 \text{ m/s})(\cos 45^\circ) + Q_C (-20 \text{ m/s})(\cos 45^\circ) - (20 \text{ m/s})(0.02513 \text{ m}^3/\text{s})] \\ F &= 1000 [(Q_B + Q_C)(20 \cos 45^\circ) + 0.5027] \end{aligned}$$

However, $Q_B + Q_C = Q_A = 0.02513 \text{ m}^3/\text{s}$. Then

$$\begin{aligned} F &= 1000 [0.02513 (20 \cos 45^\circ) + 0.5027] \\ &= 858.09 \text{ N} \approx 858 \text{ N} \end{aligned}$$

Ans.

6–9. The nozzle has a diameter of 40 mm. If it discharges water with a velocity of 20 m/s against the fixed blade, determine the horizontal force exerted by the water on the blade as a function of the blade angle θ . Plot this force (vertical axis) versus θ for $0 \leq \theta \leq 75^\circ$. Give values for increments of $\Delta\theta = 15^\circ$. The blade divides the water evenly.



SOLUTION

The discharge is

$$Q = V_A A_A = (20 \text{ m/s}) [\pi (0.02 \text{ m})^2] = 0.008\pi \text{ m}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Since this is a free flow, $p_A = p_B = p_C = 0$. Also, since the change in elevation is negligible, $V_A = V_B = V_C = 20 \text{ m/s}$ (Bernoulli's equation). The flow is steady and incompressible. Thus

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

The horizontal component of this equation gives

$$\pm \Sigma F_x = 0 + [-(V_A)_x] \rho (-V_A A_A) + (V_B)_x \rho (V_B A_B) + (V_C)_x \rho (V_C A_C)$$

$$F = (1000 \text{ kg/m}^3) [(20 \text{ m/s})(0.008\pi \text{ m}^3/\text{s}) + (20 \sin \theta \text{ m/s})Q_B + (20 \sin \theta \text{ m/s})Q_C]$$

$$F = 20(10^3) [0.008\pi + (Q_B + Q_C) \sin \theta]$$

However continuity requires that $Q_A = Q_B + Q_C$. Then,

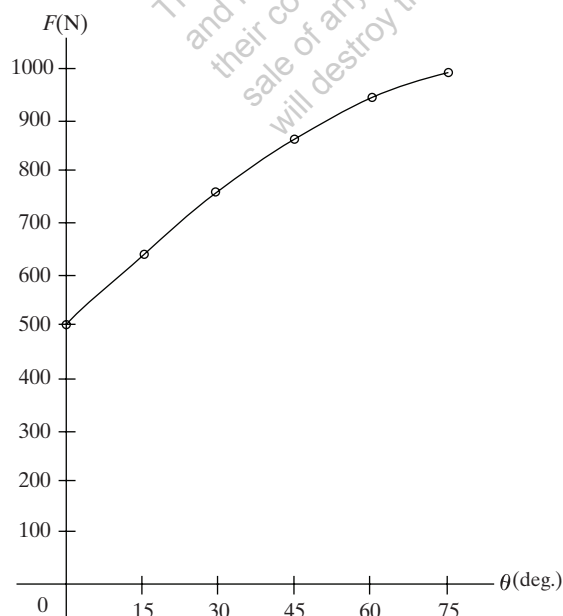
$$F = 20(10^3) [0.008\pi + (0.008\pi) \sin \theta]$$

$$F = [160\pi(1 + \sin \theta)] \text{ N where } \theta \text{ is in deg.}$$

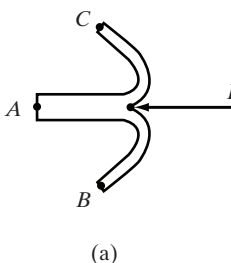
Ans.

The plot of F vs θ is shown in Fig. *b*.

$\theta(\text{deg.})$	0	15	30	45	60	75
$F(\text{N})$	503	633	754	858	938	988



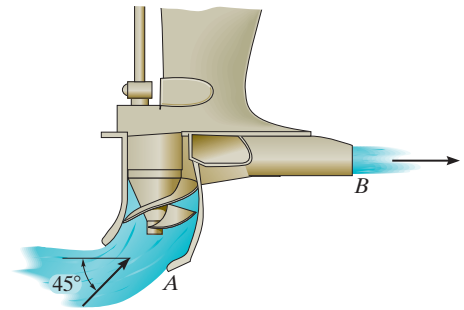
(b)



Ans:

$$F = [160\pi(1 + \sin \theta)] \text{ N}$$

6–10. A speedboat is powered by the jet drive shown. Seawater is drawn into the pump housing at the rate of $20 \text{ ft}^3/\text{s}$ through a 6-in.-diameter intake A . An impeller accelerates the water and forces it out horizontally through a 4-in.-diameter nozzle B . Determine the horizontal and vertical components of thrust exerted on the speedboat. The specific weight of seawater is $\gamma_{\text{sw}} = 64.3 \text{ lb/ft}^3$.



SOLUTION

Consider the control volume to be the jet drive and the water it contains, Fig. a .
From the discharge

$$Q = V_A A_A; \quad 20 \text{ ft}^3/\text{s} = V_A \left[\pi \left(\frac{3}{12} \text{ ft} \right)^2 \right] \quad V_A = 101.86 \text{ ft/s}$$

$$Q = V_B A_B; \quad 20 \text{ ft}^3/\text{s} = V_B \left[\pi \left(\frac{2}{12} \text{ ft} \right)^2 \right] \quad V_B = 229.18 \text{ ft/s}$$

Here the flow is steady. Applying the Linear Momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho dV + \int_{\text{cs}} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

Writing the horizontal and vertical scalar components of this equation by referring to the FBD of the control volume, Fig. a ,

$$\rightarrow \Sigma F_x = 0 + (V_A \cos 45^\circ) \rho (-V_A A_A) + V_B \rho (V_B A_B)$$

$$T_h = \left[(101.86 \text{ ft/s}) \cos 45^\circ \right] \left(\frac{64.3 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-20 \text{ ft}^3/\text{s}) + (229.18 \text{ ft/s}) \left(\frac{64.3 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (20 \text{ ft}^3/\text{s})$$

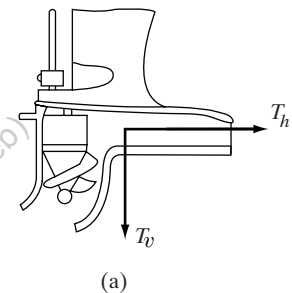
$$= 6276.55 \text{ lb} = 6.28 \text{ kip} \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + (V_A \sin 45^\circ) \rho (-V_A A_A)$$

$$-T_v = \left[(101.86 \text{ ft/s}) \sin 45^\circ \right] \left(\frac{64.3 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-20 \text{ ft}^3/\text{s})$$

$$T_v = 2876.54 \text{ lb} = 2.88 \text{ kip} \quad \text{Ans.}$$

The thrust components on the speedboat are equal and opposite to those exerted on the water.

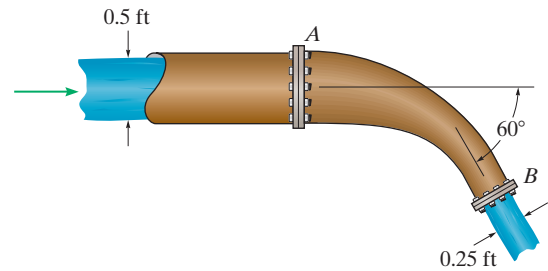


Ans:

$$T_h = 6.28 \text{ kip}$$

$$T_v = 2.88 \text{ kip}$$

6-11. Water flows out of the reducing elbow at $0.4 \text{ ft}^3/\text{s}$. Determine the horizontal and vertical components of force that are necessary to hold the elbow in place at A . Neglect the size and weight of the elbow and the water within it. The water is discharged to the atmosphere at B .



SOLUTION

$$Q = V_A A_A; \quad 0.4 \text{ ft}^3/\text{s} = V_A [\pi (0.25 \text{ ft})^2]$$

$$V_A = 2.0372 \text{ ft/s}$$

Continuity equation

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-0.4 \text{ ft}^3/\text{s} + V_B (\pi) (0.125 \text{ ft})^2 = 0$$

$$V_B = 8.149 \text{ ft/s}$$

Bernoulli equation. Neglecting elevation change

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)} + \frac{(2.037 \text{ ft/s})^2}{2} + 0 = 0 + \frac{(8.149 \text{ ft/s})^2}{2} + 0$$

$$p_A = 60.3234 \text{ lb/ft}^2$$

The free-body diagram is shown in Fig. a.

Linear Momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

$$\rightarrow \Sigma F_x = 0 + \rho Q (V_{B_x} - V_{A_x})$$

$$-F_x + (60.3234 \text{ lb/ft}^2) [(\pi) (0.25 \text{ ft})^2] = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right) (0.4 \text{ ft}^3/\text{s}) [8.149 \text{ ft/s} (\cos 60^\circ) - 2.0372 \text{ ft/s}]$$

$$F_x = 10.3 \text{ lb}$$

Ans.

$$+\uparrow \Sigma F_y = \rho Q [-V_{B_y} + 0]$$

$$-F_y = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right) (0.4 \text{ ft}^3/\text{s}) (-8.149 \text{ ft/s}) (\sin 60^\circ)$$

$$F_y = 5.47 \text{ lb}$$

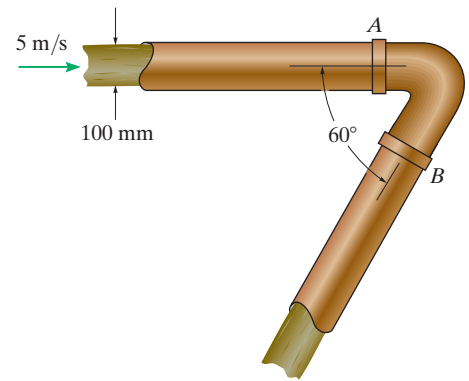
Ans.

Ans:

$$F_x = 10.3 \text{ lb}$$

$$F_y = 5.47 \text{ lb}$$

***6–12.** Oil flows through the 100-mm-diameter pipe with a velocity of 5 m/s. If the pressure in the pipe at A and B is 80 kPa, determine the x and y components of force the flow exerts on the elbow. The flow occurs in the horizontal plane. Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = VA = (5 \text{ m/s})[\pi(0.05 \text{ m})^2] \\ = 0.03927 \text{ m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. a . Here, $p_A = p_B = 80 \text{ kPa}$.

Linear Momentum. Since the flow is steady incompressible

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$+\rightarrow \Sigma F_x = 0 + (V_A)_x \rho(-Q) + (V_B)_x \rho Q$$

$$-F_x + [80(10^3) \text{ N/m}^2][\pi(0.05 \text{ m})^2] + [80(10^3) \text{ N/m}^2][\pi(0.05 \text{ m})^2] \cos 60^\circ \\ = (900 \text{ kg/m}^3)(0.03927 \text{ m}^3/\text{s})(-5 \text{ m/s} \cos 60^\circ - 5 \text{ m/s})$$

$$F_x = 1207.55 = 1.21 \text{ kN}$$

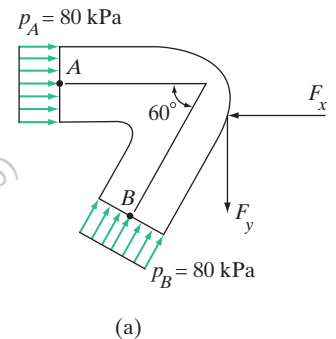
Ans.

$$+\uparrow \Sigma F_y = 0 + 0 + (V_B)_y \rho(-Q)$$

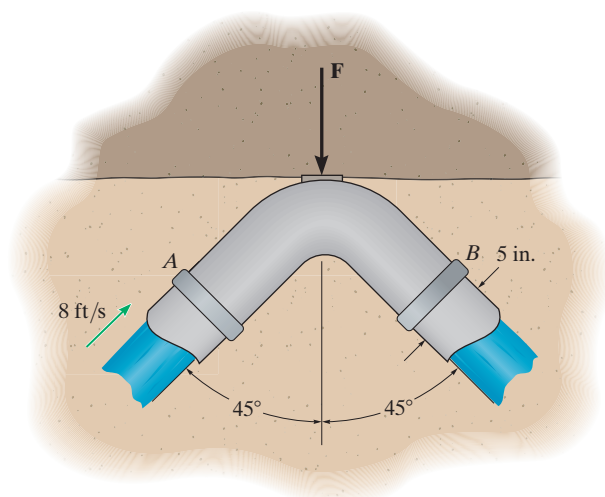
$$-F_y + [80(10^3) \text{ N/m}^2][\pi(0.05 \text{ m})^2] \sin 60^\circ = (5 \text{ m/s} \sin 60^\circ)(900 \text{ kg/m}^3)(-0.03927 \text{ m}^3/\text{s})$$

$$F_y = 697 \text{ N}$$

Ans.



6–13. The speed of water passing through the elbow on a buried pipe is $V = 8 \text{ ft/s}$. Assuming that the pipe connections at A and B do not offer any force resistance on the elbow, determine the resultant horizontal force \mathbf{F} that the soil must exert on the elbow in order to hold it in equilibrium. The pressure within the pipe at A and B is 10 psi.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = VA = (8 \text{ ft/s}) \left[\pi \left(\frac{2.5}{12} \text{ ft} \right)^2 \right] \\ = 1.091 \text{ ft}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*.

Linear Momentum. Since the flow is steady and incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

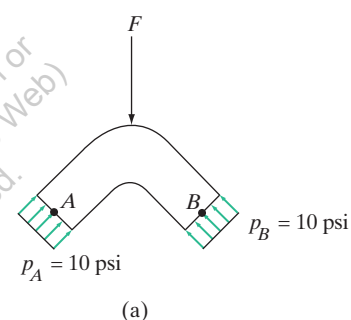
or

$$+\uparrow \Sigma F_y = 0 + (V_A)_y(\rho)(-Q) + (-V_B)_y \rho Q$$

$$2 \left[(10 \text{ lb/in}^2) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 \cos 45^\circ \left[\pi \left(\frac{2.5}{12} \text{ ft} \right)^2 \right] \right] - F = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (1.091 \text{ ft}^3/\text{s}) [-8 \text{ ft/s} \cos 45^\circ - 8 \text{ ft/s} \cos 45^\circ]$$

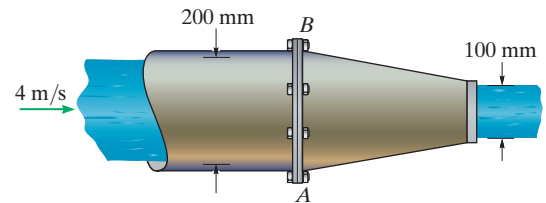
$$F = 301.60 \text{ lb} = 302 \text{ lb}$$

Ans.



Ans:
302 lb

6–14. Water flows through the 200-mm-diameter pipe at 4 m/s. If it exits into the atmosphere through the nozzle, determine the resultant force the bolts must develop at the connection *AB* to hold the nozzle onto the pipe.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$ average velocities will be used. The control volume contains the water in the nozzle as shown in Fig. *a*. Continuity requires

$$\begin{aligned}\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} &= 0 \\ 0 - V_{in} A_{in} + V_{out} A_{out} &= 0 \\ -(4 \text{ m/s}) [\pi (0.1 \text{ m})^2] + V_{out} [\pi (0.05 \text{ m})^2] &= 0 \\ V_{out} &= 16 \text{ m/s}\end{aligned}$$

Applying the Bernoulli's equation between two points on the control streamline with $p_{out} = p_{atm} = 0$,

$$\begin{aligned}\frac{p_{in}}{\gamma_w} + \frac{V_{in}^2}{2g} + z_{in} &= \frac{p_{out}}{\gamma_w} + \frac{V_{out}^2}{2g} + z_{out} \\ \frac{p_{in}}{9810 \text{ N/m}^3} + \frac{(4 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 &= 0 + \frac{(16 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 \\ p_{in} &= 120(10^3) \text{ N/m}^2\end{aligned}$$

Thus, the pressure force acting on the inlet control surface on the *FBD* of the control volume is

$$F_{in} = p_{in} A_{in} = [120(10^3) \text{ N/m}^2] [\pi (0.1 \text{ m})^2] = 3769.91 \text{ N}$$

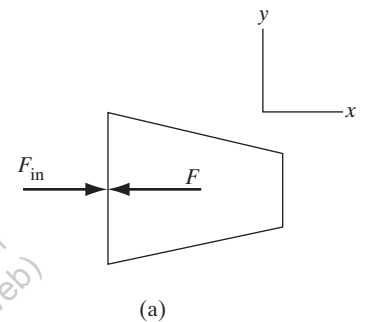
Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

Write the scalar component of this equation along *x* axis, referring to Fig. *a*

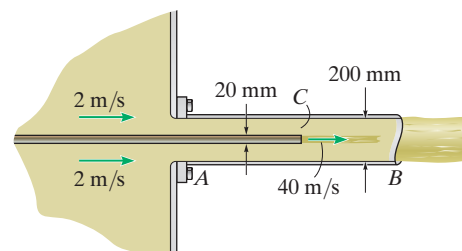
$$\begin{aligned}(\rightarrow) \Sigma F_x &= 0 + V_{out} \rho_w (V_{out} A_{out}) + V_{in} \rho_w (-V_{in} A_{in}) \\ 3769.91 \text{ N} - F &= (16 \text{ m/s})(1000 \text{ kg/m}^3)(16 \text{ m/s}) [\pi (0.05 \text{ m})^2] + (4 \text{ m/s})(1000 \text{ kg/m}^3)(-4 \text{ m/s}) [\pi (0.1 \text{ m})^2] \\ F &= 2261.95 \text{ N} = 2.26 \text{ kN}\end{aligned}$$

Ans.



Ans:
2.26 kN

6-15. The apparatus or “jet pump” used in an industrial plant is constructed by placing the tube within the pipe. Determine the increase in pressure ($P_B - P_A$) that occurs between the back A and front B of the pipe if the velocity of the flow within the 200-mm-diameter pipe is 2 m/s, and the velocity of the flow through the 20-mm-diameter tube is 40 m/s. The fluid is ethyl alcohol having a density of $\rho_{ea} = 790 \text{ kg/m}^3$. Assume the pressure at each cross section of the pipe is uniform.



SOLUTION

The flow is steady and the ethyl alcohol can be considered an ideal fluid (incompressible and inviscid) such that $\rho_{ea} = 790 \text{ kg/m}^3$. Average velocities will be used. The control volume considered is shown in Fig. *a*. Continuity requires

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - (V_A)_t(A_A)_t - (V_A)_p(A_A)_p + (V_B)_p(A_B)_p = 0$$

$$-(40 \text{ m/s})[\pi(0.01 \text{ m})^2] - (2 \text{ m/s})\left\{\pi[(0.1 \text{ m})^2 - (0.01 \text{ m})^2]\right\} + (V_B)_p[\pi(0.1 \text{ m})^2] = 0$$

$$(V_B)_p = 2.38 \text{ m/s}$$

Within the tube, $z_C = z_A$ and $V_C = V_A$, so by Bernoulli's equation, $p_C = p_A$. Furthermore, because p_C at the tube exit equals p_C in the surrounding pipe flow, which again by Bernoulli's equation equals p_A in the pipe, it follows that p_A is the same inside and outside the tube.

The pressure forces on the inlet and outlet control surfaces are

$$F_A = p_A A_A = p_A [\pi(0.1 \text{ m})^2] = 0.01 \pi p_A$$

$$F_B = p_B A_B = p_B [\pi(0.1 \text{ m})^2] = 0.01 \pi p_B$$

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

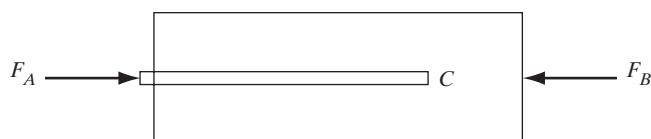
Writing the scalar component of this equation along the x axis by referring to Fig. *a*,

$$\pm \Sigma F_x = 0 + (V_B)_p \rho_{ea} (V_B)_p (A_B)_p + (V_A)_t \rho_{ea} [-(V_A)_t (A_A)_t] + (V_A)_p \rho_{ea} [-(V_A)_p (A_A)_p]$$

$$0.01 \pi p_A - 0.01 \pi p_B = (2.38 \text{ m/s})^2 (790 \text{ kg/m}^3) [\pi(0.1 \text{ m})^2] - (40 \text{ m/s})^2 (790 \text{ kg/m}^3) [\pi(0.01 \text{ m})^2]$$

$$-(2 \text{ m/s})^2 (790 \text{ kg/m}^3) \left\{ \pi[(0.1 \text{ m})^2 - (0.01 \text{ m})^2] \right\}$$

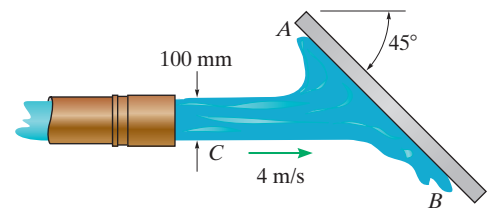
$$\Delta P = p_B - p_A = 11.29(10^3) p_a = 11.3 \text{ kPa} \quad \text{Ans.}$$



(a)

Ans:
11.3 kPa

***6–16.** The jet of water flows from the 100-mm-diameter pipe at 4 m/s. If it strikes the fixed vane and is deflected as shown, determine the normal force the jet exerts on the vane.



SOLUTION

We consider steady flow of an ideal fluid.

Bernoulli Equation. Since the water jet is a free flow, $p_A = p_B = p_C = 0$. Also, if we neglect the elevation change in the water jet, the Bernoulli equation gives

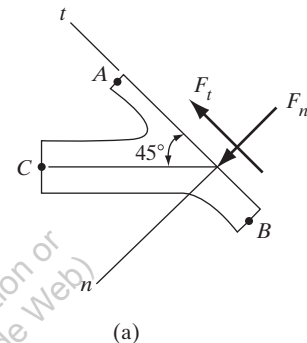
$$\begin{aligned}\frac{p_A}{\gamma} + \frac{V_A^2}{2g} &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} \\ 0 + \frac{V_A^2}{2g} &= 0 + \frac{V_B^2}{2g} = 0 + \frac{(4 \text{ m/s})^2}{2g} \\ V_A &= V_B = 4 \text{ m/s}\end{aligned}$$

Ans.

The discharge at C is

$$Q_C = V_C A_C = (4 \text{ m/s})[\pi(0.05 \text{ m})^2] = 0.03142 \text{ m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since the flow is steady incompressible.



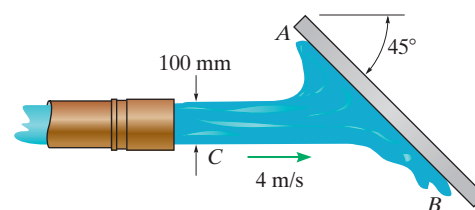
$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\begin{aligned}\Sigma F_n &= 0 + (-Q_C)(\rho)(-V_C)_n \\ F_n &= (-0.03142 \text{ m}^3/\text{s})(1000 \text{ kg/m}^3)(-4 \text{ m/s} \sin 45^\circ) \\ F_n &= 88.9 \text{ N}\end{aligned}$$

Ans.

6–17. The jet of water flows from the 100-mm-diameter pipe at 4 m/s. If it strikes the fixed vane and is deflected as shown, determine the volume flow towards *A* and towards *B* if the tangential component of the force that the water exerts on the vane is zero.



SOLUTION

We consider steady flow of an ideal fluid.

Bernoulli Equation. Since the water jet is a free flow, $p_A = p_B = p_C = 0$. Also, if we neglect the elevation change in the water jet, the Bernoulli equation gives

$$\begin{aligned}\frac{p_A}{\gamma} + \frac{V_A^2}{2g} &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} \\ 0 + \frac{V_A^2}{2g} &= 0 + \frac{V_B^2}{2g} = 0 + \frac{(4 \text{ m/s})^2}{2g} \\ V_A &= V_B = 4 \text{ m/s}\end{aligned}$$

The discharge at *C* is

$$Q_C = V_C A_C = (4 \text{ m/s})[\pi(0.05 \text{ m})^2] = 0.03142 \text{ m}^3/\text{s}$$

Continuity Equation.

$$\begin{aligned}\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \mathbf{V} \cdot d\mathbf{A} &= 0 \\ 0 - Q_C + Q_A + Q_B &= 0 \quad Q_A + Q_B = 0.03142\end{aligned}\quad (1)$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Here, it is required that $F_t = 0$. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

or

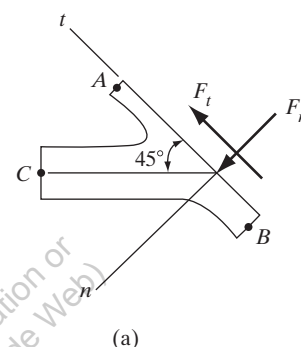
$$\Sigma F_t = \rho [Q_A (V_A)_t + Q_B (V_B)_t - Q_C (V_C)_t]$$

$$0 = (1000 \text{ kg/m}^3) [Q_A (4 \text{ m/s}) + Q_B (-4 \text{ m/s}) - 0.03142 \text{ m}^3/\text{s} (-4 \text{ m/s} \cos 45^\circ)]$$

$$Q_A - Q_B = -0.02221\quad (2)$$

Solving Eqs. (1) and (2),

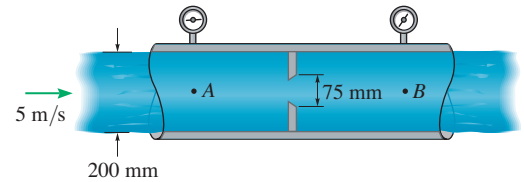
$$Q_A = 0.00460 \text{ m}^3/\text{s} \quad Q_B = 0.0268 \text{ m}^3/\text{s} \quad \text{Ans.}$$



Ans:

$$\begin{aligned}Q_A &= 0.00460 \text{ m}^3/\text{s} \\ Q_B &= 0.0268 \text{ m}^3/\text{s}\end{aligned}$$

6–18. As water flows through the pipe at a velocity of 5 m/s, it encounters the orifice plate, which has a hole in its center. If the pressure at *A* is 230 kPa, and at *B* it is 180 kPa, determine the force the water exerts on the plate.



SOLUTION

We consider steady flow of an ideal fluid.

Take the water from *A* to *B* to be the control volume.

Continuity Equation. Since the diameters of the pipe at *A* and *B* are equal, continuity requires

$$V_A = V_B = 5 \text{ m/s}$$

The free-body diagram of the control volume is shown in Fig. *a*.

Linear Momentum. The flow is steady and incompressible since points *A* and *B* are selected at a sufficient distance from the gate.

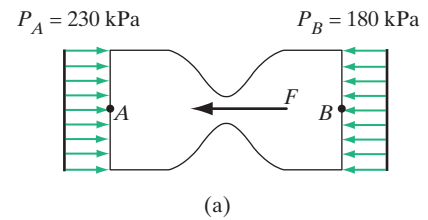
$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\pm \Sigma F_x = 0 + (V_A) \rho (-Q) + (V_B) \rho (Q)$$

$$-F + [230(10^3) \text{ N/m}^2] [\pi(0.1 \text{ m})^2] - [180(10^3) \text{ N/m}^2] [\pi(0.1 \text{ m})^2] = \rho Q(V - V) = 0$$

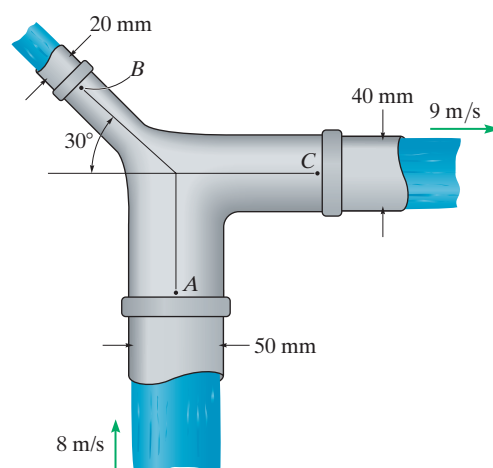
$$F = 1570.80 \text{ N} = 1.57 \text{ kN}$$

Ans.



Ans:
1.57 kN

6–19. Water enters *A* with a velocity of 8 m/s and pressure of 70 kPa. If the velocity at *C* is 9 m/s, determine the horizontal and vertical components of the resultant force that must act on the transition to hold it in place. Neglect the size of the transition.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. The average velocities will be used. The control volume contains the water in the transition as shown in Fig. *a*. The continuity condition requires

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$-(8 \text{ m/s}) [\pi (0.025 \text{ m})^2] + V_B [\pi (0.01 \text{ m})^2] + (9 \text{ m/s}) [\pi (0.02 \text{ m})^2] = 0$$

$$V_B = 14 \text{ m/s}$$

Write the Bernoulli's equation between *A* and *B*, and *A* and *C*.

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{70(10^3) \text{ N/m}^2}{9810 \text{ N/m}^3} + \frac{(8 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = \frac{p_B}{9810 \text{ N/m}^3} + \frac{(14 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_B = 4(10^3) \text{ N/m}^2$$

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C$$

$$\frac{70(10^3) \text{ N/m}^2}{9810 \text{ N/m}^3} + \frac{(8 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = \frac{p_C}{9810 \text{ N/m}^3} + \frac{(9 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_C = 61.5(10^3) \text{ N/m}^2$$

The pressure forces acting on the inlet and outlet control surfaces indicated on the *FBD* of the control volume are

$$F_A = p_A A_A = [70(10^3) \text{ N/m}^2] [\pi (0.025 \text{ m})^2] = 43.75\pi \text{ N}$$

$$F_B = p_B A_B = [4(10^3) \text{ N/m}^2] [\pi (0.01 \text{ m})^2] = 0.4\pi \text{ N}$$

$$F_C = p_C A_C = [61.5(10^3) \text{ N/m}^2] [\pi (0.02 \text{ m})^2] = 24.6\pi \text{ N}$$

6-19. Continued

Applying the Linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along x and y axes,

$$(\rightarrow) \Sigma F_x = 0 + (-V_B \cos 30^\circ)(\rho_w)(V_B A_B) + V_C \rho_w (V_C A_C)$$

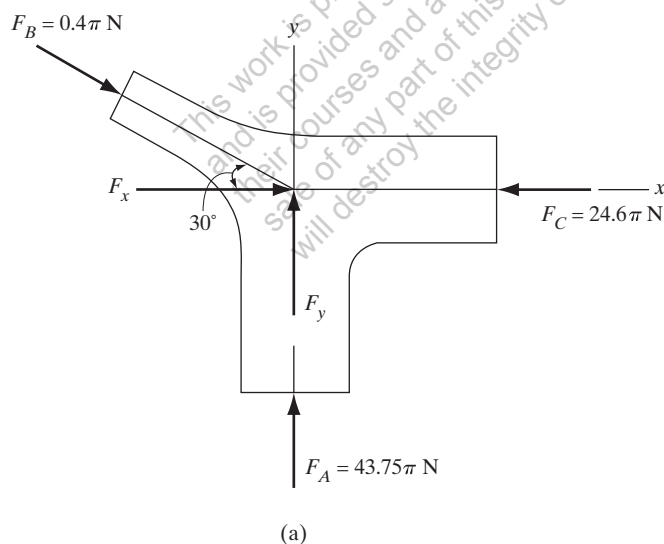
$$F_x + (0.4\pi \text{ N}) \cos 30^\circ - 24.6\pi \text{ N} = -(14 \text{ m/s})(\cos 30^\circ)(1000 \text{ kg/m}^3)(14 \text{ m/s})[\pi(0.01 \text{ m})^2] \\ + (9 \text{ m/s})(1000 \text{ kg/m}^3)(9 \text{ m/s})[\pi(0.02 \text{ m})^2]$$

$$F_x = 125 \text{ N} \rightarrow \quad \text{Ans.}$$

$$(+\uparrow) \Sigma F_y = 0 + (V_B \sin 30^\circ)(\rho_w)(V_B A_B) + V_A \rho_w (-V_A A_A)$$

$$F_y + 43.75\pi \text{ N} - (0.4\pi \text{ N}) \sin 30^\circ = (14 \text{ m/s})(\sin 30^\circ)(1000 \text{ kg/m}^3)(14 \text{ m/s})[\pi(0.01 \text{ m})^2] \\ + (8 \text{ m/s})(1000 \text{ kg/m}^3)(-8 \text{ m/s})[\pi(0.025 \text{ m})^2]$$

$$F_y = +231.69 \text{ N} \approx 232 \text{ N} \downarrow \quad \text{Ans.}$$

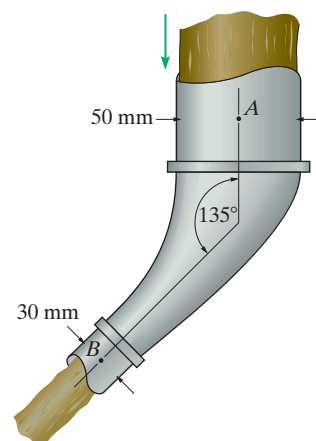


Ans:

$$F_x = 125 \text{ N}$$

$$F_y = 232 \text{ N}$$

***6–20.** Crude oil flows through the horizontal tapered 45° elbow at 0.02 m³/s. If the pressure at A is 300 kPa, determine the horizontal and vertical components of the resultant force the oil exerts on the elbow. Neglect the size of the elbow.



SOLUTION

The flow is steady and crude oil can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_{co} = 880 \text{ kg/m}^3$ average velocities will be used. The control volume considered contains the crude oil in the elbow as shown in Fig. *a*. From the discharge,

$$Q = V_A A_A; \quad 0.02 \text{ m}^3/\text{s} = V_A [\pi(0.025 \text{ m})^2] \quad V_A = 10.19 \text{ m/s}$$

$$Q = V_B A_B; \quad 0.02 \text{ m}^3/\text{s} = V_B [\pi(0.015 \text{ m})^2] \quad V_B = 28.29 \text{ m/s}$$

Applying Bernoulli's equation between A and B,

$$\frac{p_A}{\gamma_{co}} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_{co}} + \frac{V_B^2}{2g} + z_B$$

$$\frac{300(10^3) \text{ N/m}^2}{(880 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} + \frac{(10.19 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = \frac{p_B}{(880 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} + \frac{(28.29 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_B = -6.596(10^3) \text{ Pa}$$

The negative sign indicates that suction occurs at B. The pressure for acting on the inlet and outlet control surfaces indicated on the *FBD* of the control volume are

$$F_A = p_A A_A = [300(10^3) \text{ N/m}^2] [\pi(0.025 \text{ m})^2] = 589.05 \text{ N}$$

$$F_B = p_B A_B = [6.596(10^3) \text{ N/m}^2] [\pi(0.015 \text{ m})^2] = 4.663 \text{ N}$$

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_{co} dV + \int_{cs} \mathbf{V} \rho_{co} \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along *x* and *y* axis by referring to Fig. *a*

$$\begin{aligned} \rightarrow \Sigma F_x &= 0 + (-V_B \cos 45^\circ)(\rho_{co})(V_B A_B) \\ (-4.663 \text{ N}) \cos 45^\circ - F_x &= (-28.29 \text{ m/s}) \cos 45^\circ (880 \text{ kg/m}^3)(0.02 \text{ m}^3/\text{s}) \end{aligned}$$

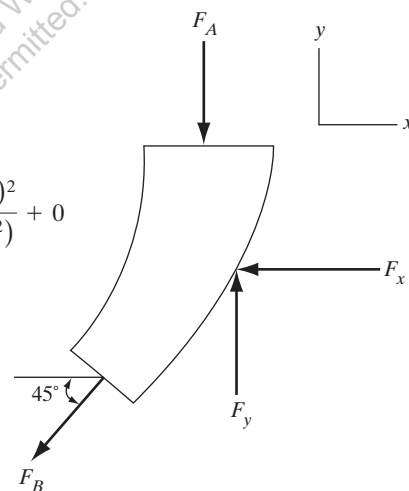
$$F_x = 349 \text{ N} \leftarrow$$

Ans.

$$+\uparrow \Sigma F_y = 0 + (-V_B \sin 45^\circ) \rho_{co} (V_B A_B) + (-V_A) \rho_{co} (-V_A A_A)$$

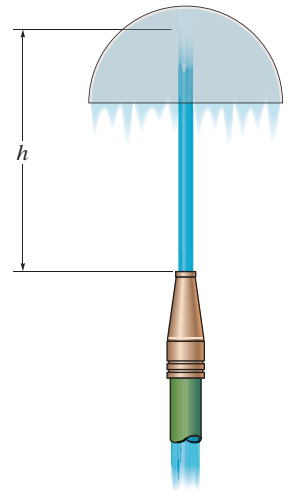
$$F_y - (4.663 \text{ N}) \sin 45^\circ - 589.05 \text{ N} = (-28.29 \text{ m/s}) \sin 45^\circ (880 \text{ kg/m}^3)(0.02 \text{ m}^3/\text{s}) + (-10.19 \text{ m/s})(880 \text{ kg/m}^3)(-0.02 \text{ m}^3/\text{s})$$

$$F_y = 419 \text{ N} \uparrow$$



(a)

6-21. The hemispherical bowl of mass m is held in equilibrium by the vertical jet of water discharged through a nozzle of diameter d . If the volumetric flow is Q , determine the height h at which the bowl is suspended. The water density is ρ_w .



SOLUTION

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid) such that its density is constant. Average velocities will be used. From the discharge the velocity of the water leaving the nozzle (point A on the control volume shown in Fig. a) is

$$Q = V_A A_A; \quad Q = V_A \left(\frac{\pi d^2}{4} \right) \quad V_A = \frac{4Q}{\pi d^2}$$

Applying Bernoulli's equation between points A and B on the central streamline with $p_A = p_B = 0$, $z_A = 0$ and $z_B = h$,

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{\left(\frac{4Q}{\pi d^2} \right)^2}{2g} + 0 = 0 + \frac{V_B^2}{2g} + h$$

$$V_B = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh} \quad (1)$$

By considering the FBD of the control volume shown in Fig. b, where B and C are the inlet and outlet control surfaces,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_w dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along the y axis realizing that by

Bernoulli's equation $V_C = V_B = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$ and $Q = V_A$,

$$+ \uparrow \Sigma F_y = 0 + V_B \rho_w (-V_B A_B) + (-V_C) \rho_w (V_C A_C)$$

$$-mg = V_B \rho_w (-Q) - V_B \rho_w Q$$

$$mg = 2\rho_w Q V_B$$

Substituting Eq. 1 into this equation

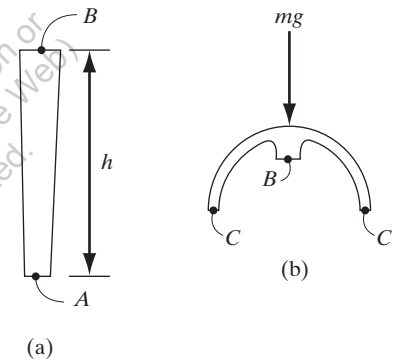
$$mg = 2\rho_w Q \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$$

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w^2 Q^2}$$

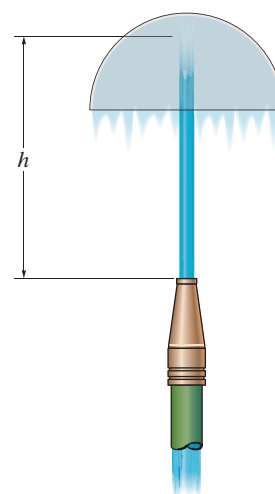
Ans.

Ans:

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w^2 Q^2}$$



6–22. The 500-g hemispherical bowl is held in equilibrium by the vertical jet of water discharged through the 10-mm-diameter nozzle. Determine the height h of the bowl as a function of the volumetric flow Q of the water through the nozzle. Plot the height h (vertical axis) versus Q for $0.5(10^{-3}) \text{ m}^3/\text{s} \leq Q \leq 1(10^{-3}) \text{ m}^3/\text{s}$. Give values for increments of $\Delta Q = 0.1(10^{-3}) \text{ m}^3/\text{s}$.



SOLUTION

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid) such that its density is constant. Average velocities will be used. From the discharge, the velocity of the water leaving the nozzle (point A on the control volume as shown in Fig. a) is

$$Q = V_A A_A; \quad Q = V_A [\pi(0.005 \text{ m})^2]$$

$$V_A = \left[\frac{40(10^3)}{\pi} Q \right] \text{ m/s}$$

Applying Bernoulli's equation between points A and B on the central streamline with $p_A = p_B = 0$, $z_A = 0$ and $z_B = h$,

$$\begin{aligned} \frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B \\ 0 + \frac{\left[\frac{40(10^3)}{\pi} Q \right]^2}{2(9.81 \text{ m/s}^2)} + 0 &= 0 + \frac{V_B^2}{2(9.81 \text{ m/s}^2)} + h \\ V_B &= \sqrt{\frac{1.6(10^9)}{\pi^2} Q^2 - 19.62 h} \end{aligned} \quad (1)$$

By considering the FBD of the fixed control volume shown in Fig. b , where B and C are the inlet and outlet control surfaces,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_w dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

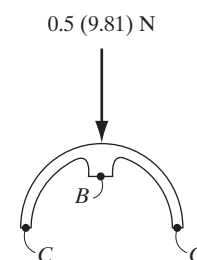
Writing the scalar component of the equation along y axis realizing that

$$V_c = V_B = \sqrt{\frac{1.6(10^9)}{\pi^2} Q^2 - 19.62 h} \text{ and } Q = V_A A_A,$$

$$+\uparrow \Sigma F_y = 0 + V_B \rho_w (-V_B A_B) + (-V_C) \rho_w (V_C A_C)$$



(a)



(b)

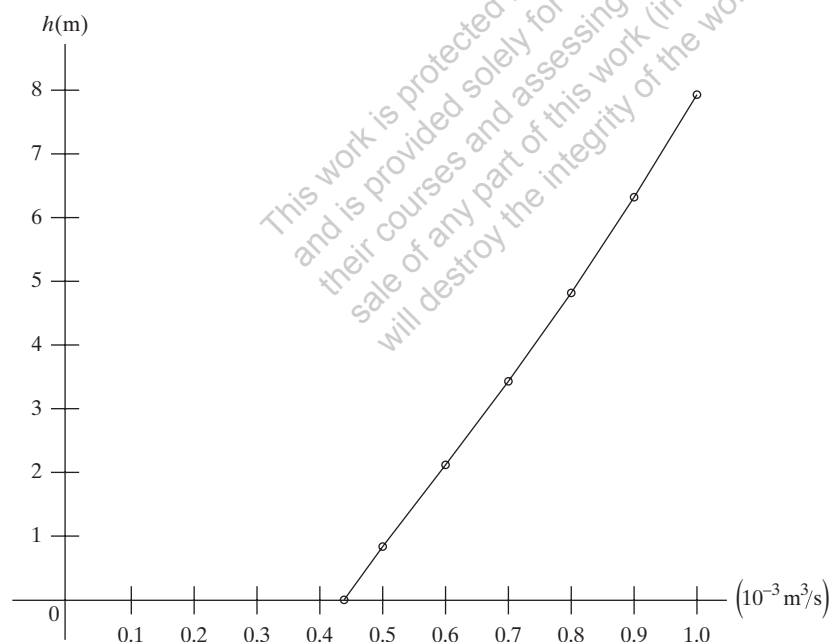
6-22. Continued

$$-0.5(9.81)\text{N} = (1000 \text{ kg/m}^3) \left[-2 \left(\sqrt{\frac{1.6(10^9)}{\pi^2}} Q^2 - 19.62 h \right) Q \right]$$

$$h = \left[\frac{8.26(10^6)Q^4 - 0.307(10^{-6})}{Q^2} \right] \text{ m, where } Q \text{ is in m}^3/\text{s} \quad \textbf{Ans.}$$

The plot of h vs. Q is shown in Fig. c

$Q(10^{-3} \text{ m}^3/\text{s})$	0.5	0.6	0.7	0.8	0.9	1.0	0.439
$h(\text{m})$	0.839	2.12	3.42	4.81	6.31	7.96	0

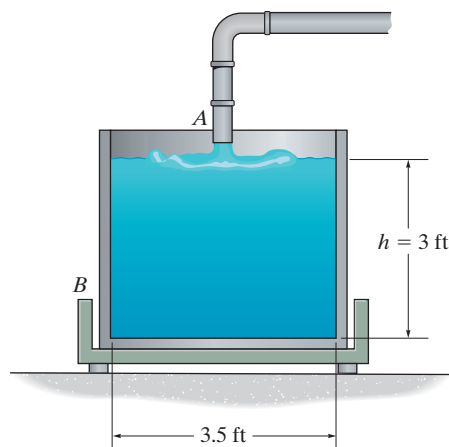


(c)

Ans:

$$h = \left[\frac{8.26(10^6)Q^4 - 0.307(10^{-6})}{Q^2} \right] \text{ m}$$

6–23. Water flows into the rectangular tank at the rate of $0.5 \text{ ft}^3/\text{s}$ from the 3-in.-diameter pipe at A . If the tank has a width of 2 ft and an empty weight of 150 lb, determine the apparent weight of the tank caused by the flow at the instant $h = 3 \text{ ft}$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_A A_A; \quad 0.5 \text{ ft}^3/\text{s} = V \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right]$$

$$V_A = 10.186 \text{ ft/s}$$

The control volume is the water in the tank. Its free-body diagram is shown in Fig. *a*. The weight of the water in the control volume is $W = \gamma_w V = (62.4 \text{ lb/ft}^3) [(3.5 \text{ ft})(2 \text{ ft})(3 \text{ ft})] = 1310.4 \text{ lb}$. Here, A is exposed to the atmosphere, $p_A = 0$.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

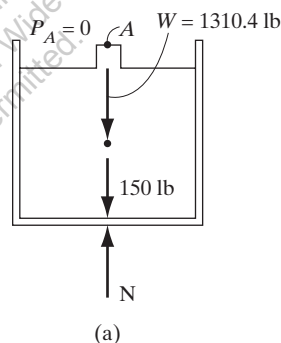
or

$$+\uparrow \Sigma F_y = 0 \Rightarrow (-V_A) \rho (-Q)$$

$$N - 150 \text{ lb} - 1310.4 \text{ lb} = \left(-10.186 \text{ ft/s} \right) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-0.5 \text{ ft}^3/\text{s})$$

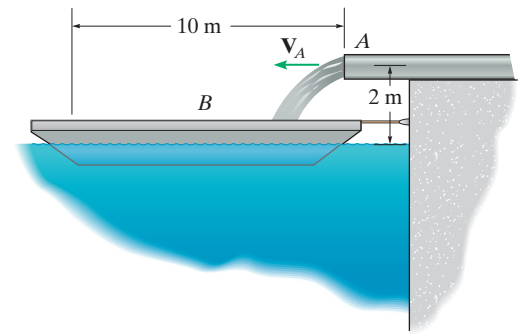
$$N = 1470 \text{ lb} = 1.47 \text{ kip}$$

Ans.



Ans:
1.47 kip

***6–24.** The barge is being loaded with an industrial waste liquid having a density of 1.2 Mg/m^3 . If the average velocity of flow out of the 100-mm-diameter pipe is $V_A = 3 \text{ m/s}$, determine the force in the tie rope needed to hold the barge stationary.



SOLUTION

We consider steady flow of an ideal fluid.

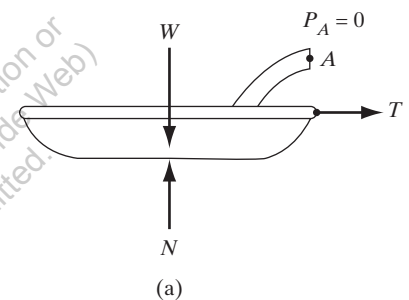
$$\begin{aligned} Q &= V_A A_A \\ &= (3 \text{ m/s}) [\pi (0.05 \text{ m})^2] \\ &= 0.023562 \text{ m}^3/\text{s} \end{aligned}$$

The control volume is the barge and its contents. Its free-body diagram is shown in Fig. *a*. Since the flow is free, $p_A = 0$.

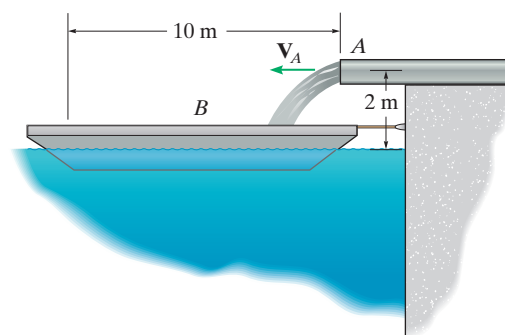
Linear Momentum. Since the flow is steady and incompressible,

$$\begin{aligned} \Sigma \mathbf{F} &= \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A} \\ \rightarrow \Sigma F_x &= 0 + (-V_A) \rho (-Q) \\ T &= (-3 \text{ m/s}) (1.2 (10^3 \text{ kg/m}^3) (-0.023562 \text{ m}^3)) \\ T &= 84.8 \text{ N} \end{aligned}$$

Ans.



6–25. The barge is being loaded with an industrial waste liquid having a density of 1.2 Mg/m^3 . Determine the maximum force in the tie rope needed to hold the barge stationary. The waste can enter the barge at any point within the 10-m region. Also, what is the speed of the waste exiting the pipe at A when this occurs? The pipe has a diameter of 100 mm.



SOLUTION

The maximum force developed in the tie rope occurs when the velocity \mathbf{V} of the flow is maximum. This happens when the flow achieves the maximum range, ie, $S_x = 10 \text{ m}$. Consider the vertical motion by referring to Fig. a .

$$(+\downarrow) S_y = (S_0)_y + (v_0)_y t + \frac{1}{2} a_c t^2; \quad 2 \text{ m} = 0 + 0 + \frac{1}{2} (9.81 \text{ m/s}^2) t^2$$

$$t = 0.6386 \text{ s}$$

The horizontal motion gives

$$(\leftarrow) S_x = (S_0)_x + (v_0)_x t; \quad 10 \text{ m} = 0 + V_A (0.6386 \text{ s})$$

$$V_A = 15.66 \text{ m/s} = 15.7 \text{ m/s}$$

Ans.

The fixed control volume considered is the barge and its contents as shown in Fig. b . Since the flow is free, $p_A = 0$. The flow is steady and incompressible. Then

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

Writing the horizontal scalar component of this equation by referring to the FBD of the control volume, Fig. b ,

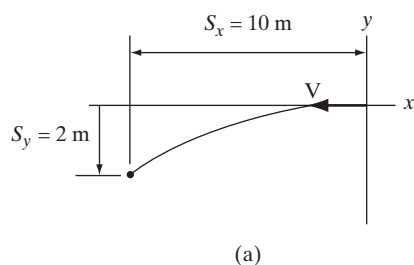
$$(\rightarrow) \Sigma F_x = 0 + (-V_A) \rho (-V_A) A$$

$$T = 0 + (-15.66 \text{ m/s}) (1200 \text{ kg/m}^3) [-(15.66 \text{ m/s}) \pi (0.05 \text{ m})^2]$$

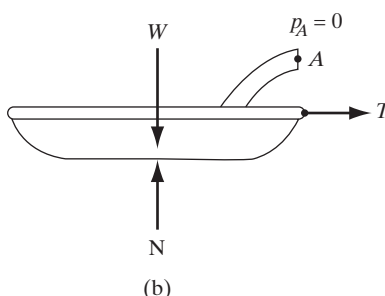
$$= 2311.43 \text{ N}$$

$$= 2.31 \text{ kN}$$

Ans.



(a)



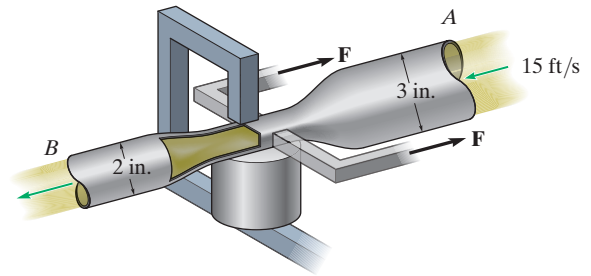
(b)

Ans:

$$V_A = 15.7 \text{ m/s}$$

$$T = 2.31 \text{ kN}$$

6-26. A nuclear reactor is cooled with liquid sodium, which is transferred through the reactor core using the electromagnetic pump. The sodium moves through a pipe at A having a diameter of 3 in., with a velocity of 15 ft/s and pressure of 20 psi, and passes through the rectangular duct, where it is pumped by an electromagnetic force giving it a 30-ft pumphead. If it emerges at B through a 2-in.-diameter pipe, determine the restraining force \mathbf{F} on each arm, needed to hold the pipe in place. Take $\gamma_{\text{Na}} = 53.2 \text{ lb/ft}^3$.



SOLUTION

The flow is steady and the liquid sodium can be considered as an ideal fluid (incompressible and inviscid) such that $\gamma_{\text{Na}} = 53.2 \text{ lb/ft}^3$. Average velocities will be used. The control volume contains the liquid in the pipe and the transition as shown in Fig. a .

Continuity requires

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho dV + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(15 \text{ ft/s}) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] + V_B \left[\pi \left(\frac{1}{12} \text{ ft} \right)^2 \right] = 0$$

$$V_B = 33.75 \text{ ft/s}$$

Applying the energy equation with $h_s = -30 \text{ ft}$ (negative sign indicates pump head),

$$p_A = \left(20 \frac{\text{lb}}{\text{in}^2} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 = 2880 \text{ lb/ft}^2 \text{ and } h_l = 0,$$

$$\frac{p_A}{\gamma_{\text{Na}}} + \frac{V_A^2}{2g} + Z_A + h_l + h_i = \frac{p_B}{\gamma_{\text{Na}}} + \frac{V_B^2}{2g} + Z_B + h_l + h_i$$

$$\frac{2880 \text{ lb/ft}^2}{53.2 \text{ lb/ft}^3} + \frac{(15 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0 + (30 \text{ ft}) - 0 = \frac{p_B}{53.2 \text{ lb/ft}^3} + \frac{(33.75 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0$$

$$p_B = 3720.90 \text{ lb/ft}^2$$

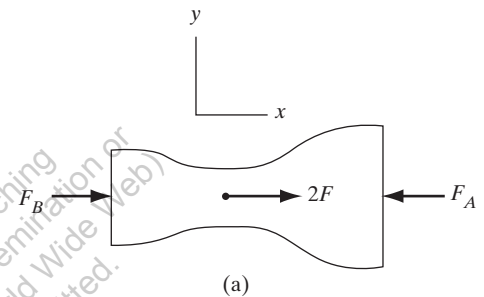
Thus, the pressure force acting on opened control surfaces at A and B are

$$F_A = p_A A_A = (2880 \text{ lb/ft}^2) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] = 141.37 \text{ lb}$$

$$F_B = p_B A_B = (3720.90 \text{ lb/ft}^2) \left[\pi \left(\frac{1}{12} \text{ ft} \right)^2 \right] = 81.18 \text{ lb}$$

Applying the linear momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho \mathbf{V} dV + \int_{\text{cs}} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$



6-26. Continued

Writing the scalar component of this equation along x axis by referring to Fig. a

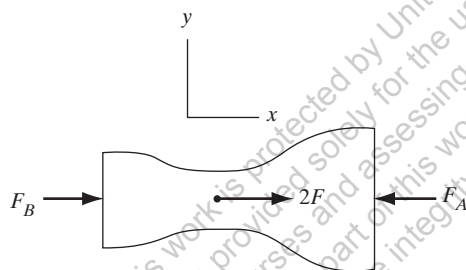
$$\left(\frac{\pm}{\rightarrow}\right) \Sigma F_x = 0 + (-V_B)\rho_{NA}(V_B A_B) + (-V_A)\rho_{NA}(-V_A A_A)$$

$$81.18 \text{ lb} - 141.37 \text{ lb} + 2F = (-33.75 \text{ ft/s})\left(\frac{53.2 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left\{(33.75 \text{ ft/s})\left[\pi\left(\frac{1}{12} \text{ ft}\right)^2\right]\right\} \\ + (-15 \text{ ft/s})\left(\frac{53.2 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left\{(-15 \text{ ft/s})\left[\pi\left(\frac{1.5}{12} \text{ ft}\right)^2\right]\right\}$$

$$F = 18.7 \text{ lb}$$

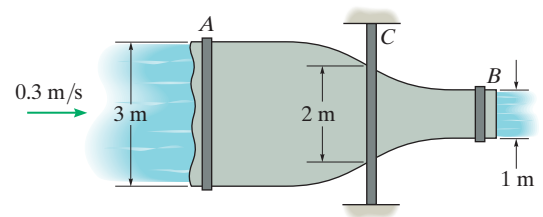
Ans.

Note: This solution assumes that the electromagnetic pump is mounted on the outside of the duct, so that the EM force of the pump on the liquid is canceled by the equal and opposite reaction force on the pump, transferred to the pipe.



Ans:
18.7 lb

6–27. Air flows through the closed duct with a uniform velocity of 0.3 m/s. Determine the horizontal force **F** that the strap must exert on the duct to hold it in place. Neglect any force at the slip joints *A* and *B*. Take $\rho_a = 1.22 \text{ kg/m}^3$.



SOLUTION

Assume the air is incompressible and non-viscous.

$$Q = V_A A_A = (0.3 \text{ m/s})(3 \text{ m})(1 \text{ m}) = 0.9 \text{ m}^3/\text{s}$$

Continuity requires

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - 0.9 \text{ m}^3/\text{s} + V_B(1 \text{ m})(1 \text{ m}) = 0$$

$$V_B = 0.9 \text{ m/s}$$

Apply the Bernoulli's equation between *A* and *B*.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1.22 \text{ kg/m}^3} + \frac{(0.3 \text{ m/s})^2}{2} + 0 = 0 + \frac{(0.9 \text{ m/s})^2}{2} + 0$$

$$p_A = 0.4392 P_a$$

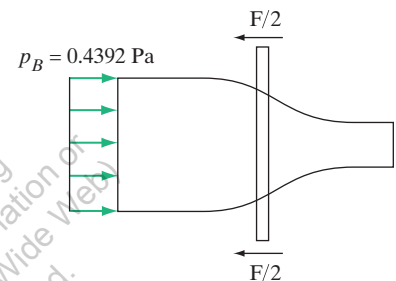
Linear Momentum equation

$$\sum F_x = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$-F + (0.4392 \text{ N/m}^2)(3 \text{ m})(1 \text{ m}) = 0 + (0.3 \text{ m/s})(1.22 \text{ kg/m}^3)(-0.9 \text{ m}^3/\text{s}) + (0.9 \text{ m/s})(1.22 \text{ kg/m}^3)(0.9 \text{ m}^3/\text{s})$$

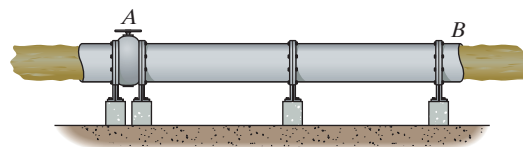
$$F = 0.659 \text{ N}$$

Ans.



Ans:
0.659 N

***6–28.** As oil flows through the 20-m-long, 200-mm-diameter pipeline, it has a constant average velocity of 2 m/s. Friction losses along the pipe cause the pressure at B to be 8 kPa less than the pressure at A . Determine the resultant friction force on this length of pipe. Take $\rho_o = 880 \text{ kg/m}^3$.



SOLUTION

Here $\Delta p = 8 \text{ kPa}$ and so the force developed by the pressure difference is

$$F_p = 8 (10^3) \text{ N/m}^2 (\pi) (0.1 \text{ m})^2 = 251.3 \text{ N}$$

The free-body diagram is shown in Fig. a .

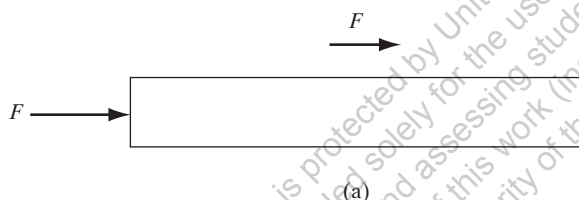
Applying the linear momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

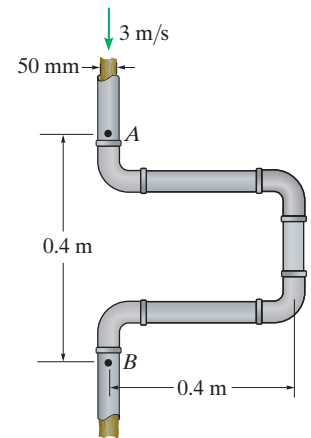
$$251.3 - F = 0 + (2 \text{ m/s})(880 \text{ kg/m}^3)(2 \text{ m/s})(\pi)(0.1 \text{ m}^2) + (2 \text{ m/s})(880 \text{ kg/m}^3)(2 \text{ m/s})(\pi)(0.1 \text{ m}^2)$$

$$F = 251 \text{ N}$$

Ans.



6–29. Oil flows through the 50-mm-diameter vertical pipe assembly such that the pressure at A is 240 kPa and the velocity is 3 m/s. Determine the horizontal and vertical components of force the pipe exerts on the U-section AB of the assembly. The assembly and the oil within it have a combined weight of 60 N. Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

Bernoulli Equation: Because the diameter is the same at A and B , $V_A = V_B = V$.
With the datum at B ,

$$\frac{p_A}{\rho} + \frac{V^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V^2}{2} + gz_B$$

$$\frac{240(10^3) \text{ Pa}}{900 \text{ Kg/m}^3} + (9.81 \text{ m/s}^2)(0.4 \text{ m}) = \frac{p_B}{900 \text{ Kg/m}^3} + 0$$

$$p_B = 243.532(10^3) \text{ Pa}$$

Linear Momentum:

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$(\rightarrow) F_x = 0 + 0 = 0$$

Ans.

$$(+\uparrow) F_y + [243.532(10^3) \text{ Pa}] \pi (0.025 \text{ m})^2 - [240(10^3) \text{ Pa}] \pi (0.025 \text{ m})^2$$

$$-60 \text{ N} = 0 + (-V) \rho (-V_A) + (+V) \rho (V_A)$$

$$F_y = 53.1 \text{ N}$$

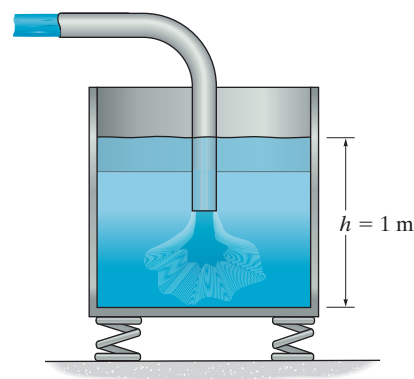
Ans.

Ans:

$$F_x = 0$$

$$F_y = 53.1 \text{ N}$$

6–30. Water flows into the tank at the rate of $0.05 \text{ m}^3/\text{s}$ from the 100-mm-diameter pipe. If the tank is 500 mm on each side, determine the compression in each of the four springs that support its corners when the water reaches a depth of $h = 1 \text{ m}$. Each spring has a stiffness of $k = 8 \text{ kN/m}$. When empty, the tank compresses each spring 30 mm.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid). Hence Average velocities are used and $\rho_w = 1000 \text{ kg/m}^3$. The control volume contains the water in the pipe and the tank and it is fixed instantaneously, Fig. *a*. From the discharge

$$Q = V_{in}A_{in}; \quad -0.05 \text{ m}^3/\text{s} = V_{in}[\pi(0.05 \text{ m})^2] \quad V_{in} = 6.366 \text{ m/s}$$

$$Q = V_{out}A_{out}; \quad 0.05 \text{ m}^3/\text{s} = V_{out}[(0.5 \text{ m})^2 - \pi(0.05 \text{ m})^2] \quad V_{out} = 0.2065 \text{ m/s}$$

Applying the linear momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar equation along the y axis by referring to the *FBD* of the control volume, Fig. *a*,

$$\begin{aligned} + \uparrow F &= 0 + V_{out}\rho_w V_{out}A_{out} + (-V_{in})\rho_w(-V_{in}A_{in}) \\ F &= (0.2065 \text{ m/s})^2(1000 \text{ kg/m}^3)[(0.5 \text{ m})^2 - \pi(0.05 \text{ m})^2] + (6.366 \text{ m/s})^2(1000 \text{ Kg/m}^3)[\pi(0.05 \text{ m})^2] \\ &= 328.63 \text{ N} \end{aligned}$$

The weight of the water in the tank at a depth of 1m is

$$W_N = \rho_w g V_W = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.5 \text{ m})^2(1 \text{ m})] = 2452.5 \text{ N}$$

The weight of the empty tank is

$$W_t = 4kx = 4[8(10^3) \text{ N/m}](0.03 \text{ m}) = 960 \text{ N}$$

Thus, the total weight is

$$W_T = W_W + W_t = 2452.5 \text{ N} + 960 \text{ N} = 3412.5 \text{ N}$$

Equilibrium of the *FBD* of the tank, Fig. *b*, requires

$$+ \uparrow \Sigma F_y = 0; \quad 4F_{sp} - 328.63 \text{ N} - 3412.5 \text{ N} = 0$$

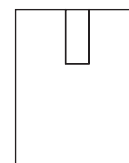
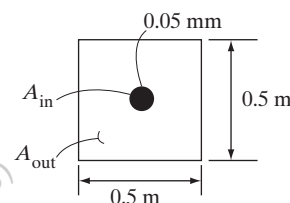
$$F_{sp} = 935.28 \text{ N}$$

Thus, the compression of the spring is

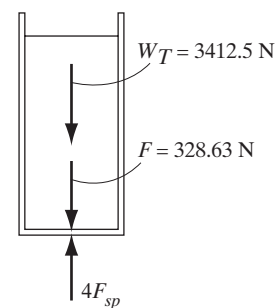
$$F_{sp} = kx; \quad 935.28 \text{ N} = [8(10^3) \text{ N/m}]x$$

$$x = 0.1169 \text{ m} = 117 \text{ mm}$$

Ans.



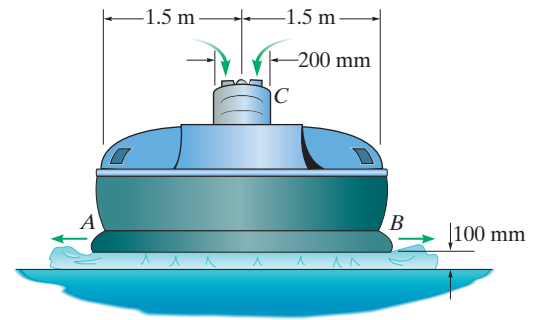
(a)



(b)

Ans:
117 mm

6–31. The 300-kg circular craft is suspended 100 mm from the ground. For this to occur, air is drawn in at 18 m/s through the 200-mm-diameter intake and discharged to the ground as shown. Determine the pressure that the craft exerts on the ground. Take $\rho_a = 1.22 \text{ kg/m}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_C A_C = (18 \text{ m/s}) [\pi (0.1 \text{ m})^2] = 0.5655 \text{ m}^3/\text{s}$$

Take the control volume to be the craft and the air inside it. Its free-body diagram is shown in Fig. *a*. Since the flow is open to the atmosphere, $p_C = 0$.

Linear Momentum. Since no air escapes from the hovercraft vertically, $V_{\text{out}} = 0$. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A}$$

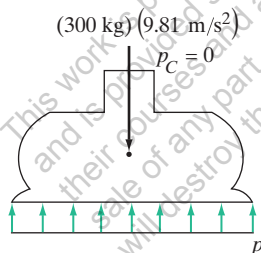
or

$$+\uparrow \Sigma F_y = 0 + (-V_C) \rho (-Q)$$

$$p [\pi (1.5 \text{ m})^2] - 300 \text{ kg} (9.81 \text{ m/s}^2) = (-18 \text{ m/s}) (1.22 \text{ kg/m}^3) (0.5655 \text{ m}^3/\text{s})$$

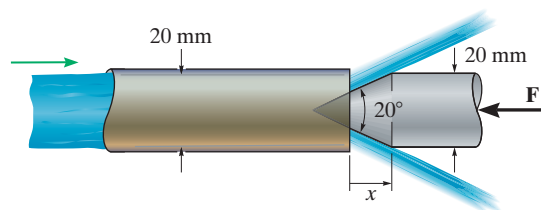
$$p = 418 \text{ Pa}$$

Ans.



Ans:
418 Pa

***6–32.** The cylindrical needle valve is used to control the flow of $0.003 \text{ m}^3/\text{s}$ of water through the 20-mm-diameter tube. Determine the force \mathbf{F} required to hold it in place when $x = 10 \text{ mm}$.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The control volume is shown in Fig. *a*. From the discharge,

$$Q = V_{\text{in}} A_{\text{in}}; \quad 0.003 \text{ m}^3/\text{s} = V_{\text{in}} [\pi (0.01 \text{ m})^2] \quad V_{\text{in}} = 9.549 \text{ m/s}$$

From the geometry shown in Fig. *b*,

$$\frac{r}{\frac{0.01 \text{ m}}{\tan 10^\circ} - 0.01 \text{ m}} = \frac{0.01 \text{ m}}{\tan 10^\circ}; \quad r = 0.008237 \text{ m}$$

Thus,

$$A_{\text{out}} = \pi [(0.01 \text{ m})^2 - (0.008237 \text{ m})^2] = 0.1010 (10^{-3}) \text{ m}^2$$

Then

$$Q = V_{\text{out}} A_{\text{out}}; \quad 0.003 \text{ m}^3/\text{s} = V_{\text{out}} [0.1010 (10^{-3}) \text{ m}^2]$$

$$V_{\text{out}} = 29.70 \text{ m/s}$$

Applying Bernoulli's equation between the center points of the inlet and outlet control surfaces where $p_{\text{out}} = p_{\text{atm}} = 0$

$$\frac{p_{\text{in}}}{\gamma_w} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} = \frac{p_{\text{out}}}{\gamma_w} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}}$$

$$\frac{p_{\text{in}}}{9810 \text{ N/m}^3} + \frac{(9.549 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = 0 + \frac{(29.70 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_{\text{in}} = 395.35 (10^3) \text{ Pa}$$

Thus, the pressure force exerted on the inlet control surface is

$$F_{\text{in}} = p_{\text{in}} A_{\text{in}} = [395.35 (10^3) \text{ N/m}^2] [\pi (0.01 \text{ m})^2] = 124.20 \text{ N}$$

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho_w dV + \int_{\text{cs}} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component along x axis by referring to the *FBD* of the control volume shown in Fig. *a*

$$\rightarrow \Sigma F_x = 0 + V_{\text{out}} \rho_w V_{\text{out}} A_{\text{out}} + V_{\text{in}} \rho_w (-V_{\text{in}} A_{\text{in}})$$

***6-32. Continued**

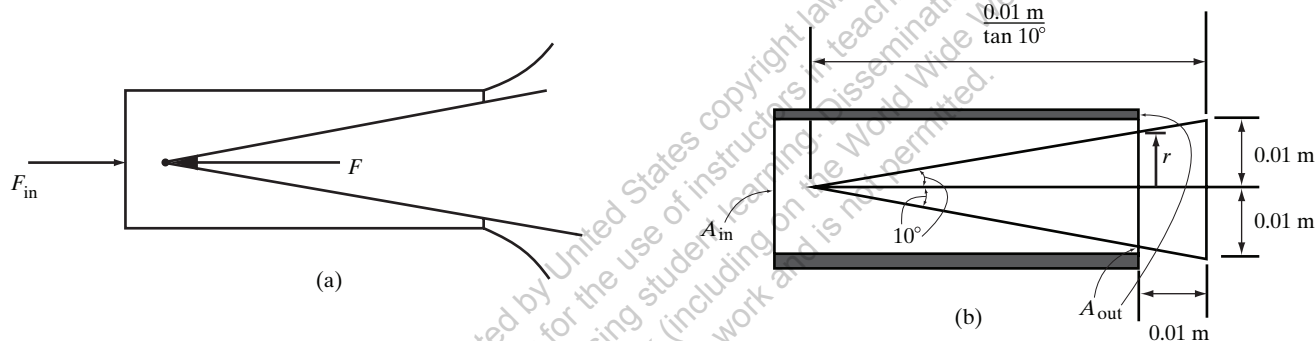
However, $Q = V_{\text{out}}A_{\text{out}} = V_{\text{in}}A_{\text{in}} = 0.003 \text{ m}^3/\text{s}$

$$124.20 \text{ N} - F = (29.70 \text{ m/s})(1000 \text{ kg/m}^3)(0.003 \text{ m}^3/\text{s}) + (9.549 \text{ m/s})(1000 \text{ kg/m}^3)(-0.003 \text{ m}^3/\text{s})$$

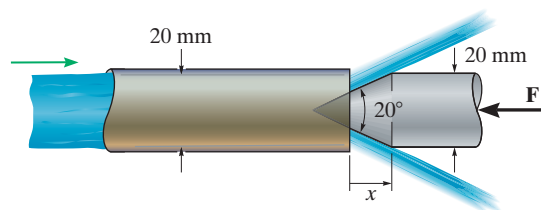
$$F = 63.76 \text{ N} = 63.8 \text{ N}$$

Ans.

Note: For simplicity, the effect of the slight deflection of the stream, away from the central axis, has been neglected. If it were accounted for, F would be slightly ($< 2\%$) larger.



6–33. The cylindrical needle valve is used to control the flow of $0.003 \text{ m}^3/\text{s}$ of water through the 20-mm-diameter tube. Determine the force \mathbf{F} required to hold it in place for any position x of closure of the valve.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The control volume is shown in Fig. *a*. From the discharge,

$$Q = V_{\text{in}} A_{\text{in}}; \quad 0.003 \text{ m}^3/\text{s} = V_{\text{in}} [\pi(0.01 \text{ m})^2] \quad V_{\text{in}} = 9.549 \text{ m/s}$$

From the geometry shown in Fig. *b*,

$$\frac{r}{\frac{0.01 \text{ m}}{\tan 10^\circ} - x} = \frac{0.01 \text{ m}}{\tan 10^\circ}; \quad r = 0.01 \text{ m} - (\tan 10^\circ)x$$

Thus,

$$A_{\text{out}} = \pi[(0.01 \text{ m})^2 - (0.01 \text{ m} - (\tan 10^\circ)x)^2] = 0.01108x - 0.09768x^2$$

Then

$$Q = V_{\text{out}} A_{\text{out}}; \quad 0.003 \text{ m}^3/\text{s} = V_{\text{out}} (0.01108x - 0.09768x^2)$$

$$V_{\text{out}} = \left(\frac{1}{3.693x - 32.559x^2} \right) \text{ m/s}$$

Applying the energy equation between the center points of the inlet and outlet control surfaces, where $p_{\text{out}} = p_{\text{atm}} = 0$.

$$\frac{p_{\text{in}}}{\gamma_w} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma_w} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

$$\frac{p_{\text{in}}}{9810 \text{ N/m}^3} + \frac{(9.549 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 + 0 = 0 + \frac{\left(\frac{1}{3.693x - 32.559x^2} \right)^2}{2(9.81 \text{ m/s}^2)} + 0 + 0 + 0$$

$$p_{\text{in}} \left[\frac{500}{(3.693x - 32.559x^2)^2} - 45.595(10^3) \right] \text{ Pa}$$

Thus, the pressure force on the inlet control surface is

$$\begin{aligned} F_{\text{in}} = p_{\text{in}} A_{\text{in}} &= \left[\frac{500}{(3.693x - 32.559x^2)^2} - 45.595(10^3) \right] [\pi(0.01 \text{ m})^2] \\ &= \frac{0.05\pi}{(3.693x - 32.559x^2)^2} - 14.324 \end{aligned}$$

6-33. Continued

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{CV} \mathbf{V} \rho_w dV + \int_{CS} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component along the x axis by referring to the *FBD* of the control volume shown in Fig. *a*,

$$\rightarrow \Sigma F_x = 0 + V_{out} \rho_w V_{out} A_{out} + V_{in} \rho_w (-V_{in} A_{in})$$

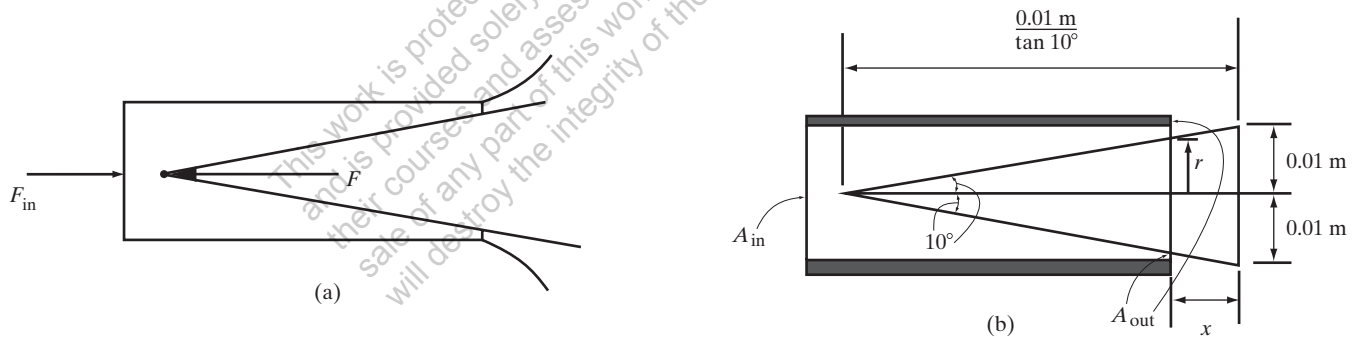
$$\text{However, } Q = V_{out} A_{out} = V_{in} A_{in} = 0.003 \text{ m}^3/\text{s}$$

$$\frac{0.05\pi}{(3.693x - 32.559x^2)^2} - 14.324 - F = \left(\frac{1}{3.693x - 32.559x^2} \right) (1000 \text{ kg/m}^3) (0.003 \text{ m}^3/\text{s})$$

$$+ (9.549 \text{ m/s}) (1000 \text{ kg/m}^3) (-0.003 \text{ m}^3/\text{s})$$

$$F = \left[\frac{97.7x^2 - 11.1x + 0.157}{(3.69x - 32.6x^2)^2} + 14.3 \right] \text{ N} \quad \text{Ans.}$$

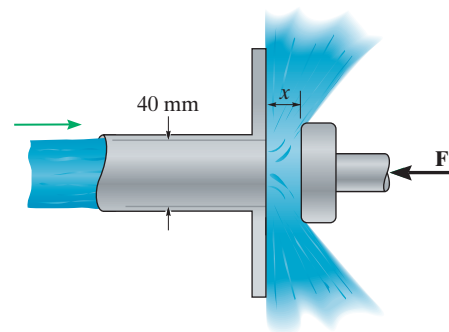
Note: As in the preceding problem, the slight effect of the 10° deflection of the stream has been neglected.



Ans:

$$F = \left[\frac{97.7x^2 - 11.1x + 0.157}{(3.69x - 32.6x^2)^2} + 14.3 \right] \text{ N}$$

6-34. The disk valve is used to control the flow of $0.008 \text{ m}^3/\text{s}$ of water through the 40-mm-diameter tube. Determine the force \mathbf{F} required to hold the valve in place for any position x of closure of the valve.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The control volume is shown in Fig. *a*. From the discharge,

$$Q = V_{\text{in}} A_{\text{in}}; \quad 0.008 \text{ m}^3/\text{s} = V_{\text{in}} [\pi(0.02 \text{ m})^2] \quad V_{\text{in}} = 6.366 \text{ m/s}$$

The cross-sectional area of the outlet control surfaces is

$$A_{\text{out}} = 2\pi(0.02 \text{ m})x = (0.04\pi x) \text{ m}^2$$

Then

$$Q = V_{\text{out}} A_{\text{out}}; \quad 0.008 \text{ m}^3/\text{s} = V_{\text{out}} (0.04\pi x)$$

$$V_{\text{out}} = \left(\frac{0.06366}{x} \right) \text{ m/s}$$

Applying Bernoulli's equation between the center points of the inlet and outlet control surfaces, where $p_{\text{out}} = p_{\text{atm}} = 0$.

$$\frac{p_{\text{in}}}{\gamma_w} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} = \frac{p_{\text{out}}}{\gamma_w} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}}$$

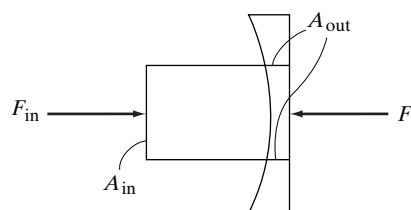
$$\frac{p_{\text{in}}}{9810 \text{ N/m}^3} + \frac{(6.366 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = 0 + \frac{\left(\frac{0.06366}{x} \right)^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_{\text{in}} = \left[\frac{2.026}{x^2} - 20.264(10^3) \right] \text{ Pa}$$

Thus, the pressure force on the inlet control surface is

$$F_{\text{in}} = p_{\text{in}} A_{\text{in}} = \left[\frac{2.026}{x^2} - 20.264(10^3) \right] [\pi(0.02 \text{ m})^2]$$

$$= \left[\frac{2.546(10^{-3})}{x^2} - 25.465 \right] \text{ N}$$



(a)

6-34. Continued

Applying the linear momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_w dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along x axis by referring to the *FBD* of the control volume shown in Fig. *a*

$$\rightarrow \Sigma F_x = 0 + V_{in} \rho_w (-V_{in} A_{in})$$

However, $Q = V_{in} A_{in} = 0.008 \text{ m}^3/\text{s}$. Thus

$$\frac{2.546(10^{-3})}{x^2} - 25.465 - F = (6.366 \text{ m/s})(1000 \text{ kg/m}^3)(-0.008 \text{ m}^3/\text{s})$$

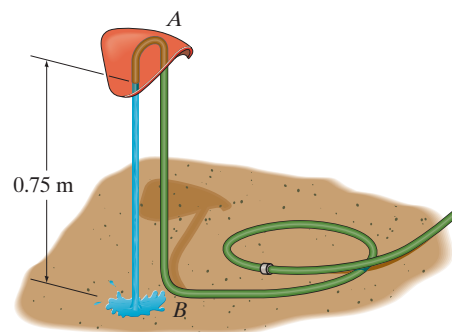
$$F = \left[\frac{2.55(10^{-3})}{x^2} + 25.5 \right] \text{ N}$$

Ans.

Ans:

$$F = \left[\frac{2.55(10^{-3})}{x^2} + 25.5 \right] \text{ N}$$

6–35. The toy sprinkler consists of a cap and a rigid tube having a diameter of 20 mm. If water flows through the tube at $0.7(10^{-3}) \text{ m}^3/\text{s}$, determine the vertical force the wall of the tube must support at B . Neglect the weight of the sprinkler head and the water within the curved segment of the tube. The weight of the tube and water within the vertical segment AB is 4 N.



SOLUTION

$$Q = VA$$

$$0.7(10^{-3}) \text{ m}^3/\text{s} = V(\pi)(0.01 \text{ m})^2$$

$$V = 2.228 \text{ m/s}$$

Since the hose has a constant diameter, continuity requires $V_A = V_B = V = 2.228 \text{ m/s}$

Applying Bernoulli's equation between A and B , with the datum of B ,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{V^2}{2g} + 0.75 \text{ m} = \frac{p_B}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} + \frac{V^2}{2g} + 0$$

$$p_B = 7357.5 \text{ Pa}$$

The free-body diagram of the control volume is shown in Fig a . Applying the linear momentum equation in the vertical direction, for steady flow

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

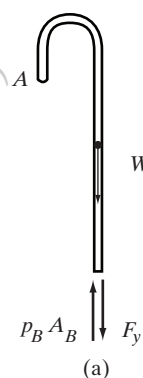
$$+\uparrow \Sigma F_y = 0 + (-V_A)\rho(Q_A) + V_B\rho(-Q_B)$$

$$+\uparrow \Sigma F_y = -2 V \rho Q$$

$$(7357.5 \text{ N/m}^2)(\pi)(0.01 \text{ m})^2 - F_y - 4 \text{ N} = -2(2.228 \text{ m/s})(1000 \text{ kg/m}^3)(0.7(10^{-3}) \text{ m}^3/\text{s})$$

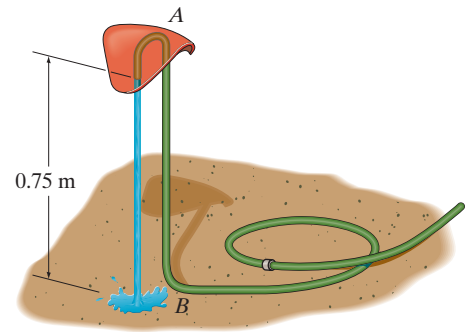
$$F_y = 1.43 \text{ N}$$

Ans.



Ans:
1.43 N

***6-36.** The toy sprinkler consists of a cap and a rigid tube having a diameter of 20 mm. Determine the flow through the tube such that it creates a vertical force of 6 N in the tube at B . Neglect the weight of the sprinkler head and the water within the curved segment of the tube. The weight of the tube and water within the vertical segment AB is 4 N.



SOLUTION

Since the hose has a constant diameter, continuity requires $V_A = V_B = V$. Applying Bernoulli's equation between A and B , with the datum at B ,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{V^2}{2g} + 0.75 \text{ m} = \frac{p_B}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} + \frac{V^2}{2g} + 0$$

$$p_B = 7357.5 \text{ Pa}$$

The free-body diagram of the control volume is shown in Fig. a . Applying the linear momentum equation in the vertical direction for steady flow,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$+\uparrow \Sigma F_y = 0 + (-V_A)\rho Q_A + V_B \rho(-Q_B)$$

$$+\uparrow \Sigma F_y = -2V\rho Q$$

$$(7357.5 \text{ N/m}^2)(\pi)(0.01 \text{ m})^2 - 4 - 6 = -2(V)(1000 \text{ kg/m}^3)(V)(\pi)(0.01 \text{ m})^2$$

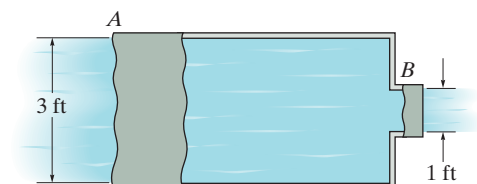
$$V = 3.4981 \text{ m/s}$$

$$Q = VA = (3.4981 \text{ m/s})(\pi)(0.01 \text{ m})^2 = 1.10(10^{-3}) \text{ m}^3/\text{s}$$

Ans.



6–37. Air flows through the 1.5-ft-wide rectangular duct at 900 ft³/min. Determine the horizontal force acting on the end plate *B* of the duct. Take $\rho_a = 0.00240$ slug/ft³.



SOLUTION

$$Q = 900 \text{ ft}^3/\text{min} (1 \text{ min.}/60 \text{ s}) = 15 \text{ ft}^3/\text{s}$$

$$V_A = \frac{15 \text{ ft}^3/\text{s}}{(3 \text{ ft})(1.5 \text{ ft})} = 3.33 \text{ ft/s}$$

$$V_B = \frac{15 \text{ ft}^3/\text{s}}{(1 \text{ ft})(1.5 \text{ ft})} = 10 \text{ ft/s}$$

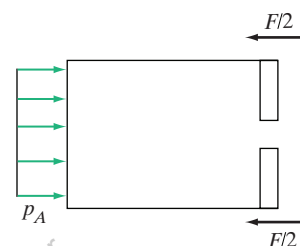
Apply Bernoulli's equation between *A* and *B*.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{0.00240 \text{ slug/ft}^3} + \frac{(3.33 \text{ ft/s})^2}{2} + 0 = 0 + \frac{(10 \text{ ft/s})^2}{2} + 0$$

$$p_A = 0.10667 \text{ lb/ft}^2$$

Using the free-body diagram, Fig. *a* the linear momentum equation becomes



$$\sum F_x = \frac{\partial}{\partial t} \int_{cv} V_x \rho dV + \int_{cs} V_x \rho \mathbf{V} \cdot d\mathbf{A}$$

$$(0.10667 \text{ lb/ft}^2)(3 \text{ ft})(1.5 \text{ ft}) - F =$$

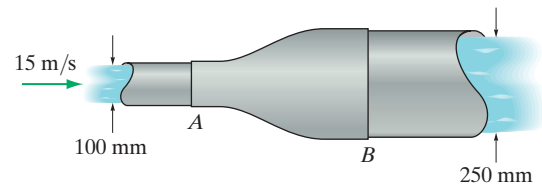
$$0 + (3.33 \text{ ft/s})(0.00240 \text{ slug/ft}^3)(-15 \text{ ft}^3/\text{s}) + (10 \text{ ft/s})(0.00240 \text{ slug/ft}^3)(15 \text{ ft}^3/\text{s})$$

$$F = 0.24 \text{ lb}$$

Ans.

Ans:
0.24 lb

6–38. Air at a temperature of 30°C flows through the expansion fitting such that its velocity at A is 15 m/s and the absolute pressure is 250 kPa. If no heat or frictional loss occurs, determine the resultant force needed to hold the fitting in place.



SOLUTION

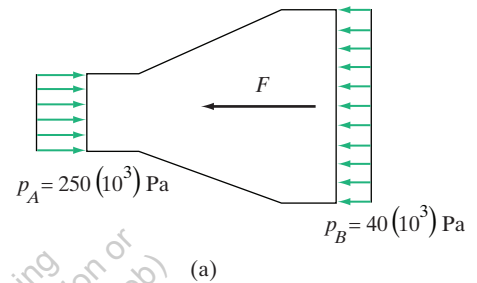
Using the ideal gas law with $R = 286.9 \text{ J/kg} \cdot \text{K}$ for air (Appendix A),

$$p_A = \rho_A R T_A; \quad 250(10^3) \text{ N/m}^2 = \rho_A (286.9 \text{ J/kg} \cdot \text{K})(273 + 30) \text{ K}$$

$$\rho_A = 2.8759 \text{ kg/m}^3$$

$$p_B = \rho_B R T_B; \quad p_B = \rho_B (286.9 \text{ J/kg} \cdot \text{K})(273 + 30) \text{ K}$$

$$\rho_B = [11.5034(10^{-6})p_B] \text{ kg/m}^3 \quad (1)$$



Consider the fixed control volume to be the air contained in the expansion fitting as shown in Fig. a. Continuity requires

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 + \rho_A(-V_A A_A) + \rho_B(V_B A_B) = 0$$

$$(2.8759 \text{ kg/m}^3) \{ -(15 \text{ m/s}) [\pi(0.05 \text{ m})^2] \} + [11.5034(10^{-6})p_B] \{ V_B [\pi(0.125 \text{ m})^2] \} = 0$$

$$V_B = \left[\frac{0.6(10^6)}{p_B} \right] \text{ m/s} \quad (2)$$

Since the fitting remains horizontal, $z_A = z_B = z$. The energy equation gives

$$\frac{p_A}{\gamma_A} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_B} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{250(10^3) \text{ N/m}^2}{(2.8759 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} + \frac{(15 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + z + 0 = \frac{p_B}{[11.5034(10^{-6})p_B](9.81 \text{ m/s}^2)} + \frac{\left[\frac{0.6(10^6)}{p_B} \right]^2}{2(9.81 \text{ m/s}^2)} + z + 0 + 0$$

$$p_B = 40(10^3) \text{ Pa}$$

Substituting this result into Eqs. (1) and (2)

$$\rho_B = 0.4601 \text{ kg/m}^3 \quad V_B = 15 \text{ m/s}$$

Since the flow is steady,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

6–38. Continued

Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume, Fig. *a*,

$$\pm \Sigma F_x = 0 + V_A \rho_A (-V_A A_A) + V_B \rho_B (V_B A_B)$$

$$[250(10^3) \text{ N/m}^2] [\pi(0.05 \text{ m})^2] - F - [40(10^3) \text{ N/m}^2] [\pi(0.125 \text{ m})^2]$$

$$= (15 \text{ m/s})(2.8759 \text{ kg/m}^3) \{ -(15 \text{ m/s}) [\pi(0.05 \text{ m})^2] \} + (15 \text{ m/s})(0.4601 \text{ kg/m}^3) \{ (15 \text{ m/s}) [\pi(0.125 \text{ m})^2] \}$$

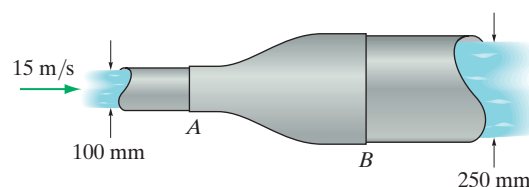
$$F = 0$$

Ans.

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

Ans:
 $F = 0$

6–39. Air at a temperature of 30°C flows through the expansion fitting such that its velocity at *A* is 15 m/s and the pressure is 250 kPa. If heat and frictional loss due to the expansion causes the temperature and absolute pressure of the air at *B* to become 20°C and 750 Pa, determine the resultant force needed to hold the fitting in place.



SOLUTION

Using the ideal gas law with $R = 286.9 \text{ J/kg} \cdot \text{K}$ for air (Appendix A),

$$p_A = \rho_A R T_A; \quad 250(10^3) \text{ N/m}^2 = \rho_A (286.9 \text{ J/kg} \cdot \text{K})(273 + 30) \text{ K}$$

$$\rho_A = 2.8759 \text{ kg/m}^3$$

$$p_B = \rho_B R T_B; \quad 750(10^3) \text{ N/m}^2 = \rho_B (286.9 \text{ J/kg} \cdot \text{K})(273 + 20) \text{ K}$$

$$\rho_B = 0.08922 \text{ kg/m}^3 \quad (1)$$

Consider the fixed control volume to be the water contained in the expansion fitting as shown in Fig. *a*. The continuity requires

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 + \rho_A (-V_A A_A) + \rho_B (V_B A_B) = 0$$

$$(2.8759 \text{ kg/m}^3) \{ -(15 \text{ m/s}) [\pi(0.05 \text{ m})^2] \} + [0.08922 \text{ kg/m}^3] \{ V_B [\pi(0.125 \text{ m})^2] \} = 0$$

$$V_B = 77.36 \text{ m/s}$$

Since the flow is steady,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume, Fig. *a*

$$\pm \Sigma F_x = 0 + V_A \rho_A (-V_A A_A) + V_B \rho_B (V_B A_B)$$

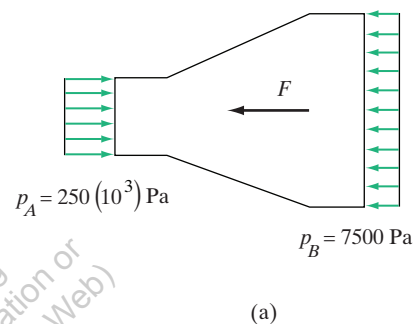
$$[250(10^3) \text{ N/m}^2] [\pi(0.05 \text{ m})^2] - F - (750(10^3) \text{ N/m}^2) [\pi(0.125 \text{ m})^2]$$

$$= (15 \text{ m/s})(2.8759 \text{ kg/m}^3) \{ -(15 \text{ m/s}) [\pi(0.05 \text{ m})^2] \}$$

$$+ (77.36 \text{ m/s})(0.08922 \text{ kg/m}^3) \{ (77.36 \text{ m/s}) [\pi(0.125 \text{ m})^2] \}$$

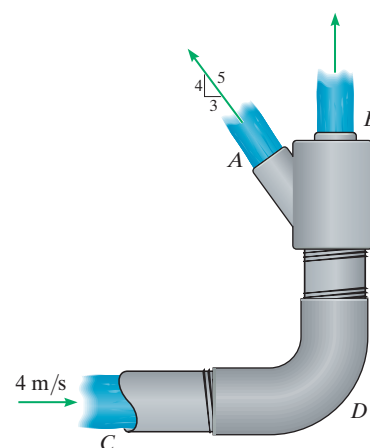
$$F = 1.57 \text{ kN}$$

Ans.



Ans:
1.57 kN

***6–40.** Water flows through the pipe C at 4 m/s . Determine the horizontal and vertical components of force exerted by elbow D necessary to hold the pipe assembly in equilibrium. Neglect the size and weight of the pipe and the water within it. The pipe has a diameter of 60 mm at C , and at A and B the diameters are 20 mm .



SOLUTION

Assume water is incompressible. We have steady flow.

$$Q = 4 \text{ m/s} (\pi) (0.03 \text{ m})^2 = 0.011310 \text{ m}^3/\text{s}$$

Continuity requires

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - 4 \text{ m/s} (\pi) (0.03 \text{ m})^2 + V_A (\pi) (0.01 \text{ m})^2 + V_B (\pi) (0.01 \text{ m})^2 = V_A + V_B = 36 \quad (1)$$

Bernoulli Equation.

$$\begin{aligned} \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C &= \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A \\ \frac{p_C}{1000 \text{ kg/m}^3} + \frac{(4 \text{ m/s})^2}{2} + 0 &= 0 + \frac{V_A^2}{2} + 0 \\ V_A^2 &= 16 + 0.002 p_C \quad (2) \end{aligned}$$

$$\begin{aligned} \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C &= \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B \\ \frac{p_C}{1000 \text{ kg/m}^3} + \frac{(4 \text{ m/s})^2}{2} + 0 &= 0 + \frac{V_B^2}{2} + 0 \\ V_B^2 &= 16 + 0.002 p_C \quad (3) \end{aligned}$$

From Eqs. (2) and (3), $V_A = V_B$. From Eq. (1),

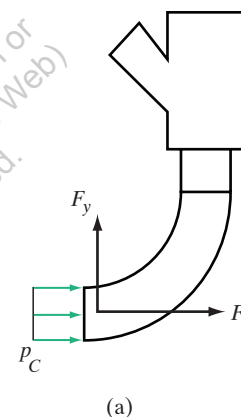
$$V_A = V_B = 18 \text{ m/s}$$

Thus

$$(18 \text{ m/s})^2 = 16 + 0.002 p_C$$

$$p_C = 154 \text{ kPa}$$

The free-body diagram is shown in Fig. a .



***6–40. Continued**

Linear momentum.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\rightarrow \Sigma F_x = 0 + (V_C)(\rho)(-V_C A_C) + \left(-V_A \frac{3}{5}\right) \rho V_A A_A + 0$$

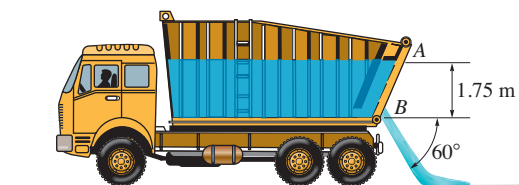
$$F_x + 154 (10^3)(\pi)(0.03 \text{ m})^2 = (4 \text{ m/s})(1000 \text{ kg/m}^3)(-4 \text{ m/s})(\pi)(0.03 \text{ m})^2 \\ - (18 \text{ m/s})\left(\frac{3}{5}\right)(1000 \text{ kg/m}^3)(18 \text{ m/s})(\pi)(0.01 \text{ m})^2$$

$$F_x = -542 \text{ N} = 542 \text{ N} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + V_A \left(\frac{4}{5}\right) \rho V_A A_A + V_B \rho V_B A_B$$

$$F_y = 18 \text{ m/s} \left(\frac{4}{5}\right)(1000 \text{ kg/m}^3)(18 \text{ m/s})(\pi)(0.01 \text{ m})^2 + 18 \text{ m/s} (1000 \text{ kg/m}^3)(18 \text{ m/s})(\pi)(0.01 \text{ m})^2 \\ F_y = 183 \text{ N} \uparrow \quad \textbf{Ans.}$$

6-41. The truck dumps water on the ground such that it flows from the truck through a 100-mm-wide opening at an angle of 60° . The length of the opening is 2 m. Determine the friction force that all the wheels of the truck must exert on the ground to keep the truck from moving at the instant the water depth in the truck is 1.75 m.



SOLUTION

We consider steady flow of an ideal fluid.

Bernoulli Equation. Since A and B are exposed to the atmosphere, $p_A = p_B = 0$. Since the water discharges from a large reservoir, $V_A \cong 0$. If the datum is at B , $z_A = 1.75$ m and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + 0 + 1.75 \text{ m} = 0 + \frac{V_B^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$V_B = 5.860 \text{ m/s}$$

The discharge at B is

$$Q_B = V_B A_B = (5.860 \text{ m/s})(2 \text{ m})(0.1 \text{ m})$$

$$= 1.172 \text{ m}^3/\text{s}$$

Take the control volume to be the dump truck and its contents. Its free-body diagram is shown in Fig. *a*.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

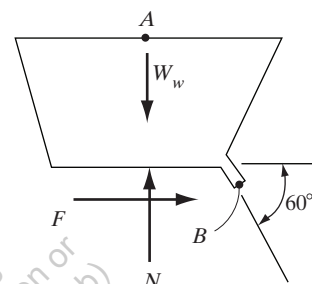
or

$$\sum F_x = (V_B)_x \rho (Q_B)$$

$$F = (5.860 \text{ m/s} \cos 60^\circ)(1000 \text{ kg/m}^3)(1.172 \text{ m}^3/\text{s})$$

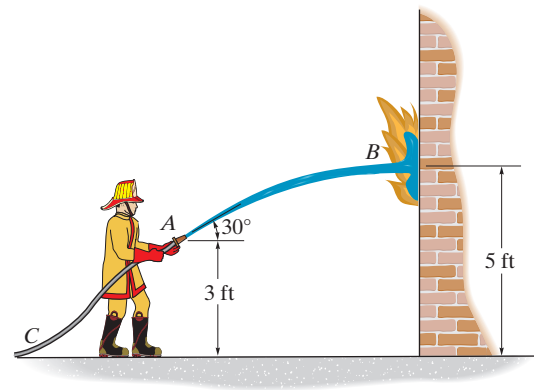
$$F = 3.43 \text{ kN}$$

Ans.



Ans:
3.43 kN

6-42. The fireman sprays a 2-in.-diameter jet of water from a hose at the burning building. If the water is discharged at $1.5 \text{ ft}^3/\text{s}$, determine the magnitude of the velocity of the water when it splashes on the wall. Also, find the normal reaction of both the fireman's feet on the ground. He has a weight of 180 lb. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_A A_A$$

$$1.5 \text{ ft}^3/\text{s} = V_A \left[\pi \left(\frac{1}{12} \text{ ft} \right)^2 \right]$$

$$V_A = 68.75 \text{ ft/s}$$

Bernoulli Equation. Since the water jet from A and B is free flow, $p_A = p_B = 0$. If the datum is at A, $z_A = 0$ and $z_B = 5 \text{ ft} - 3 \text{ ft} = 2 \text{ ft}$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(68.75 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0 = 0 + \frac{V_B^2}{2(32.2 \text{ ft/s}^2)} + 2 \text{ ft}$$

$$V_B = 67.81 \text{ ft/s} = 67.8 \text{ ft/s} \quad \text{Ans.}$$

Take the control volume to be the fireman and hose CA and the water within it. Its free-body diagram is shown in Fig. a. Here, the pressure at C, p_C , acts horizontally.

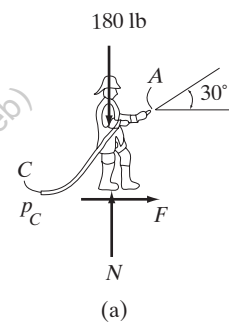
Linear Momentum. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho(Q)$$

$$N - 180 \text{ lb} = 68.75 \text{ ft/s} \sin 30^\circ \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (1.5 \text{ ft}^3/\text{s})$$

$$N = 279.93 \text{ lb} = 280 \text{ lb} \quad \text{Ans.}$$

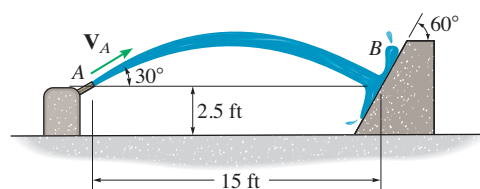


Ans:

$$V_B = 67.8 \text{ ft/s}$$

$$N = 280 \text{ lb}$$

6–43. The fountain sprays water in the direction shown. If the water is discharged at 30° from the horizontal, and the cross-sectional area of the water stream is approximately 2 in^2 , determine the normal force the water exerts on the wall at B .



SOLUTION

We consider steady flow of an ideal fluid.

Motion of Water Jet. Consider the horizontal motion by referring to Fig. *a*.

$$\pm s_x = (s_o)_x + (v_o)_x t$$

$$15 \text{ ft} = 0 + (V_A \cos 30^\circ)t$$

Referring to Fig. *a*, vertical motion gives

$$+\uparrow s_y = (s_o)_y + (v_o)_y t + \frac{1}{2}at^2$$

$$0 = 0 + (V_A \sin 30^\circ)t + \frac{1}{2}(-32.2 \text{ ft/s}^2)t^2$$

Solving Eqs. (1) and (2) yields

$$V_A = 23.62 \text{ ft/s} \quad t = 0.7334 \text{ s}$$

Bernoulli Equation. Since the water jet from A and B is free flow, $p_A = p_B = 0$. If the datum passes through A and B , $z_A = z_B = 0$.

$$\begin{aligned} \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \\ 0 + \frac{(23.62 \text{ ft/s})^2}{2g} + 0 &= 0 + \frac{V_B^2}{2g} + 0 \\ V_B &= 23.62 \text{ ft/s} \end{aligned}$$

The discharge of the flow is

$$Q = V_A A_A = (23.62 \text{ ft/s}) \left[(2 \text{ in}^2) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \right] = 0.3280 \text{ ft}^3/\text{s}$$

Take the control volume to be the portion of water striking the wall. Its free-body diagram is shown in Fig. *b*.

Linear Momentum. Here, \mathbf{V}_B is perpendicular to the wall. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

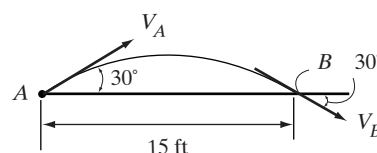
$$\Sigma F_n = 0 + (-V_B) \rho (-Q)$$

$$F_n = (23.62 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.3280 \text{ ft}^3/\text{s})$$

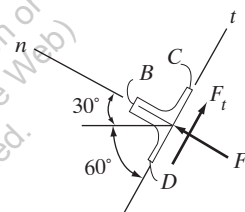
$$F_n = 15.0 \text{ lb}$$

Ans.

Ans:
15.0 lb

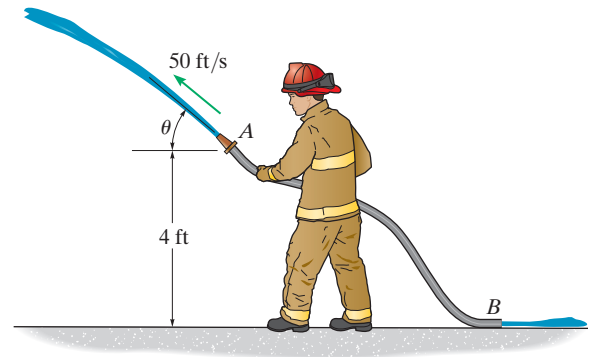


(a)



(b)

***6-44.** The 150-lb fireman is holding a hose that has a nozzle diameter of 1 in. If the nozzle velocity of the water is 50 ft/s, determine the resultant normal force acting on both the man's feet at the ground when $\theta = 30^\circ$. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



SOLUTION

The discharge of the flow is

$$Q_A = V_A A_A$$

$$Q_A = (50 \text{ ft/s}) \left[\pi \left(\frac{0.5}{12} \text{ ft} \right)^2 \right]$$

$$Q_A = 0.2727 \text{ ft}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

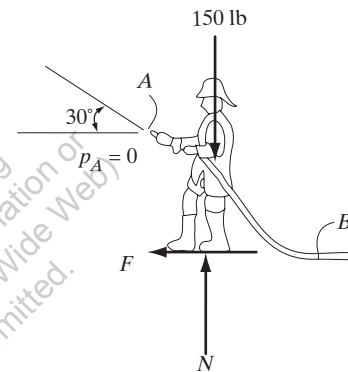
or

$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho Q$$

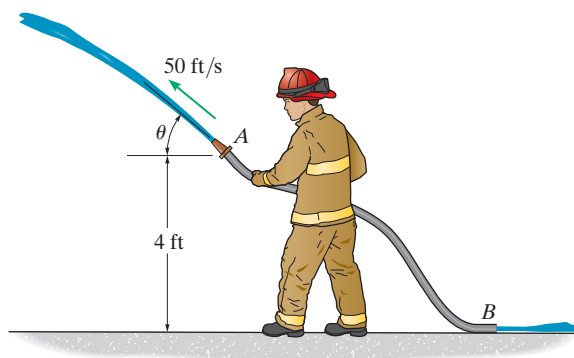
$$N - 150 \text{ lb} = 50 \text{ ft/s} \sin 30^\circ \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.2727 \text{ ft}^3/\text{s})$$

$$N = 163 \text{ lb}$$

Ans.



6-45. The 150-lb fireman is holding a hose that has a nozzle diameter of 1 in. If the velocity of the water is 50 ft/s, determine the resultant normal force acting on both the man's feet at the ground as a function of θ . Plot this normal reaction (vertical axis) versus θ for $0^\circ < \theta < 30^\circ$. Give values for increments of $\Delta\theta = 5^\circ$. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



SOLUTION

The discharge of the flow is

$$Q = V_A A_A = (50 \text{ ft/s}) \left[\pi \left(\frac{0.5}{12} \text{ ft} \right)^2 \right] = 0.2727 \text{ ft}^3/\text{s}$$

Here the flow is steady. Applying linear momentum equation.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

Writing the vertical scalar component of this equation by referring to the *FBD* of the control volume shown in Fig. *a*.

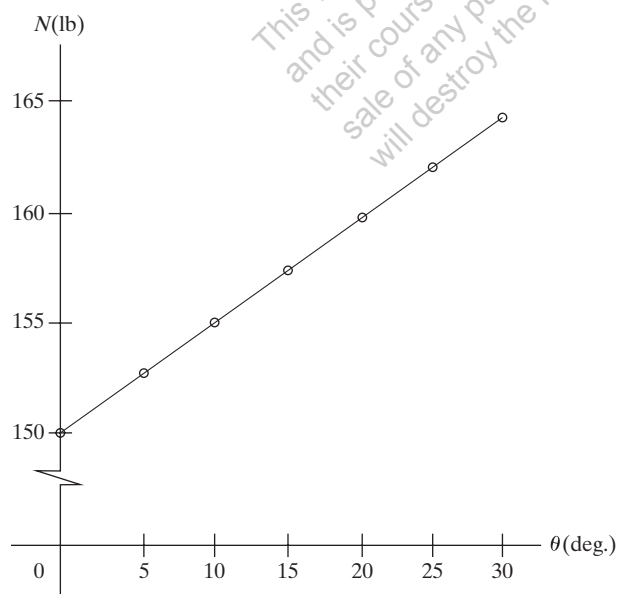
$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho (V_A A_A) + 0$$

$$N - 150 \text{ lb} = \left[(50 \text{ ft/s}) \sin \theta \right] \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.2727 \text{ ft}^3/\text{s})$$

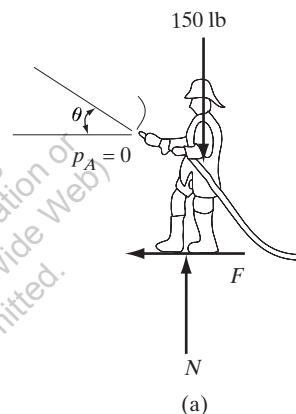
$$N = (150 + 26.4 \sin \theta) \text{ lb where } \theta \text{ is in deg.} \quad \mathbf{Ans.}$$

The plot of N vs θ is shown in Fig. *a*

$\theta(\text{deg.})$	0	5	10	15	20	25	30
$N(\text{lb})$	150	152.30	154.59	156.84	159.04	161.17	163.21



(b)

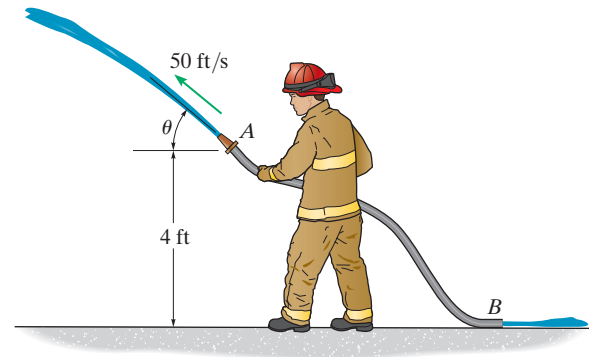


Note: See solution 6-44 regarding the effects of hose tension.

Ans:

$$N = (150 + 26.4 \sin \theta) \text{ lb}$$

6–46. The 150-lb fireman is holding a hose that has a nozzle diameter of 1 in. If the velocity of the water is 50 ft/s, determine the resultant normal force acting on both the man's feet at the ground if he holds the hose directly over his head at $\theta = 90^\circ$. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



SOLUTION

The flow is

$$Q = V_A A_A = (50 \text{ ft/s}) \left[\pi \left(\frac{0.5}{12} \text{ ft} \right)^2 \right] = 0.2727 \text{ ft}^3/\text{s}$$

Linear momentum

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

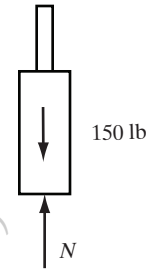
or

$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho(Q)$$

$$N - 150 \text{ lb} = (50 \text{ ft/s}) \left(\frac{62.4 \text{ lb}/\text{ft}^3}{32.2 \text{ ft}/\text{s}^2} \right) (0.2727 \text{ ft}^3/\text{s})$$

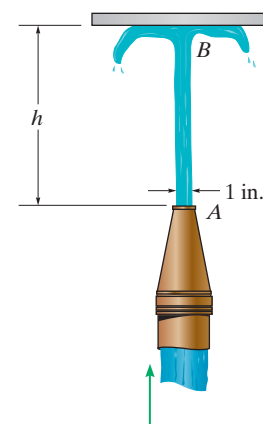
$$N = 176 \text{ lb}$$

Ans.



Ans:
176 lb

6–47. Water at A flows out of the 1-in.-diameter nozzle at 8 ft/s and strikes the 0.5-lb plate. Determine the height h above the nozzle at which the plate can be supported by the water jet.



SOLUTION

We consider steady flow of an ideal fluid.

Discharge.

$$Q = V_A A_A = (8 \text{ ft/s}) \left[\pi \left(\frac{0.5}{12} \text{ ft} \right)^2 \right] = 0.04363 \text{ ft}^3/\text{s}$$

Take the control volume of the plate and portion of water striking it. Its free-body diagram is shown in Fig. a . Since the jet has free flow, the pressure at any point is zero gauge.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$+\downarrow \Sigma F_y = 0 + (-V_B) \rho (-Q)$$

$$0.5 \text{ lb} = (-V_B) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-0.04363 \text{ ft}^3/\text{s})$$

$$V_B = 5.913 \text{ ft/s}$$

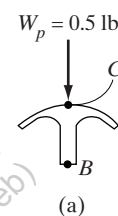
Bernoulli Equation. If the datum coincides with the horizontal line through A , $z_B = h$ and $z_A = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(8 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0 = 0 + \frac{(5.913 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + h$$

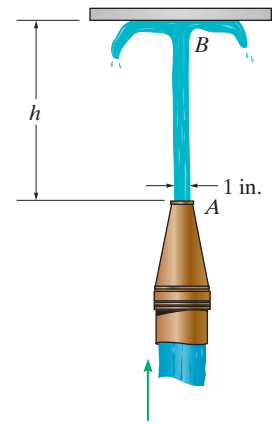
$$h = 0.4508 \text{ ft} = 0.451 \text{ ft}$$

Ans.



Ans:
0.451 ft

***6–48.** Water at A flows out of the 1-in.-diameter nozzle at 18 ft/s. Determine the weight of the plate that can be supported by the water jet $h = 2$ ft above the nozzle.



SOLUTION

We consider steady flow of an ideal fluid.

Bernoulli Equation. Since the jet is free flow, the pressure at any point is zero gauge. If the datum passes through A , $z_A = 0$ and $z_B = 2$ ft.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(18 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0 = 0 + \frac{V_B^2}{2(32.2 \text{ ft/s}^2)} + 2 \text{ ft}$$

$$V_B = 13.97 \text{ ft/s}$$

The discharge is

$$Q = V_A A_A = (18 \text{ ft/s}) \left[\pi \left(\frac{0.5}{12} \text{ ft} \right)^2 \right] = 0.09817 \text{ ft}^3/\text{s}$$

Take the control volume as the plate and a portion of water striking it. Its free-body diagram is shown in Fig. a .

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho \mathbf{V} dV + \int_{cs} \mathbf{V} \rho \mathbf{V} dA$$

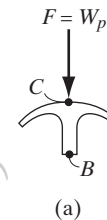
or

$$+\downarrow \Sigma F_y = 0 + (-V_B) \rho (-Q)$$

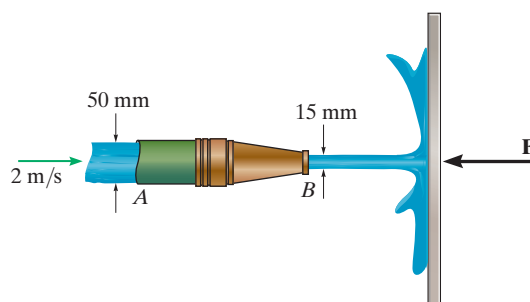
$$W_p = (-13.97 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-0.09817 \text{ ft}^3/\text{s})$$

$$W_p = 2.658 \text{ lb} = 2.66 \text{ lb}$$

Ans.



6-49. Water flows through the hose with a velocity of 2 m/s. Determine the force \mathbf{F} needed to keep the circular plate moving to the right at 2 m/s.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the plate and a portion of water striking it.

Continuity Equation.

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2 \text{ m/s}) [\pi (0.025 \text{ m})^2] + V_B [\pi (0.0075 \text{ m})^2] = 0$$

$$V_B = 22.22 \text{ m/s}$$

Relative Velocity. Relative to the control volume, the velocity at B is

$$\stackrel{+}{\rightarrow} V_{f/cs} = V_f - V_{cv} = 22.22 \text{ m/s} - 2 \text{ m/s} = 20.22 \text{ m/s}$$

Thus, the flow onto the plate is

$$Q_{f/cs} = V_{f/cs} A_B = (20.22 \text{ m/s}) [\pi (0.0075 \text{ m})^2] = 0.003574 \text{ m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

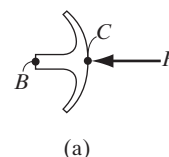
or

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0 + (V_{f/cs})_B \rho (-Q_{f/cs})$$

$$-F = (20.22 \text{ m/s}) (1000 \text{ kg/m}^3) (-0.003574 \text{ m}^3/\text{s})$$

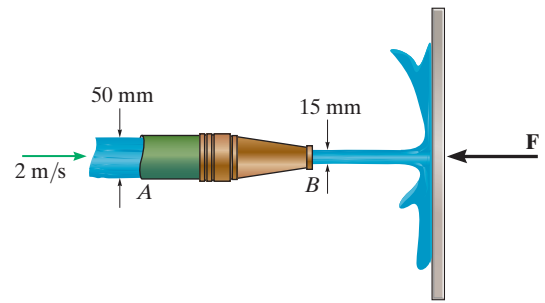
$$F = 72.3 \text{ N}$$

Ans.



Ans:
72.3 N

6–50. Water flows through the hose with a velocity of 2 m/s. Determine the force \mathbf{F} needed to keep the circular plate moving to the left at 2 m/s.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the plate and a portion of water striking it.

Continuity Equation.

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2 \text{ m/s}) [\pi (0.025 \text{ m})^2] + V_B [\pi (0.0075 \text{ m})^2] = 0$$

$$V_B = 22.22 \text{ m/s}$$

Relative Velocity. Relative to the control volume, the velocity at B is

$$\pm (V_{f/cs})_B = V_f - V_{cs} = 22.22 \text{ m/s} - (-2 \text{ m/s}) = 24.22 \text{ m/s}$$

Thus, the relative flow onto the plate is

$$Q_{f/cs} = (V_{f/cs})_B A_B = (24.22 \text{ m/s}) [\pi (0.0075 \text{ m})^2] = 0.004280 \text{ m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a ,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

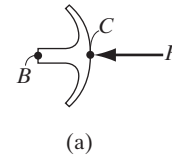
or

$$-F = 0 + (V_{f/cs})_B \rho (-Q_{f/cs})$$

$$-F = (24.22 \text{ m/s}) (1000 \text{ kg/m}^3) (-0.004280 \text{ m}^3/\text{s})$$

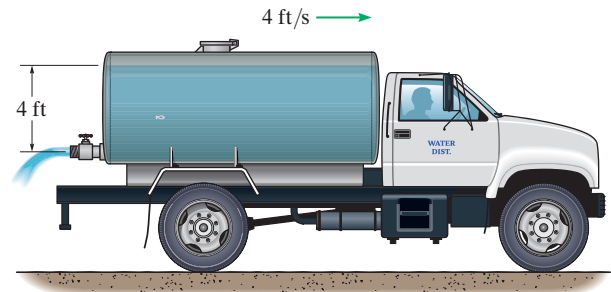
$$F = 104 \text{ N}$$

Ans.



Ans:
104 N

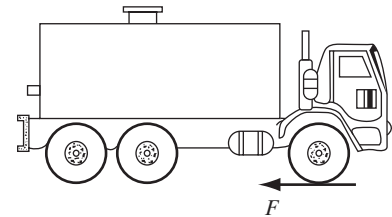
6-51. The large water truck releases water at the rate of $45 \text{ ft}^3/\text{min}$ through the 3-in.-diameter pipe. If the depth of the water in the truck is 4 ft, determine the frictional force the road has to exert on the tires to prevent the truck from rolling. How much force does the water exert on the truck if the truck is moving forward at a constant velocity of 4 ft/s and the flow is maintained at $45 \text{ ft}^3/\text{min}$?



SOLUTION

We consider steady flow of an ideal fluid.

For the case when the truck is required to be stationary, the control volume is the entire truck and its contents. Here the flow is steady. The *FBD* of the control volume is shown in Fig. *a*.



The discharge is

$$Q = \left(45 \frac{\text{ft}^3}{\text{min}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 0.75 \text{ ft}^3/\text{s}$$

Thus, the velocity at the outlet is

$$Q = V_{\text{out}} A_{\text{out}}; \quad 0.75 \text{ ft}^3/\text{s} = V_{\text{out}} \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] \quad V_{\text{out}} = 15.28 \text{ ft/s}$$

Applying the linear momentum equation by referring to Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho_w dV + \int_{\text{cs}} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along *x* axis,

$$\leftarrow \Sigma F_x = 0 + V_{\text{out}} \rho_w V_{\text{out}} A_{\text{out}}$$

$$F = (15.28 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.75 \text{ ft}^3/\text{s}) = 22.2 \text{ lb} \quad \text{Ans.}$$

For the case when the truck is moving with a constant velocity, the same control volume is considered, but it moves with this constant velocity. Then, the flow measured relative to the control volume is steady. From the discharge, the relative velocity at the outlet is

$$Q = (V_{\text{out/cs}}) A_{\text{out}}; \quad 0.75 \text{ ft}^3/\text{s} = V_{\text{out/cs}} \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right]$$

$$V_{\text{out/cs}} = 15.28 \text{ ft/s}$$

Applying the linear momentum equation by referring to Fig. *a*, but this time using the relative velocity,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V}_{w/cs} \rho_w dV + \int_{\text{cs}} \mathbf{V}_{w/cs} \rho_w \mathbf{V}_{w/cs} \cdot d\mathbf{A}$$

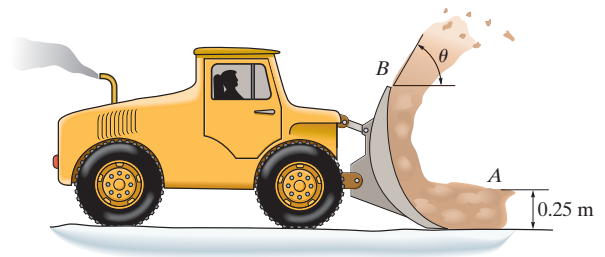
Applying the scalar component of this equation along *x* axis,

$$\leftarrow \Sigma F_x = 0 + (V_{\text{out/cs}}) (\rho_w) (V_{\text{out/cs}} A_{\text{out}})$$

$$F = (15.28 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.75 \text{ ft}^3/\text{s}) = 22.2 \text{ lb} \quad \text{Ans.}$$

Ans:
22.2 lb

***6-52.** A plow located on the front of a truck scoops up a liquid slush at the rate of $12 \text{ ft}^3/\text{s}$ and throws it off perpendicular to its motion, $\theta = 90^\circ$. If the truck is traveling at a constant speed of 14 ft/s , determine the resistance to motion caused by the shoveling. The specific weight of the slush is $\gamma_s = 5.5 \text{ lb/ft}^3$.



SOLUTION

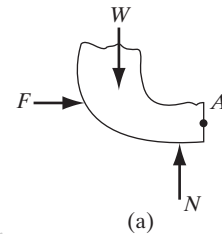
We consider steady flow of an ideal fluid.

Take the slush in contact with the blade as the control volume.

Relative Velocity. Since the slush is at rest before it enters control volume, then the velocity at A relative to control volume is

$$\rightarrow (V_{f/cs})_A = V_f - V_{cs} = 0 - 14 \text{ ft/s} = 14 \text{ ft/s} \leftarrow$$

Linear Momentum. Here, $Q_{f/cs} = 12 \text{ ft}^3/\text{s}$ and $(V_{f/cs})_B = (V_{f/cs})_A = 14 \text{ ft/s}$. Referring to the free-body diagram of the control volume in Fig. a ,



$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

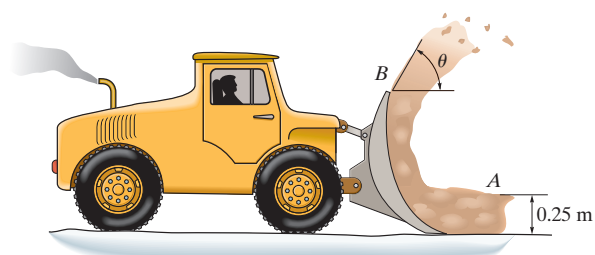
or

$$\rightarrow \Sigma F_x = 0 + (-V_A) \rho (-Q)$$

$$\begin{aligned} -F &= (-14 \text{ ft/s}) \left(\frac{5.5 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-12 \text{ ft}^3/\text{s}) \\ F &= -28.69 \text{ lb} = 28.7 \text{ lb} \leftarrow \end{aligned}$$

Ans.

6-53. The truck is traveling forward at 5 m/s, shoveling a liquid slush that is 0.25 m deep. If the slush has a density of 125 kg/m³ and is thrown upwards at an angle of $\theta = 60^\circ$ from the 3-m-wide blade, determine the traction force of the wheels on the road necessary to maintain the motion. Assume that the slush is thrown off the shovel at the same rate as it enters the shovel.



SOLUTION

We consider steady flow of an ideal fluid.

Take the slush in contact with the blade as the control volume.

Relative Velocity. Since the slush is at rest before it enters the control volume, then the velocity at A relative to the control volume is

$$\vec{V}_{f/cs} = V_f - V_{cs} = 0 - 5 \text{ m/s} = 5 \text{ m/s} \leftarrow$$

Thus, the flow rate of snow onto the shovel is

$$Q_{f/cs} = V_{f/cs} A_A = (5 \text{ m/s}) [0.25 \text{ m}(3 \text{ m})] = 3.75 \text{ m}^3/\text{s}$$

Linear Momentum. Here, $(V_{f/cs})_B = (V_{f/cs})_A = 5 \text{ ft/s}$. Referring to the free-body diagram of the control volume in Fig. a ,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

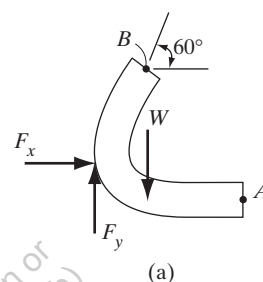
or

$$\vec{\Sigma} F_x = 0 + (-V_{f/cs})_{A,x} \rho (-Q_{f/cs}) + (V_{f/cs})_{B,x} \rho (Q_{f/cs})$$

$$F_x = 0 + (-5 \text{ m/s})(125 \text{ kg/m}^3)(-3.75 \text{ m}^3/\text{s}) + (5 \text{ m/s} \cos 60^\circ)(125 \text{ kg/m}^3)(3.75 \text{ m}^3/\text{s})$$

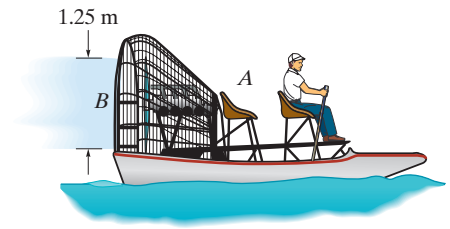
$$F_x = 3.52 \text{ kN}$$

Ans.



Ans:
3.52 kN

6-54. The boat is powered by the fan, which develops a slipstream having a diameter of 1.25 m. If the fan ejects air with an average velocity of 40 m/s, measured relative to the boat, and the boat is traveling with a constant velocity of 8 m/s, determine the force the fan exerts on the boat. Assume that the air has a constant density of $\rho_a = 1.22 \text{ kg/m}^3$ and that the entering air at A is essentially at rest relative to the ground.



SOLUTION

We consider steady flow of an ideal fluid.

Relative Velocity. Since the air is at rest before it enters the control volume, then the inlet velocity relative to the control volume is

$$\pm (V_{f/cs})_A = V_f - V_{cs} = 0 - 8 \text{ m/s} = 8 \text{ m/s} \leftarrow$$

The outlet velocity relative to the control volume is $(V_{f/cv})_{out} = 40 \text{ m/s}$. Then, the flow of air in and out of the fan is

$$Q_{f/cs} = (V_{f/cs})_B A_B = (40 \text{ m/s}) [\pi (0.625 \text{ m})^2] = 49.09 \text{ m}^3/\text{s}$$

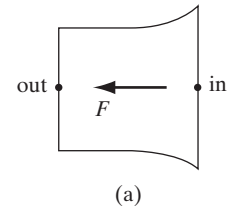
Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

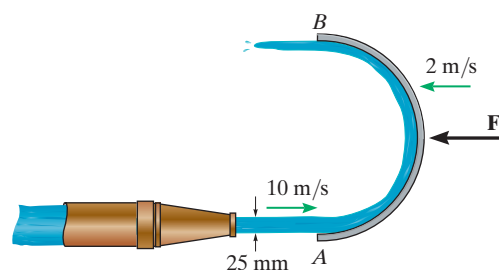
$$\begin{aligned} \leftarrow \Sigma F_x &= 0 + (V_{f/cs})_A \rho (-Q_{f/cs}) + (V_{f/cs})_B \rho (Q_{f/cs}) \\ &= (1.22 \text{ kg/m}^3) [(8 \text{ m/s})(-49.09 \text{ m}^3/\text{s}) + (40 \text{ m/s})(49.09 \text{ m}^3/\text{s})] \\ &= 1.92 \text{ kN} \end{aligned}$$

Ans.



Ans:
1.92 kN

6-55. A 25-mm-diameter stream flows at 10 m/s against the blade and is deflected 180° as shown. If the blade is moving to the left at 2 m/s, determine the horizontal force F of the blade on the water.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the blade.

Relative Velocity. Relative to the control volume, the velocity at A is

$$\xrightarrow{+} (V_{f/cs})_A = V_f - V_{cs} = 10 \text{ m/s} - (-2 \text{ m/s}) = 12 \text{ m/s} \rightarrow$$

Thus, the flow rate onto the vane is

$$Q_{f/cs} = (V_{f/cs})_A A_A = (12 \text{ m/s}) [\pi(0.0125 \text{ m})^2] = 0.005890 \text{ m}^3/\text{s}$$

Linear Momentum. Here, $(V_{f/cs})_B = (V_{f/cs})_A = 12 \text{ m/s}$ (Bernoulli equation).

Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

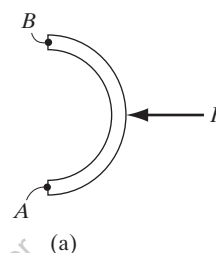
or

$$\xrightarrow{+} \Sigma F_x = 0 + (V_{f/cs})_A \rho (-Q_{f/cs}) + (-V_{f/cs})_B \rho (Q_{f/cs})$$

$$-F = (1000 \text{ kg/m}^3) [(12 \text{ m/s})(-0.005890 \text{ m}^3/\text{s}) + (-12 \text{ m/s})(0.005890 \text{ m}^3/\text{s})]$$

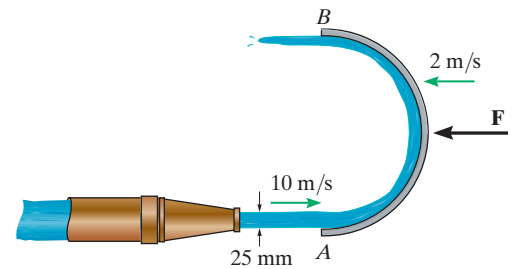
$$F = 141 \text{ N}$$

Ans.



Ans:
141 N

***6-56.** Solve Prob. 6-55 if the blade is moving *to the right* at 2 m/s. At what speed must the blade be moving to the right to reduce the force F to zero?



SOLUTION

Consider the control volume as the water on the blade. The velocity of the water at A relative to the control volume is

$$(\rightarrow)(V_{f/cs})_A = 10 \text{ m/s} - 2 \text{ m/s} = 8 \text{ m/s} \rightarrow$$

To satisfy the Bernoulli's equation, $(V_{f/cs})_B = 8 \text{ m/s} \leftarrow$ for small elevations. The flow is steady relative to control volume.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V}_{f/cv} \rho dV + \int_{cs} \mathbf{V}_{f/cs} \rho \mathbf{V}_{f/cs} \cdot d\mathbf{A}$$

Writing the horizontal scalar component of this equation by referring to the FBD of the control volume shown in Fig. a

$$\rightarrow \Sigma F_x = 0 + (V_{f/cs})_A \rho [-(V_{f/cs})_A A_A] + [-(V_{f/cs})_B] \rho [(V_{f/cs})_B A_B]$$

However, $Q = (V_{f/cs})_A A_A = (V_{f/cs})_B A_B$ and $(V_{f/cs})_B = (V_{f/cs})_A$. Then

$$-F = -2\rho (V_{f/cs})_A [(V_{f/cs})_A A_A]$$

$$F = 2\rho (V_{f/cs})_A^2 A_A \quad (1)$$

$$F = 2(1000 \text{ kg/m}^3) (8 \text{ m/s})^2 [\pi(0.0125 \text{ m})^2]$$

$$= 62.8 \text{ N}$$

Ans.

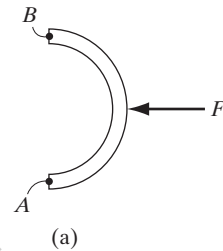
By inspecting Eq (1), $F = 0$ if $(V_{f/cs})_A = 0$. Then

$$\rightarrow (V_{f/cs})_A = V_w - V_b$$

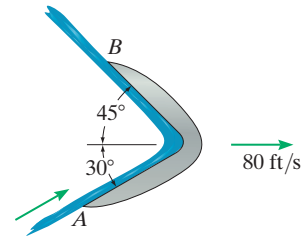
$$0 = 10 \text{ m/s} - V_b$$

$$V_b = 10 \text{ m/s} \rightarrow$$

Ans.



6-57. The vane is moving at 80 ft/s when a jet of water having a velocity of 150 ft/s enters at A. If the cross-sectional area of the jet is 1.5 in², and it is diverted as shown, determine the horsepower developed by the water on the blade. 1 hp = 550 ft · lb/s.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the blade.

Relative Velocity. Applying the relative velocity equation to determine the velocity relative to the vane, $V_{A/cs}$, and the angle θ , of the jet in a stationary frame,

$$\mathbf{V}_{A/cs} = \mathbf{V}_A - \mathbf{V}_{cs}$$

$$(\rightarrow) \quad V_{A/cs} \cos 30^\circ = 150 \cos \theta - 80 \quad (1)$$

$$(+\uparrow) \quad V_{A/cs} \sin 30^\circ = 150 \sin \theta \quad (2)$$

Solving Eqs. (1) and (2),

$$\theta = 14.53^\circ$$

$$V_{A/cs} = 75.29 \text{ ft/s}$$

Here, $(V_{f/cs})_A = V_{A/cs} = 75.29 \text{ ft/s}$. Thus, the relative flow rate at the vane is

$$Q_{f/cs} = (V_{f/cs})_A A = (75.29 \text{ ft/s}) \left[1.5 \text{ in}^2 \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \right] = 0.7842 \text{ ft}^3/\text{s}$$

Linear Momentum. Here, $(V_{f/cs})_A = V_{A/cs} = 75.29 \text{ ft/s}$ (Bernoulli equation).

Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\rightarrow \Sigma F_x = 0 + (V_{A/cs})_x \rho (-Q_{f/cs}) + (-V_{B/cs})_x \rho (Q_{f/cs})$$

$$-F_x = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) [(75.29 \text{ ft/s} \cos 30^\circ)(-0.7842 \text{ ft}^3/\text{s}) + (-75.29 \text{ ft/s} \cos 45^\circ)(0.7842 \text{ ft}^3/\text{s})]$$

$$F_x = 179.99 \text{ lb}$$

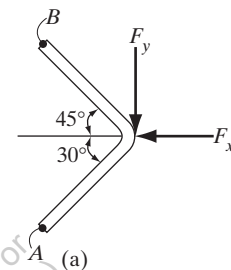
Thus, the power of the water jet can be determined from

$$\dot{W} = \mathbf{F} \cdot \mathbf{V} = F_x V = (179.99 \text{ lb})(80 \text{ ft/s})$$

$$= \left(14399.40 \frac{\text{ft} \cdot \text{lb}}{\text{s}} \right) \left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}} \right)$$

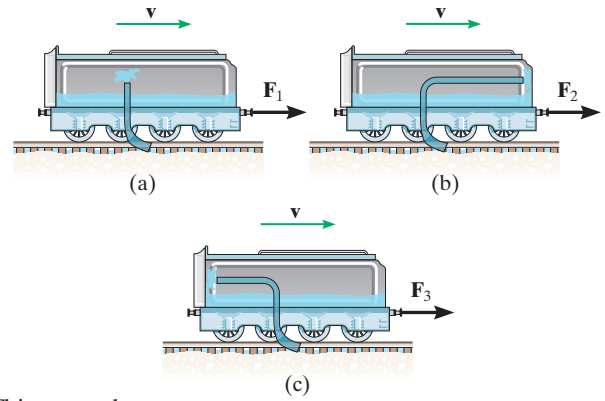
$$= 26.2 \text{ hp}$$

Ans.



Ans:
26.2 hp

6–58. The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity \mathbf{v} for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w .



SOLUTION

The control volume considered consists of the car and the scoop. This control volume has only inlet control surface (the scoop) but no outlet control surface. Since this same control volume can be used for cases a , b , and c , $F_1 = F_2 = F_3 = F$. Here,

$$\dot{m}_a = \rho_w VA \quad \dot{m}_f = 0 \quad V_e = 0 \quad \frac{dV_{cv}}{dt} = 0 \text{ (constant velocity)}$$

Along x axis,

$$\sum F_x = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$F = 0 + \rho_w VAV = \rho_w AV^2$$

Therefore

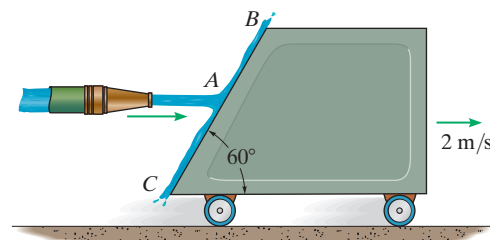
$$F_1 = F_2 = F_3 = \rho_w AV^2$$

Ans.

Ans:

$$F_1 = F_2 = F_3 = \rho_w AV^2$$

6-59. Flow from the water stream strikes the inclined surface of the cart. Determine the power produced by the stream if, due to rolling friction, the cart moves to the right with a constant velocity of 2 m/s. The discharge from the 50-mm-diameter nozzle is $0.04 \text{ m}^3/\text{s}$. One-fourth of the discharge flows down the incline, and three-fourths flows up the incline.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as a portion of water striking the cart.

Relative Velocity. The velocity of the jet at A is

$$V_A = \frac{Q}{A_A} = \frac{0.04 \text{ m}^3/\text{s}}{\pi(0.025 \text{ m})^2} = 20.37 \text{ m/s}$$

Thus, the velocity at A relative to the control volume is

$$\vec{V}_{A/cs} = V_A - V_{cs} = 20.37 \text{ m/s} - 2 \text{ m/s} = 18.37 \text{ m/s}$$

Here, $V_{B/cs} = V_{C/cs} = V_{A/cs} = 18.37 \text{ m/s}$ can be determined using the Bernoulli equation and neglecting the elevation change. Thus, the relative flow at A, B, and C are

$$Q_{A/cs} = V_{A/cs} A_A = (18.37 \text{ m/s})[\pi(0.025 \text{ m})^2] = 0.03607 \text{ m}^3/\text{s}$$

$$Q_{B/cs} = \frac{3}{4}(Q_{A/cs}) = \frac{3}{4}(0.03607 \text{ m}^3/\text{s}) = 0.02705 \text{ m}^3/\text{s}$$

$$Q_{C/cs} = \frac{1}{4}(Q_{A/cs}) = \frac{1}{4}(0.03607 \text{ m}^3/\text{s}) = 0.009018 \text{ m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

or

$$\vec{\Sigma F}_x = \rho [Q_{B/cs}(V_{B/cs})_x + Q_{C/cs}(V_{C/cs})_x - Q_{A/cs}(V_{A/cs})_x]$$

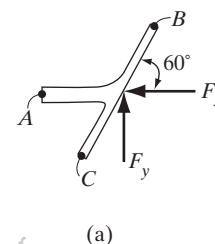
$$-F_x = (1000 \text{ kg/m}^3) [(0.02705 \text{ m}^3/\text{s})(18.37 \text{ m/s} \cos 60^\circ) + (0.009018 \text{ m}^3/\text{s})(-18.37 \text{ m/s} \cos 60^\circ) - (0.03607 \text{ m}^3/\text{s})(18.37 \text{ m/s})]$$

$$F_x = 497.04 \text{ N}$$

Thus, the power of the jet stream can be determined from

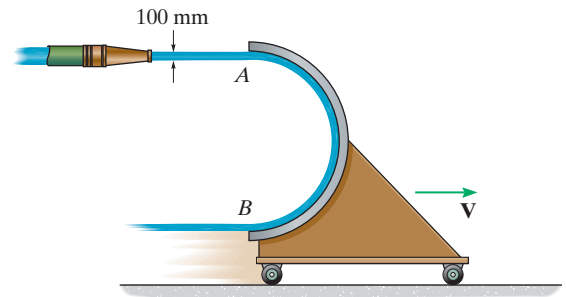
$$\begin{aligned} \dot{W} &= \mathbf{F} \cdot \mathbf{V} = F_x V = (497.04 \text{ N})(2 \text{ m/s}) \\ &= 994.09 \text{ W} = 994 \text{ W} \end{aligned}$$

Ans.



Ans:
994 W

***6–60.** Water flows at $0.1 \text{ m}^3/\text{s}$ through the 100-mm-diameter nozzle and strikes the vane on the 150-kg cart, which is originally at rest. Determine the velocity of the cart 3 seconds after the jet strikes the vane.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the cart.

Relative Velocity. The velocity of the jet at A is

$$V_A = \frac{Q}{A_A} = \frac{0.1 \text{ m}^3/\text{s}}{\pi(0.05 \text{ m})^2} = 12.73 \text{ m/s}$$

Thus, the velocity at A relative to the control volume is

$$\pm \quad V_{A/cs} = V_A - V_{cs} = 12.73 - V \rightarrow$$

Here, $(V_{f/cs})_A = (V_{f/cs})_B = V_{A/cs}$. Thus, the relative flow rate onto the vane is

$$Q_{f/cs} = (V_{f/cs})_A A_A = (12.73 - V)[\pi(0.05 \text{ m})^2] = 2.5(10^{-3})\pi(12.73 - V)$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\begin{aligned} \pm \Sigma F_x &= 0 + (-V_{f/cs})_B \rho (Q_{f/cs}) + (V_{f/cs})_A \rho (-Q_{f/cs}) \\ -F &= 1000 \text{ kg/m}^3 [-(12.73 - V)(2.5(10^{-3})\pi(12.73 - V)) + (12.73 - V)(-2.5(10^{-3})\pi(12.73 - V))] \\ F &= 5\pi(12.73 - V)^2 \end{aligned}$$

Equation of Motion. Referring to the free-body diagram of the cart in Fig. b,

$$\pm \Sigma F_x = ma; \quad 5\pi(12.73 - V)^2 = (150 \text{ kg}) \left(\frac{dV}{dt} \right)$$

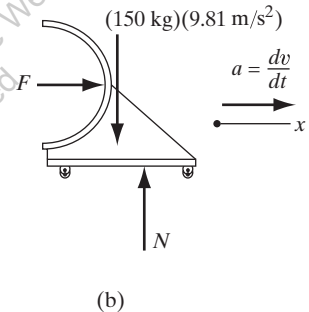
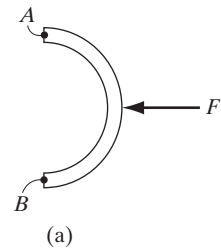
$$\int_0^{3s} dt = \frac{30}{\pi} \int_0^V \frac{dV}{(12.73 - V)^2}$$

$$t|_0^{3s} = \frac{30}{\pi} \left(\frac{1}{12.73 - V} \right) \bigg|_0^V$$

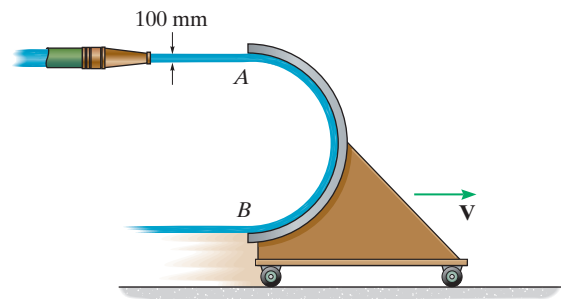
$$3 = \frac{30}{\pi} \left(\frac{1}{12.73 - V} - \frac{1}{12.73} \right)$$

$$V = 10.19 \text{ m/s} = 10.2 \text{ m/s}$$

Ans.



6-61. Water flows at $0.1 \text{ m}^3/\text{s}$ through the 100-mm-diameter nozzle and strikes the vane on the 150-kg cart, which is originally at rest. Determine the acceleration of the cart when it attains a velocity of 2 m/s .



SOLUTION

The velocity of the jet at A can be determined from the discharge.

$$Q = V_A A_A; \quad 0.1 \text{ m}^3/\text{s} = V_A [\pi(0.05 \text{ m})^2] \quad V_A = 12.73 \text{ m/s}$$

The velocity at A relative to the control volume is

$$\pm \rightarrow (V_{f/cs})_A = V_A - V_{cs} = (12.73 - V) \text{ m/s} \rightarrow$$

To satisfy Bernoulli's equation $(V_{f/cs})_B = (12.73 - V) \text{ m/s} \leftarrow$ for small equations
The flow is steady relative to control volume.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V}_{f/cv} \rho dV + \int_{cs} \mathbf{V}_{f/cs} \rho \mathbf{V}_{f/cs} \cdot d\mathbf{A}$$

Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume shown in Fig. *a*,

$$\pm \rightarrow \Sigma F_x = 0 + (V_{f/cs})_A \rho [-(V_{f/cs})_A A_A] + [-(V_{f/cs})_B] \rho [(V_{f/cs})_B A_B]$$

However, $Q = (V_{f/cs})_A A_A = (V_{f/cs})_B A_B$ and $(V_{f/cs})_B = (V_{f/cs})_A$. Then

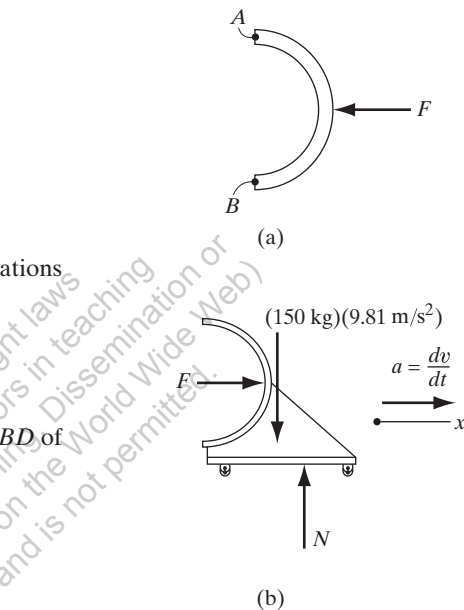
$$\begin{aligned} -F &= -2\rho(V_{f/cs})_A [(V_{f/cs})_A A_A] \\ F &= 2\rho(V_{f/cs})_A^2 A_A \\ F &= 2(1000 \text{ kg/m}^3)(12.73 - V)^2 [\pi(0.05 \text{ m})^2] \\ &= [5\pi(12.73 - V)^2] \text{ N} \end{aligned}$$

Referring to the *FBD* of the cart Fig. *b*,

$$\begin{aligned} \pm \rightarrow \Sigma F_x &= ma; \quad 5\pi(12.73 - V)^2 = 150a \\ a &= \left[\frac{\pi}{30}(12.73 - V)^2 \right] \text{ m/s}^2 \end{aligned}$$

When $V = 2 \text{ m/s}$,

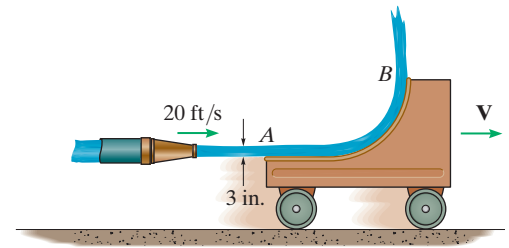
$$a = \frac{\pi}{30}(12.73 - 2)^2 = 12.06 \text{ m/s}^2 = 12.1 \text{ m/s}^2$$



Ans.

Ans:
 12.1 m/s^2

6-62. Determine the rolling resistance on the wheels if the cart moves to the right with a constant velocity of $V_c = 4 \text{ ft/s}$ when the vane is struck by the water jet. The jet flows from the nozzle at 20 ft/s and has a diameter of 3 in.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the cart.

Relative Velocity. The velocity at A relative to the control volume is

$$\pm \rightarrow V_{A/cs} = V_A - V_{cv} = 20 \text{ ft/s} - 4 \text{ ft/s} = 16 \text{ ft/s}$$

Here, $(V_{f/cs})_{in} = (V_{f/cs})_B = V_{A/cs} = 16 \text{ ft/s}$ (Bernoulli equation). Thus, the relative flow rate onto the vane is

$$Q_{f/cs} = (V_{f/cs})_A A_A = (16 \text{ ft/s}) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] = 0.7856 \text{ ft}^3/\text{s}$$

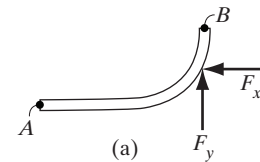
Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a ,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

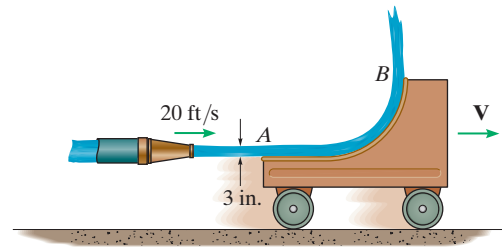
$$\begin{aligned} \pm \rightarrow \Sigma F_x &= 0 + (V_{A/cs}) \rho (-Q_{f/cs}) \\ -F_x &= (16 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-0.7854 \text{ ft}^3/\text{s}) \\ F_x &= 24.35 \text{ lb} = 24.4 \text{ lb} \end{aligned}$$

Ans.



Ans:
24.4 lb

6-63. Determine the velocity of the 50-lb cart in 3 s starting from rest if a stream of water, flowing from the nozzle at 20 ft/s, strikes the vane and is deflected upwards. The stream has a diameter of 3 in. Neglect the rolling resistance of the wheels.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the cart.

Relative Velocity. The velocity of the jet at A relative to the control volume is

$$\pm \quad V_{A/cs} = V_A - V_{cs} = (20 - V) \text{ ft/s}$$

Here, $(V_{f/cs})_{in} = (V_{f/cs})_B = V_{A/cs}$. Thus, the relative flow rate onto the vane is

$$Q_{f/cs} = (V_{f/cs})_A A_A = (20 - V) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] = 0.015625\pi(20 - V)$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\pm \quad \Sigma F_x = 0 + (V_{A/cs})\rho(-Q_{f/cs})$$

$$-F_x = (20 - V) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.015625\pi(20 - V))$$

$$F_x = 0.09513(20 - V)^2$$

Equation of Motion. Referring to the free-body diagram of the cart in Fig. b,

$$\pm \quad \Sigma F_x = ma; \quad 0.09513(20 - V)^2 = \left(\frac{50 \text{ lb}}{32.2 \text{ ft/s}^2} \right) \frac{dV}{dt}$$

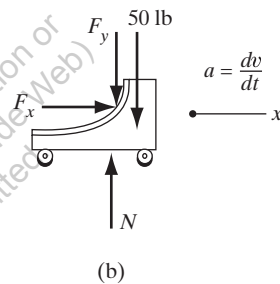
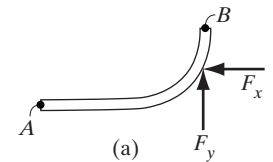
$$0.06126 \int_0^{3 \text{ s}} dt = \int_0^V \frac{dV}{(20 - V)^2}$$

$$0.06126(t) \Big|_0^{3 \text{ s}} = \left(\frac{1}{20 - V} \right) \Big|_0^V$$

$$0.1838 = \left(\frac{1}{20 - V} \right) - \frac{1}{20}$$

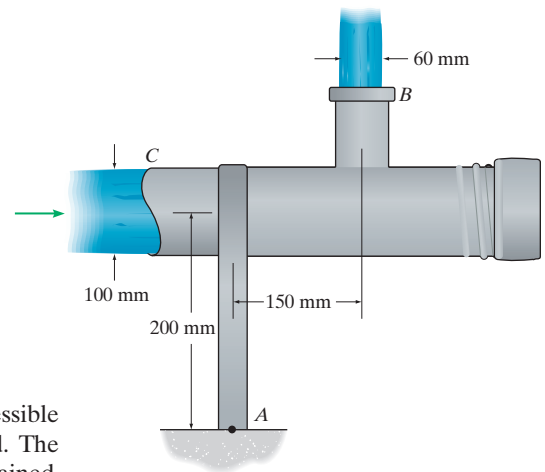
$$V = 15.72 \text{ ft/s} = 15.7 \text{ ft/s}$$

Ans.



Ans:
15.7 ft/s

***6-64.** Water flows through the Tee fitting at $0.02 \text{ m}^3/\text{s}$. If the water exits the fitting at B to the atmosphere, determine the horizontal and vertical components of force, and the moment that must be exerted on the fixed support at A , in order to hold the fitting in equilibrium. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The control volume consists of the fitting the fixed support and the water contained. From the discharge,

$$Q = V_C A_C; \quad 0.02 \text{ m}^3/\text{s} = V_A [\pi(0.05 \text{ m})^2] \quad V_C = 2.546 \text{ m/s}$$

$$Q = V_B A_B; \quad 0.02 \text{ m}^3/\text{s} = V_B [\pi(0.03 \text{ m})^2] \quad V_B = 7.074 \text{ m/s}$$

Applying Bernoulli's equation between C and B , with $p_B = p_{\text{atm}} = 0$.

$$\frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_C}{9810 \text{ N/m}^3} + \frac{(2.546 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = 0 + \frac{(7.074 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_C = 21.775(10^3) \text{ N/m}^2$$

Then the pressure force on inlet control surface C is

$$F_C = p_C A_C = [21.775(10^3) \text{ N/m}^2] [\pi(0.05 \text{ m})^2] = 171.02 \text{ N}$$

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

writing the scalar component of this equation along the x and y axes by referring to the free-body diagram, Fig. a

$$(\rightarrow) \Sigma F_x = 0 + V_C \rho_w (-V_C A_C)$$

$$A_x + 171.02 \text{ N} = (2.546 \text{ m/s})(1000 \text{ kg/m}^3)(-0.02 \text{ m}^3/\text{s})$$

$$A_x = -221.95 \text{ N} = 222 \text{ N} \leftarrow$$

Ans.

$$(+\uparrow) \Sigma F_y = 0 + V_B \rho_w (V_B A_B)$$

$$A_y = (7.074 \text{ m/s})(1000 \text{ kg/m}^3)(0.02 \text{ m}^3/\text{s})$$

$$= 141.47 \text{ N} = 141 \text{ N}$$

Ans.

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

writing the scalar component of this equation about point A by referring to Fig. a ,

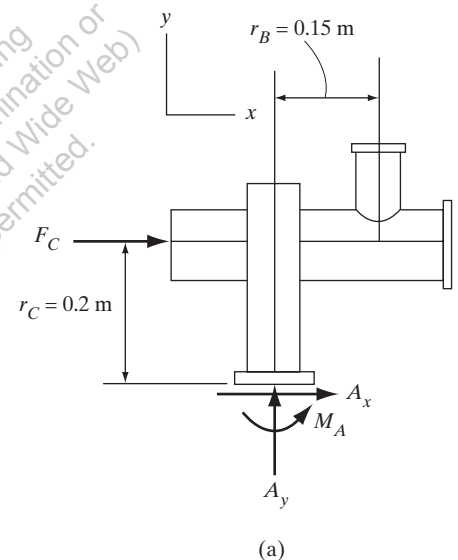
$$\zeta \downarrow + \Sigma M_A = 0 + r_B V_B \rho_w (V_B A_B) - r_C V_C \rho_w (-V_C A_C)$$

$$M_A - (171.02 \text{ N})(0.2 \text{ m}) = (0.15 \text{ m})(7.074 \text{ m/s})(1000 \text{ kg/m}^3)(0.02 \text{ m}^3/\text{s})$$

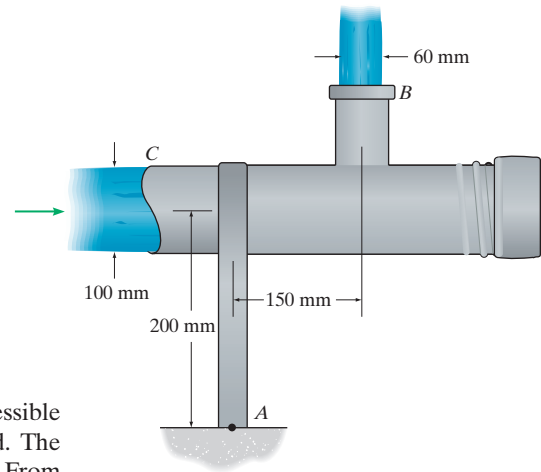
$$-(0.2 \text{ m})(2.546 \text{ m/s})(1000 \text{ kg/m}^3)(-0.02 \text{ m}^3/\text{s})$$

$$M_A = 65.61 \text{ N} \cdot \text{m} = 65.6 \text{ N} \cdot \text{m}$$

Ans.



6–65. Water flows through the Tee fitting at $0.02 \text{ m}^3/\text{s}$. If the pipe at B is extended and the pressure in the pipe at B is 75 kPa , determine the horizontal and vertical components of force, and the moment that must be exerted on the fixed support at A , to hold the fitting in equilibrium. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The control volume consists of the fitting, fixed support and the contained water. From the discharge,

$$Q = V_C A_C; \quad 0.02 \text{ m}^3/\text{s} = V_C [\pi(0.05 \text{ m})^2] \quad V_C = 2.546 \text{ m/s}$$

$$Q = V_B A_B; \quad 0.02 \text{ m}^3/\text{s} = V_B [\pi(0.03 \text{ m})^2] \quad V_B = 7.074 \text{ m/s}$$

Applying Bernoulli's equation between A and B ,

$$\frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_C}{9810 \text{ N/m}^3} + \frac{(2.546 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = \frac{75(10^3) \text{ N/m}^2}{9810 \text{ N/m}^3} + \frac{(7.074 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_C = 96.775(10^3) \text{ N/m}^2$$

Then the pressure forces on the inlet and outlet control surfaces at C and B are

$$F_C = p_C A_C = [96.775(10^3) \text{ N/m}^2] [\pi(0.05 \text{ m})^2] = 760.07 \text{ N}$$

$$F_B = p_B A_B = [75(10^3) \text{ N/m}^2] [\pi(0.03 \text{ m})^2] = 212.06 \text{ N}$$

Applying linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_w dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

writing the scalar component of this equation along the x and y axes by referring to the free-body diagram, Fig. a ,

$$(\rightarrow) \Sigma F_x = 0 + V_A \rho_w (-V_A A_A)$$

$$A_x + 760.07 \text{ N} = (2.546 \text{ m/s})(1000 \text{ kg/m}^3)(-0.02 \text{ m}^3/\text{s})$$

$$A_x = -811 \text{ N} = 811 \text{ N} \leftarrow$$

Ans.

$$(+\uparrow) \Sigma F_y = 0 + V_B \rho_w (V_B A_B)$$

$$A_y - 212.06 \text{ N} = (7.074 \text{ m/s})(1000 \text{ kg/m}^3)(0.02 \text{ m}^3/\text{s})$$

$$A_y = 353.53 \text{ N} = 354 \text{ N} \uparrow$$

Ans.

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho_w dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

writing the scalar component of this equation about point A by referring to Fig. a ,

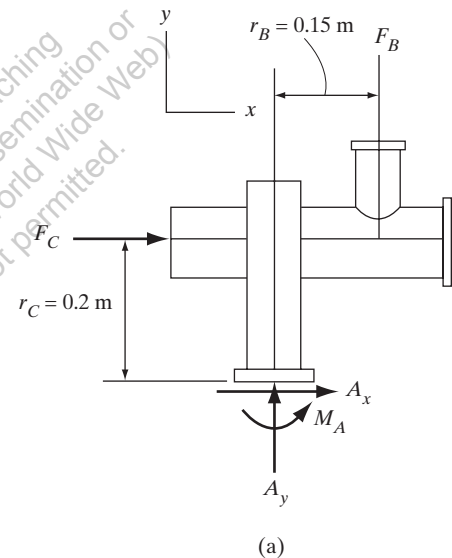
$$\zeta \downarrow + \Sigma M_A = 0 + r_B V_B \rho_w (V_B A_B) - r_C V_C \rho_w (-V_C A_C)$$

$$M_A - (760.07 \text{ N})(0.2 \text{ m}) - (212.06 \text{ N})(0.15 \text{ m}) = (0.15 \text{ m})(7.074 \text{ m/s})(1000 \text{ kg/m}^3)(0.02 \text{ m}^3/\text{s})$$

$$- (0.2 \text{ m})(2.546 \text{ m/s})(1000 \text{ kg/m}^3)(-0.02 \text{ m}^3/\text{s})$$

$$M_A = 215.22 \text{ N} \cdot \text{m} = 215 \text{ N} \cdot \text{m} \curvearrowright$$

Ans.



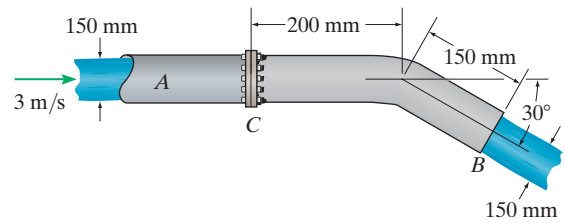
Ans:

$$A_x = 811 \text{ N}$$

$$A_y = 354 \text{ N}$$

$$M_A = 215 \text{ N} \cdot \text{m}$$

6-66. Water flows into the bend fitting with a velocity of 3 m/s. If the water exists at B into the atmosphere, determine the horizontal and vertical components of force, and the moment at C , needed to hold the fitting in place. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The control volume consists of the bend fitting and the contained water. The discharge is

$$Q = V_C A_C = (3 \text{ m/s}) [\pi (0.075 \text{ m})^2] = 0.016875\pi \text{ m}^3/\text{s}$$

The water exits at B into the atmosphere. Then $p_B = p_{\text{atm}} = 0$. Since the diameter of the bend fitting is constant, $V_B = V_C = 3 \text{ m/s}$ and the elevation change is small. Therefore $p_C = p_B = 0$. As a result, no pressure force acting on the control volume. The FBD of the control volume is shown in Fig. a . Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho_w dV + \int_{\text{cs}} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar components of this equation along the x and y axes by referring to the free-body diagram, Fig. a ,

$$(\pm) \Sigma F_x = 0 + V_B \cos 30^\circ \rho_w (V_B A_B) + V_C \rho_w (-V_C A_C)$$

$$C_x = [(3 \text{ m/s}) \cos 30^\circ] (1000 \text{ kg/m}^3) (0.016875\pi \text{ m}^3/\text{s}) + (3 \text{ m/s}) (1000 \text{ kg/m}^3) (-0.016875\pi \text{ m}^3/\text{s})$$

$$= -21.31 \text{ N} = 21.3 \text{ N} \leftarrow \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + (-V_B \sin 30^\circ) (\rho_w) (V_B A_B)$$

$$-C_y = [-(3 \text{ m/s}) \sin 30^\circ] (1000 \text{ kg/m}^3) (0.016875\pi \text{ m}^3/\text{s})$$

$$C_y = 79.52 \text{ N} = 79.5 \text{ N} \downarrow \quad \text{Ans.}$$

Applying the angular momentum equation,

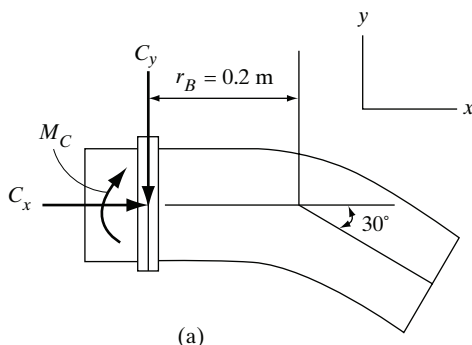
$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho_w dV + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

writing the scalar component of this equation about point C by referring to Fig. a ,

$$\zeta + \Sigma M_C = 0 + (-r_B V_B \sin 30^\circ) \rho_w (V_B A_B)$$

$$-M_C = -(0.2 \text{ m}) [(3 \text{ m/s}) \sin 30^\circ] (1000 \text{ kg/m}^3) (0.016875\pi \text{ m}^3/\text{s})$$

$$M_C = 15.9 \text{ N} \cdot \text{m} \curvearrowright \quad \text{Ans.}$$



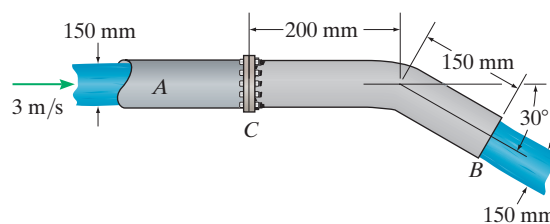
Ans:

$$C_x = 21.3 \text{ N}$$

$$C_y = 79.5 \text{ N}$$

$$M_C = 15.9 \text{ N} \cdot \text{m}$$

6–67. Water flows into the bend fitting with a velocity of 3 m/s. If the water at B exits into a tank having a gage pressure of 10 kPa, determine the horizontal and vertical components of force, and the moment at C , needed to hold the fitting in place. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocities will be used. The fixed control volume consists of the bend fitting and the contained water. Since the diameter of the pipe is constant, $V_B = V_A = 3 \text{ m/s}$. Also the change in elevation is negligible, $p_A = p_B = 10 \text{ kPa}$, to satisfy Bernoulli's equation. Then

$$F_A = F_B = [10(10^3) \text{ N/m}^2] [\pi(0.075 \text{ m})^2] = 56.25\pi \text{ N}$$

Also, the discharge is

$$Q = V_A A_A = V_B A_B = (3 \text{ m/s}) [\pi(0.075 \text{ m})^2] = 0.016875\pi \text{ m}^3/\text{s}$$

The *FBD* of the control volume is shown in Fig. *a*. Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_w dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along x and y axes by referring to the *FBD*, Fig. *a*

$$\begin{aligned} (\rightarrow) \Sigma F_x &= 0 + V_B \cos 30^\circ \rho_w (V_B A_B) + V_A p_w (-V_A A_A) \\ 56.25\pi \text{ N} - [(56.25\pi \text{ N}) \cos 30^\circ] - C_x &= [(3 \text{ m/s}) \cos 30^\circ] (1000 \text{ kg/m}^3) (0.016875\pi \text{ m}^3/\text{s}) \\ &\quad + (3 \text{ m/s}) (1000 \text{ kg/m}^3) (-0.016875\pi \text{ m}^3/\text{s}) \end{aligned}$$

$$C_x = 44.98 \text{ N} = 45.0 \text{ N} \leftarrow \quad \text{Ans.}$$

$$\begin{aligned} +\uparrow \Sigma F_y &= 0 + (-V_B \sin 30^\circ) (\rho_w) (V_B A_B) \\ (56.25\pi \text{ N}) \sin 30^\circ - C_y &= [-(3 \text{ m/s}) \sin 30^\circ] (1000 \text{ kg/m}^3) (0.016875\pi \text{ m}^3/\text{s}) \end{aligned}$$

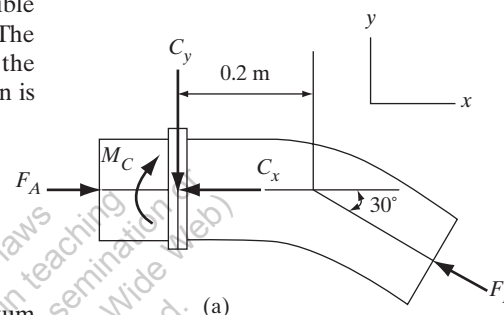
$$C_y = 167.88 \text{ N} = 168 \text{ N} \downarrow \quad \text{Ans.}$$

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho_w dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation about point C by referring to the *FBD*, Fig. *a*,

$$\begin{aligned} \zeta + \Sigma M_C &= 0 + (-r_B V_B \sin 30^\circ) \rho_w (V_B A_B) \\ [(56.25\pi \text{ N}) \sin 30^\circ] (0.2 \text{ m}) - M_C &= -(0.2 \text{ m}) [(3 \text{ m/s}) \sin 30^\circ] (1000 \text{ kg/m}^3) (0.016875\pi \text{ m}^3/\text{s}) \\ M_C &= 33.58 \text{ N} \cdot \text{m} = 33.6 \text{ N} \cdot \text{m} \curvearrowright \quad \text{Ans.} \end{aligned}$$



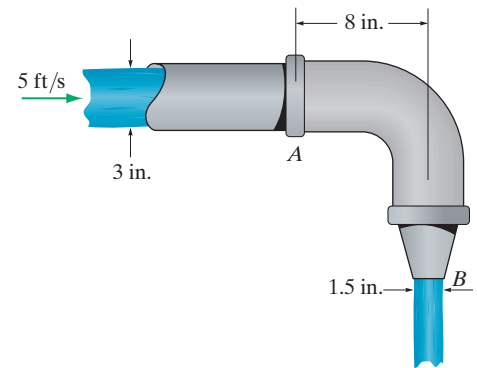
Ans:

$$C_x = 45.0 \text{ N}$$

$$C_y = 168 \text{ N}$$

$$M_C = 33.6 \text{ N} \cdot \text{m}$$

***6–68.** Water flows into the pipe with a velocity of 5 ft/s. Determine the horizontal and vertical components of force, and the moment at A, needed to hold the elbow in place. Neglect the weight of the elbow and the water within it.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the elbow and the water within it.

$$\begin{aligned} Q &= V_A A_A \\ &= (5 \text{ ft/s}) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] \\ &= 0.2454 \text{ ft}^3/\text{s} \end{aligned}$$

Continuity Equation.

$$\begin{aligned} \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \mathbf{V} \cdot d\mathbf{A} &= 0 \\ 0 - V_A A_A + V_B A_B &= 0 \\ -0.2454 \text{ ft}^3/\text{s} + V_B \left[\pi \left(\frac{0.75}{12} \text{ ft} \right)^2 \right] &= 0 \\ V_B &= 20 \text{ ft/s} \end{aligned}$$

Applying the Bernoulli equation between A and B,

$$\begin{aligned} \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A &= \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B \\ \frac{p_A}{\left(\frac{62.4 \text{ lb ft}^3}{32.2 \text{ ft/s}^2} \right)} + \frac{(5 \text{ ft/s})^2}{2} + 0 &= 0 + \frac{(20 \text{ ft/s})^2}{2} + 0 \\ p_A &= 363.354 \text{ lb/ft}^2 = 2.523 \text{ lb/in}^2 \end{aligned}$$

The free-body diagram of the control volume is shown in Fig. a. Here, water is discharged into the atmosphere at B. Therefore, $p_B = 0$.

Linear Momentum. Referring to Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{A}$$

$$\rightarrow \Sigma F_x = \rho Q[(V_B)_x - (V_A)_x];$$

$$-A_x + 2.523 \text{ lb/in}^2 [\pi (1.5 \text{ in.})^2] = \left(\frac{62.4}{32.2} \text{ slug/ft}^3 \right) (0.2454 \text{ ft}^3/\text{s}) (0 - 5 \text{ ft/s})$$

$$A_x = 20.2 \text{ lb} \leftarrow$$

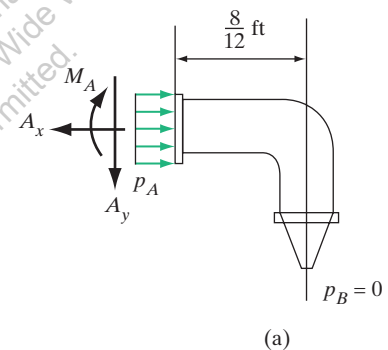
Ans.

$$+\uparrow \Sigma F_y = \rho Q[(V_B)_y - (V_A)_y];$$

$$-A_y = \left(\frac{62.4}{32.2} \text{ slug/ft}^3 \right) (0.2454 \text{ ft}^3/\text{s}) (-20 \text{ ft/s} - 0)$$

$$A_y = 9.51 \text{ lb} \downarrow$$

Ans.



***6–68. Continued**

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\zeta + \Sigma M_A = \Sigma \rho Q V d; \quad -M_A = \left(\frac{62.4}{32.2} \text{ slug/ft}^3 \right) (0.2454 \text{ ft}^3/\text{s}) \left[\left(-\frac{8}{12} \text{ ft} \right) (20 \text{ ft/s}) - 0 \right]$$

$$M_A = 6.34 \text{ lb}\cdot\text{ft} \curvearrowright$$

Ans.

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

6-69. Continued

Applying the Angular Momentum equation

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho_w dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

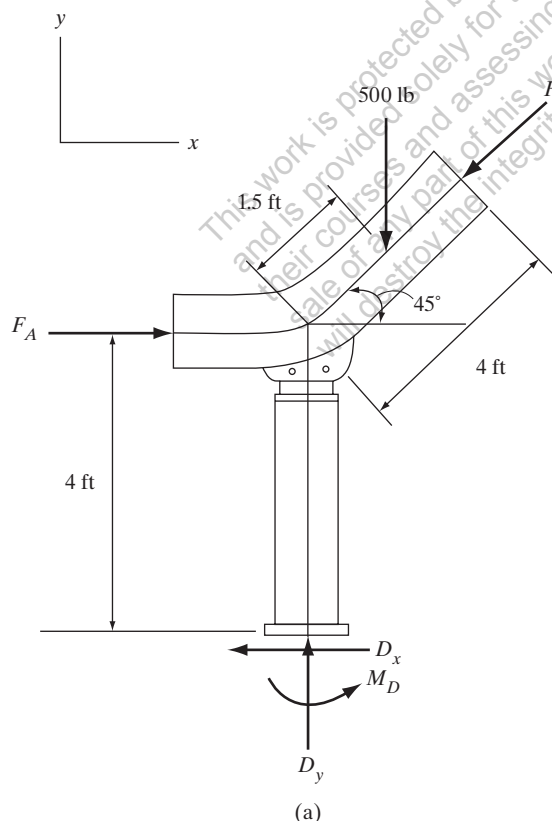
Writing the scalar component of this equation about D by referring to the FBD

$$\zeta + \Sigma M_D = 0 + (-r_A V_A) \rho_w (-V_A A_A) + (-r_B V_B \cos 45^\circ) \rho_w (V_B A_B)$$

$$M_D + [(1559.84 \text{ lb}) \cos 45^\circ](4 \text{ ft}) - (1696.46 \text{ lb})(4 \text{ ft}) - (500 \text{ lb})[(1.5 \text{ ft}) \cos 45^\circ]$$

$$= -(4 \text{ ft})(63.66 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-50 \text{ ft}^3/\text{s}) + (-4 \text{ ft}) \left[(63.66 \text{ ft/s}) \cos 45^\circ \right] \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (50 \text{ ft}^3/\text{s})$$

$$M_D = 10136.8 \text{ lb} \cdot \text{ft} = 10.1 \text{ kip} \cdot \text{ft} \quad \mathbf{Ans.}$$



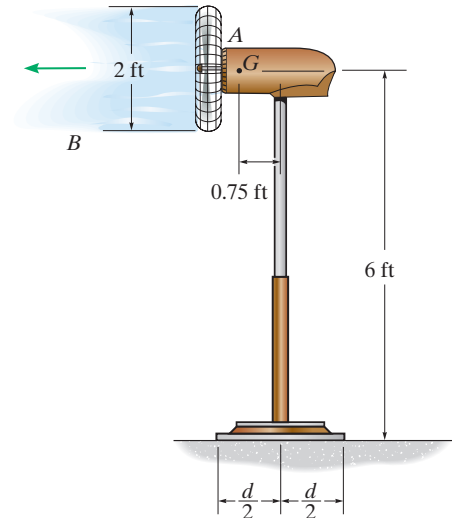
Ans:

$$D_x = 2.40 \text{ kip}$$

$$D_y = 5.96 \text{ kip}$$

$$M_D = 10.1 \text{ kip} \cdot \text{ft}$$

6–70. The fan blows air at $6000 \text{ ft}^3/\text{min}$. If the fan has a weight of 40 lb and a center of gravity at G , determine the smallest diameter d of its base so that it will not tip over. Assume the airstream through the fan has a diameter of 2 ft . The specific weight of the air is $\gamma_a = 0.076 \text{ lb/ft}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = \left(\frac{6000 \text{ ft}^3}{\text{min}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 100 \text{ ft}^3/\text{s}$$

Then,

$$Q = V_B A_B; \quad 100 \text{ ft}^3/\text{s} = V_B [\pi (1 \text{ ft})^2]$$

$$V_B = 31.83 \text{ ft/s}$$

Take the control volume as the fan and air passing through it. The free-body diagram of the control volume is shown in Fig. *a*. Here, tipping will occur about point C .

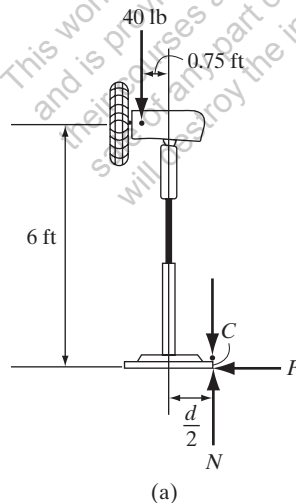
Angular Momentum. Air is sucked into the fan at A from a large source of still air, $V_A \approx 0$. Referring to Fig. *a*,

$$\Sigma \mathbf{M}_c = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\zeta + 40 \text{ lb} \left(0.75 \text{ ft} + \frac{d}{2} \right) = \left(\frac{0.076 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (100 \text{ ft}^3/\text{s}) [6 \text{ ft} (31.83 \text{ ft/s}) - 0]$$

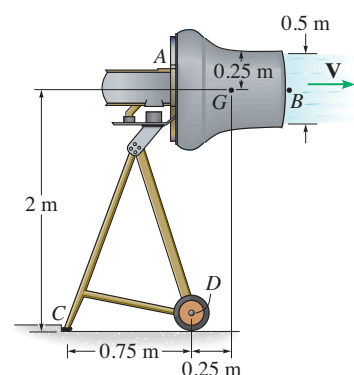
$$d = 0.7539 \text{ ft} \approx 0.754 \text{ ft}$$

Ans.



Ans:
0.754 ft

6-71. When operating, the air-jet fan discharges air with a speed of $V = 18 \text{ m/s}$ into a slipstream having a diameter of 0.5 m . If the air has a density of 1.22 kg/m^3 , determine the horizontal and vertical components of reaction at C , and the vertical reaction at each of the two wheels, D . The fan and motor have a mass of 25 kg and a center of mass at G . Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume to be the fan and the air passing through it.

$$Q = V_B A_B = (18 \text{ m/s}) [\pi (0.25 \text{ m})^2] = 3.5343 \text{ m}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*.

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$2N_D(0.75 \text{ m}) - 25 \text{ kg}(9.81 \text{ m/s}^2)(1 \text{ m}) = 0 + (1.22 \text{ kg/m}^3)(3.5343 \text{ m}^3/\text{s})[(-2 \text{ m})(18 \text{ m/s}) - 0]$$

$$N_D = 60.02 \text{ N} = 60.0 \text{ N}$$

Ans.

Linear Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\pm \Sigma F_x = 0 + V_B \rho Q$$

$$C_x = (18 \text{ m/s})(1.22 \text{ kg/m}^3)(3.5343 \text{ m}^3/\text{s})$$

$$C_x = 77.6 \text{ N}$$

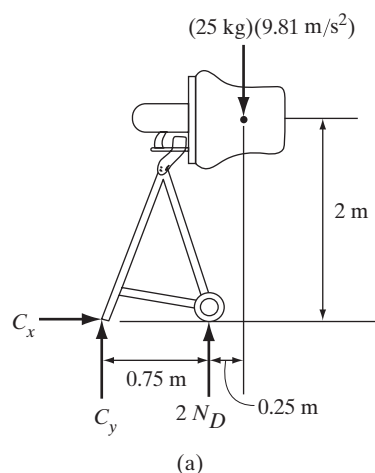
Ans.

$$+\uparrow \Sigma F_y = 0 + 0$$

$$C_y + 2(60.02 \text{ N}) - 25 \text{ kg}(9.81 \text{ m/s}^2) = (1.22 \text{ kg/m}^3)(3.5343 \text{ m}^3/\text{s})(0 - 0)$$

$$C_y = 125.22 \text{ N} = 125 \text{ N}$$

Ans.



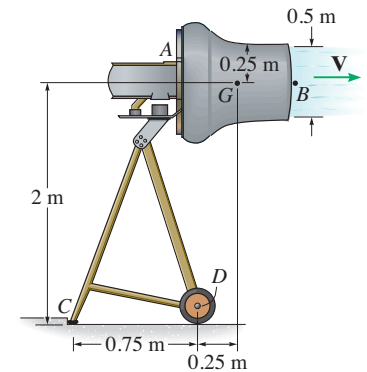
Ans:

$$N_D = 60.0 \text{ N}$$

$$C_x = 77.6 \text{ N}$$

$$C_y = 125 \text{ N}$$

***6-72.** If the air has a density of 1.22 kg/m^3 , determine the maximum speed V that the air-jet fan can discharge air into the slipstream having a diameter of 0.5 m at B , so that the fan does not topple over. The fan and motor have a mass of 25 kg and a center of mass at G . Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



SOLUTION

Consider the control volume to be the fan and the air passing through it, Fig. *a*. Since the inlet A and outlet B are opened to the atmosphere, $p_A = p_B = 0$. The free-body diagram of the control volume is shown in Fig. *a*. Here, if the fan is about to topple about C , $N_D = 0$. Applying the angular momentum equation

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

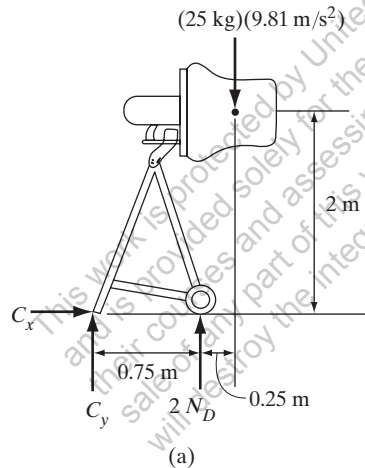
And writing the scalar component of the equation about C by referring to the *FBD*,

$$\zeta + \Sigma M_C = 0 + (-r_B V_B)(\rho_a)(V_B A_B)$$

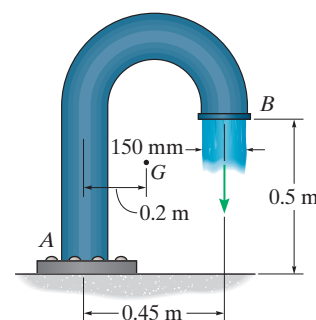
$$-(25 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) = -[(2 \text{ m})V_B](1.22 \text{ kg/m}^3)V_B[\pi(0.25 \text{ m})^2]$$

$$V_B = 22.63 \text{ m/s} = 22.6 \text{ m/s}$$

Ans.



6-73. Water flows through the curved pipe at a speed of 5 m/s. If the diameter of the pipe is 150 mm, determine the horizontal and vertical components of the resultant force, and the moment acting on the coupling at A. The weight of the pipe and the water within it is 450 N, having a center of gravity at G.



SOLUTION

Take the control volume as the pipe and the water within it.

$$Q_A = V_A A_A = (5 \text{ m/s}) [\pi (0.075 \text{ m})^2] \\ = 0.08836 \text{ m}^3/\text{s}$$

Bernoulli Equation, where $V_A = V_B$. Datum at A, the free-body diagram of the control volume is shown in Fig. a. Here, water is discharged into the atmosphere at B. Therefore, $p_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B \\ \frac{p_A}{1000 \text{ kg/m}^3} + \frac{V^2}{2} + 0 = 0 + \frac{V^2}{2} + (9.81 \text{ m/s}^2)(0.5 \text{ m}) \\ p_A = 4905 \text{ Pa}$$

Linear Momentum. Referring to Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\pm \Sigma F_x = 0 + pQ[(V_B)_x - (V_A)_x];$$

$$A_x = pQ(0 - 0) = 0$$

Ans.

$$+ \uparrow \Sigma F_y = 0 + pQ[(V_B)_y - (V_A)_y];$$

$$-A_y + [4905 \text{ N/m}^2] [\pi (0.075 \text{ m})^2] = 450 \text{ N} = (1000 \text{ kg/m}^3)(0.08836 \text{ m}^3/\text{s})(-5 \text{ m/s} - 5 \text{ m/s})$$

$$A_y = 520 \text{ N}$$

Ans.

Angular Momentum. Referring to Fig. a,

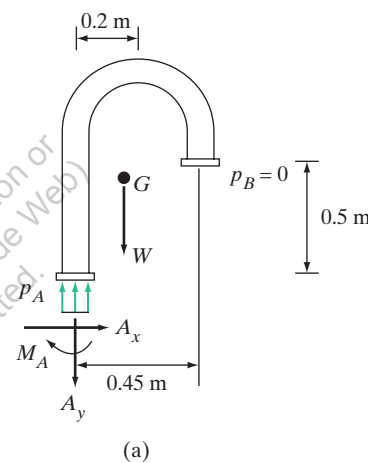
$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\zeta + \Sigma M_D = 0 + \Sigma pQVd;$$

$$-M_A - (450 \text{ N})(0.2 \text{ m}) = (1000 \text{ kg/m}^3)(0.08836 \text{ m}^3/\text{s})[(-0.45 \text{ m})(5 \text{ m/s}) - 0]$$

$$M_A = 109 \text{ N} \cdot \text{m}$$

Ans.



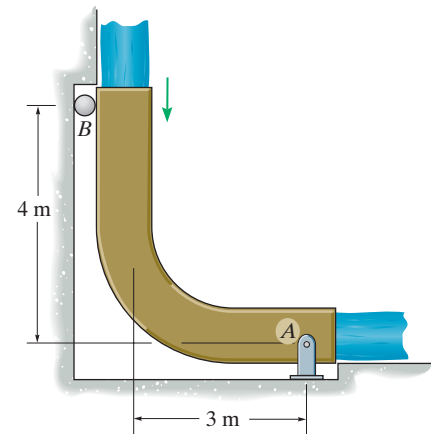
Ans:

$$A_x = 0$$

$$A_y = 520 \text{ N}$$

$$M_A = 109 \text{ N} \cdot \text{m}$$

6-74. The chute is used to divert the flow of water. If the flow is $0.4 \text{ m}^3/\text{s}$ and it has a cross-sectional area of 0.03 m^2 , determine the horizontal and vertical force components at the pin A , and the horizontal force at the roller B , necessary for equilibrium. Neglect the weight of the chute and the water on it.



SOLUTION

Take the control volume as the chute and the water on it.

$$Q = VA; \quad 0.4 \text{ m}^3/\text{s} = V(0.03 \text{ m}^2)$$

$$V = 13.33 \text{ m/s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Here, $p_A = p_B = 0$ since points A and B are exposed to the atmosphere,

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} \mathbf{r} \times \mathbf{V} \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\zeta + \Sigma M_A = 0 + \Sigma \rho Q V d;$$

$$-B_x(4 \text{ m}) = (1000 \text{ kg/m}^3)(0.4 \text{ m}^3/\text{s})[0 - 3 \text{ m}(13.33 \text{ m/s})]$$

$$B_x = 4000 \text{ N} = 4 \text{ kN}$$

Ans.

Linear Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$+\rightarrow \Sigma F_x = 0 + (V_A) \rho Q$$

$$4000 \text{ N} + A_x = (13.33 \text{ m/s})(1000 \text{ kg/m}^3)(0.4 \text{ m}^3/\text{s})$$

$$A_x = 1.33 \text{ kN}$$

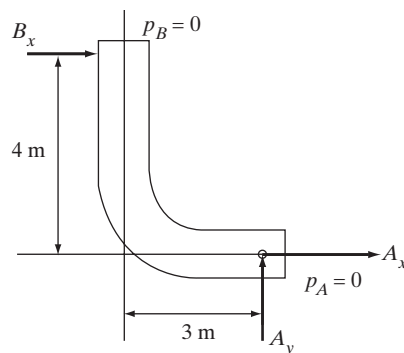
Ans.

$$+\uparrow \Sigma F_y = 0 + V_B \rho Q$$

$$A_y = (13.33 \text{ m/s})(1000 \text{ kg/m}^3)(0.4 \text{ m}^3/\text{s})$$

$$A_y = 5.33 \text{ kN}$$

Ans.



(a)

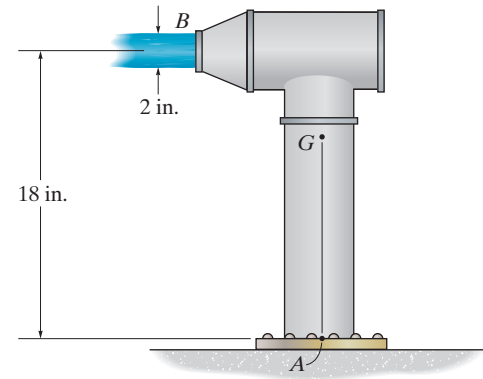
Ans:

$$B_x = 4 \text{ kN}$$

$$A_x = 1.33 \text{ kN}$$

$$A_y = 5.33 \text{ kN}$$

6-75. Water flows through A at 400 gal/min and is discharged to the atmosphere through the reducer at B . Determine the horizontal and vertical components of force, and the moment acting on the coupling at A . The vertical pipe has an inner diameter of 3 in. Assume the assembly and the water within it has a weight of 40 lb and a center of gravity at G . $1 \text{ ft}^3 = 7.48 \text{ gal}$.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\gamma_w = 62.4 \text{ lb/ft}^3$. Average velocities will be used. The control volume consists of the vertical pipe, reducer and the contained water as shown in Fig. a . The discharge is

$$Q = \left(400 \frac{\text{gal}}{\text{min}} \right) \left(\frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 0.8913 \text{ ft}^3/\text{s}$$

Thus,

$$Q = V_A A_A; \quad 0.8913 \text{ ft}^3/\text{s} = V_A \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] \quad V_A = 18.16 \text{ ft/s}$$

$$Q = V_B A_B; \quad 0.8913 \text{ ft}^3/\text{s} = V_B \left[\pi \left(\frac{1}{12} \text{ ft} \right)^2 \right] \quad V_B = 40.85 \text{ ft/s}$$

Applying Bernoulli's equation between points A and B with $p_B = p_{\text{atm}} = 0$ and $z_B = 1.5 \text{ ft}$,

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_A}{62.4 \text{ lb/ft}^3} + \frac{(18.16 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0 = 0 + \frac{(40.85 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 1.5 \text{ ft}$$

$$p_A = 1391.28 \text{ lb/ft}^2$$

Then the pressure force acting on the inlet control surface A , indicated in the FBD of the control volume, is

$$F_A = p_A A_A = (1391.28 \text{ lb/ft}^2) \left[\pi \left(\frac{1.5}{12} \text{ ft} \right)^2 \right] = 68.28 \text{ lb}$$

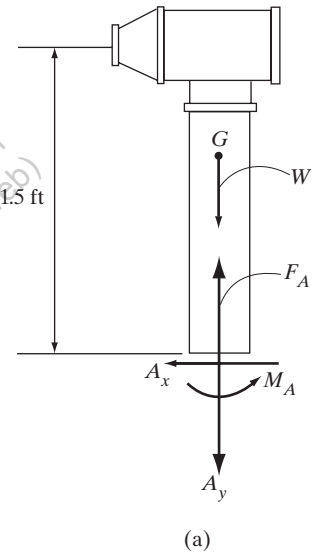
Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho_w dV + \int_{cs} \mathbf{V} \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar components of this equation along the x and y axes by referring to Fig. a

$$\begin{aligned} \sum F_x &= 0 + (-V_B) \rho_w (V_B A_B) \\ -A_x &= (-40.85 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.8913 \text{ ft}^3/\text{s}) \\ A_x &= 70.56 \text{ lb} = 70.6 \text{ lb} \end{aligned}$$

Ans.



6-75. Continued

$$+\uparrow \Sigma F_y = 0 + V_A \rho_w (-V_A A_A)$$

$$-40 \text{ lb} + 68.29 \text{ lb} - A_y = (18.16 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (-0.8913 \text{ ft}^3/\text{s})$$

$$A_y = 59.65 \text{ lb} = 59.7 \text{ lb}$$

Ans.

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho_w dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation about point A,

$$\zeta + \Sigma M_A = 0 + r_{AB} V_B (\rho_w V_B A_B)$$

$$M_A = (1.5 \text{ ft}) (40.85 \text{ ft/s}) \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \right) (0.8913 \text{ ft}^3/\text{s})$$

$$= 105.84 \text{ lb} \cdot \text{ft} = 106 \text{ lb} \cdot \text{ft}$$

Ans.

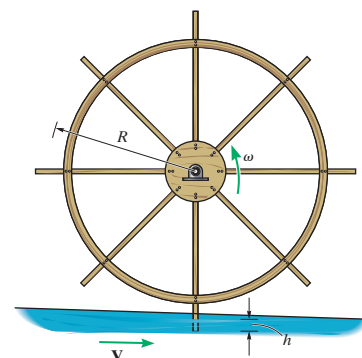
Ans:

$$A_x = 70.6 \text{ lb}$$

$$A_y = 59.7 \text{ lb}$$

$$M_A = 106 \text{ lb} \cdot \text{ft}$$

***6-76.** The waterwheel consists of a series of flat plates that have a width b and are subjected to the impact of water to a depth h , from a stream that has an average velocity of V . If the wheel is turning at ω , determine the power supplied to the wheel by the water.



SOLUTION

Using a fixed control volume, with water entering on the left with velocity V and exiting on the right with (x -component) velocity ωR (the speed of the plates), we apply the angular momentum equation:

$$\zeta + \Sigma M_{\text{hub}} = \frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

$$-T = 0 + RV\rho_w(-VA) + R\omega R\rho_w(VA)$$

where T is the torque or moment exerted by the water on the wheel and $-T$ is the torque exerted by the wheel on the water. So then, since $A = bh$,

$$T = \rho_w b h R V (V - \omega R)$$

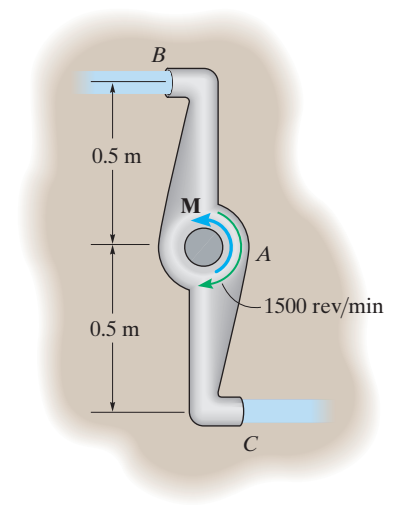
and since $\dot{W} = T\omega$,

$$P = \rho_w b h \omega R V (V - \omega R)$$

Ans.

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

6-77. Air enters into the hollow propeller tube at A with a mass flow of 3 kg/s and exits at the ends B and C with a velocity of 400 m/s , measured relative to the tube. If the tube rotates at 1500 rev/min , determine the frictional torque M on the tube.



SOLUTION

The flow is periodic hence it can be considered steady in the mean. The air is assumed to be an ideal fluid (incompressible and inviscid) such that its density is constant. Average velocities will be used. The control volume consists of the hollow propeller and the contained air. Its *FBD* is shown in Fig. *a*

The velocity of point B (or C) is

$$V_B = \omega r = \left[\left(1500 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \right] (0.5 \text{ m}) = 25\pi \text{ m/s} \rightarrow$$

Thus, the velocity of the air ejected from B (or C) is

$$V_a = V_B + V_{a/B}$$

$$\left(\leftarrow \right) V_a = (-25\pi \text{ m/s}) + (400 \text{ m/s}) = 321.46 \text{ m/s} \leftarrow$$

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

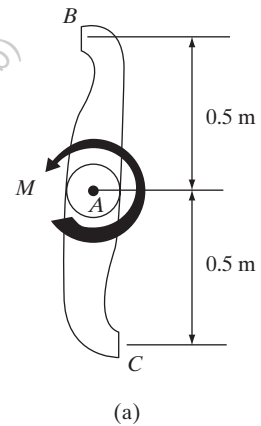
Writing the scalar component about point A ,

$$\zeta + \Sigma M_A = 0 + 2[r_{AB} V_B \rho_a V_{BA_B}]$$

Here $\rho_a V_B A_B = \dot{m}_B = 1.5 \text{ kg/s}$. Then

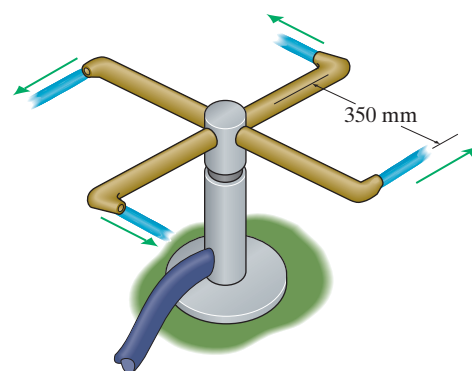
$$\begin{aligned} M &= 2(0.5 \text{ m})(321.46 \text{ m/s})(1.5 \text{ kg/s}) \\ &= 482.19 \text{ N} \cdot \text{m} = 482 \text{ N} \cdot \text{m} \end{aligned}$$

Ans.



Ans:
482 N · m

6-78. The lawn sprinkler consists of four arms that rotate in the horizontal plane. The diameter of each nozzle is 10 mm, and the water is supplied through the hose at $0.008 \text{ m}^3/\text{s}$ and is ejected horizontally, through the four arms. Determine the torque required to hold the arms from rotating.



SOLUTION

We consider steady flow of an ideal fluid relative to the control volume.

Take the control volume as the sprinkler and the water within it. Due to symmetry and the continuity condition, the discharge from each nozzle is $Q = (0.008 \text{ m}^3/\text{s})/4 = 0.002 \text{ m}^3/\text{s}$.

$$Q = VA; \quad 0.002 \text{ m}^3/\text{s} = V[\pi(0.005 \text{ m})^2]$$

$$V = 25.46 \text{ m/s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Here, water is discharged to the atmosphere at the nozzle, $p = 0$.

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

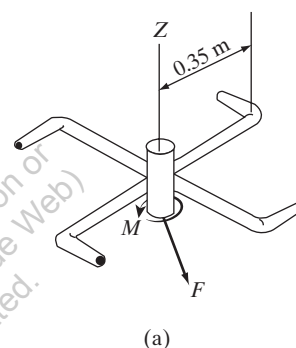
or

$$\zeta + \Sigma M_A = \Sigma \rho Q V d;$$

$$M = 4[(1000 \text{ kg/m}^3)(0.002 \text{ m}^3/\text{s})][0.35 \text{ m}(25.46 \text{ m/s}) - 0]$$

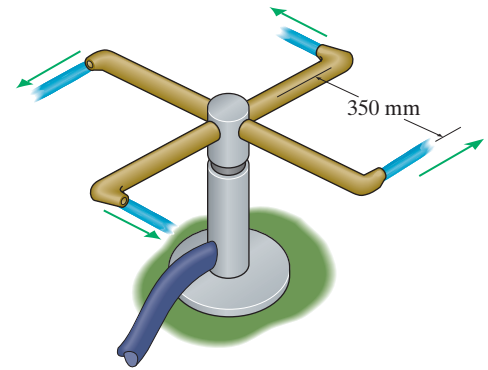
$$= 71.30 \text{ N} \cdot \text{m} = 71.3 \text{ N} \cdot \text{m}$$

Ans.



Ans:
 $71.3 \text{ N} \cdot \text{m}$

6-79. The lawn sprinkler consists of four arms that rotate in the horizontal plane. The diameter of each nozzle is 10 mm, and the water is supplied through the hose at $0.008 \text{ m}^3/\text{s}$ and is ejected horizontally, through the four arms. Determine the steady-state angular velocity of the arms. Neglect friction.



SOLUTION

We consider steady flow of an ideal fluid relative to the control volume.

Take the control volume as the sprinkler and the water within it. Due to symmetry and the continuity condition, the discharge from each nozzle is $Q = (0.008 \text{ m}^3/\text{s})/4 = 0.002 \text{ m}^3/\text{s}$.

$$Q = V_{f/n}A; \quad 0.002 \text{ m}^3/\text{s} = V_{f/n}[\pi(0.005 \text{ m})^2]$$

$$V_{f/n} = 25.46 \text{ m/s}$$

The velocity of the nozzle is

$$V_n = \omega r = \omega(0.35 \text{ m}) = 0.35 \omega$$

Thus, the velocity of the flow can be determined from

$$\mathbf{V}_f = \mathbf{V}_n + \mathbf{V}_{f/n}$$

$$V_f = -0.35\omega + 25.46$$

The free-body diagram of the control volume is shown in Fig. *a*. Here, water is discharged to the atmosphere at the nozzle, $p = 0$.

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A}$$

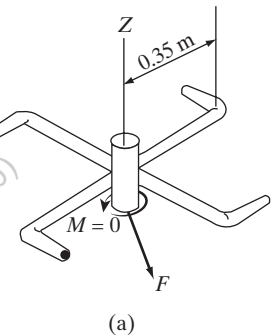
or

$$0 + \Sigma M_A = \Sigma \rho Q dV;$$

$$0 = 4[(1000 \text{ kg/m}^3)(0.002 \text{ m}^3/\text{s})][0.35 \text{ m}(-0.35\omega + 25.46)]$$

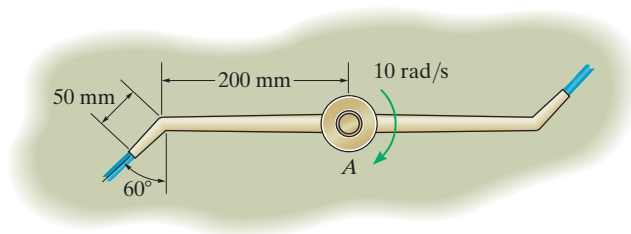
$$\omega = 72.76 \text{ rad/s} = 72.8 \text{ rad/s}$$

Ans.



Ans:
72.8 rad/s

***6–80.** The 5-mm-diameter arms of a rotating lawn sprinkler have the dimensions shown. Water flows out relative to the arms at 6 m/s, while the arms are rotating at 10 rad/s. Determine the frictional torsional resistance at the bearing A, and the speed of the water as it emerges from the nozzles, as measured by a fixed observer.



SOLUTION

Referring to the geometry shown in Fig. a, the cosine and sine laws give

$$r = \sqrt{50^2 + 200^2 - 2(50)(200) \cos 150^\circ} = 244.6 \text{ mm}$$

$$\frac{\sin \alpha}{0.05 \text{ m}} = \frac{\sin 150^\circ}{0.2446 \text{ m}}; \quad \alpha = 5.867^\circ$$

Then

$$\beta = 180^\circ - 150^\circ - 5.867^\circ = 24.133^\circ$$

Thus, the velocity of the tip of the arm is

$$V_t = \omega r = (10 \text{ rad/s})(0.2446 \text{ m}) = 2.446 \text{ m/s} \uparrow$$

Referring to the velocity vector diagram shown in Fig. b, the relative velocity equation gives

$$\mathbf{V}_w = \mathbf{V}_t + \mathbf{V}_{w/t}$$

$$\begin{bmatrix} (V_w)_x \\ (V_w)_y \end{bmatrix} = \begin{bmatrix} 2.446 \text{ m/s} \\ \uparrow \end{bmatrix} + \begin{bmatrix} 6 \text{ m/s} \\ \swarrow 24.133^\circ \end{bmatrix}$$

$$\begin{aligned} (+\rightarrow) - (V_w)_x &= -(6 \text{ m/s}) \cos 24.133^\circ & (V_w)_x &= 5.476 \text{ m/s} \leftarrow \\ (+\uparrow) - (V_w)_y &= 2.446 \text{ m/s} - (6 \text{ m/s}) \sin 24.133^\circ & (V_w)_y &= 0.007339 \text{ m/s} \downarrow \end{aligned}$$

The magnitude of \mathbf{V}_w is

$$\begin{aligned} V_w &= \sqrt{(V_w)_x^2 + (V_w)_y^2} = \sqrt{(5.476 \text{ m/s})^2 + (0.007339 \text{ m/s})^2} \\ &= 5.476 \text{ m/s} = 5.48 \text{ m/s} \end{aligned}$$

Ans.

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocity will be used. The control volume consists of the entire arm and the contained water as shown in Fig. a.

Applying the angular momentum equation,

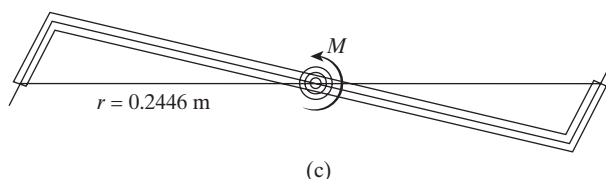
$$\Sigma \mathbf{M} = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho_w dV + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation about point A, by referring to the FBD of the control volume, Fig. a,

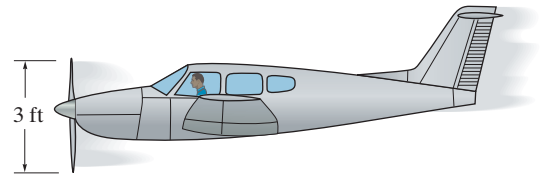
$$\zeta + \Sigma M_A = 0 + r(V_w)_y \rho_w (V_{w/t} A)$$

$$\begin{aligned} M &= (0.2446 \text{ m})(0.007339 \text{ m/s})(1000 \text{ kg/m}^3) \{ (6 \text{ m/s}) [\pi(0.0025 \text{ m})^2] \} \\ &= 2.1145(10^{-4}) \text{ N} \cdot \text{m} \\ &= 0.211 \text{ mN} \cdot \text{m} \end{aligned}$$

Ans.



6–81. The airplane is flying at 250 km/h through still air as it discharges 350 m³/s of air through its 1.5-m-diameter propeller. Determine the thrust on the plane and the ideal efficiency of the propeller. Take $\rho_a = 1.007 \text{ kg/m}^3$.



SOLUTION

The average velocity of the air flow through the propeller (control volume) is

$$Q = VA; \quad 350 \text{ m}^3/\text{s} = V[\pi(0.75 \text{ m})^2]$$

$$V = 198.06 \text{ m/s}$$

$$\text{Here, } V_1 = \left(250 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 69.44 \text{ m/s}$$

$$V = \frac{V_1 + V_2}{2}; \quad 198.06 \text{ m/s} = \frac{(69.44 \text{ m/s}) + V_2}{2}$$

$$V_2 = 326.67 \text{ m/s}$$

The ideal efficiency is

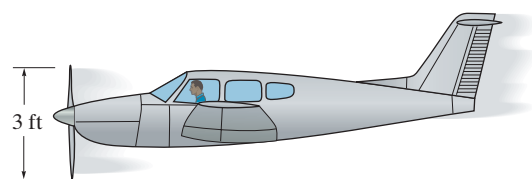
$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(69.44 \text{ m/s})}{69.44 \text{ m/s} + 326.67 \text{ m/s}} = 0.3506 = 0.351 \quad \text{Ans.}$$

The thrust of the propeller is

$$\begin{aligned} F &= \frac{\rho\pi R^2}{2}(V_2^2 - V_1^2) \\ &= \frac{(1.007 \text{ kg/m}^3)(\pi)(0.75 \text{ m})^2}{2}[(326.67 \text{ m/s})^2 - (69.44 \text{ m/s})^2] \\ &= 90.66(10^3) \text{ N} = 90.7 \text{ kN} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} \text{Ans:} \\ e &= 0.351 \\ F &= 90.7 \text{ kN} \end{aligned}$$

6-82. The airplane travels at 400 ft/s through still air. If the air flows through the propeller at 560 ft/s, measured relative to the plane, determine the thrust on the plane and the ideal efficiency of the propeller. Take $\rho_a = 2.15(10^{-3})$ slug/ft³.



SOLUTION

The propeller and air within it is the control volume. We consider steady flow of an ideal fluid relative to the control volume.

Here, $V_1 = 400$ ft/s and $V = 560$ ft/s.

$$V = \frac{V_1 + V_2}{2}, \quad 560 \text{ ft/s} = \frac{400 \text{ ft/s} + V_2}{2}$$

$$V_2 = 720 \text{ ft/s}$$

The ideal efficiency is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(400 \text{ ft/s})}{400 \text{ ft/s} + 720 \text{ ft/s}} = 0.7143 = 0.714$$

Ans.

The thrust of the propeller is

$$F = \frac{\rho \pi R^2}{2} (V_2^2 - V_1^2)$$

$$= \frac{(2.15(10^{-3}) \text{ slug/ft}^3)(\pi)(1.5 \text{ ft})^2}{2} [(720 \text{ ft/s})^2 - (400 \text{ ft/s})^2]$$

$$= 2723.38 \text{ lb} = 2.72 \text{ kip}$$

Ans.

Ans:

$$e = 0.714$$

$$F = 2.72 \text{ kip}$$

6-83. A boat has a 250-mm-diameter propeller that discharges $0.6 \text{ m}^3/\text{s}$ of water as the boat travels at 35 km/h in still water. Determine the thrust developed by the propeller on the boat.

SOLUTION

The propeller and water within is the control volume. The average velocity of the water through the propeller is

$$Q = VA; \quad 0.6 \text{ m}^3/\text{s} = V[\pi(0.125 \text{ m})^2]$$

$$V = 12.22 \text{ m/s}$$

$$\text{Here, } V_1 = \left(35 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 9.722 \text{ m/s}$$

$$V = \frac{V_1 + V_2}{2}; \quad 12.22 \text{ m/s} = \frac{9.722 \text{ m/s} + V_2}{2}$$

$$V_2 = 14.72 \text{ m/s}$$

The thrust of the propeller is

$$F = \frac{\rho \pi R^2}{2} (V_2^2 - V_1^2)$$

$$= \frac{(1000 \text{ kg/m}^3)(\pi)(0.125 \text{ m})^2}{2} [(14.72 \text{ m/s})^2 - (9.722 \text{ m/s})^2]$$

$$= 3.001(10^3) \text{ N} = 3.00 \text{ kN}$$

Ans.

Ans:
300 kN

***6-84.** A ship has a 2.5-m-diameter propeller with an ideal efficiency of 40%. If the thrust developed by the propeller is 1.5 MN, determine the constant speed of the ship in still water and the power that must be supplied to the propeller to operate it.

SOLUTION

The propeller and water within it is the control volume.

The ideal efficiency is

$$e = \frac{2V_1}{V_1 + V_2}; \quad 0.4 = \frac{2V_1}{V_1 + V_2} \quad V_2 = 4V_1 \quad (1)$$

The thrust of the propeller is

$$F = \frac{\rho \pi R^2}{2} (V_2^2 - V_1^2); \quad 1.5(10^6) \text{ N} = \frac{(1000 \text{ kg/m}^3)(\pi)(1.25 \text{ m})^2}{2} (V_2^2 - V_1^2) \quad (2)$$

$$V_2^2 - V_1^2 = 611.15$$

Solving Eqs. (1) and (2) yields

$$V_1 = 6.383 \text{ m/s} = 6.38 \text{ m/s}$$

$$V_2 = 25.53 \text{ m/s}$$

Ans.

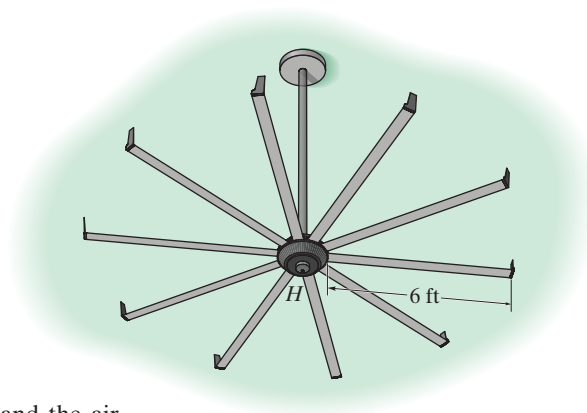
The power output is

$$\dot{W}_{\text{out}} = FV_1 = [1.5(10^6) \text{ N}](6.383 \text{ m/s}) = 9.575(10^6) \text{ W} = 9.575 \text{ MW}$$

Thus, the power supply to the propeller is

$$\dot{W}_{\text{in}} = \frac{P_{\text{out}}}{e} = \frac{9.575 \text{ MW}}{0.4} = 23.94 \text{ MW} = 23.9 \text{ MW} \quad \text{Ans.}$$

6–85. The fan is used to circulate air within a large industrial building. The blade assembly weighs 200 lb and consists of 10 blades, each having a length of 6 ft. Determine the power that must be supplied to the motor to lift the assembly off its bearings and allow it to freely turn without friction. What is the downward air velocity for this to occur? Neglect the size of the hub H . Take $\rho_a = 2.36(10^{-3})$ slug/ft³.



SOLUTION

The blade and air within it is the control volume. The flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 2.36(10^{-3})$ slug/ft³. Average velocities will be used. To lift the blade assembly off the bearings, the thrust must be equal to the weight of the assembly, i.e., $F = 200$ lb. Since the air enters the blade assembly from the surroundings which is at rest, $V_1 = 0$.

$$F = \frac{\rho_a \pi R^2}{2} (V_2^2 - V_1^2); \quad 200 \text{ lb} = \frac{2.36(10^{-3}) \text{ slug/ft}^3 [\pi(6 \text{ ft})^2]}{2} (V_2^2 - 0)$$

$$V_2 = 38.71 \text{ ft/s} = 38.7 \text{ ft/s}$$

Ans.

$$V = \frac{V_1 + V_2}{2} = \frac{0 + 38.71 \text{ ft/s}}{2} = 19.36 \text{ ft/s}$$

The power required by the motor is

$$\begin{aligned} \dot{W} &= FV = (200 \text{ lb})(19.36 \text{ ft/s}) \\ &= \left(3871.22 \frac{\text{ft} \cdot \text{lb}}{\text{s}} \right) \left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}} \right) \\ &= 7.04 \text{ hp} \end{aligned}$$

Ans.

Ans:

$$\begin{aligned} V_2 &= 38.7 \text{ ft/s} \\ \dot{W} &= 7.04 \text{ hp} \end{aligned}$$

6-86. The 12-Mg helicopter is hovering over a lake as the suspended bucket collects 5 m³ of water used to extinguish a fire. Determine the power required by the engine to hold the filled water bucket over the lake. The horizontal blade has a diameter of 14 m. Take $\rho_a = 1.23 \text{ kg/m}^3$.



SOLUTION

The helicopter, bucket, water, and air within the helicopter blade is the control volume. The flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 1.23 \text{ kg/m}^3$. Average velocities will be used. To maintain the hovering, the thrust produced by the rotor blade must be equal to the weight of the helicopter and the water. Thus,

$$\begin{aligned} F &= [12(10^3) \text{ kg}](9.81 \text{ m/s}^2) + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}^3) \\ &= 166.77(10^3) \text{ N} \end{aligned}$$

Since the air enters the blade from the surroundings, which is at rest, $V_1 = 0$.

$$F = \frac{\rho_a \pi R^2}{2} (V_2^2 - V_1^2); \quad 166.77(10^3) \text{ N} = \frac{(1.23 \text{ kg/m}^3) [\pi (7 \text{ m})^2]}{2} (V_2^2 - 0)$$

$$V_2 = 41.97 \text{ m/s}$$

$$V = \frac{V_1 + V_2}{2} = \frac{0 + 41.97 \text{ m/s}}{2} = 20.985 \text{ m/s}$$

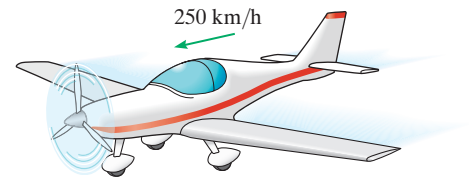
Thus, the power required by the engine is

$$\begin{aligned} \dot{W} &= FV = [166.77(10^3) \text{ N}](20.985 \text{ m/s}) \\ &= 3.4997(10^6) \text{ W} \\ &= 3.50 \text{ MW} \end{aligned}$$

Ans.

Ans:
3.50 MW

6-87. The airplane has a constant speed of 250 km/h in still air. If it has a 2.4-m-diameter propeller, determine the force acting on the plane if the speed of the air behind the propeller, measured relative to the plane, is 750 km/h. Also, what is the ideal efficiency of the propeller, and the power produced by the propeller? Take $\rho_a = 0.910 \text{ kg/m}^3$.



SOLUTION

The airplane moves in the still air and the control volume is attached to the airplane, which is travelling with a constant velocity of $\left(250 \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 69.44 \text{ m/s}$. Then the inlet velocity is $V_1 = 69.44 \text{ m/s}$. Relative to the control volume, the flow is steady. The air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 0.910 \text{ kg/m}^3$. Average velocities will be used. The outlet velocity is $V_2 = \left(750 \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 208.33 \text{ m/s}$. The thrust on the plane is

$$\begin{aligned} F &= \frac{\rho_a \pi R^2}{2} (V_2^2 - V_1^2); \\ &= \frac{(0.910 \text{ kg/m}^3) [\pi (1.2 \text{ m})^2]}{2} [(208.33 \text{ m/s})^2 - (69.44 \text{ m/s})^2] \\ &= 79.41(10^3) \text{ N} \end{aligned}$$

The power generated by the propeller is

$$\begin{aligned} \dot{W}_0 &= FV_1 = [79.41(10^3) \text{ N}] (69.44 \text{ m/s}) \\ &= 5.515(10^6) \text{ W} = 5.51 \text{ MW} \end{aligned}$$

Ans.

The efficiency of the propeller is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(69.44 \text{ m/s})}{69.44 \text{ m/s} + 208.33 \text{ m/s}} = 0.5$$

Ans.

Ans:

$$F = 79.4 \text{ kN}$$

$$\dot{W} = 5.51 \text{ MW}$$

$$e = 0.5$$

***6–88.** The 12-kg fan develops a breeze of 10 m/s using a 0.8-m-diameter blade. Determine the smallest dimension d for the support so that the fan does not tip over. Take $\rho_a = 1.20 \text{ kg/m}^3$.

SOLUTION

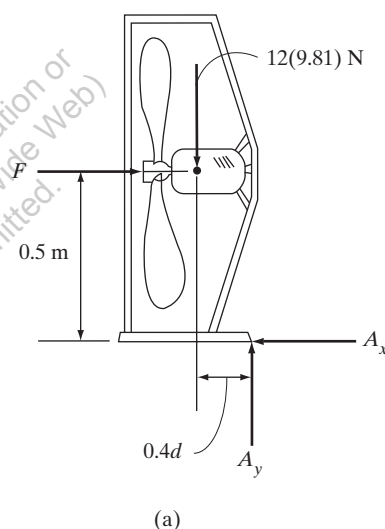
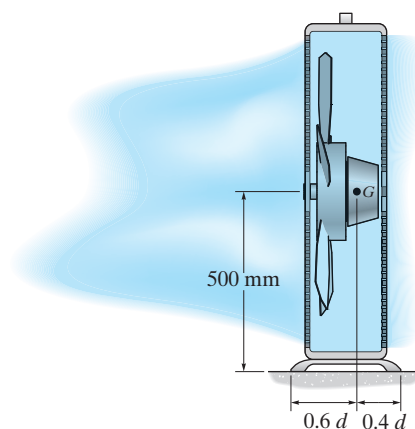
Take the fan and air within it as the control volume. The flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 1.20 \text{ kg/m}^3$. Average velocities can be used. Since the air enters the blade from the surroundings which is at rest, $V_1 = 0$. Here, $V_2 = 10 \text{ m/s}$.

$$\begin{aligned} F &= \frac{\rho_a \pi R^2}{2} (V_2^2 - V_1^2) \\ &= \frac{(1.20 \text{ kg/m}^3) [\pi (0.4 \text{ m})^2]}{2} [(10 \text{ m/s})^2 - 0] \\ &= 9.6\pi \text{ N} \end{aligned}$$

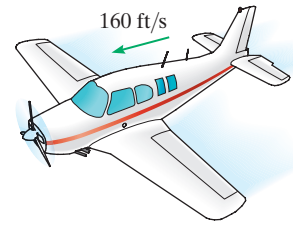
Referring to the FBD of the fan shown in Fig. a , and writing the moment equation of equilibrium about point A ,

$$\begin{aligned} \zeta + \Sigma M_A &= 0; & [12(9.81) \text{ N}](0.4 d) - (9.6\pi \text{ N})(0.5 \text{ m}) &= 0 \\ d &= 0.320 \text{ m} = 320 \text{ mm} \end{aligned}$$

Ans.



6–89. The airplane is flying at 160 ft/s in still air at an altitude of 10 000 ft. The 7-ft-diameter propeller moves the air at 10 000 ft³/s. Determine the power required by the engine to turn the propeller, and the thrust on the plane.



SOLUTION

Take the propeller and air within it as the control volume. Since the airplane moves in the still air and the control volume is attached to the airplane, which is travelling with a constant velocity of the 160 ft/s, then the inlet velocity is $V_1 = 160$ ft/s.

Relative to the control volume, the flow is steady. The air can be considered as an ideal fluid (incompressible and inviscid) such that at an altitude of 10,000 ft, $\rho_a = 1.754(10^{-3})$ slug/ft³. Average velocity will be used. From the discharge

$$Q = VA; \quad 10\,000 \text{ ft}^3/\text{s} = V[\pi(3.5 \text{ ft})^2] \quad V = 259.84 \text{ ft/s}$$

$$V = \frac{V_1 + V_2}{2}; \quad 259.84 \text{ ft/s} = \frac{160 \text{ ft/s} + V_2}{2} \quad V_2 = 359.69 \text{ ft/s}$$

The thrust on the plane is

$$\begin{aligned} F &= \frac{\rho_a \pi R^2}{2} (V_2^2 - V_1^2) \\ &= \frac{[1.754(10^{-3}) \text{ slug/ft}^3][\pi(3.5 \text{ ft})^2]}{2} [(359.69 \text{ ft/s})^2 - (160 \text{ ft/s})^2] \\ &= 3.503(10^3) \text{ lb} = 3.50 \text{ kip} \end{aligned}$$

Ans.

The power required to turn the propeller is

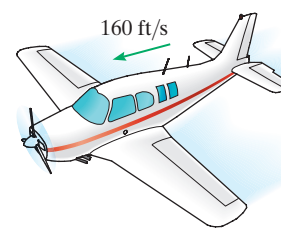
$$\begin{aligned} \dot{W}_i &= FV = [3.503(10^3) \text{ lb}](259.84 \text{ ft/s}) \\ &= [910.22(10^3) \text{ ft} \cdot \text{lb/s}] \left[\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}} \right] \\ &= 1655 \text{ hp} \end{aligned}$$

Ans.

Ans:

$$\begin{aligned} F &= 3.50 \text{ kip} \\ \dot{W} &= 1655 \text{ hp} \end{aligned}$$

6-90. The airplane is flying at 160 ft/s in still air at an altitude of 10 000 ft. The 7-ft-diameter propeller moves the air at 10 000 ft³/s. Determine the propeller's ideal efficiency, and the pressure difference between the front and back of the blades.



SOLUTION

Take the propeller and air within it as the control volume. Since the airplane moves in the still air and the control volume is attached to the airplane, which is travelling with a constant velocity of 160 ft/s, then the inlet velocity is $V_1 = 160$ ft/s.

Relative to the control volume the flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that at an altitude of 10,000 ft, $\rho_a = 1.754(10^{-3})$ slug/ft³. Average velocities will be used. From the discharge

$$Q = VA; \quad 10\,000 \text{ ft}^3/\text{s} = V[\pi(3.5 \text{ ft})^2] \quad V = 259.84 \text{ ft/s}$$

$$V = \frac{V_1 + V_2}{2}; \quad 259.84 \text{ ft/s} = \frac{160 \text{ ft/s} + V_2}{2} \quad V_2 = 359.69 \text{ ft/s}$$

The ideal efficiency of the propeller is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(160 \text{ ft/s})}{160 \text{ ft/s} + 359.69 \text{ ft/s}} = 0.616 \quad \text{Ans.}$$

The pressure difference is

$$\begin{aligned} \Delta p &= p_4 - p_3 = \rho_a V(V_2 - V_1) \\ &= [1.754(10^{-3}) \text{ slug/ft}^3](259.84 \text{ ft/s})[359.69 \text{ ft/s} - (160 \text{ ft/s})] \\ &= \left(91.01 \frac{\text{lb}}{\text{ft}^2}\right) \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2 = 0.632 \text{ psi} \quad \text{Ans.} \end{aligned}$$

Ans:

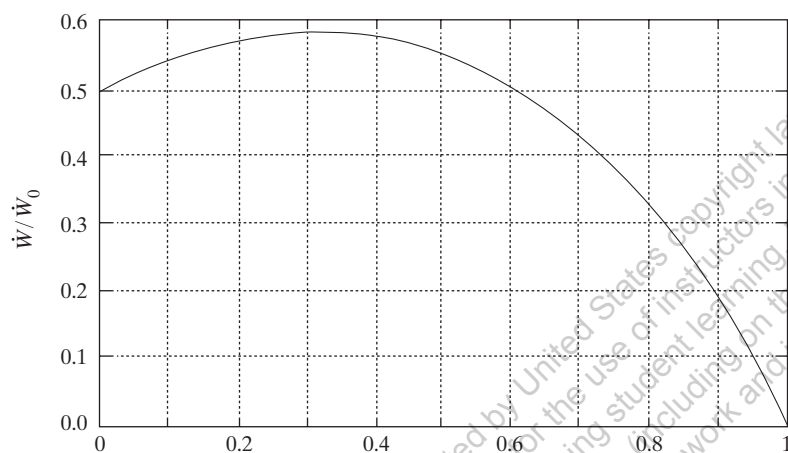
$$e = 0.616$$

$$\Delta p = 0.632 \text{ psi}$$

6-91. Plot Eq. 6-15 and show that the maximum efficiency of a wind turbine is 59.3% as stated by Betz's law.

SOLUTION

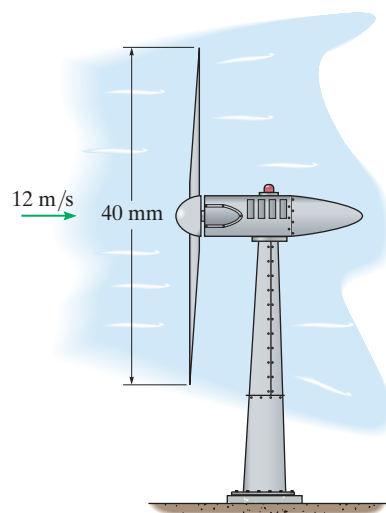
$$e_{\text{turb}} = \frac{1}{2} \left[1 - \left(\frac{V_2}{V_1} \right)^2 \right] \left[1 + \left(\frac{V_2}{V_1} \right) \right]$$



$$\frac{\dot{W}}{\dot{W}_0} = 0.593 = 59.3\%$$

when $\frac{V_2}{V_1} = \frac{1}{3}$.

***6-92.** The wind turbine has a rotor diameter of 40 m and an ideal efficiency of 50% in a 12 m/s wind. If the density of the air is $\rho_a = 1.22 \text{ kg/m}^3$, determine the thrust on the blade shaft, and the power withdrawn by the blades.



SOLUTION

$$e = \frac{1}{2} \left[1 - \left(\frac{V_2}{V_1} \right)^2 \right] \left[1 + \frac{V_2}{V_1} \right]$$

Solving the cubic equation with $e = 0.5$, we find $V_2/V_1 = 0.6180$ as the nonzero solution. Then $V_2 = 0.6180(12 \text{ m/s}) = 7.416 \text{ m/s}$ and

$$V = \frac{V_1 + V_2}{2} = \frac{12 \text{ m/s} + 7.416 \text{ m/s}}{2} = 9.708 \text{ m/s}$$

The thrust on the blades is

$$\begin{aligned} F &= \frac{\rho_a \pi R^2}{2} (V_2^2 - V_1^2) \\ &= \frac{(1.22 \text{ kg/m}^3) \pi (20 \text{ m})^2}{2} [(12 \text{ m/s})^2 - (7.416 \text{ m/s})^2] \\ &= 68.220(10^3) \text{ N} = 68.2 \text{ kN} \end{aligned}$$

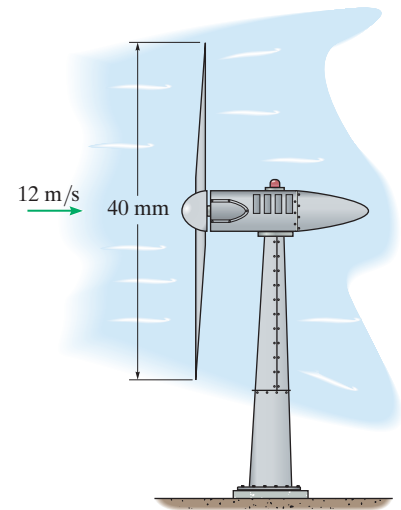
Ans.

The power withdrawn by the blades is

$$\begin{aligned} \dot{W} &= FV = [68.220(10^3) \text{ N}](9.708 \text{ m/s}) \\ &= 662.3(10^3) \text{ W} \\ &= 662 \text{ kW} \end{aligned}$$

Ans.

6–93. The wind turbine has a rotor diameter of 40 m and an efficiency of 50% in a 12 m/s wind. If the density of the air is $\rho_a = 1.22 \text{ kg/m}^3$, determine the difference between the pressure just in front of and just behind the blades. Also find the mean velocity of the air passing through the blades.



SOLUTION

$$e = \frac{1}{2} \left[1 - \left(\frac{V_2}{V_1} \right)^2 \right] \left[1 + \frac{V_2}{V_1} \right]$$

Solving the cubic equation with $e = 0.5$, we find $V_2/V_1 = 0.6180$ as the nonzero solution. Then $V_2 = 0.6180(12 \text{ m/s}) = 7.416 \text{ m/s}$ and

$$V = \frac{V_1 + V_2}{2} = \frac{12 \text{ m/s} + 7.416 \text{ m/s}}{2} = 9.71 \text{ m/s}$$

Ans.

The thrust on the blades is

$$\begin{aligned} F &= \frac{\rho_a \pi R^2}{2} (V_1^2 - V_2^2) \\ &= \frac{(1.22 \text{ kg/m}^3) \pi (20 \text{ m})^2}{2} [(12 \text{ m/s})^2 - (7.416 \text{ m/s})^2] \\ &= 68.220(10^3) \text{ N} \end{aligned}$$

The pressure difference is

$$\Delta p = \frac{F}{A} = \frac{68.220(10^3) \text{ N}}{\pi (20 \text{ m})^2} = 54.3 \text{ Pa}$$

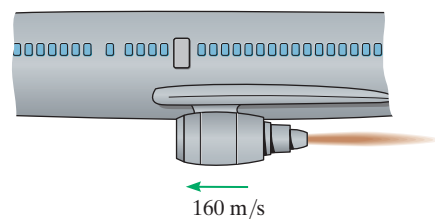
Ans.

Ans:

$$V = 9.71 \text{ m/s}$$

$$\Delta p = 54.3 \text{ Pa}$$

6–94. The jet engine on a plane flying at 160 m/s in still air draws in air at standard atmospheric temperature and pressure through a 0.5-m-diameter inlet. If 2 kg/s of fuel is added and the mixture leaves the 0.3-m-diameter nozzle at 600 m/s, measured relative to the engine, determine the thrust provided by the turbojet.



SOLUTION

From Appendix A, at standard atmospheric pressure and temperature (15° C), the density of air is $\rho_a = 1.23 \text{ kg/m}^3$. Thus,

$$\dot{m}_a = \rho_a VA = (1.23 \text{ kg/m}^3)(160 \text{ m/s})[\pi(0.25 \text{ m})^2] = 38.64 \text{ kg/s}$$

The thrust of the turbojet is

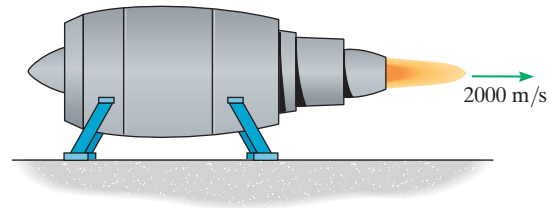
$$\begin{aligned} T &= (\dot{m}_a + \dot{m}_f)V_e - \dot{m}_a V_{cv} \\ &= (38.64 \text{ kg/s} + 2 \text{ kg/s})(600 \text{ m/s}) - (38.64 \text{ kg/s})(160 \text{ m/s}) \\ &= 18.20(10^3) \text{ N} = 18.2 \text{ kN} \end{aligned}$$

Ans.

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

Ans:
18.2 kN

6-95. The jet engine is mounted on the stand while it is being tested. Determine the horizontal force that the engine exerts on the supports, if the fuel–air mixture has a mass flow of 11 kg/s and the exhaust has a velocity of 2000 m/s.



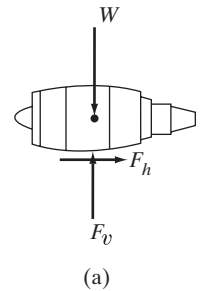
SOLUTION

Take the control volume as the engine and the fluid within it. We consider steady flow of an ideal fluid. Since the turbojet is at rest in still air, $\frac{dV_{cv}}{dt} = 0$, $V_{cv} = 0$, and $\dot{m}_a = 0$. Referring to the free-body diagram of the turbojet in Fig. a,

$$\begin{aligned} (\pm) \Sigma F_x &= m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e \\ -F_h &= 0 + 0 - (0 + 11 \text{ kg/s})(2000 \text{ m/s}) \\ F_h &= 22 \text{ kN} \end{aligned}$$

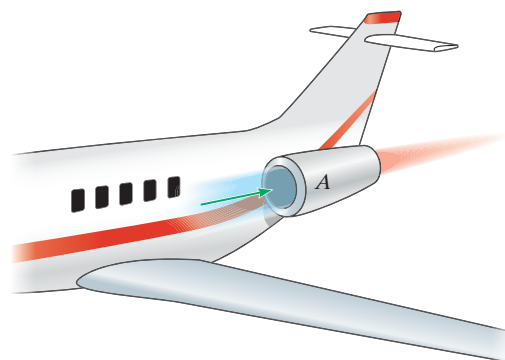
Ans.

This is the magnitude of the force the supports exert on the engine, and therefore also the magnitude of the equal and opposite force the engine exerts on the supports.



Ans:
22 kN

***6-96.** The jet plane has a constant velocity of 750 km/h. Air enters its engine nacelle at A having a cross-sectional area of 0.8 m^2 . Fuel is mixed with the air at $\dot{m}_f = 2.5 \text{ kg/s}$ and is exhausted into the ambient air with a velocity of 900 m/s, measured relative to the plane. Determine the force the engine exerts on the wing of the plane. Take $\rho_a = 0.850 \text{ kg/m}^3$.



SOLUTION

The control volume is considered to be the entire engine and its contents which move with a constant velocity. The flow, measured relative to the control volume, is steady. Here, $V_{cv} = \left(750 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 208.33 \text{ m/s}$, $\dot{m}_f = 2.5 \text{ kg/s}$ and $V_e = 900 \text{ m/s}$.

Thus,

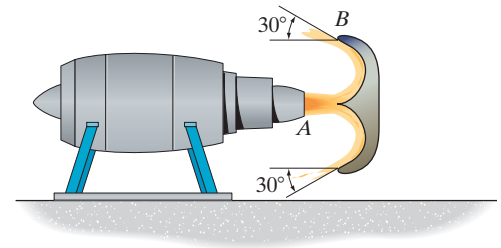
$$\dot{m}_a = \rho_a V_{cv} A_A = (0.850 \text{ kg/m}^3)(208.33 \text{ m/s})(0.8 \text{ m}^2) = 141.67 \text{ kg/s}$$

The thrust developed is

$$\begin{aligned} T &= -[\dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e] \\ &= -[(141.67 \text{ kg/s})(208.33 \text{ m/s}) - (141.67 \text{ kg/s} + 2.5 \text{ kg/s})(900 \text{ m/s})] \\ &= 100.24(10^3) \text{ N} = 100 \text{ kN} \end{aligned}$$

Ans.

6–97. The jet engine is mounted on the stand while it is being tested with the braking deflector in place. If the exhaust has a velocity of 800 m/s and the pressure just outside the nozzle is assumed to be atmospheric, determine the horizontal force that the supports exert on the engine. The fuel–air mixture has a flow of 11 kg/s.



SOLUTION

Under test conditions, with the pressure just outside the nozzle assumed to be atmospheric, the deflector is irrelevant since it is not attached to the engine. Since the engine is at rest in still air, $dV_{cv}/dt = 0$ and $V_{cv} = 0$, so that the support reaction force F , which points rightward, is given by

$$(\pm) \Sigma F_x = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$-F = 0 + 0 - (11 \text{ kg/s})(800 \text{ m/s})$$

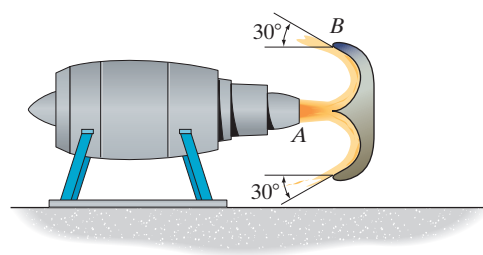
$$F = 8800 \text{ N} = 8.80 \text{ kN}$$

Ans.

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

Ans:
8.80 kN

6–98. If an engine of the type shown in Prob. 6–97 is attached to a jet plane, and it operates the braking deflector with the conditions stated in that problem, determine the speed of the plane in 5 seconds after it lands with a touch-down velocity of 30 m/s. The plane has a mass of 8 Mg. Neglect rolling friction from the landing gear.



SOLUTION

Assume that the fuel is only a small fraction of the fuel-air mixture, so that $\dot{m}_a \approx \dot{m}_a + \dot{m}_f = 11 \text{ kg/s}$. Then the force equation for the whole plane, of mass m_p , is

$$\left(\pm \right) \Sigma F_x = m_p \frac{dV}{dt} + \dot{m}_a V - (\dot{m}_a + \dot{m}_f) V_e$$

$$0 = (8000 \text{ kg}) \frac{dV}{dt} + (11 \text{ kg/s}) V - (11 \text{ kg/s})(-800 \text{ m/s} - V) \cos 30^\circ$$

$$-8000 \frac{dV}{dt} = 11[V(1 + \cos 30^\circ) + 800 \cos 30^\circ]$$

$$-8000 \frac{dV}{dt} = 11(1.8660V + 692.82)$$

$$-\int_{30 \text{ m/s}}^V \frac{8000}{1.866 V + 692.82} dV = \int_0^5 11 dt$$

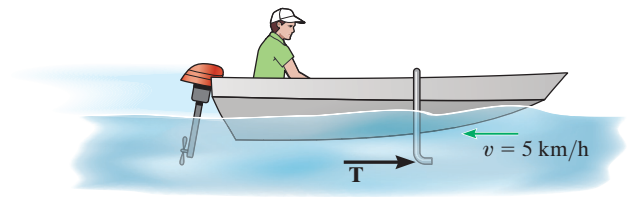
$$-\frac{8000}{1.866} \ln \left(\frac{1.866V + 692.82}{748.80} \right) = 55$$

$$V = 24.9 \text{ m/s}$$

Ans.

Ans:
24.9 m/s

6–99. The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured relative to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection.



SOLUTION

Consider the boat, tube, and water within it as the moving control volume. We consider steady flow of an ideal fluid relative to the control volume.

$$\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}$$

$$v_{D/t} = (70) \left(\frac{1000}{3600} \right) = 19.444 \text{ m/s}$$

$$\Sigma F_x = m \frac{dV}{dt} + v_{D/i} \frac{dm_i}{dt}$$

$$T = 0 + 19.444(0.5) = 9.72 \text{ N}$$

Ans.

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

Ans:
9.72 N

***6-100.** The jet is traveling at a constant velocity of 400 m/s in still air, while consuming fuel at the rate of 1.8 kg/s and ejecting it at 1200 m/s relative to the plane. If the engine consumes 1 kg of fuel for every 50 kg of air that passes through the engine, determine the thrust produced by the engine and the efficiency of the engine.



SOLUTION

The control volume considered is the entire airplane and its contents which moves with a constant velocity. We consider steady flow of an ideal fluid. The flow measured relative to the control volume is steady. Here,

$$V_{cv} = 400 \text{ m/s}, \dot{m}_f = 1.8 \text{ kg/s}, \dot{m}_a = 50 (1.8 \text{ kg/s})$$

$$= 90 \text{ kg/s and } V_e = 1200 \text{ m/s}$$

$$T = -[\dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e]$$

$$= -[(90 \text{ kg/s})(400 \text{ m/s}) - (90 \text{ kg/s} + 1.8 \text{ kg/s})(1200 \text{ m/s})]$$

$$= 74.16(10^3) \text{ N} = 74.2 \text{ kN}$$

Ans.

The useful power output of the engine is

$$\dot{W}_0 = TV = [74.16(10^3) \text{ N}](400 \text{ m/s}) = 29.664(10^6) \text{ W}$$

Some of the power produces the kinetic energy per unit time of the exhaust fuel-air mixture. Its velocity relative to the ground is $V_{\text{mix}} = V_e - V_{cv} = 1200 \text{ m/s} - 400 \text{ m/s} = 800 \text{ m/s}$. Thus, the power loss is

$$\dot{W}_l = \frac{1}{2}(\dot{m}_a + \dot{m}_f) V_{\text{mix}}^2$$

$$= \frac{1}{2}(90 \text{ kg/s} + 1.8 \text{ kg/s})(800 \text{ m/s})^2$$

$$= 29.376(10^6) \text{ W}$$

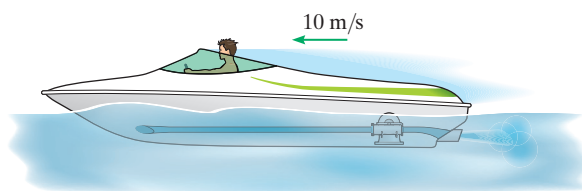
The efficiency of the engine is

$$e = \frac{\dot{W}_0}{\dot{W}_0 + P_l} = \frac{29.664(10^6) \text{ W}}{29.664(10^6) \text{ W} + 29.376(10^6) \text{ W}}$$

$$= 0.502$$

Ans.

6-101. The jet boat takes in water through its bow at $0.03 \text{ m}^3/\text{s}$, while traveling in still water with a constant velocity of 10 m/s . If the water is ejected from a pump through the stern at 30 m/s , measured relative to the boat, determine the thrust developed by the engine. What would be the thrust if the $0.03 \text{ m}^3/\text{s}$ of water were taken in along the sides of the boat, perpendicular to the direction of motion? If the efficiency is defined as the work done per unit time divided by the energy supplied per unit time, then determine the efficiency for each case.



SOLUTION

The control volume considered is the entire boat and its contents, which moves with a constant velocity. The flow, measured relative to the control volume, is steady. Water is considered to be incompressible. Here, $V_{cv} = 10 \text{ m/s}$, $\dot{m}_f = 0$, $\dot{m}_w = \rho Q = (1000 \text{ kg/m}^3)(0.03 \text{ m}^3/\text{s}) = 30 \text{ kg/s}$, and $V_e = 30 \text{ m/s}$. The thrust is

$$\begin{aligned} T_1 &= -[\dot{m}_w V_{cv} - (\dot{m}_w + \dot{m}_f) V_e] \\ &= -[(30 \text{ kg/s})(10 \text{ m/s}) - (30 \text{ kg/s} + 0)(30 \text{ m/s})] \\ &= 600 \text{ N} \end{aligned}$$

Ans.

If the intake of water is perpendicular to the direction of motion, $V_{cv} = 0$. Then

$$\begin{aligned} T_2 &= [\dot{m}_w V_{cv} - (\dot{m}_w + \dot{m}_f) V_e] \\ &= -[(30 \text{ kg/s})(0) - (30 \text{ kg/s} + 0)(30 \text{ m/s})] \\ &= 900 \text{ N} \end{aligned}$$

Ans.

The power output for both cases can be determined from

$$(\dot{W}_o)_1 = T_1 V = (600 \text{ N})(10 \text{ m/s}) = 6000 \text{ W}$$

$$(\dot{W}_o)_2 = T_2 V = (900 \text{ N})(10 \text{ m/s}) = 9000 \text{ W}$$

Some of the power produces the kinetic energy per unit time of the ejected water. Its velocity relative to ground is $V = V_e - V_{cv} = 30 \text{ m/s} - 10 \text{ m/s} = 20 \text{ m/s}$. For both cases, the power loss in the same and is

$$\dot{W}_l = \frac{1}{2}(\dot{m}_w + \dot{m}_f) V^2 = \frac{1}{2}(30 \text{ kg/s} + 0)(20 \text{ m/s})^2 = 6000 \text{ W} \quad \textbf{Ans.}$$

Thus, the efficiency for each case is

$$e_1 = \frac{(\dot{W}_o)_1}{(\dot{W}_o)_1 + (\dot{W}_o)_l} = \frac{6000 \text{ W}}{6000 \text{ W} + 6000 \text{ W}} = 0.5 \quad \textbf{Ans.}$$

$$e_2 = \frac{(\dot{W}_o)_2}{(\dot{W}_o)_2 + (\dot{W}_o)_l} = \frac{9000 \text{ W}}{9000 \text{ W} + 6000 \text{ W}} = 0.6 \quad \textbf{Ans.}$$

Ans:

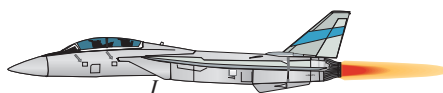
$$T_1 = 600 \text{ N}$$

$$T_2 = 900 \text{ N}$$

$$e_1 = 0.5$$

$$e_2 = 0.6$$

6–102. The 10-Mg jet plane has a constant speed of 860 km/h when it is flying horizontally. Air enters the intake *I* at the rate of 40 m³/s. If the engine burns fuel at the rate of 2.2 kg/s, and the gas (air and fuel) is exhausted relative to the plane with a speed of 600 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that the air has a constant density of $\rho_a = 1.22 \text{ kg/m}^3$.



SOLUTION

Take the plane and its contents as the control volume. We consider steady flow of an ideal fluid.

$$V_{cv} = \left(860 \frac{\text{km}}{\text{h}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = 238.89 \text{ m/s} \text{ and } \dot{m}_a = \rho Q = (1.22 \text{ kg/m}^3)(40 \text{ m}^3/\text{s}) = 48.8 \text{ kg/s}$$

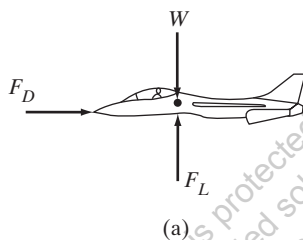
Since the airplane is traveling with constant speed, $\frac{dV_{cv}}{dt} = 0$. Referring to the free-body diagram of the jet plane in Fig. *a*,

$$(\pm) \Sigma F_x = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$-F_D = 0 + (48.8 \text{ kg/s})(238.89 \text{ m/s}) - (48.8 \text{ kg/s} + 2.2 \text{ kg/s})(600 \text{ m/s})$$

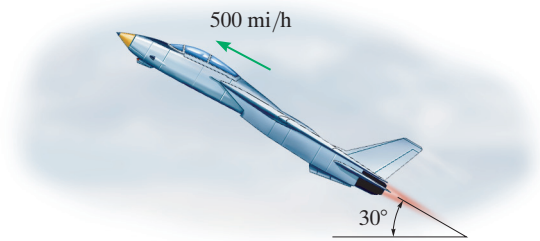
$$F_D = 18.94(10^3) \text{ N} = 18.9 \text{ kN}$$

Ans.



Ans:
18.9 kN

6-103. The jet is traveling at a speed of 500 mi/h, 30° above the horizontal. If the fuel is being spent at 10 lb/s, and the engine takes in air at 900 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 4000 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.07v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. Take 1 mi = 5280 ft.



SOLUTION

The control volume considered is the entire jet and its contents as shown in Fig. *a* which is accelerating. We consider steady flow of an ideal fluid relative to the control volume. Here,

$$V_{cv} = \left(500 \frac{\text{mi}}{\text{h}} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 733.33 \text{ ft/s}$$

$$F_D = 0.07V_{cv}^2 = 0.07(733.33^2) = 37\,644.44 \text{ lb}$$

$$\dot{m}_a = \frac{900 \text{ lb/s}}{32.2 \text{ ft/s}^2} = 27.9503 \text{ slug/s}$$

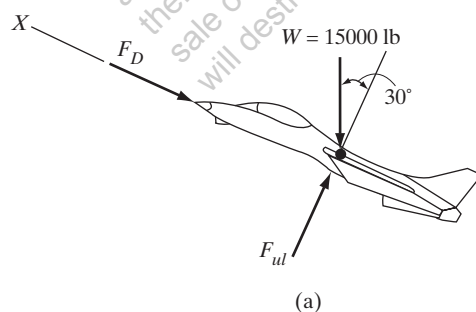
$$\dot{m}_f = \frac{10 \text{ lb/s}}{32.2 \text{ ft/s}^2} = 0.3106 \text{ slug/s}$$

$$V_e = 4000 \text{ ft/s}$$

Referring to the FBD of the control volume, Fig. *a*,

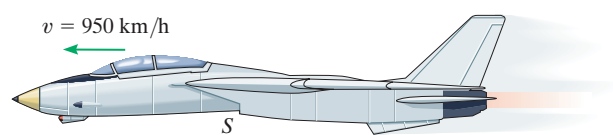
$$\begin{aligned} \sum F_x &= m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e \\ -(15000 \text{ lb}) \sin 30^\circ - 37644.44 \text{ lb} &= \left(\frac{15000 \text{ lb}}{32.2 \text{ ft/s}^2} \right) a_{cv} + (27.9503 \text{ slug/s})(733.33 \text{ ft/s}) \\ -(27.9503 \text{ slug/s} + 0.3106 \text{ slug/s})(4000 \text{ ft/s}) & \\ a_{cv} &= 101.76 \text{ ft/s}^2 = 102 \text{ ft/s}^2 \end{aligned}$$

Ans.



Ans:
102 ft/s²

***6–104.** The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops *S* at the rate of 50 m³/s. If the engine burns fuel at the rate of 0.4 kg/s, and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m³.



SOLUTION

The control volume considered is the entire jet and its contents as shown in Fig. *a*. We consider steady flow of an ideal fluid relative to the control volume. Here

$$V_{cv} = \left(950 \frac{\text{km}}{\text{h}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 263.89 \text{ m/s}$$

$$\dot{m}_a = \rho_a Q = (1.22 \text{ kg/m}^3)(50 \text{ m}^3/\text{s}) = 61 \text{ kg/s}$$

$$\dot{m}_f = 0.4 \text{ kg/s}$$

$$V_e = 450 \text{ m/s}$$

Ans.

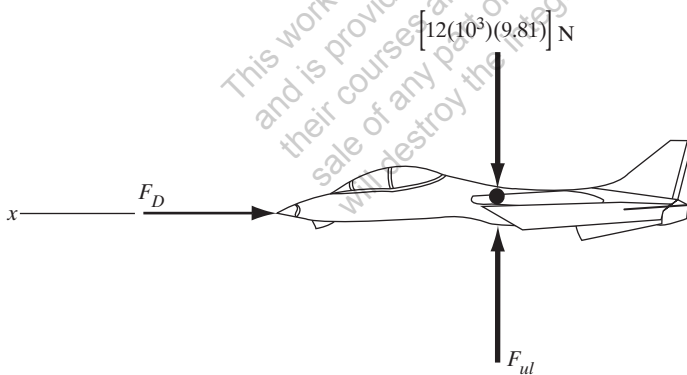
Referring to the FBD of the control volume, Fig. *a*, with $\frac{dV_{cv}}{dt} = 0$, since the jet travels with a constant velocity, we have

$$\sum F_x = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$-F_D = 0 + (61 \text{ kg/s})(263.89 \text{ m/s}) - (61 \text{ kg/s} + 0.4 \text{ kg/s})(450 \text{ m/s})$$

$$F_D = 11.53(10^3) \text{ N} = 11.5 \text{ kN}$$

Ans.



(a)

6–105. A commercial jet aircraft has a mass of 150 Mg and is cruising at a constant speed of 850 km/h in level flight ($\theta = 0^\circ$). If each of the two engines draws in air at a rate of 1000 kg/s and ejects it with a velocity of 900 m/s relative to the aircraft, determine the maximum angle θ at which the aircraft can fly with a constant speed of 750 km/h. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cV^2$, where c is a constant to be determined. The engines are operating with the same power in both cases. Neglect the amount of fuel consumed.



SOLUTION

The control volume considered is the entire plane and its contents as shown in Fig. *a*, which is accelerating. We consider steady flow of an ideal fluid relative to the control volume. Here,

$$V_{cv} = \left(850 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 236.11 \text{ m/s } (\theta = 0^\circ)$$

$$V_{cv} = \left(750 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 208.33 \text{ m/s}$$

$$\dot{m}_a = 2(1000 \text{ kg/s}) = 2000 \text{ kg/s}$$

$$\dot{m}_f = 0 \text{ (negligible)}$$

$$V_e = 900 \text{ m/s}$$

Referring to the FBD of the control volume, Fig. *a*, along the x axis with $\frac{dV_{cv}}{dt} = 0$ (constant velocity), we have

$$\Sigma F_x = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$\{-[150(10^3)(9.81) \text{ N}]\} \sin \theta - c(208.33 \text{ m/s})^2 = 0 + (2000 \text{ kg/s})(208.33 \text{ m/s}) - (2000 \text{ kg/s} + 0)(900 \text{ m/s})$$

$$1.4715(10^6) \sin \theta + 43.403(10^3)c = 1.3833(10^6) \quad (1)$$

For level flight, $\theta = 0^\circ$. Then

$$-c(236.11 \text{ m/s})^2 = 0 + (2000 \text{ kg/s})(236.11 \text{ m/s}) - (2000 \text{ kg/s} + 0)(900 \text{ m/s})$$

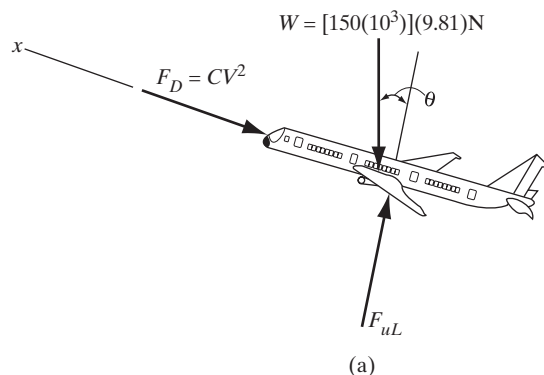
$$c = 23.817$$

Substituting this result into Eq. (1),

$$1.4715(10^6) \sin \theta + 43.403(10^3)(23.817) = 1.3833(10^6)$$

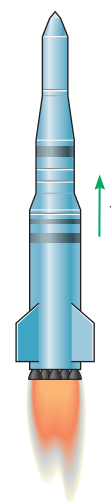
$$\theta = 13.74^\circ = 13.7^\circ$$

Ans.



Ans:
13.7°

6–106. A missile has a mass of 1.5 Mg (without fuel). If it consumes 500 kg of solid fuel at a rate of 20 kg/s and ejects it with a velocity of 2000 m/s relative to the missile, determine the velocity and acceleration of the missile at the instant all the fuel has been consumed. Neglect air resistance and the variation of gravity with altitude. The missile is launched vertically starting from rest.



SOLUTION

The control volume consists of the missile and its contents as shown in Fig. *a*, which is accelerating upward. We consider steady flow of an ideal fluid relative to the control volume. The mass of the control volume as a function of time t is

$$\begin{aligned} M &= M_o - \dot{m}_f t = [(1.5 + 0.5)(10^3) \text{ kg}] - (20 \text{ kg/s})t \\ &= [2(10^3) - 20t] \text{ kg} \end{aligned}$$

Referring to the FBD of the control volume, Fig. *a*, with $\dot{m}_a = 0$ and $V_e = 2000 \text{ m/s}$,

$$\begin{aligned} +\uparrow \Sigma F_y &= m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e \\ -[2(10^3) - 20t](9.81) \text{ N} &= \{ [2(10^3) - 20t] \text{ kg} \} \frac{dV}{dt} + 0 - (0 + 20 \text{ kg/s})(2000 \text{ m/s}) \\ \frac{dV}{dt} &= \frac{40(10^3)}{2(10^3) - 20t} - 9.81 \quad (1) \end{aligned}$$



Integrating this equation with the initial condition $V = 0$ at $t = 0$,

$$\begin{aligned} \int_0^V dV &= \int_0^t \left(\frac{40(10^3)}{2(10^3) - 20t} - 9.81 \right) dt \\ V &= [-2(10^3) \ln [2(10^3) - 20t] - 9.81t] \bigg|_0^t \\ V &= 2(10^3) \ln \left[\frac{2(10^3)}{2(10^3) - 20t} \right] - 9.81t \quad (2) \end{aligned}$$

The time required to consume all the fuel is

$$t = \frac{m_f}{\dot{m}_f} = \frac{500 \text{ kg}}{20 \text{ kg/s}} = 25 \text{ s}$$

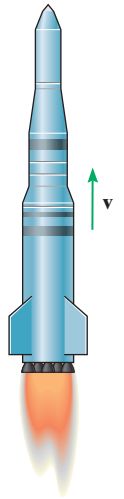
Substituting this result into Eqs. (1) and (2)

$$a = \frac{40(10^3)}{2(10^3) - 20(25)} - 9.81 = 16.9 \text{ m/s}^2 \quad \text{Ans.}$$

$$V = 2(10^3) \ln \left[\frac{2(10^3)}{2(10^3) - 20(25)} \right] - 9.81(25) = 330 \text{ m/s} \quad \text{Ans.}$$

Ans:
330 m/s

6-107. The rocket has a weight of 65 000 lb, including the solid fuel. Determine the constant rate at which the fuel must be burned, so that its thrust gives the rocket a speed of 200 ft/s in 10 s starting from rest. The fuel is expelled from the rocket at a speed of 3000 ft/s relative to the rocket. Neglect air resistance and the variation of gravity with altitude.



SOLUTION

The control volume considered consists of the rocket and its contents as shown in Fig. *a*, which is accelerating upwards. We consider steady flow of an ideal fluid relative to the control volume. The mass of the control volume as a function of time *t* is

$$M = M_o - \dot{m}_f t = \frac{65000 \text{ lb}}{32.2 \text{ ft/s}^2} - \dot{m}_f t = (2018.63 - \dot{m}_f t) \text{ slug}$$

Referring to the *FBD* of the control volume, Fig. *a* with $\dot{m}_a = 0$ and $V_e = 3000 \text{ ft/s}$,

$$+\uparrow \Sigma F_y = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$-(2018.63 - \dot{m}_f t)(32.2) = (2018.63 - \dot{m}_f t) \frac{dV}{dt} + 0 - (0 + \dot{m}_f)(3000 \text{ ft/s})$$

$$\frac{dV}{dt} = \frac{3000 \dot{m}_f}{2018.63 - \dot{m}_f t} - 32.2$$

Integrating this equation with the initial condition $V = 0$ at $t = 0$ and the requirement $V = 200 \text{ ft/s}$ at $t = 10 \text{ s}$,

$$\int_0^{200 \text{ ft/s}} dV = \int_0^{10 \text{ s}} \left(\frac{3000 \dot{m}_f}{2018.63 - \dot{m}_f t} - 32.2 \right) dt$$

$$200 = \left[-3000 \ln(2018.63 - \dot{m}_f t) - 32.2 t \right]_0^{10 \text{ s}}$$

$$200 = 3000 \ln \left(\frac{2018.63}{2018.63 - 10 \dot{m}_f} \right) - 322$$

$$\ln \left(\frac{2018.63}{2018.63 - 10 \dot{m}_f} \right) = 0.174$$

$$\frac{2018.63}{2018.63 - 10 \dot{m}_f} = e^{0.174}$$

$$\dot{m}_f = 32.2 \text{ slug/s}$$

Ans.



(a)

$$W = (2018.63 - \dot{m}_f t)(32.2) \text{ lb}$$

Ans:
32.2 slug/s

***6–108.** The rocket is traveling upwards at 300 m/s and discharges 50 kg/s of fuel with a velocity of 3000 m/s measured relative to the rocket. If the exhaust nozzle has a cross-sectional area of 0.05 m², determine the thrust of the rocket.



SOLUTION

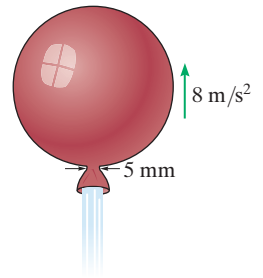
Take the rocket and its contents as the control volume.

The thrust **T** needed to overcome W , F_D , and $m \frac{dV_{cv}}{dt}$ is

$$\begin{aligned} T &= \dot{m}_f V_e \\ &= (50 \text{ kg/s})(3000 \text{ m/s}) \\ &= 150(10^3) \text{ N} = 150 \text{ kN} \quad \mathbf{Ans.} \end{aligned}$$

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted.

6–109. The balloon has a mass of 20 g (empty) and it is filled with air having a temperature of 20°C. If it is released, it begins to accelerate upwards at 8 m/s². Determine the initial mass flow of air from the stem. Assume the balloon is a sphere having a radius of 300 mm.



SOLUTION

The control volume considered is the balloon and the air contained within it, Fig. *a*. The initial flow measured relative to the accelerated control volume is treated as approximately steady. At $T = 20^\circ\text{C}$, $\rho_a = 1.202 \text{ kg/m}^3$. The initial mass and weight of the balloon are

$$\begin{aligned} m &= m_b + m_a = 0.02 \text{ kg} + (1.202 \text{ kg/m}^3) \left[\frac{4}{3} \pi (0.3 \text{ m})^3 \right] \\ &= 0.1559 \text{ kg} \end{aligned}$$

$$W = m_b g = (0.02 \text{ kg})(9.81 \text{ m/s}^2) = 0.1962 \text{ N}$$

We neglect the weight of the air inside because it is counter-acted by buoyancy. Thus,

$$\Sigma \mathbf{F} = m \frac{d\mathbf{V}_{cv}}{dt} + \frac{\partial}{\partial t} \int_{cv} \mathbf{V}_{f/cv} \rho_a dV + \int_{cs} \mathbf{V}_{f/cs} (\rho_a \mathbf{V}_{f/cs}) dA$$

Writing the scalar components of this equation along the y axis by referring to the FBD of the control volume, Fig. *a*.

$$+\uparrow \Sigma F_y = m \frac{dV_{cv}}{dt} + 0 + (-V_e) \rho_a (V_e A_e)$$

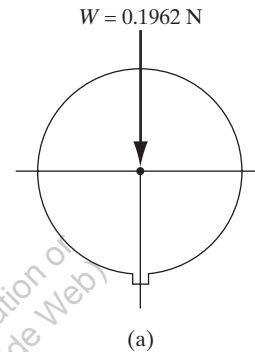
$$-0.1962 \text{ N} = (0.1559 \text{ kg})(8 \text{ m/s}^2) - (1.202 \text{ kg/m}^3) [\pi (0.0025 \text{ m})^2] V_e^2$$

$$V_e = 247.33 \text{ m/s}$$

Thus, the initial mass flow is

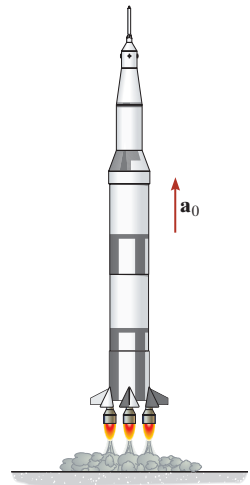
$$\begin{aligned} \dot{m}_e &= \rho_a V_e A_e = (1.202 \text{ kg/m}^3)(247.33 \text{ m/s}) [\pi (0.0025 \text{ m})^2] \\ &= 0.00584 \text{ kg/s} \end{aligned}$$

Ans.



Ans:
0.00584 kg/s

6-110. The rocket has an initial total mass m_0 , including the fuel. When it is fired, it ejects a mass flow of \dot{m}_e with a velocity of v_e measured relative to the rocket. As this occurs, the pressure at the nozzle, which has a cross-sectional area A_e , is p_e . If the drag force on the rocket is $F_D = ct$, where t is the time and c is a constant, determine the velocity of the rocket if the acceleration due to gravity is assumed to be constant.



SOLUTION

The control volume considered is the entire rocket and its contents, which accelerates upward. We consider steady flow of an ideal fluid relative to the control volume. The FBD of the control volume is shown in Fig. *a*. Here, the mass of the rocket as a function of time t is $m = m_0 - \dot{m}_e t$. Thus, the weight of the rocket as a function of time t is $W = mg = (m_0 - \dot{m}_e t)g$. The gage pressure force on the nozzle is $F_e = p_e A_e$.

$$\Sigma \mathbf{F} = m \frac{d\mathbf{V}_{cv}}{dt} + \frac{\partial}{\partial t} \int_{cv} \mathbf{V}_{f/cs} \rho dV + \int_{cv} \mathbf{V}_{f/cs} \rho \mathbf{V}_{f/cs} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along the y axis by referring to Fig. *a*,

$$+\uparrow \Sigma F_y = (m_0 - \dot{m}_e t) \frac{dV}{dt} + 0 + (-V_e)(\rho_e V_e A_e)$$

Here, $\dot{m}_e = \rho_e V_e A_e$. Then

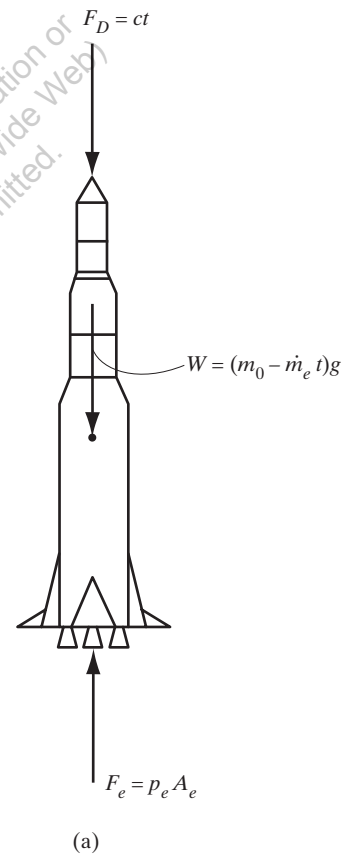
$$\rho_e A_e - ct - (m_0 - \dot{m}_e t)g = (m_0 - \dot{m}_e t) \frac{dV}{dt} - \dot{m}_e V_e$$

$$\frac{dV}{dt} = \frac{\dot{m}_e V_e}{m_0 - \dot{m}_e t} + \frac{p_e A_e}{m_0 - \dot{m}_e t} - \frac{ct}{m_0 - \dot{m}_e t} - g$$

Integrating this equation with the initial condition $V = 0$ at $t = 0$,

$$\begin{aligned} \int_0^V dV &= \int_0^t \left(\frac{\dot{m}_e V_e}{m_0 - \dot{m}_e t} + \frac{p_e A_e}{m_0 - \dot{m}_e t} - \frac{ct}{m_0 - \dot{m}_e t} - g \right) dt \\ V &= \left\{ -V_e \ln(m_0 - \dot{m}_e t) - \frac{p_e A_e}{\dot{m}_e} \ln(m_0 - \dot{m}_e t) - \left[\frac{ct}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2} \ln(m_0 - \dot{m}_e t) \right] - gt \right\} \bigg|_0^t \\ &= V_e \ln \left(\frac{m_0}{m_0 - \dot{m}_e t} \right) + \frac{p_e A_e}{\dot{m}_e} \ln \left(\frac{m_0}{m_0 - \dot{m}_e t} \right) + \frac{ct}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2} \ln \frac{m_0}{m_0 - \dot{m}_e t} - gt \\ &= \left(V_e + \frac{p_e A_e}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2} \right) \ln \left(\frac{m_0}{m_0 - \dot{m}_e t} \right) + \left(\frac{c}{\dot{m}_e} - g \right) t \end{aligned}$$

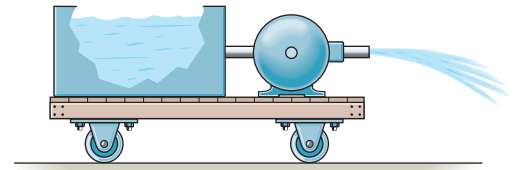
Ans.



Ans:

$$V = \left(V_e + \frac{p_e A_e}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2} \right) \ln \left(\frac{m_0}{m_0 - \dot{m}_e t} \right) + \left(\frac{c}{\dot{m}_e} - g \right) t$$

6-111. The cart has a mass M and is filled with water that has an initial mass m_0 . If a pump ejects the water through a nozzle having a cross-sectional area A , at a constant rate of v_0 relative to the cart, determine the velocity of the cart as a function of time. What is the maximum speed of the cart, assuming all the water can be pumped out? The frictional resistance to forward motion is F . The density of the water is ρ .



SOLUTION

The control volume considered is the entire cart assembly as shown in Fig. *a* which is accelerating. Here, the mass flow rate of the water is

$$\dot{m}_f = \rho V_e A$$

Thus, the mass of the control volume as a function of time t is

$$m = (M + m_0) - \dot{m}_e t = m + m_0 - \rho V_e A t$$

Referring to the FBD of the control volume, Fig. *a* with $\dot{m}_a = 0$,

$$\pm \Sigma F_x = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$-F = (M + m_0 - \rho V_e A t) \frac{dV}{dt} + 0 - (0 + \rho V_e A) V_e$$

$$\frac{dV}{dt} = \frac{\rho V_e^2 A - F}{(M + m_0) - \rho V_e A t}$$

Integrating this equation with the initial condition $V = 0$ at $t = 0$,

$$\int_0^V dV = \int_0^t \left[\frac{\rho V_e^2 A - F}{(M + m_0) - \rho V_e A t} \right] dt$$

$$V = -\frac{\rho V_e^2 A - F}{\rho V_e A} \left[\ln(M + m_0 - \rho V_e A t) \right] \Big|_0^t$$

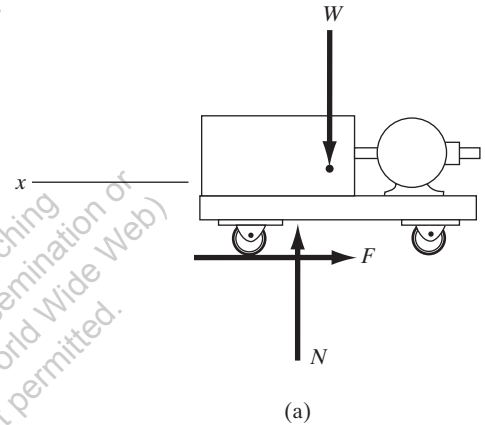
$$= \frac{\rho V_e^2 A - F}{\rho V_e A} \ln \left(\frac{M + m_0}{M + m_0 - \rho V_e A t} \right)$$

$$t_{\text{empty}} = \frac{m_0}{\dot{m}_e} = \frac{m_0}{\rho V_e A} \text{ so}$$

$$V_{\text{max}} = \frac{\rho V_e^2 A - F}{\rho V_e A} \ln \left(\frac{M + m_0}{M} \right)$$

Ans.

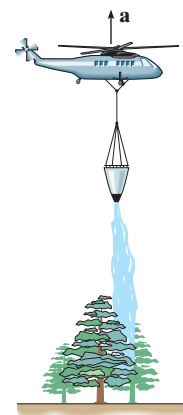
Ans.



Ans:

$$V_{\text{max}} = \frac{\rho V_e^2 A - F}{\rho V_e A} \ln \left(\frac{M + m_0}{M} \right)$$

***6-112.** The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration of the helicopter as the water is being released.



SOLUTION

The control volume considered consists of the helicopter and the bucket containing water as shown in Fig. *a*, which is accelerating upward. We consider steady flow of an ideal fluid relative to the control volume. The initial mass of the control volume is

$$M_0 = 10(10^3) \text{ kg} + 0.5(10^3) \text{ kg} = 10.5(10^3) \text{ kg}$$

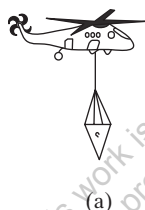
Since the helicopter is hovering before the water is released, its weight and the water's initial weight are balanced by the uplift generated by the rotor blade. Therefore, they are not shown in the FBD of the control volume, Fig. *a*. Referring to the FBD of the control volume with $\dot{m}_a = 0$, $\dot{m}_f = 50 \text{ kg/s}$, $V_e = 10 \text{ m/s}$,

$$+\uparrow \Sigma F_y = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

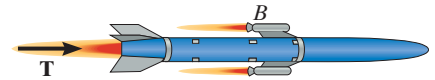
$$0 = [10.5(10^3) \text{ kg}] \frac{dV}{dt} + 0 - (0 + 50 \text{ kg/s})(10 \text{ m/s})$$

$$a_0 = \frac{dV}{dt} = 0.0476 \text{ m/s}^2 \uparrow$$

Ans.



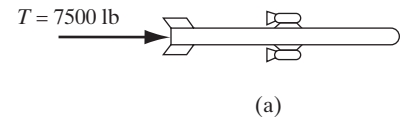
6-113. The missile has an initial total weight of 8000 lb. The constant horizontal thrust provided by the jet engine is $T = 7500$ lb. Additional thrust is provided by *two* rocket boosters B . The propellant in each booster is burned at a constant rate of 80 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the jet engine can be neglected, determine the velocity of the missile after the 3-s burn time of the boosters. The initial velocity of the missile is 375 ft/s. Neglect drag resistance.



SOLUTION

Take the missile and its contents as the control volume. We consider steady flow of an ideal fluid relative to the control volume.

At any instant t , the total mass of the missile is $m = m_0 - \dot{m}_f t$. Referring to the free-body diagram of the missile in Fig. *a*.



$$\begin{aligned}\pm \Sigma F &= m \frac{dV_{cv}}{dt} - \dot{m}_f V_e \\ T &= (m_0 - \dot{m}_f t) \frac{dV}{dt} - \dot{m}_f V_e \\ \frac{dV}{dt} &= \frac{T + \dot{m}_f V_e}{m_0 - \dot{m}_f t}\end{aligned}$$

Integrating gives

$$\begin{aligned}\int_{V_0}^V dV &= \int_0^t \left(\frac{T + \dot{m}_f V_e}{m_0 - \dot{m}_f t} \right) dt \\ V \Big|_{V_0}^V &= - \frac{T + \dot{m}_f V_e}{\dot{m}_f} \ln(m_0 - \dot{m}_f t) \Big|_0^t \\ V &= \frac{T + \dot{m}_f V_e}{\dot{m}_f} \left[\ln \left(\frac{m_0}{m_0 - \dot{m}_f t} \right) \right] + V_0\end{aligned}$$

$$\text{Here, } m_0 = \frac{8000 \text{ lb}}{32.2 \text{ ft/s}^2} = 248.45 \text{ slug}$$

$$\dot{m}_f = 2 \left(\frac{80 \text{ lb/s}}{32.2 \text{ ft/s}^2} \right) = 4.969 \text{ slug/s}$$

$$V_e = 3000 \text{ ft/s}$$

$$t = 3 \text{ s} \quad T = 7500 \text{ lb} \quad V_0 = 375 \text{ ft/s}$$

Substituting these values into the expression of V ,

$$V = \left(\frac{7500 \text{ lb} + (4.969 \text{ slug/s})(3000 \text{ ft/s})}{4.969 \text{ slug/s}} \right) \ln \left(\frac{248.45 \text{ slug}}{248.45 \text{ slug} - 4.969 \text{ slug/s}(3 \text{ s})} \right) + 375 \text{ ft/s}$$

$$V = 654.02 \text{ ft/s} = 654 \text{ ft/s}$$

Ans.

Ans:
654 ft/s

6-114. The rocket has an initial mass m_0 , including the fuel. For the comfort of the crew, it must maintain a constant upward acceleration a_0 . If the fuel is expelled from the rocket at a relative speed v_e , determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.

SOLUTION

The control volume considered is the entire rocket and its contents as shown in Fig. *a*, which accelerates upward. We consider steady flow of an ideal fluid relative to the control volume. The FBD of the control volume is shown in Fig. *a*. Here, \dot{m}_f is a function of time t . Also, \dot{m}_f is the negative of the rate of change of the rocket's mass m .

Thus, $\dot{m}_f = -\frac{dm}{dt}$. Applying Eq. (6-16) with $\dot{m}_a = 0$,

$$+\uparrow \Sigma F_y = m \frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f) V_e$$

$$-mg = ma_0 + 0 - \left(0 - \frac{dm}{dt} V_e \right)$$

$$\frac{dm}{dt} = -\frac{m(a_0 + g)}{V_e}$$

$$\int_{m_0}^m \frac{dm}{m} = -\frac{a_0 + g}{V_e} \int_0^t dt$$

$$\ln \frac{m}{m_0} = -\left(\frac{a_0 + g}{V_e} \right) t$$

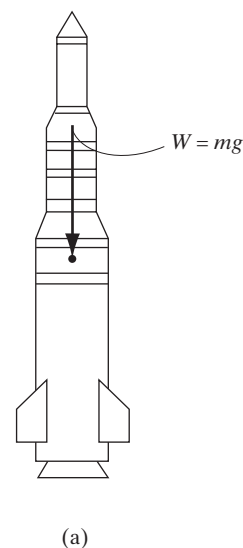
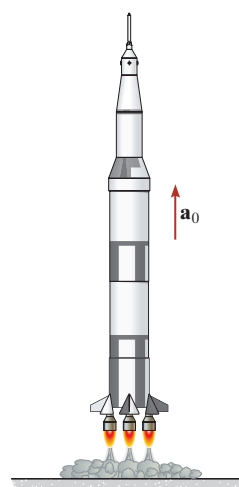
$$\frac{m}{m_0} = e - \left(\frac{a_0 + g}{V_e} \right) t$$

$$m = m_0 e - \left(\frac{a_0 + g}{V_e} \right) t$$

Substitute this result into Eq (1)

$$\dot{m}_f = -\frac{dm}{dt} = \frac{m_0}{V_e} (a_0 + g) e - \left(\frac{a_0 + g}{V_e} \right) t$$

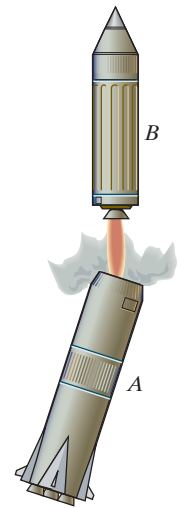
Ans.



Ans:

$$\dot{m}_f = -\frac{dm}{dt} = \frac{m_0}{V_e} (a_0 + g) e^{-(a_0 + g)t/V_e}$$

6–115. The second stage *B* of the two-stage rocket weighs 2500 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 800 lb. If it is consumed at the rate of 75 lb/s, and ejected with a relative velocity of 6000 ft/s, determine the acceleration of the second stage *B* just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravity and air resistance.



SOLUTION

Take the second stage of the rocket and its contents as the control volume. We consider steady flow of an ideal fluid relative to the control volume. When second stage is fired, the total mass is $m = \frac{2500 \text{ lb} + 800 \text{ lb}}{32.2 \text{ ft/s}^2} = 102.48 \text{ slug}$. Since the effect of gravity and air resistance can be neglected, $\Sigma F_y = 0$.

$$\Sigma F_y = m \frac{dV_{cv}}{dt} - \dot{m}_f V_e$$

$$0 = (102.48 \text{ slug}) \frac{dV}{dt} - \left(\frac{75}{32.2} \text{ slug/s} \right) (6000 \text{ ft/s})$$

$$a = \frac{dV}{dt} = 136.36 \text{ ft/s}^2 = 136 \text{ ft/s}^2$$

Ans.

Just before all the fuel is consumed, $m = \frac{2500 \text{ lb}}{32.2 \text{ ft/s}^2} = 77.64 \text{ slug}$

$$\Sigma F_y = m \frac{dV}{dt} - \dot{m}_f V_e$$

$$0 = (77.64 \text{ slug}) \frac{dV}{dt} - \left(\frac{75}{32.2} \text{ slug/s} \right) (6000 \text{ ft/s})$$

$$a = \frac{dV}{dt} = 180 \text{ ft/s}^2$$

Ans.

Ans:

When second stage is fired, $a = 136 \text{ ft/s}^2$.

Just before all the fuel is consumed, $a = 180 \text{ ft/s}^2$.