1–1. Represent each of the following quantities with combinations of units in the correct SI form, using an appropriate prefix: (a) $\text{GN} \cdot \mu\text{m}$, (b) $\text{kg}/\mu\text{m}$, (c) N/ks^2, (d) $\text{kN}/\mu\text{s}$.

SOLUTION

a) $\text{GN} \cdot \mu\text{m} = (10^9)\text{N}(10^{-6})\text{m} = 10^3\ \text{N} \cdot \text{m} = \text{kN} \cdot \text{m}$ **Ans.**

b) $\text{kg}/\mu\text{m} = (10^3)\text{g}/(10^{-6})\text{m} = 10^9\ \text{g}/\text{m} = \text{Gg}/\text{m}$ **Ans.**

c) $\text{N}/\text{ks}^2 = \text{N}/(10^3\ \text{s})^2 = 10^{-6}\ \text{N}/\text{s}^2 = \mu\text{N}/\text{s}^2$ **Ans.**

d) $\text{kN}/\mu\text{s} = (10^3)\text{N}/(10^{-6})\text{s} = 10^9\ \text{N}/\text{s} = \text{GN}/\text{s}$ **Ans.**

Ans:
a) $\text{kN} \cdot \text{m}$
b) Gg/m
c) $\mu\text{N}/\text{s}^2$
d) GN/s

1–2. Evaluate each of the following to three significant figures, and express each answer in SI units using an appropriate prefix: (a) $(425\text{ mN})^2$, (b) $(67\,300\text{ ms})^2$, (c) $\left[723(10^6)\right]^{1/2}$ mm.

SOLUTION

a) $(425\text{ mN})^2 = \left[425(10^{-3})\text{ N}\right]^2 = 0.181\text{ N}^2$ **Ans.**

b) $(67\,300\text{ ms})^2 = \left[67.3(10^3)(10^{-3})\text{ s}\right]^2 = 4.53(10^3)\text{ s}^2$ **Ans.**

c) $\left[723(10^6)\right]^{1/2}\text{ mm} = \left[723(10^6)\right]^{1/2}(10^{-3})\text{ m} = 26.9\text{ m}$ **Ans.**

Ans:
a) 0.181 N^2
b) $4.53(10^3)\text{ s}^2$
c) 26.9 m

1–3. Evaluate each of the following to three significant figures, and express each answer in SI units using an appropriate prefix: (a) 749 μm/63 ms, (b) (34 mm)(0.0763 Ms)/263 mg, (c) (4.78 mm)(263 Mg).

SOLUTION

a) $749\ \mu\text{m}/63\ \text{ms} = 749(10^{-6})\ \text{m}/63(10^{-3})\ \text{s} = 11.88(10^{-3})\ \text{m/s}$

$= 11.9\ \text{mm/s}$ **Ans.**

b) $(34\ \text{mm})(0.0763\ \text{Ms})/263\ \text{mg} = \left[34(10^{-3})\ \text{m}\right]\left[0.0763(10^{6})\ \text{s}\right]/\left[263(10^{-6})(10^{3})\ \text{g}\right]$

$= 9.86(10^{6})\ \text{m}\cdot\text{s/kg} = 9.86\ \text{Mm}\cdot\text{s/kg}$ **Ans.**

c) $(4.78\ \text{mm})(263\ \text{Mg}) = \left[4.78(10^{-3})\ \text{m}\right]\left[263(10^{6})\ \text{g}\right]$

$= 1.257(10^{6})\ \text{g}\cdot\text{m} = 1.26\ \text{Mg}\cdot\text{m}$ **Ans.**

Ans:
a) 11.9 mm/**s**
b) 9.86 Mm · s/kg
c) 1.26 Mg · m

***1–4.** Convert the following temperatures: (a) 20°C to degrees Fahrenheit, (b) 500 K to degrees Celsius, (c) 125°F to degrees Rankine, (d) 215°F to degrees Celsius.

SOLUTION

a) $T_C = \frac{5}{9}(T_F - 32)$

$20°\text{C} = \frac{5}{9}(T_F - 32)$

$T_F = 68.0°\text{F}$ **Ans.**

b) $T_K = T_C + 273$

$500\text{ K} = T_C + 273$

$T_C = 227°\text{C}$ **Ans.**

c) $T_R = T_F + 460$

$T_R = 125°\text{F} + 460 = 585°\text{R}$ **Ans.**

d) $T_C = \frac{5}{9}(T_F - 32)$

$T_C = \frac{5}{9}(215°\text{F} - 32) = 102°\text{C}$ **Ans.**

1–5. Mercury has a specific weight of 133 kN/m^3 when the temperature is 20°C. Determine its density and specific gravity at this temperature.

SOLUTION

$$\gamma = \rho g$$

$$133(10^3)\ N/m^3 = \rho_{Hg}(9.81\ m/s^2)$$

$$\rho_{Hg} = 13\,558\ kg/m^3 = 13.6\ Mg/m^3$$ **Ans.**

$$S_{Hg} = \frac{\rho_{Hg}}{\rho_w} = \frac{13\,558\ kg/m^3}{1000\ kg/m^3} = 13.6$$ **Ans.**

Ans:
$\rho_{Hg} = 13.6\ Mg/m^3$
$S_{Hg} = 13.6$

1–6. The fuel for a jet engine has a density of $1.32\ \text{slug/ft}^3$. If the total volume of fuel tanks A is $50\ \text{ft}^3$, determine the weight of the fuel when the tanks are completely full.

SOLUTION

The specific weight of the fuel is

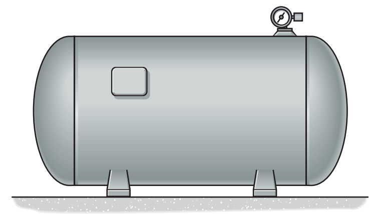
$$\gamma = \rho g = \left(1.32\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right) = 42.504\ \text{lb/ft}^3$$

Then, the weight of the fuel is

$$W = \gamma \forall = \left(42.504\ \text{lb/ft}^3\right)\left(50\ \text{ft}^3\right) = 2.13\left(10^3\right)\ \text{lb} = 2.13\ \text{kip} \qquad \textbf{Ans.}$$

Ans:
$\gamma = 42.5\ \text{lb/ft}^3$
$W = 2.13\ \text{kip}$

1–7. If air within the tank is at an absolute pressure of 680 kPa and a temperature of 70°C, determine the weight of the air inside the tank. The tank has an interior volume of 1.35 m^3.



SOLUTION

From the table in Appendix A, the gas constant for air is $R = 286.9\ \text{J/kg}\cdot\text{K}$.

$$p = \rho R T$$

$$680\left(10^3\right)\ \text{N/m}^2 = \rho(286.9\ \text{J/kg}\cdot\text{K})(70^\circ + 273)\ \text{K}$$

$$\rho = 6.910\ \text{kg/m}^3$$

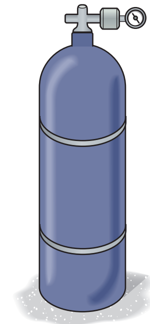
The weight of the air in the tank is

$$W = \rho g \forall = \left(6.910\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.35\ \text{m}^3\right)$$

$$= 91.5\ \text{N}$$ **Ans.**

Ans:
91.5 N

***1–8.** The bottle tank has a volume of 1.12 m^3 and contains oxygen at an absolute pressure of 12 MPa and a temperature of 30°C. Determine the mass of oxygen in the tank.



SOLUTION

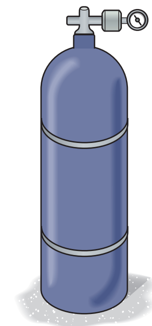
From the table in Appendix A, the gas constant for oxygen is $R = 259.8 \text{ J/kg}\cdot\text{K}$.

$$p = \rho R T$$
$$12\left(10^6\right) \text{ N/m}^2 = \rho(259.8 \text{ J/kg}\cdot\text{K})(30° + 273) \text{ K}$$
$$\rho = 152.44 \text{ kg/m}^3$$

The mass of oxygen in the tank is

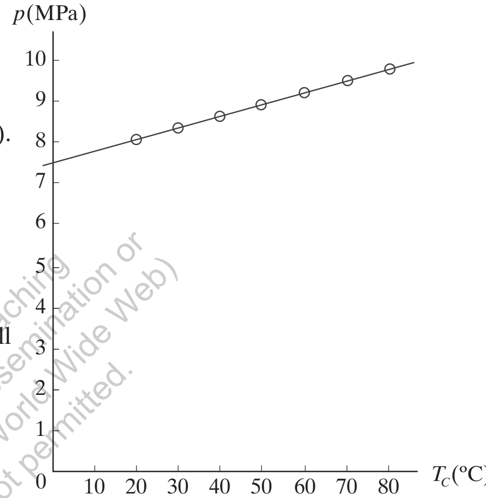
$$m = \rho V = \left(152.44 \text{ kg/m}^3\right)\left(0.12 \text{ m}^3\right)$$
$$= 18.3 \text{ kg}$$ **Ans.**

1–9. The bottle tank has a volume of 0.12 m^3 and contains oxygen at an absolute pressure of 8 MPa and temperature of 20°C. Plot the variation of the pressure in the tank (vertical axis) versus the temperature for $20°\text{C} \le T \le 80°\text{C}$. Report values in increments of $\Delta T = 10°\text{C}$.



SOLUTION

T_C(°C)	20	30	40	50	60	70	80
p(MPa)	8.00	8.27	8.55	8.82	9.09	9.37	9.64



(a)

From the table in Appendix A, the gas constant for oxygen is $R = 259.8\ \text{J}/(\text{kg}\cdot\text{K})$. For $T = (20°\text{C} + 273)\ \text{K} = 293\ \text{K}$,

$$p = \rho RT$$
$$8\left(10^6\right)\ \text{N/m}^2 = \rho\left[259.8\ \text{J}/(\text{kg}\cdot\text{K})\right](293\ \text{K})$$
$$\rho = 105.10\ \text{kg/m}^3$$

Since the mass and volume of the oxygen in the tank remain constant, its density will also be constant.

$$p = \rho RT$$
$$p = \left(105.10\ \text{kg/m}^3\right)\left[259.8\ \text{J}/(\text{kg}\cdot\text{K})\right](T_C + 273)$$
$$p = (0.02730\ T_C + 7.4539)\left(10^6\right)\ \text{Pa}$$
$$p = (0.02730T_C + 7.4539)\ \text{MPa where } T_C \text{ is in °C.}$$

The plot of p vs T_C is shown in Fig. *a*.

Ans:
$p = (0.0273\ T_c + 7.45)$ MPa, where T_c is in C°

1–10. Determine the specific weight of carbon dioxide when the temperature is 100°C and the absolute pressure is 400 kPa.

SOLUTION

From the table in Appendix A, the gas constant for carbon dioxide is $R = 188.9\ \text{J/kg}\cdot\text{K}$.

$$p = \rho R T$$

$$400\left(10^3\right)\ \text{N/m}^2 = \rho(188.9\ \text{J/kg}\cdot\text{K})(100^\circ + 273)\ \text{K}$$

$$\rho = 5.677\ \text{kg/m}^3$$

The specific weight of carbon dioxide is

$$\gamma = \rho g = \left(5.677\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)$$

$$= 55.7\ \text{N/m}^3$$

Ans.

Ans:
$55.7\ \text{N/m}^3$

1–11. Determine the specific weight of air when the temperature is 100°F and the absolute pressure is 80 psi.

SOLUTION

From the table in Appendix A, the gas constant for the air is $R = 1716\ \text{ft}\cdot\text{lb/slug}\cdot\text{R}$.

$$p = \rho R T$$

$$80\ \text{lb/in}^2\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = \rho(1716\ \text{ft}\cdot\text{lb/slug}\cdot\text{R})(100^\circ + 460)\ \text{R}$$

$$\rho = 0.01200\ \text{slug/ft}^3$$

The specific weight of the air is

$$\gamma = \rho g = \left(0.01200\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)$$

$$= 0.386\ \text{lb/ft}^3 \qquad \textbf{Ans.}$$

Ans:
$0.386\ \text{lb/ft}^3$

***1–12.** Dry air at 25°C has a density of 1.23 kg/m^3. But if it has 100% humidity at the same pressure, its density is 0.65% less. At what temperature would dry air produce this same density?

SOLUTION

For both cases, the pressures are the same. Applying the ideal gas law with $\rho_1 = 1.23\ \text{kg/m}^3$, $\rho_2 = \left(1.23\ \text{kg/m}^3\right)(1 - 0.0065) = 1.222005\ \text{kg/m}^3$ and $T_1 = (25°\text{C} + 273) = 298\ \text{K}$,

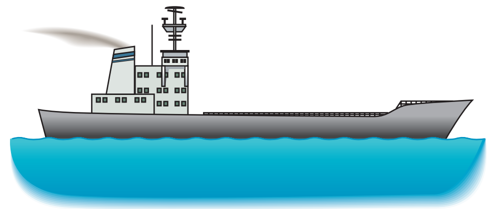
$$p = \rho_1 R T_1 = \left(1.23\ \text{kg/m}^3\right) R\,(298\ \text{K}) = 366.54\ \text{R}$$

Then

$$p = \rho_2 R T_2; \quad 366.54\ R = \left(1.222005\ \text{kg/m}^3\right) R(T_C + 273)$$

$$T_C = 26.9°\text{C} \qquad \textbf{Ans.}$$

1–13. The tanker carries $1.5(10^6)$ barrels of crude oil in its hold. Determine the weight of the oil if its specific gravity is 0.940. Each barrel contains 42 gallons, and there are $7.48\ \text{gal/ft}^3$.



SOLUTION

The specific weight of the oil is

$$\gamma_o = S_o\gamma_w = 0.940\left(62.4\ \text{lb/ft}^3\right) = 58.656\ \text{lb/ft}^3$$

Weight of one barrel of oil:

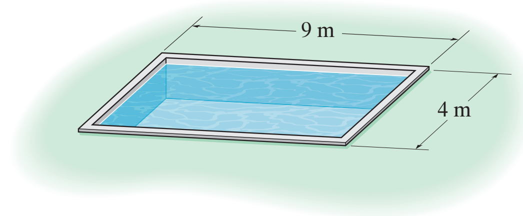
$$W_b = \gamma_o V = \left(58.656\ \text{lb/ft}^3\right)(42\ \text{gal/bl})\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)$$
$$= 329.4\ \text{lb/bl}$$

Total weight:

$$W = 1.5\left(10^6\right)\text{bl}(329.4\ \text{lb/bl})$$
$$= 494\left(10^6\right)\text{lb}$$ **Ans.**

Ans:
$494\left(10^6\right)$ lb

1–14. Water in the swimming pool has a measured depth of 3.03 m when the temperature is 5°C. Determine its approximate depth when the temperature becomes 35°C. Neglect losses due to evaporation.



SOLUTION

From Appendix A, at $T_1 = 5°\text{C}$, $(\rho_w)_1 = 1000.0\ \text{kg/m}^3$. The volume of the water is $\forall = Ah$. Thus, $\forall_1 = (9\ \text{m})(4\ \text{m})(3.03\ \text{m})$. Then

$$(\rho_w)_1 = \frac{m}{\forall_1}; \qquad 1000.0\ \text{kg/m}^3 = \frac{m}{36\ \text{m}^2(3.03\ \text{m})}$$

$$m = 109.08(10^3)\ \text{kg}$$

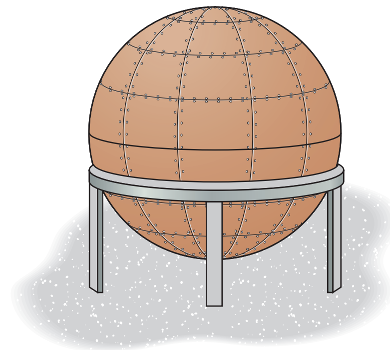
At $T_2 = 35°\text{C}$, $(\rho_w)_2 = 994.0\ \text{kg/m}^3$. Then

$$(\rho_w)_2 = \frac{m}{\forall_2}; \qquad 994.0\ \text{kg/m}^3 = \frac{109.08(10^3)}{(36\ \text{m}^2)h}$$

$$h = 3.048\ \text{m} = 3.05\ \text{m}$$ **Ans.**

Ans:
3.05 m

1–15. The tank contains air at a temperature of 15°C and an absolute pressure of 210 kPa. If the volume of the tank is 5 m^3 and the temperature rises to 30°C, determine the mass of air that must be removed from the tank to maintain the same pressure.



SOLUTION

For $T_1 = (15 + 273)\text{ K} = 288\text{ K}$ and $R = 286.9\text{ J/kg}\cdot\text{K}$ for air, the ideal gas law gives

$$p_1 = \rho_1 R T_1; \qquad 210\left(10^3\right)\text{ N/m}^2 = \rho_1(286.9\text{ J/kg}\cdot\text{K})(288\text{ K})$$
$$\rho_1 = 2.5415\text{ kg/m}^3$$

Thus, the mass of air at T_1 is

$$m_1 = \rho_1 V = \left(2.5415\text{ kg/m}^3\right)\left(5\text{ m}^3\right) = 12.70768\text{ kg}$$

For $T_2 = (273 + 30)\text{ K} = 303\text{ K}$ and $R = 286.9\text{ J/kg}\cdot\text{K}$

$$p_2 = \rho_1 R T_2; \qquad 210(10^3)\text{ N/m}^2 = \rho_2(286.9\text{ J/kg}\cdot\text{K})(303\text{ K})$$
$$\rho_2 = 2.4157\text{ kg/m}^3$$

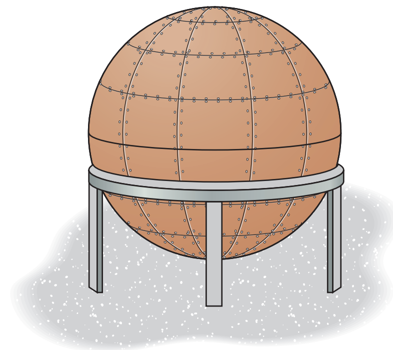
Thus, the mass of air at T_2 is $m_2 = \rho_2 V = \left(2.4157\text{ kg/m}^3\right)\left(5\text{ m}^3\right) = 12.07886\text{ kg}$

Finally, the mass of air that must be removed is

$$\Delta m = m_1 - m_2 = 12.70768\text{ kg} - 12.07886\text{ kg} = 0.629\text{ kg} \qquad \textbf{Ans.}$$

Ans:
0.629 kg

***1–16.** The tank contains 2 kg of air at an absolute pressure of 400 kPa and a temperature of 20°C. If 0.6 kg of air is added to the tank and the temperature rises to 32°C, determine the pressure in the tank.



SOLUTION

For $T_1 = 20 + 273 = 293$ K, $p_1 = 400$ kPa and $R = 286.9$ J/kg·K for air, the ideal gas law gives

$$p_1 = \rho_1 R T_1; \qquad 400(10^3)\ \text{N/m}^2 = \rho_1(286.9\ \text{J/kg}\cdot\text{K})(293\ \text{K})$$
$$\rho_1 = 4.7584\ \text{kg/m}^3$$

Since the volume is constant. Then

$$\cancel{V} = \frac{m_1}{\rho_1} = \frac{m_2}{\rho_2};\ \rho_2 = \frac{m_2}{m_1}\rho_1$$

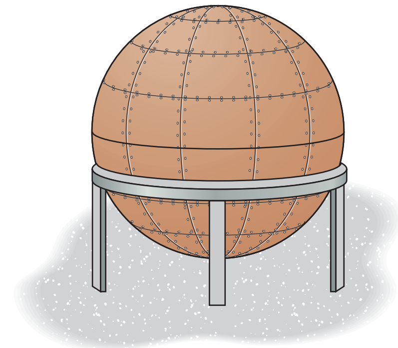
Here $m_1 = 2$ kg and $m_2 = (2 + 0.6)$ kg $= 2.6$ kg

$$\rho_2 = \left(\frac{2.6\ \text{kg}}{2\ \text{kg}}\right)\left(4.7584\ \text{kg/m}^3\right) = 6.1859\ \text{kg/m}^3$$

Again applying the ideal gas law with $T_2 = (32 + 273)$ K $= 305$ K

$$p_2 = \rho_2 R T_2 = \left(6.1859\ \text{kg/m}^3\right)(286.9\ \text{J/kg}\cdot\text{k})(305\ \text{K}) = 541.30\left(10^3\right)\ \text{Pa}$$
$$= 541\ \text{kPa} \qquad \textbf{Ans.}$$

1–17. The tank initially contains carbon dioxide at an absolute pressure of 200 kPa and temperature of 50°C. As more carbon dioxide is added, the pressure is increasing at 25 kPa/min. Plot the variation of the pressure in the tank (vertical axis) versus the temperature for the first 10 minutes. Report the values in increments of two minutes.



SOLUTION

p(kPa)	200	225	250	275	300	325
T_C(°C)	50.00	90.38	130.75	171.12	211.50	251.88

From the table in Appendix A, the gas constant for carbon dioxide is $R = 188.9\ \mathrm{J/(kg \cdot K)}$. For $T = (50°\mathrm{C} + 273)\ \mathrm{K} = 323\ \mathrm{K}$,

$$p = \rho R T$$

$$200\left(10^3\right)\ \mathrm{N/m^2} = \rho\left[188.9\ \mathrm{J/(kg \cdot K)}\right](323\ \mathrm{K})$$

$$\rho = 3.2779\ \mathrm{kg/m^3}$$

Since the mass and the volume of carbon dioxide in the tank remain constant, its density will also be constant.

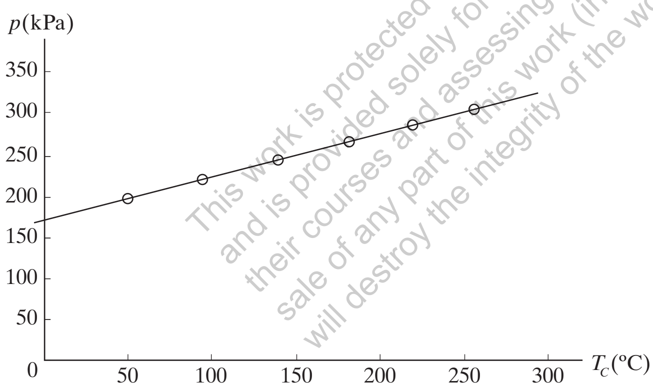
$$p = \rho R T$$

$$p = \left(3.2779\ \mathrm{kg/m^3}\right)\left[188.9\ \mathrm{J/(kg \cdot K)}\right](T_C + 273)\ \mathrm{K}$$

$$p = (0.6192\ T_C + 169.04)\left(10^3\right)\ \mathrm{Pa}$$

$$p = (0.6192\ T_C + 169.04)\ \mathrm{kPa}\ \text{where } T_C \text{ is in °C}$$

The plot of p vs T_C is shown in Fig. *a*



(a)

Ans:
$p = (0.619\ T_c + 169)$ kPa, where T_c is in C°

1–18. Kerosene has a specific weight of $\gamma_k = 50.5\ \text{lb/ft}^3$ and benzene has a specific weight of $\gamma_b = 56.2\ \text{lb/ft}^3$. Determine the amount of kerosene that should be mixed with 8 lb of benzene so that the combined mixture has a specific weight of $\gamma = 52.0\ \text{lb/ft}^3$.

SOLUTION

The volumes of benzene and kerosene are given by

$$\gamma_b = \frac{W_b}{V_b}; \qquad 56.2\ \text{lb/ft}^3 = \frac{8\ \text{lb}}{V_b} \qquad V_b = 0.1423\ \text{ft}^3$$

$$\gamma_k = \frac{W_k}{V_k}; \qquad 50.5\ \text{lb/ft}^3 = \frac{W_k}{V_k} \qquad V_k = 0.019802\ W_k$$

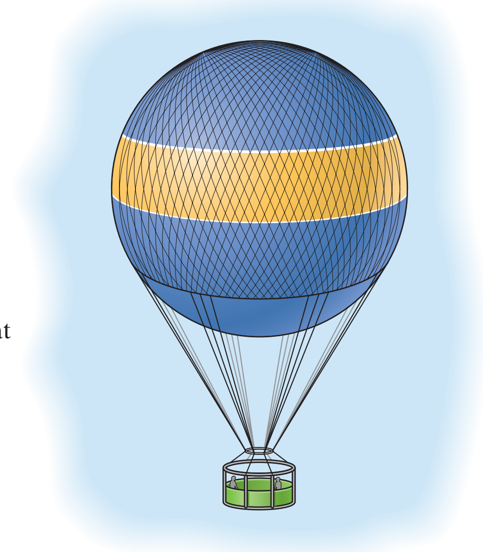
The specific weight of mixture is

$$\gamma = \frac{W_m}{V_m}; \qquad 52.0\ \text{lb/ft}^3 = \frac{W_k + 8\ \text{lb}}{0.1423\ \text{ft}^3 + 0.019802\ W_k}$$

$$W_k = 20.13\ \text{lb} = 20.1\ \text{lb} \qquad \textbf{Ans.}$$

Ans:
20.1 lb

1–19. The 8-m-diameter spherical balloon is filled with helium that is at a temperature of 28°C and a pressure of 106 kPa. Determine the weight of the helium contained in the balloon. The volume of a sphere is $\forall = \frac{4}{3}\pi r^3$.



SOLUTION

For Helium, the gas constant is $R = 2077\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law at $T = (28 + 273)\ \text{K} = 301\ \text{K}$,

$$p = \rho R T; \qquad 106(10^3)\ \text{N/m}^2 = \rho(2077\ \text{J/kg}\cdot\text{K})(301\ \text{K})$$
$$\rho = 0.1696\ \text{kg/m}^3$$

Here

$$\forall = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4\ \text{m})^3 = \frac{256}{3}\pi\ \text{m}^3$$

Then, the mass of the helium is

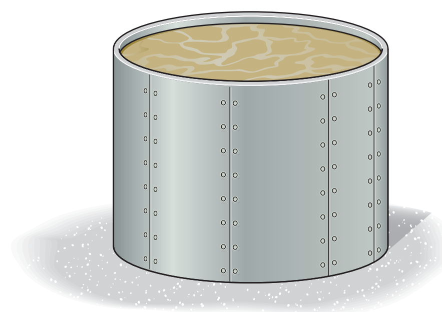
$$M = \rho\forall = \left(0.1696\ \text{kg/m}^3\right)\left(\frac{256}{3}\pi\ \text{m}^3\right) = 45.45\ \text{kg}$$

Thus,

$$W = mg = (45.45\ \text{kg})(9.81\ \text{m/s}^2) = 445.90\ \text{N} = 446\ \text{N} \qquad \textbf{Ans.}$$

Ans:
446 N

***1–20.** Kerosene is mixed with 10 ft^3 of ethyl alcohol so that the volume of the mixture in the tank becomes 14 ft^3. Determine the specific weight and the specific gravity of the mixture.



SOLUTION

From Appendix A,

$$\rho_k = 1.58 \text{ slug/ft}^3$$
$$\rho_{ea} = 1.53 \text{ slug/ft}^3$$

The volume of kerosene is

$$\forall_k = 14 \text{ ft}^3 - 10 \text{ ft}^3 = 4 \text{ ft}^3$$

Then the total weight of the mixture is therefore

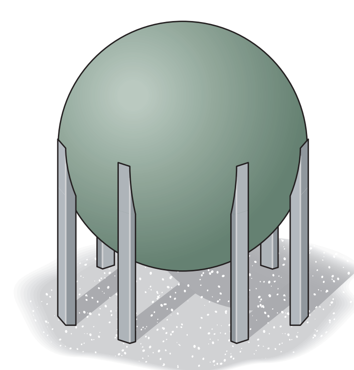
$$\begin{aligned} W &= \rho_k g \forall k + \rho_{ea} g \forall_{ea} \\ &= (1.58 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(4 \text{ ft}^3) + (1.53 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(10 \text{ ft}^3) \\ &= 696.16 \text{ lb} \end{aligned}$$

The specific weight and specific gravity of the mixture are

$$\gamma_m = \frac{W}{\forall} = \frac{696.16 \text{ lb}}{14 \text{ ft}^3} = 49.73 \text{ lb/ft}^3 = 49.7 \text{ lb/ft}^3 \qquad \textbf{Ans.}$$

$$S_m = \frac{\gamma_m}{\gamma_w} = \frac{49.73 \text{ lb/ft}^3}{62.4 \text{ lb/ft}^3} = 0.797 \qquad \textbf{Ans.}$$

1–21. The tank is fabricated from steel that is 20 mm thick. If it contains carbon dioxide at an absolute pressure of 1.35 MPa and a temperature of 20°C, determine the total weight of the tank. The density of steel is 7.85 Mg/m^3, and the inner diameter of the tank is 3 m. *Hint*: The volume of a sphere is $V = \left(\frac{4}{3}\right)\pi r^3$.



SOLUTION

From the table in Appendix A, the gas constant for carbon dioxide is $R = 188.9\ \text{J/kg}\cdot\text{K}$.

$$p = \rho RT$$

$$1.35\left(10^6\right)\ \text{N/m}^2 = \rho_{co}(188.9\ \text{J/kg}\cdot\text{K})(20° + 273)\ \text{K}$$

$$\rho_{co} = 24.39\ \text{kg/m}^3$$

Then, the total weight of the tank is

$$W = \rho_{st}\, g\, V_{st} + \rho_{co}\, g\, V_{co}$$

$$W = \left[7.85\left(10^3\right)\ \text{kg/m}^3\right]\left(9.81\ \text{m/s}^2\right)\left(\frac{4}{3}\right)(\pi)\left[\left(\frac{3.04}{2}\text{m}\right)^3 - \left(\frac{3.00}{2}\text{m}\right)^3\right] + \left(24.39\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(\frac{4}{3}\right)(\pi)\left(\frac{3.00}{2}\text{m}\right)^3$$

$$W = 47.5\ \text{kN}$$ **Ans.**

Ans:
47.5 kN

1–22. What is the increase in the density of helium when the pressure changes from 230 kPa to 450 kPa while the temperature *remains constant* at 20°C? This is called an *isothermal process*.

SOLUTION

Applying the ideal gas law with $T_1 = (20 + 273)\text{ K} = 293\text{ K}$, $p_1 = 230\text{ kPa}$ and $R = 2077\text{ J/(kg}\cdot\text{k)}$,

$$p_1 = \rho_1 R T_1; \qquad 230\left(10^3\right)\text{ N/m}^2 = \rho_1(2077\text{ J/(kg}\cdot\text{K)})(293\text{ K})$$
$$\rho_1 = 0.3779\text{ kg/m}^3$$

For $p_2 = 450\text{ kPa}$ and $T_2 = (20 + 273)\text{ K} = 293\text{ K}$,

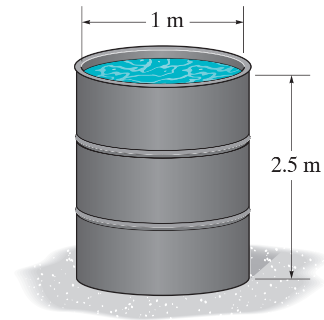
$$p_2 = \rho_2 R T_2; \qquad 450\left(10^3\right)\text{ N/m}^2 = \rho_2(2077\text{ J/(kg}\cdot\text{k)})(293\text{ K})$$
$$\rho_2 = 0.7394\text{ kg/m}^3$$

Thus, the change in density is

$$\Delta\rho = \rho_2 - \rho_1 = 0.7394\text{ kg/m}^3 - 0.3779\text{ kg/m}^3 = 0.3615\text{ kg/m}^3$$
$$= 0.362\text{ kg/m}^3 \qquad \textbf{Ans.}$$

Ans:
0.362 kg/m^3

1–23. The container is filled with water at a temperature of 25°C and a depth of 2.5 m. If the container has a mass of 30 kg, determine the combined weight of the container and the water.



SOLUTION

From Appendix A, $\rho_w = 997.1\ \text{kg/m}^3$ at $T = 25°\text{C}$. Here the volume of water is

$$\forall = \pi r^2 h = \pi(0.5\ \text{m})^2(2.5\ \text{m}) = 0.625\pi\ \text{m}^3$$

Thus, the mass of water is

$$M_w = \rho_w \forall = 997.1\ \text{kg/m}^3\left(0.625\pi\ \text{m}^3\right) = 1957.80\ \text{kg}$$

The total mass is

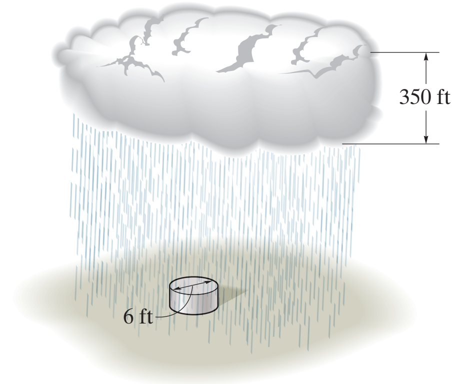
$$M_T = M_w + M_c = (1957.80 + 30)\ \text{kg} = 1987.80\ \text{kg}$$

Then the total weight is

$$W_T = M_T g = (1987.80\ \text{kg})\left(9.81\ \text{m/s}^2\right) = 19500\ \text{N} = 19.5\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
19.5 kN

***1–24.** The rain cloud has an approximate volume of 6.50 mile^3 and an average height, top to bottom, of 350 ft. If a cylindrical container 6 ft in diameter collects 2 in. of water after the rain falls out of the cloud, estimate the total weight of rain that fell from the cloud. 1 mile = 5280 ft.



SOLUTION

The volume of rain water collected is $\forall_w = \pi(3\text{ ft})^2\left(\frac{2}{12}\text{ ft}\right) = 1.5\pi\text{ ft}^3$. Then, the weight of the rain water is $W_w = \gamma_w \forall_w = \left(62.4\text{ lb/ft}^3\right)\left(1.5\pi\text{ ft}^3\right) = 93.6\pi\text{ lb}$. Here, the volume of the overhead cloud that produced this amount of rain is

$$\forall_c' = \pi(3\text{ ft})^2(350\text{ ft}) = 3150\pi\text{ ft}^3$$

Thus,

$$\gamma_c = \frac{W}{\forall_c'} = \frac{93.6\pi\text{ lb}}{3150\pi\text{ ft}^3} = 0.02971\text{ lb/ft}^3$$

Then

$$W_c = \gamma_c \forall_c = \left(0.02971\,\frac{\text{lb}}{\text{ft}^3}\right)\left[(6.50)\left(\frac{5280^3\text{ ft}^3}{1}\right)\right]$$
$$= 28.4\left(10^9\right)\text{ lb} \qquad \textbf{Ans.}$$

1–25. If 4 m^3 of helium at 100 kPa of absolute pressure and 20°C is subjected to an absolute pressure of 600 kPa while the temperature remains constant, determine the new density and volume of the helium.

SOLUTION

From the table in Appendix A, the gas constant for helium is $R = 2077\ \text{J/kg}\cdot\text{K}$,

$$p_1 = \rho_1 R T_1$$
$$100\left(10^3\right)\ \text{N/m}^3 = \rho(2077\ \text{J/kg}\cdot\text{K})(20^\circ + 273)\ \text{K}$$
$$\rho_1 = 0.1643\ \text{kg/m}^3$$

$$T_1 = T_2$$
$$\frac{p_1}{p_2} = \frac{\rho_1 R T_1}{\rho_2 R T_2}$$
$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2}$$
$$\frac{100\ \text{kPa}}{600\ \text{kPa}} = \frac{0.1643\ \text{kg/m}^3}{\rho_2}$$
$$\rho_2 = 0.9859\ \text{kg/m}^3 = 0.986\ \text{kg/m}^3 \qquad \textbf{Ans.}$$

The mass of the helium is

$$m = \rho_1 V_1 = \left(0.1643\ \text{kg/m}^3\right)\left(4\ \text{m}^3\right) = 0.6573\ \text{kg}$$

Since the mass of the helium is constant, regardless of the temperature and pressure,

$$m = \rho_2 V_2$$
$$0.6573\ \text{kg} = \left(0.9859\ \text{kg/m}^3\right) V_2$$
$$V_2 = 0.667\ \text{m}^3 \qquad \textbf{Ans.}$$

Ans:
$\rho_2 = 0.986\ \text{kg/m}^3$, $V_2 = 0.667\ \text{m}^3$

1–26. Water at 20°C is subjected to a pressure increase of 44 MPa. Determine the percent increase in its density. Take $E_{\mathcal{V}} = 2.20\,\text{GPa}$.

SOLUTION

$$\frac{\Delta\rho}{\rho_1} = \frac{m/\mathcal{V}_2 - m/\mathcal{V}_1}{m/\mathcal{V}_1} = \frac{\mathcal{V}_1}{\mathcal{V}_2} - 1$$

To find $\mathcal{V}_1/\mathcal{V}_2$, use $E_{\mathcal{V}} = -d_p/(d\mathcal{V}/\mathcal{V})$.

$$\frac{d\mathcal{V}}{\mathcal{V}} = -\frac{dp}{E_{\mathcal{V}}}$$

$$\int_{\mathcal{V}_1}^{\mathcal{V}_2} \frac{d\mathcal{V}}{\mathcal{V}} = -\frac{1}{E_{\mathcal{V}}}\int_{p_1}^{p_2} dp$$

$$\ln\left(\frac{\mathcal{V}_1}{\mathcal{V}_2}\right) = \frac{1}{E_{\mathcal{V}}}\Delta p$$

$$\frac{\mathcal{V}_1}{\mathcal{V}_2} = e^{\Delta p/E_{\mathcal{V}}}$$

So, since the bulk modulus of water at 20°C is $E_{\mathcal{V}} = 2.20$ GPa,

$$\frac{\Delta\rho}{\rho_1} = e^{\Delta p/E_{\mathcal{V}}} - 1$$

$$= e^{(44\text{ MPa})/2.20\text{ GPa})} - 1$$

$$= 0.0202 = 2.02\% \qquad \textbf{Ans.}$$

Ans:
2.02%

1–27. A solid has a specific weight of 280 lb/ft^3. When a pressure of 800 psi is applied, the specific weight increases to 295 lb/ft^3. Determine the approximate bulk modulus.

SOLUTION

$$\gamma = 280 \text{ lb/ft}^3$$

$$E_{\forall} = -\frac{dp}{\dfrac{d\forall}{\forall}}$$

Since

$$\forall = \frac{W}{\gamma},\; d\forall = -W\frac{d\gamma}{\gamma^2}$$

Thus

$$E_{\forall} = \frac{-dp}{\left[-W\dfrac{d\gamma}{\gamma^2}\Big/(W/\gamma)\right]} = \frac{dp}{\dfrac{d\gamma}{\gamma}}$$

Therefore

$$E_{\forall} = \frac{800 \text{ lb/in}^2}{\left(\dfrac{295 \text{ lb/ft}^3 - 280 \text{ lb/ft}^3}{280 \text{ lb/ft}^3}\right)} = 14.9\left(10^3\right) \text{ lb/in}^2 \qquad \textbf{Ans.}$$

Note: The answer is approximate due to using $\gamma = \gamma_i$. More precisely,

$$E_{\forall} = \frac{\displaystyle\int dp}{\displaystyle\int \frac{d\gamma}{\gamma}} = \frac{800}{\ln(295/280)} = 15.3\left(10^3\right) \text{ lb/in}^2$$

Ans:
$14.9\left(10^3\right) \text{ lb/in}^2$

***1–28.** If the bulk modulus for water at 70°F is 319 kip/in², determine the change in pressure required to reduce its volume by 0.3%.

SOLUTION

Use $E_{\forall} = -dp/(d\forall/\forall)$.

$$dp = -E_{\forall}\frac{d\forall}{\forall}$$

$$\Delta p = \int_{p_i}^{p_f} dp = -E_{\forall}\int_{\forall_i}^{\forall_f}\frac{d\forall}{\forall}$$

$$= -(319 \text{ kip/in}^2)\ln\left(\frac{\forall - 0.03\forall}{\forall}\right)$$

$$= 0.958 \text{ kip/in}^2 \text{ (ksi)}$$ **Ans.**

1–29. Sea water has a density of 1030 kg/m^3 at its surface, where the absolute pressure is 101 kPa. Determine its density at a depth of 7 km, where the absolute pressure is 70.4 MPa. The bulk modulus is 2.33 GPa.

SOLUTION

Since the pressure at the surface is 101 kPa, then $\Delta_p = 70.4 - 0.101 = 70.3$ MPa.

Here, the mass of seawater is constant.

$$\text{M} = \rho_0 \forall_0 = \rho \forall$$

$$\rho = \rho_0 \left(\frac{\forall_0}{\forall}\right)$$

To find $\forall_0/\forall$, use $E_\forall = -dp/(d\forall/\forall)$.

$$\int \frac{d\forall}{\forall} = -\frac{1}{E_\forall} \int dp$$

$$\ln\left(\frac{\forall}{\forall_0}\right) = -\frac{1}{E_\forall} \Delta p$$

$$\frac{\forall_0}{\forall} = e^{\Delta p/E_\forall}$$

So,

$$\begin{aligned} \rho &= \rho_0 e^{\Delta p/E_\forall} \\ &= (1030 \text{ kg/m}^3) e^{(70.3 \text{ MPa}/2.33 \text{ GPa})} \\ &= 1061.55 \text{ kg/m}^3 \\ &= 1.06(10^3) \text{ kg/m}^3 \end{aligned}$$ **Ans.**

Ans:
$1.06(10^3) \text{ kg/m}^3$

1–30. The specific weight of sea water at its surface is $63.6\ \text{lb/ft}^3$, where the absolute pressure is $14.7\ \text{lb/in}^2$. If at a point deep under the water the specific weight is $66.2\ \text{lb/ft}^3$, determine the absolute pressure in lb/in^2 at this point. Take $E_V = 48.7(10^6)\ \text{lb/ft}^2$.

SOLUTION

Use $E_V = -dp/(dV/V)$ and the fact that since the mass and therefore the weight of the seawater is assumed to be constant, $mg = \gamma_1 V_1 = \gamma_2 V_2$, so that $V_2/V_1 = \gamma_1/\gamma_2$.

$$\int dp = -E_V \int \frac{dV}{V}$$

$$\Delta p = -E_V \ln\left(\frac{V_2}{V_1}\right)$$

$$= -E_V \ln\left(\frac{\gamma_1}{\gamma_2}\right)$$

$$p = p_0 + \Delta p$$

$$= 14.7\ \text{lb/in}^2 - \left[48.7(10^6)\ \text{lb/ft}^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \ln\left(\frac{63.6\ \text{lb/ft}^3}{66.2\ \text{lb/ft}^3}\right)$$

$$= 13.6(10^3)\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$13.6(10^3)$ psi

1–31. A 2-kg mass of oxygen is held at a constant temperature of 50° and an absolute pressure of 220 kPa. Determine its bulk modulus.

SOLUTION

$$E_{\forall} = -\frac{dp}{d\forall/\forall} = -\frac{dp\forall}{d\forall}$$

$$p = \rho RT$$

$$dp = d\rho RT$$

$$E_{\forall} = -\frac{d\rho RT\forall}{d\forall} = -\frac{d\rho p\forall}{\rho d\forall}$$

$$\rho = \frac{m}{\forall}$$

$$d\rho = -\frac{md\forall}{\forall^2}$$

$$E_{\forall} = \frac{md\forall p\forall}{\forall^2(m/\forall)d\forall} = P = 220 \text{ kPa}$$ **Ans.**

Note: This illustrates a general point. For an ideal gas, the isothermal (constant-temperature) bulk modulus equals the absolute pressure.

Ans:
220 kPa

***1–32.** At a particular temperature the viscosity of an oil is $\mu = 0.354\ \mathrm{N\cdot s/m^2}$. Determine its kinematic viscosity. The specific gravity is $S_o = 0.868$. Express the answer in SI and FPS units.

SOLUTION

The density of the oil can be determined from

$$\rho_o = S_o\rho_w = 0.868\left(1000\ \mathrm{kg/m^3}\right) = 868\ \mathrm{kg/m^3}$$

$$\upsilon_o = \frac{\mu_o}{\rho_o} = \frac{0.354\ \mathrm{N\cdot s/m^2}}{868\ \mathrm{kg/m^3}} = 0.4078\left(10^{-3}\right)\mathrm{m^2/s} = 0.408\left(10^{-3}\right)\mathrm{m^2/s} \qquad \textbf{Ans.}$$

In FPS units,

$$\upsilon_o = \left[0.4078\left(10^{-3}\right)\frac{\mathrm{m^2}}{\mathrm{s}}\right]\left[\frac{1\ \mathrm{ft}}{0.3048\ \mathrm{m}}\right]^2$$

$$= 4.39\left(10^{-3}\right)\mathrm{ft^2/s} \qquad \textbf{Ans.}$$

1–33. The kinematic viscosity of kerosene is $\upsilon = 2.39(10^{-6})\ \text{m}^2/\text{s}$. Determine its viscosity in FPS units. At the temperature considered, kerosene has a specific gravity of $S_k = 0.810$.

SOLUTION

The density of kerosene is

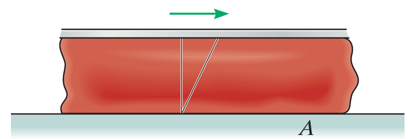
$$\rho_k = S_k \rho_w = 0.810(1000\ \text{kg/m}^3) = 810\ \text{kg/m}^3$$

Then,

$$\begin{aligned}\mu_k &= \upsilon\rho_k = \left[2.39(10^{-6})\ \text{m}^2/\text{s}\right](810\ \text{kg/m}^3)\\ &= \left[1.9359(10^{-3})\ \text{N}\cdot\text{s/m}^2\right]\left(\frac{\text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\\ &= 40.4(10^{-6})\ \text{lb}\cdot\text{s/ft}^2\end{aligned}$$ **Ans.**

Ans:
$40.4(10^{-6})\ \text{lb}\cdot\text{s/ft}^2$

1–34. An experimental test using human blood at $T = 30°\text{C}$ indicates that it exerts a shear stress of $\tau = 0.15\ \text{N/m}^2$ on surface A, where the measured velocity gradient at the surface is $16.8\ \text{s}^{-1}$. Since blood is a non-Newtonian fluid, determine its *apparent viscosity* at the surface.



SOLUTION

Here $\dfrac{du}{dy} = 16.8\ \text{s}^{-1}$ and $\tau = 0.15\ \text{N/m}^2$. Thus

$$\tau = \mu_a \frac{du}{dy}; \qquad 0.15\ \text{N/m}^2 = \mu_a\left(16.8\ \text{s}^{-1}\right)$$

$$\mu_a = 8.93\left(10^{-3}\right)\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Realize that blood is a non-Newtonian fluid. For this reason, we are calculating the *apparent* viscosity.

Ans:
$8.93\left(10^{-3}\right)\ \text{N}\cdot\text{s/m}^2$

1–35. Two measurements of shear stress on a surface and the rate of change in shear strain at the surface for a fluid have been determined by experiment to be $\tau_1 = 0.14\ \text{N/m}^2$, $(du/dy)_1 = 13.63\ \text{s}^{-1}$ and $\tau_2 = 0.48\ \text{N/m}^2$, $(du/dy)_2 = 153\ \text{s}^{-1}$. Classify the fluid as Newtonian or non-Newtonian.

SOLUTION

Applying Newton's Law of viscosity,

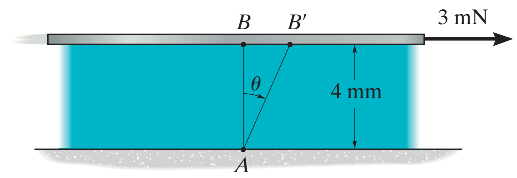
$$\tau_1 = \mu_1\left(\frac{du}{dy}\right)_1; \qquad 0.14\ \text{N/m}^2 = \mu_1\left(13.63\ \text{s}^{-1}\right) \qquad \mu_1 = 0.01027\ \text{N}\cdot\text{s/m}^2$$

$$\tau_2 = \mu_2\left(\frac{du}{dy}\right)_2; \qquad 0.48\ \text{N/m}^2 = \mu_2\left(153\ \text{s}^{-1}\right) \qquad \mu_2 = 0.003137\ \text{N}\cdot\text{s/m}^2$$

Since $\mu_1 \neq \mu_2$ then μ is not constant. It is an apparent viscosity. The fluid is non-Newtonian. **Ans.**

Ans:
non-Newtonian

***1–36.** When the force of 3 mN is applied to the plate the line AB in the liquid remains straight and has an angular rate of rotation of 0.2 rad/s. If the surface area of the plate in contact with the liquid is 0.6 m^2, determine the approximate viscosity of the liquid.



SOLUTION

The shear stress acting on the fluid contact surface is

$$\tau = \frac{P}{A} = \frac{3(10^{-3})\text{ N}}{0.6\text{ m}^2} = 5(10^{-3})\,\frac{\text{N}}{\text{m}^2}$$

Since line AB' is a straight line, the velocity distribution will be linear. Here, the velocity gradient is a constant.

The velocity of the plate is

$$U = a\dot{\theta} = (0.004\text{ m})(0.2\text{ rad/s}) = 0.8(10^{-3})\text{ m/s}$$

Then,

$$\tau = \mu\frac{du}{dy}$$

$$5(10^{-3})\text{ N/m}^2 = \mu\left(\frac{0.8(10^{-3})\text{ m/s}}{0.004\text{ m}}\right)$$

$$\mu = 0.025\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

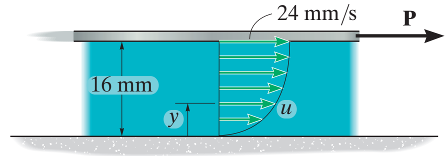
Alternatively,

$$\tau = \mu\frac{d\theta}{dt}$$

$$5(10^{-3})\text{ N/m}^2 = \mu(0.2\text{ rad/s})$$

$$\mu = 0.025\text{ N}\cdot\text{s/m}^2$$

1–37. When the force **P** is applied to the plate, the velocity profile for a Newtonian fluid that is confined under the plate is approximated by $u = (12y^{1/4})$ mm/s, where y is in mm. Determine the shear stress within the fluid at $y = 8$ mm. Take $\mu = 0.5(10^{-3})\ \text{N}\cdot\text{s/m}^2$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = 12y^{1/4}$$

$$\frac{du}{dy} = 3y^{-3/4}$$

At $y = 8$ mm,

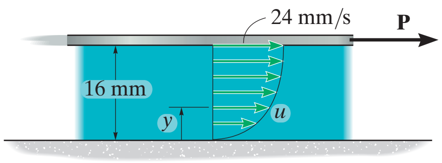
$$\tau = \mu\frac{du}{dy} = 0.5(10^{-3})\ \text{N}\cdot\text{s/m}^2\left[3(8\ \text{mm})^{-3/4}\ \text{s}^{-1}\right]$$

$$\tau = 0.315\ \text{mPa}$$ **Ans.**

Note: When $y = 0$, $\dfrac{du}{dy} \rightarrow \infty$, so that $\tau \rightarrow \infty$. Hence the equation can not be used at this point.

Ans:
0.315 mPa

1–38. When the force **P** is applied to the plate, the velocity profile for a Newtonian fluid that is confined under the plate is approximated by $u = (12y^{1/4})$ mm/s, where y is in mm. Determine the minimum shear stress within the fluid. Take $\mu = 0.5(10^{-3})\ \text{N}\cdot\text{s/m}^2$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = 12y^{1/4}$$

$$\frac{du}{dy} = 3y^{-3/4}$$

The velocity gradient is smallest when $y = 16$ mm. Thus,

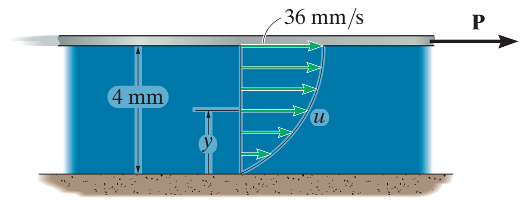
$$\tau_{\min} = \mu\frac{du}{dy} = \left[0.5(10^{-3})\ \text{N}\cdot\text{s/m}^2\right]\left[3(16\ \text{mm})^{-3/4}\ \text{s}^{-1}\right]$$

$$\tau_{\min} = 0.1875\ \text{mPa}$$ **Ans.**

Note: When $y = 0$, $\frac{du}{dy} \rightarrow \infty$, so, that $\tau \rightarrow \infty$. Hence the equation can not be used at this point.

Ans:
0.1875 mPa

1–39. The velocity profile for a thin film of a Newtonian fluid that is confined between a plate and a fixed surface is defined by $u = (10y - 0.25y^2)$ mm/s, where y is in mm. Determine the shear stress that the fluid exerts on the plate and on the fixed surface. Take $\mu = 0.532\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = \left(10y - 0.25y^2\right)\ \mathrm{mm/s}$$

$$\frac{du}{dy} = (10 - 0.5y)\ \mathrm{s^{-1}}$$

At the plate

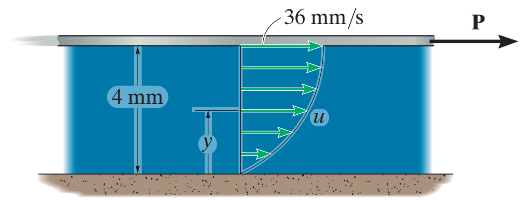
$$\tau_p = \mu\frac{du}{dy} = \left(0.532\ \mathrm{N \cdot s/m^2}\right)\left[10 - 0.5(4\ \mathrm{mm})\ \mathrm{s^{-1}}\right] = 4.26\ \mathrm{Pa} \qquad \textbf{Ans.}$$

At the fixed surface

$$\tau_{fs} = \mu\frac{du}{dy} = \left(0.532\ \mathrm{N \cdot s/m^2}\right)\left[(10 - 0)\ \mathrm{s^{-1}}\right] = 5.32\ \mathrm{Pa} \qquad \textbf{Ans.}$$

Ans:
$\tau_p = 4.26$ Pa, $\tau_{fs} = 5.32$ Pa

***1–40.** The velocity profile for a thin film of a Newtonian fluid that is confined between the plate and a fixed surface is defined by $u = (10y - 0.25y^2)$ mm/s, where y is in mm. Determine the force **P** that must be applied to the plate to cause this motion. The plate has a surface area of 5000 mm^2 in contact with the fluid. Take $\mu = 0.532\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = \left(10y - 0.25y^2\right)\ \mathrm{mm/s}$$

$$\frac{du}{dy} = (10 - 0.5y)\ \mathrm{s^{-1}}$$

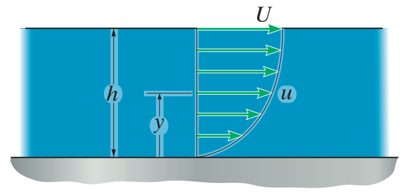
At the plate

$$\tau_p = \mu\frac{du}{dy} = \left(0.532\ \mathrm{N \cdot s/m^2}\right)\left[10 - 0.5(4\ \mathrm{mm})\right]\mathrm{s^{-1}} = 4.256\ \mathrm{Pa}$$

$$P = \tau_p A = \left[(4.256)\ \mathrm{N/m^2}\right]\left[5000\left(10^{-6}\right)\mathrm{m^2}\right]$$

$$= 21.3\ \mathrm{mN}$$ **Ans.**

1–41. The velocity profile of a Newtonian fluid flowing over a fixed surface is approximated by $u = U\sin\left(\frac{\pi}{2h}y\right)$. Determine the shear stress in the fluid at $y = h$ and at $y = h/2$. The viscosity of the fluid is μ.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = U\sin\left(\frac{\pi}{2h}y\right)$$

$$\frac{du}{dy} = U\left(\frac{\pi}{2h}\right)\cos\left(\frac{\pi}{2h}y\right)$$

At $y = h$,

$$\tau = \mu\frac{du}{dy} = \mu U\left(\frac{\pi}{2h}\right)\cos\frac{\pi}{2h}(h)$$

$$\tau = 0; \qquad \textbf{Ans.}$$

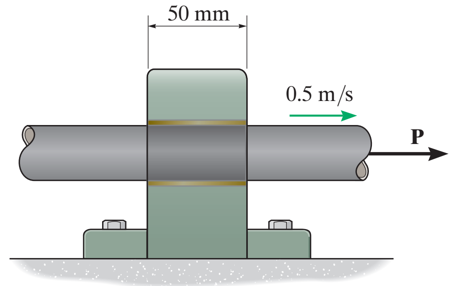
At $y = h/2$,

$$\tau = \mu\frac{du}{dy} = \mu U\left(\frac{\pi}{2h}\right)\cos\frac{\pi}{2h}\left(\frac{h}{2}\right)$$

$$\tau = \frac{0.354\pi\mu U}{h} \qquad \textbf{Ans.}$$

Ans:
At $y = h, \tau = 0;$
At $y = h/2, \quad \tau = \frac{0.354\pi\mu U}{h}$

1–42. If a force of $P = 2\text{ N}$ causes the 30-mm-diameter shaft to slide along the lubricated bearing with a constant speed of 0.5 m/s, determine the viscosity of the lubricant and the constant speed of the shaft when $P = 8\text{ N}$. Assume the lubricant is a Newtonian fluid and the velocity profile between the shaft and the bearing is linear. The gap between the bearing and the shaft is 1 mm.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{2\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = \mu\left(\frac{0.5\text{ m/s}}{0.001\text{ m}}\right)$$

$$\mu = 0.8498\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Thus,

$$\frac{8\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = \left(0.8488\text{ N}\cdot\text{s/m}^2\right)\left(\frac{v}{0.001\text{ m}}\right)$$

$$v = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

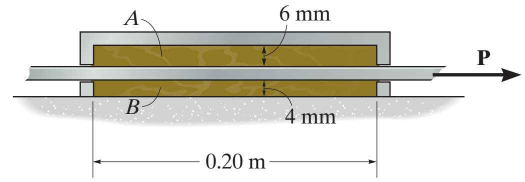
Also, by proportion,

$$\frac{\left(\frac{2\text{ N}}{A}\right)}{\left(\frac{8\text{ N}}{A}\right)} = \frac{\mu\left(\frac{0.5\text{ m/s}}{t}\right)}{\mu\left(\frac{v}{t}\right)}$$

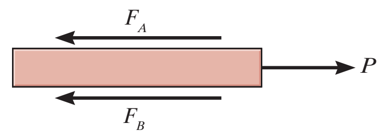
$$v = \frac{4}{2}\text{ m/s} = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\mu = 0.849\text{ N}\cdot\text{s/m}^2$
$v = 2.00\text{ m/s}$

1–43. The 0.15-m-wide plate passes between two layers, A and B, of oil that has a viscosity of $\mu = 0.04\ \text{N}\cdot\text{s/m}^2$. Determine the force **P** required to move the plate at a constant speed of 6 mm/s. Neglect any friction at the end supports, and assume the velocity profile through each layer is linear.



SOLUTION



(a)

The oil is a Newtonian fluid.

Considering the force equilibrium along the x axis, Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_A - F_B = 0$$
$$P = F_A + F_B$$

Since the velocity distribution is linear, the velocity gradient will be constant.

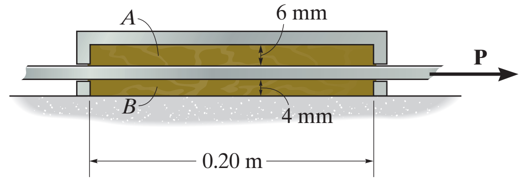
$$\tau_A = \mu \frac{du}{dy} = \left(0.04\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{6\ \text{mm/s}}{6\ \text{mm}}\right) = 0.04\ \text{Pa}$$

$$\tau_B = \mu \frac{du}{dy} = \left(0.04\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{6\ \text{mm/s}}{4\ \text{mm}}\right) = 0.06\ \text{Pa}$$

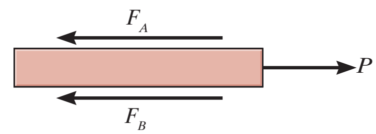
$$P = \left(0.04\ \text{N/m}^2\right)(0.2\ \text{m})(0.15\ \text{m}) + \left(0.06\ \text{N/m}^2\right)(0.2\ \text{m})(0.15\ \text{m})$$
$$= 3.00\ \text{mN} \qquad \textbf{Ans.}$$

Ans:
3.00 mN

***1–44.** The 0.15-m-wide plate passes between two layers A and B of different oils, having viscosities of $\mu_A = 0.03\ \text{N}\cdot\text{s/m}^2$ and $\mu_B = 0.01\ \text{N}\cdot\text{s/m}^2$. Determine the force **P** required to move the plate at a constant speed of 6 mm/s. Neglect any friction at the end supports, and assume the velocity profile through each layer is linear.



SOLUTION



(a)

The oil is a Newtonian fluid.

Considering the force equilibrium along the x axis, Fig. a,

$$\Sigma F_x = 0; \qquad P - F_A - F_B = 0$$

$$P = F_A + F_B$$

Since the velocity distribution is linear, the velocity gradient will be constant.

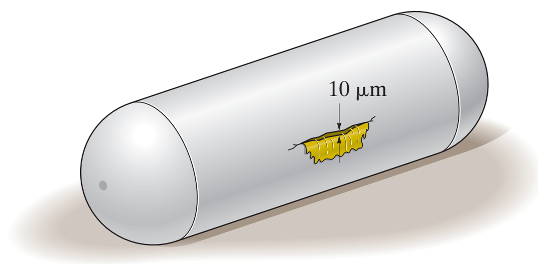
$$\tau_A = \mu\frac{du}{dy} = \left(0.03\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{6\ \text{mm/s}}{6\ \text{mm}}\right) = 0.03\ \text{Pa}$$

$$\tau_B = \mu\frac{du}{dy} = \left(0.01\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{6\ \text{mm/s}}{4\ \text{mm}}\right) = 0.015\ \text{Pa}$$

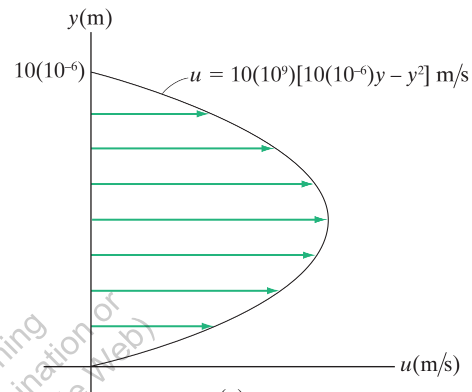
$$P = \left(0.03\ \text{N/m}^2\right)(0.2\ \text{m})(0.15\ \text{m}) + \left(0.015\ \text{N/m}^2\right)(0.2\ \text{m})(0.15\ \text{m})$$

$$= 1.35\ \text{mN} \qquad \textbf{Ans.}$$

1–45. The tank containing gasoline has a long crack on its side that has an average opening of 10 μm. The velocity through the crack is approximated by the equation $u = 10(10^9)\ [10(10^{-6}y - y^2)]$ m/s, where y is in meters, measured upward from the bottom of the crack. Find the shear stress at the bottom, at $y = 0$ and the location y within the crack where the shear stress in the gasoline is zero. Take $\mu_g = 0.317(10^{-3})\ \text{N}\cdot\text{s/m}^2$.



SOLUTION



(a)

Gasoline is a Newtonian fluid.

The rate of change of shear strain as a function of y is

$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right]\ \text{s}^{-1}$$

At the surface of crack, $y = 0$ and $y = 10(10^{-6})$ m. Then

$$\left.\frac{du}{dy}\right|_{y=0} = 10(10^9)\left[10(10^{-6}) - 2(0)\right] = 100(10^3)\ \text{s}^{-1}$$

or

$$\left.\frac{du}{dy}\right|_{y=10\,(10^{-6})\ \text{m}} = 10(10^9)\left\{10(10^{-6}) - 2\left[10(10^{-6})\right]\right\} = -100(10^3)\ \text{s}^{-1}$$

Applying Newton's law of viscosity,

$$\tau_{y=0} = \mu_g \left.\frac{du}{dy}\right|_{y=0} = \left[0.317(10^{-3})\ \text{N}\cdot\text{s/m}^2\right]\left[100(10^3)\ \text{s}^{-1}\right] = 31.7\ \text{N/m}^2 \qquad \textbf{Ans.}$$

$\tau = 0$ when $\dfrac{du}{dy} = 0$. Thus

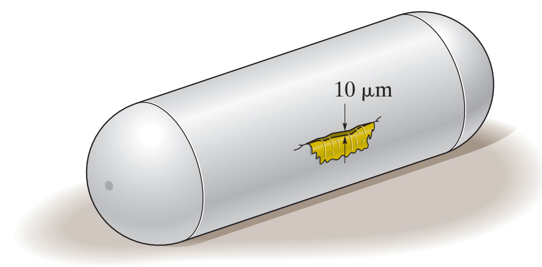
$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right] = 0$$

$$10(10^{-6}) - 2y = 0$$

$$y = 5(10^{-6})\ \text{m} = 5\ \mu\text{m} \qquad \textbf{Ans.}$$

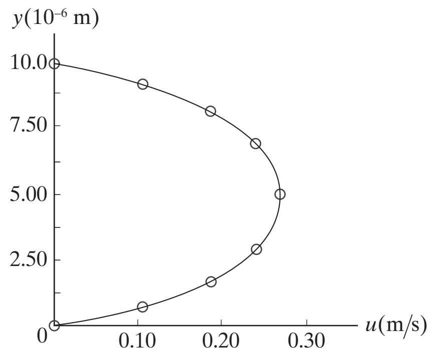
Ans:
$\tau_{y=0} = 31.7\ \text{N/m}^2$
$\tau = 0$ when $y = 5\ \mu\text{m}$

1–46. The tank containing gasoline has a long crack on its side that has an average opening of 10 μm. If the velocity profile through the crack is approximated by the equation $u = 10(10^9)[10(10^{-6}y - y^2)]$ m/s, where y is in meters, plot both the velocity profile and the shear stress distribution for the gasoline as it flows through the crack. Take $\mu_g = 0.317(10^{-3})\ \mathrm{N \cdot s/m^2}$.



SOLUTION

$y(10^{-6}\,\mathrm{m})$	0	1.25	2.50	3.75	5.00
$u(\mathrm{m/s})$	0	0.1094	0.1875	0.2344	0.250
	6.25	7.50	8.75	10.0	
	0.2344	0.1875	0.1094	0	



(a)

Gasoline is a Newtonian fluid. The rate of change of shear strain as a function of y is

$$\frac{du}{dy} = 10(10^9)[10(10^{-6}) - 2y]\ \mathrm{s^{-1}}$$

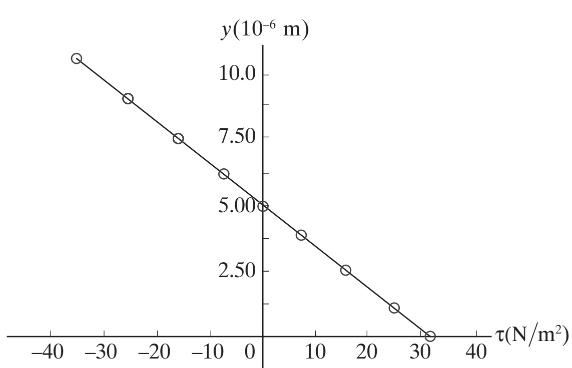
Applying Newton's law of viscoscity,

$$\tau = \mu\frac{du}{dy} = [0.317(10^{-3})\ \mathrm{N \cdot s/m^2}]\{10(10^9)[10(10^{-6}) - 2y]\ \mathrm{s^{-1}}\}$$

$$\tau = 3.17(10^6)[10(10^{-6}) - 2y]\ \mathrm{N/m^2}$$

The plots of the velocity profile and the shear stress distribution are shown in Fig. *a* and *b* respectively.

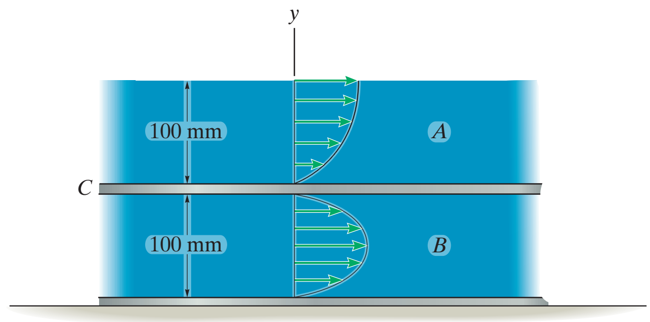
$y(10^{-6}\,\mathrm{m})$	0	1.25	2.50	3.75	5.00
$\tau(\mathrm{N/m^2})$	31.70	23.78	15.85	7.925	0
	6.25	7.50	8.75	10.0	
	−7.925	−15.85	−23.78	−31.70	



(b)

Ans:
$y = 1.25\,(10^{-6})$ m, $u = 0.109$ m/s, $\tau = 23.8\ \mathrm{N/m^2}$

1–47. Water at A has a temperature of 15°C and flows along the top surface of the plate C. The velocity profile is approximated as $\mu_A = 10\sin(2.5\pi y)$ m/s, where y is in meters. Below the plate the water at B has a temperature of 60°C and a velocity profile of $u_B = 4(10^3)(0.1y - y^2)$, where y is in meters. Determine the resultant force per unit length of plate C the flow exerts on the plate due to viscous friction. The plate is 3 m wide.



SOLUTION

Water is a Newtonian fluid.

Water at A, $T = 15°\text{C}$. From Appendix A $\mu = 1.15(10^{-3})\ \text{N}\cdot\text{s/m}^2$. Here

$$\frac{du_A}{dy} = 10\left(\frac{5\pi}{2}\right)\cos\left(\frac{5\pi}{2}y\right) = \left(25\pi\cos\frac{5\pi}{2}y\right)\ \text{s}^{-1}$$

At surface of plate C, $y = 0$. Then

$$\left.\frac{du_A}{dy}\right|_{y=0} = 25\pi\cos\left[\frac{5\pi}{2}(0)\right] = 25\pi\ \text{s}^{-1}$$

Applying Newton's law of viscosity

$$\tau_A\big|_{y=0} = \mu\left.\frac{du_A}{dy}\right|_{y=0} = \left[1.15(10^{-3})\ \text{N}\cdot\text{s/m}^2\right]\left(25\pi\ \text{s}^{-1}\right) = 0.02875\pi\ \text{N/m}^2$$

Water at B, $T = 60°\text{C}$. From Appendix A $\mu = 0.470(10^{-3})\ \text{N}\cdot\text{s/m}^2$. Here

$$\frac{du_B}{dy} = \left[4(10^3)(0.1 - 2y)\right]\ \text{s}^{-1}$$

At the surface of plate C, $y = 0.1$ m. Then

$$\left.\frac{du_B}{dy}\right|_{y=0.1\text{ m}} = 4(10^3)\left[0.1 - 2(0.1)\right] = -400\ \text{s}^{-1}$$

Applying Newton's law of viscosity,

$$\tau_B\big|_{y=0.1\text{ m}} = \mu\left.\frac{du_B}{dy}\right|_{y=0.1\text{ m}} = \left[0.470(10^{-3})\ \text{N}\cdot\text{s/m}^2\right](400\ \text{s}^{-1}) = 0.188\ \text{N/m}^2$$

Here, the area per unit length of plate is $A = 3$ m. Thus

$$F = (\tau_A + \tau_B)A = (0.02875\pi\ \text{N/m}^2 + 0.188\ \text{N/m}^2)(3\ \text{m})$$
$$= 0.835\ \text{N/m} \qquad \textbf{Ans.}$$

Ans:
0.835 N/m

***1–48.** Determine the constants B and C in Andrade's equation for water if it has been experimentally determined that $\mu = 1.00(10^{-3})\ \mathrm{N\cdot s/m^2}$ at a temperature of 20°C and that $\mu = 0.554(10^{-3})\ \mathrm{N\cdot s/m^2}$ at 50°C.

SOLUTION

The Andrade's equation is

$$\mu = Be^{C/T}$$

At $T = (20 + 273)\ \mathrm{K} = 293\ \mathrm{K}$, $\mu = 1.00(10^{-3})\ \mathrm{N\cdot s/m^2}$. Thus

$$1.00(10^{-3})\ \mathrm{N\cdot s/m^2} = Be^{C/293\ \mathrm{K}}$$
$$\ln\left[1.00(10^{-3})\right] = \ln\left(Be^{C/293}\right)$$
$$-6.9078 = \ln B + \ln e^{C/293}$$
$$-6.9078 = \ln B + C/293$$
$$\ln B = -6.9078 - C/293 \qquad \textbf{(1)}$$

At $T = (50 + 273)\ \mathrm{K} = 323\ \mathrm{K}$, $\mu = 0.554(10^{-3})\ \mathrm{N\cdot s/m^2}$. Thus,

$$0.554(10^{-3})\ \mathrm{N\cdot s/m^2} = Be^{C/323}$$
$$\ln\left[0.554(10^{-3})\right] = \ln\left(Be^{C/323}\right)$$
$$-7.4983 = \ln B + \ln e^{C/323}$$
$$-7.4983 = \ln B + \frac{C}{323}$$
$$\ln B = -7.4983 - \frac{C}{323} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

$$-6.9078 - \frac{C}{293} = -7.4983 - \frac{C}{323}$$
$$0.5906 = 0.31699(10^{-3})\,C$$
$$C = 1863.10 = 1863\ \mathrm{K} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1)

$$B = 1.7316(10^{-6})\ \mathrm{N\cdot s/m^2}$$
$$= 1.73(10^{-6})\ \mathrm{N\cdot s/m^2} \qquad \textbf{Ans.}$$

1–49. The viscosity of water can be determined using the empirical Andrade's equation with the constants $B = 1.732(10^{-6})\ \text{N} \cdot \text{s/m}^2$ and $C = 1863\ \text{K}$. With these constants, compare the results of using this equation with those tabulated in Appendix A for temperatures of $T = 10°\text{C}$ and $T = 80°\text{C}$.

SOLUTION

The Andrade's equation for water is

$$\mu = 1.732(10^{-6})e^{1863/T}$$

At $T = (10 + 273)\ \text{K} = 283\ \text{K}$,

$$\mu = 1.732(10^{-6})e^{1863\ \text{K}/283\ \text{K}} = 1.25(10^{-3})\ \text{N} \cdot \text{s/m}^2 \qquad \textbf{Ans.}$$

From the Appendix at $T = 10°\text{C}$,

$$\mu = 1.31(10^{-3})\ \text{N} \cdot \text{s/m}^2$$

At $T = (80 + 273)\ \text{K} = 353\ \text{K}$,

$$\mu = 1.732(10^{-6})e^{1863\ \text{K}/353\ \text{K}} = 0.339(10^{-3})\ \text{N} \cdot \text{s/m}^2 \qquad \textbf{Ans.}$$

From the Appendix at $T = 80°\text{C}$,

$$\mu = 0.356(10^{-3})\ \text{N} \cdot \text{s/m}^2$$

Ans:
At $T = 283\ \text{K}$, $\mu = 1.25(10^{-3})\ \text{N} \cdot \text{s/m}^2$
At $T = 353\ \text{K}$, $\mu = 0.339(10^{-3})\ \text{N} \cdot \text{s/m}^2$

1–50. Determine the constants B and C in the Sutherland equation for air if it has been experimentally determined that at standard atmospheric pressure and a temperature of 20°C, $\mu = 18.3(10^{-6})\ \text{N}\cdot\text{s/m}^2$, and at 50°C, $\mu = 19.6(10^{-6})\ \text{N}\cdot\text{s/m}^2$.

SOLUTION

The Sutherland equation is

$$\mu = \frac{BT^{3/2}}{T + C}$$

At $T = (20 + 273)\ \text{K} = 293\ \text{K}$, $\mu = 18.3(10^{-6})\ \text{N}\cdot\text{s/m}^2$. Thus,

$$18.3(10^{-6})\ \text{N}\cdot\text{s/m}^2 = \frac{B(293^{3/2})}{293\ \text{K} + C}$$

$$B = 3.6489(10^{-9})(293 + C) \qquad \textbf{(1)}$$

At $T = (50 + 273)\ \text{K} = 323\ \text{K}$, $\mu = 19.6(10^{-6})\ \text{N}\cdot\text{s/m}^2$. Thus

$$19.6(10^{-6})\ \text{N}\cdot\text{s/m}^2 = \frac{B(323^{3/2})}{323\ \text{K} + C}$$

$$B = 3.3764(10^{-9})(323 + C) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$B = 1.36(10^{-6})\ \text{N}\cdot\text{s/}(\text{m}^2\,\text{K}^{\frac{1}{2}})$ $\qquad C = 78.8\ \text{K}$ **Ans.**

Ans:
$B = 1.36\ (10^{-6})\ \text{N}\cdot\text{s/}(\text{m}^2\cdot\text{K}^{\frac{1}{2}})$, $C = 78.8\ \text{K}$

1–51. The constants $B = 1.357(10^{-6})\ \mathrm{N \cdot s/(m^2 \cdot K^{1/2})}$ and $C = 78.84\ \mathrm{K}$ have been used in the empirical Sutherland equation to determine the viscosity of air at standard atmospheric pressure. With these constants, compare the results of using this equation with those tabulated in Appendix A for temperatures of $T = 10°\mathrm{C}$ and $T = 80°\mathrm{C}$.

SOLUTION

The Sutherland Equation for air at standard atmospheric pressure is

$$\mu = \frac{1.357(10^{-6})T^{3/2}}{T + 78.84}$$

At $T = (10 + 273)\ \mathrm{K} = 283\ \mathrm{K}$,

$$\mu = \frac{1.357(10^{-6})(283^{3/2})}{283 + 78.84} = 17.9(10^{-6})\ \mathrm{N \cdot s/m^2} \qquad \textbf{Ans.}$$

From Appendix A at $T = 10°\mathrm{C}$,

$$\mu = 17.6(10^{-6})\ \mathrm{N \cdot s/m^2}$$

At $T = (80 + 273)\ \mathrm{K} = 353\ \mathrm{K}$,

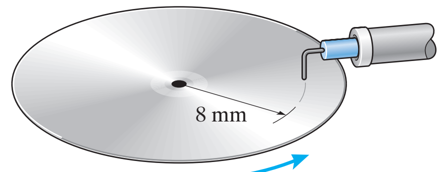
$$\mu = \frac{1.357(10^{-6})(353^{3/2})}{353 + 78.84} = 20.8(10^{-6})\ \mathrm{N \cdot s/m^2} \qquad \textbf{Ans.}$$

From Appendix A at $T = 80°\mathrm{C}$,

$$\mu = 20.9(10^{-6})\ \mathrm{N \cdot s/m^2}$$

Ans:
Using the Sutherland equation,
at $T = 283\ \mathrm{K}$, $\mu = 17.9(10^{-6})\ \mathrm{N \cdot s/m^2}$
at $T = 353\ \mathrm{K}$, $\mu = 20.8(10^{-6})\ \mathrm{N \cdot s/m^2}$

***1–52.** The read–write head for a hand-held music player has a surface area of 0.04 mm^2. The head is held 0.04 μm above the disk, which is rotating at a constant rate of 1800 rpm. Determine the torque **T** that must be applied to the disk to overcome the frictional shear resistance of the air between the head and the disk. The surrounding air is at standard atmospheric pressure and a temperature of 20°C. Assume the velocity profile is linear.



SOLUTION

Here Air is a Newtonian fluid.

$$\omega = \left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 60\pi\text{ rad/s.}$$

Thus, the velocity of the air on the disk is $U = \omega r = (60\pi)(0.008) = 0.48\pi$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. *a*,

$$\frac{du}{dy} = \frac{U}{t} = \frac{0.48\pi\text{ m/s}}{0.04(10^{-6})\text{m}} = 12(10^6)\pi\text{ s}^{-1}$$

For air at $T = 20°\text{C}$ and standard atmospheric pressure, $\mu = 18.1(10^{-6})\text{ N}\cdot\text{s/m}^2$ (Appendix A). Applying Newton's law of viscosity,

$$\tau = \mu\frac{du}{dy} = \left[18.1(10^{-6})\text{ N}\cdot\text{s/m}^2\right]\left[12(10^6)\pi\text{ s}^{-1}\right] = 217.2\pi\text{ N/m}^2$$

Then, the drag force produced is

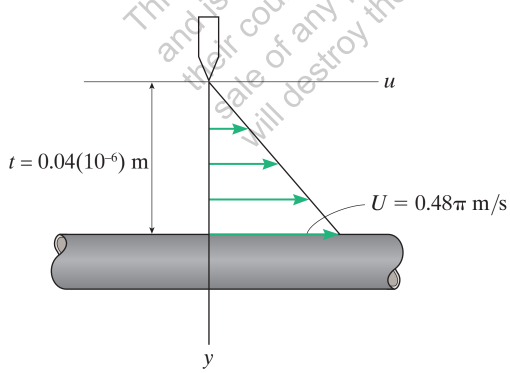
$$F_D = \tau A = (217.2\pi\text{ N/m}^2)\left(\frac{0.04}{1000^2}\text{ m}^2\right) = 8.688(10^{-6})\pi\text{ N}$$

The moment equilibrium about point O requires

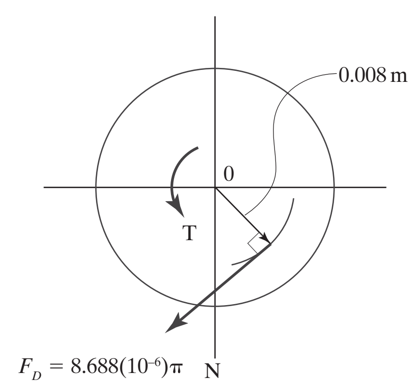
$$\zeta + \Sigma M_O = 0; \qquad T - \left[8.688(10^{-6})\pi\text{ N}\right](0.008\text{ m}) = 0$$

$$T = 0.218(10^{-6})\text{ N}\cdot\text{m}$$

$$= 0.218\ \mu\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

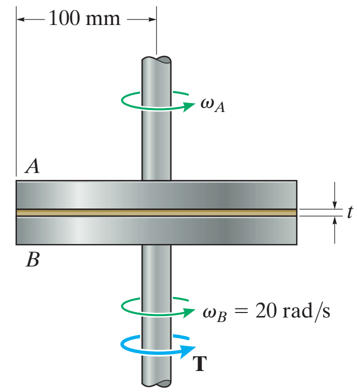


(a)



(b)

1–53. Disks A and B rotate at a constant rate of $\omega_A = 50$ rad/s and $\omega_B = 20$ rad/s respectively. Determine the torque **T** required to sustain the motion of disk B. The gap, $t = 0.1$ mm, contains SAE 10 oil for which $\mu = 0.02\ \text{N}\cdot\text{s/m}^2$. Assume the velocity profile is linear.

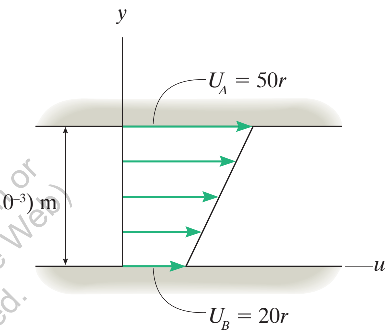


SOLUTION

Oil is a Newtonian fluid.

The velocities of the oil on the surfaces of disks A and B are $U_A = \omega_A r = (50r)$ m/s and $U_B = \omega_B r = (20r)$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. a,

$$\frac{du}{dy} = \frac{U_A - U_B}{t} = \frac{50r - 20r}{0.1\left(10^{-3}\right)\ \text{m}} = 300\left(10^3\right)r\ \text{s}^{-1}$$



(a)

Applying Newton's Law of viscosity,

$$\tau = \mu\frac{du}{dy} = \left(0.02\ \text{N}\cdot\text{s/m}^2\right)\left[300\left(10^3\right)r\right] = (6000r)\ \text{N/m}^2$$

The shaded differential element shown in Fig. b has an area of $dA = 2\pi r\,dr$. Thus, $dF = \tau dA = (6000r)(2\pi r\,dr) = 12\left(10^3\right)\pi r^2\,dr$. Moment equilibrium about point O in Fig. b requires

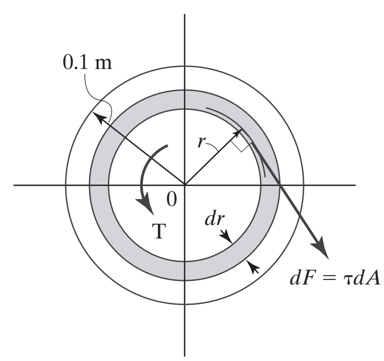
$\zeta + \Sigma M_O = 0;$ $\qquad T - \int r\,dF = 0$

$$T - \int_0^{0.1\text{ m}} r\left[12\left(10^3\right)\pi r^2\,dr\right] = 0$$

$$T = \int_0^{0.1\text{ m}} 12\left(10^3\right)\pi r^3\,dr$$

$$= 12\left(10^3\right)\pi\left(\frac{r^4}{4}\right)\Bigg|_0^{0.1\text{ m}}$$

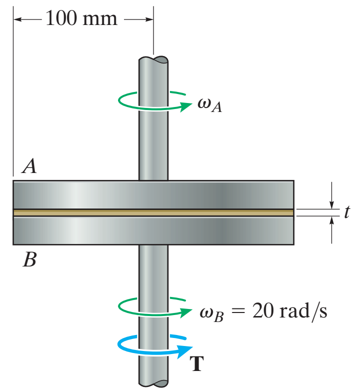
$$= 0.942\ \text{N}\cdot\text{m}$$ **Ans.**



(b)

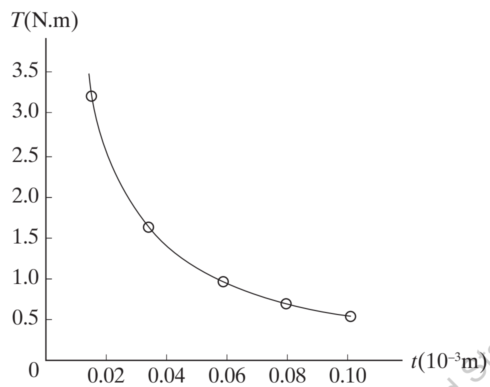
Ans:
$0.942\ \text{N}\cdot\text{m}$

1–54. If disk A is stationary, $\omega_A = 0$ and disk B rotates at $\omega_B = 20$ rad/s, determine the torque $\mathbf{T}$ required to sustain the motion. Plot your results of torque (vertical axis) versus the gap thickness for $0 \le t \le 0.1$ m. The gap contains SAE10 oil for which $\mu = 0.02\ \text{N}\cdot\text{s/m}^2$. Assume the velocity profile is linear.

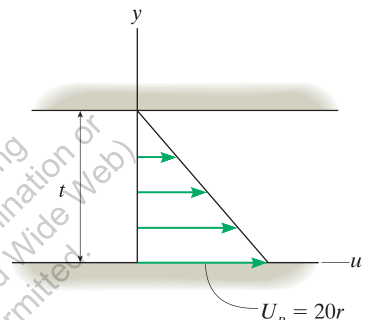


SOLUTION

$t(10^{-3})$ m	0	0.02	0.04	0.06	0.08	0.10
$T(\text{N}\cdot\text{m})$	∞	3.14	1.57	1.05	0.785	0.628



(c)



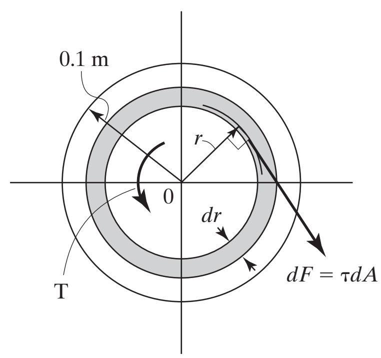
(a)

Oil is a Newtonian fluid. The velocities of the oil on the surfaces of disks A and B are $U_A = \omega_A r = 0$ and $U_B = \omega_B r = (20r)$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. *a*,

$$\frac{du}{dy} = \frac{U_A - U_B}{t} = \frac{0 - 20r}{t} = \left(-\frac{20r}{t}\right)\text{s}^{-1}$$

Applying Newton's law of viscosity,

$$\tau = \mu\left|\frac{du}{dy}\right| = \left(0.02\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{20r}{t}\right) = \left(\frac{0.4r}{t}\right)\text{N/m}^2$$



(b)

1–54. (continued)

The shaded differential element shown in Fig. *b* has an area of $dA = 2\pi r\, dr$. Thus, $dF = \tau dA = \left(\frac{0.4r}{t}\right)(2\pi r\, dr) = \left(\frac{0.8\pi}{t}\right)r^2\, dr$. Moment equilibrium about point O in Fig. *b* requires

$$\circlearrowleft + \Sigma M_O = 0; \qquad T - \int r\, dF = 0$$

$$T - \int_0^{0.1\text{ m}} r\left[\left(\frac{0.8\pi}{t}\right)r^2 dr\right] = 0$$

$$T = \int_0^{0.1\text{ m}} \left(\frac{0.8\pi}{t}\right)r^3\, dr$$

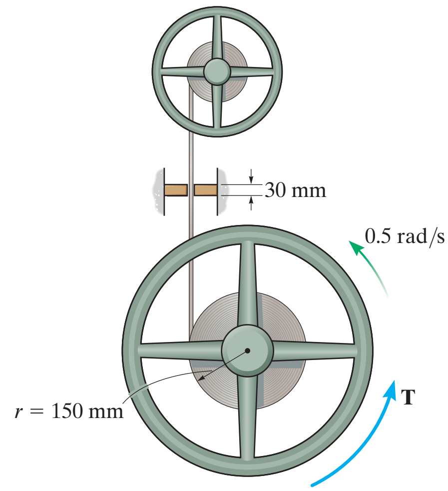
$$T = \left(\frac{0.8\pi}{t}\right)\left(\frac{r^4}{4}\right)\Bigg|_0^{0.1\text{ m}}$$

$$T = \left[\frac{20\left(10^{-6}\right)\pi}{t}\right]\text{N}\cdot\text{m} \qquad \text{where } t \text{ is in m} \qquad \textbf{Ans.}$$

The plot of T vs t is shown Fig. *c*.

Ans:
$T = \left[\frac{20\left(10^{-6}\right)\pi}{t}\right]\text{N}\cdot\text{m}$, where t is in m

1–55. The tape is 10 mm wide and is drawn through an applicator, which applies a liquid coating (Newtonian fluid) that has a viscosity of $\mu = 0.830\ \text{N}\cdot\text{s/m}^2$ to each side of the tape. If the gap between each side of the tape and the applicator's surface is 0.8 mm, determine the torque **T** at the instant $r = 150$ mm that is needed to rotate the wheel at 0.5 rad/s. Assume the velocity profile within the liquid is linear.



SOLUTION

Considering the moment equilibrium of the wheel, Fig. *a*,

$$\Sigma M_A = 0; \qquad T - P(0.15\text{ m}) = 0$$

Since the velocity distribution is linear, the velocity gradient will be constant.

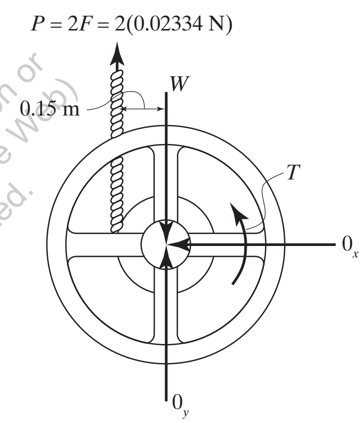
$$P = \tau(2A) = \mu(2A)\frac{du}{dy}$$

$$P = \left(0.830\ \text{N}\cdot\text{s/m}^2\right)(2)(0.03\text{ m})(0.01\text{ m})\left(\frac{0.5\text{ rad/s}(0.15\text{ m})}{0.0008\text{ m}}\right)$$

$$P = 0.04669\text{ N}$$

Thus

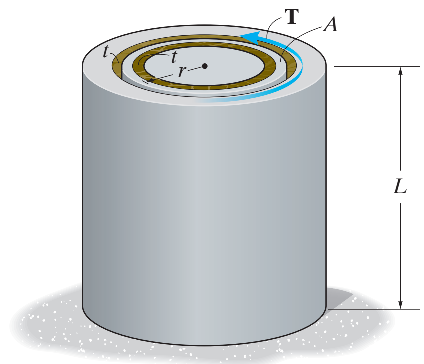
$$T = (0.04669\text{ N})(0.15\text{ m}) = 7.00\text{ mN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

Ans:
$7.00\ \text{mN}\cdot\text{m}$

***1–56.** The very thin tube A of mean radius r and length L is placed within the fixed circular cavity as shown. If the cavity has a small gap of thickness t on each side of the tube, and is filled with a Newtonian liquid having a viscosity μ, determine the torque **T** required to overcome the fluid resistance and rotate the tube with a constant angular velocity of ω. Assume the velocity profile within the liquid is linear.

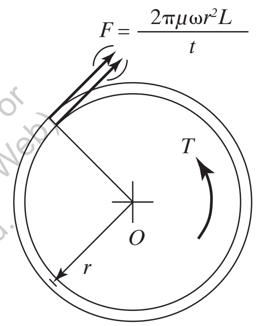


SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant.

$$\tau = \mu\frac{du}{dy}$$

$$= \mu\frac{(\omega r)}{t}$$



(a)

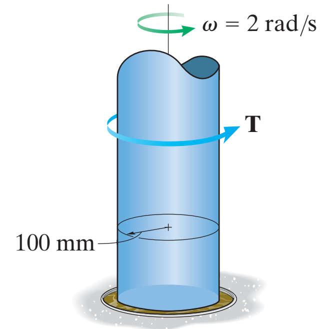
Considering the moment equilibrium of the tube, Fig. *a*,

$$\Sigma M = 0; \qquad T - 2\tau A r = 0$$

$$T = 2(\mu)\frac{(\omega r)}{t}(2\pi r L)r$$

$$T = \frac{4\pi\mu\omega r^3 L}{t} \qquad \textbf{Ans.}$$

1–57. The shaft rests on a 2-mm-thin film of oil having a viscosity of $\mu = 0.0657\ \mathrm{N \cdot s/m^2}$. If the shaft is rotating at a constant angular velocity of $\omega = 2$ rad/s, determine the shear stress in the oil at $r = 50$ mm and $r = 100$ mm. Assume the velocity profile within the oil is linear.



SOLUTION

Oil is a Newtonian fluid. Since the velocity distribution is linear, the velocity gradient will be constant.

At $r = 50$ mm,

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \left(0.0657\ \mathrm{N \cdot s/m^2}\right)\left(\frac{(2\ \mathrm{rad/s})(50\ \mathrm{mm})}{2\ \mathrm{mm}}\right)$$

$$\tau = 3.28\ \mathrm{Pa} \qquad \textbf{Ans.}$$

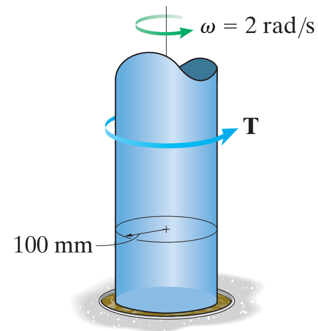
At $r = 100$ mm,

$$\tau = \left(0.0657\ \mathrm{N \cdot s/m^2}\right)\left(\frac{(2\ \mathrm{rad/s})(100\ \mathrm{mm})}{2\ \mathrm{mm}}\right)$$

$$\tau = 6.57\ \mathrm{Pa} \qquad \textbf{Ans.}$$

Ans:
At $r = 50$ mm, $\tau = 3.28$ Pa
At $r = 100$ mm, $\tau = 6.57$ Pa

1–58. The shaft rests on a 2-mm-thin film of oil having a viscosity of $\mu = 0.0657\ \mathrm{N\cdot s/m^2}$. If the shaft is rotating at a constant angular velocity of $\omega = 2$ rad/s, determine the torque **T** that must be applied to the shaft to maintain the motion. Assume the velocity profile within the oil is linear.



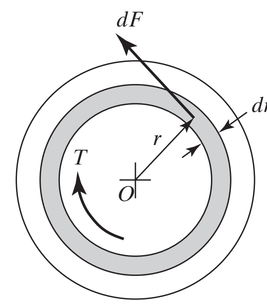
SOLUTION

Oil is a Newtonian fluid. Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus,

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

Thus, the shear force the oil exerts on the differential element of area $dA = 2\pi r\,dr$ shown shaded in Fig. *a* is

$$dF = \tau dA = \left(\frac{\mu \omega r}{t}\right)(2\pi r\,dr) = \frac{2\pi\mu\omega}{t} r^2 dr$$



(a)

Considering the moment equilibrium of the shaft, Fig. *a*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad \int_r dF - T = 0$$

$$T = \int_r dF = \frac{2\pi\mu\omega}{t}\int_0^R r^3 dr$$

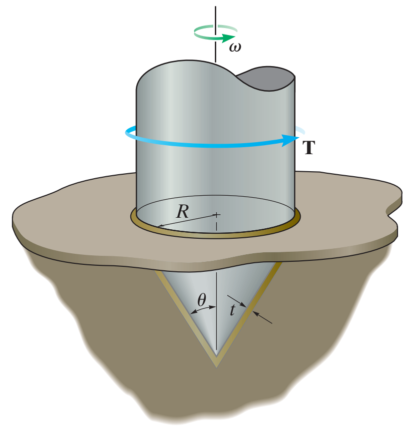
$$= \frac{2\pi\mu\omega}{t}\left(\frac{r^4}{4}\right)\Bigg|_0^R = \frac{\pi\mu\omega R^4}{2t}$$

Substituting,

$$T = \frac{\pi\left(0.0657\,\frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\right)(2\ \mathrm{rad/s})(0.1\ \mathrm{m})^4}{2(0.002\ \mathrm{m})} = 10.32\left(10^{-3}\right)\ \mathrm{N\cdot m} = 10.3\ \mathrm{mN\cdot m} \qquad \textbf{Ans.}$$

Ans:
$10.3\ \mathrm{mN\cdot m}$

1–59. The conical bearing is placed in a lubricating Newtonian fluid having a viscosity μ. Determine the torque **T** required to rotate the bearing with a constant angular velocity of ω. Assume the velocity profile along the thickness t of the fluid is linear.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus,

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

From the geometry shown in Fig. a,

$$z = \frac{r}{\tan\theta} \qquad dz = \frac{dr}{\tan\theta} \qquad \textbf{(1)}$$

Also, from the geometry shown in Fig. b,

$$dz = ds\cos\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{dr}{\tan\theta} = ds\cos\theta \qquad ds = \frac{dr}{\sin\theta}$$

The area of the surface of the differential element shown shaded in Fig. a is $dA = 2\pi r ds = \dfrac{2\pi}{\sin\theta} r dr$. Thus, the shear force the oil exerts on this area is

$$dF = \tau dA = \left(\frac{\mu\omega r}{t}\right)\left(\frac{2\pi}{\sin\theta} r dr\right) = \frac{2\pi\mu\omega}{t\sin\theta} r^2 dr$$

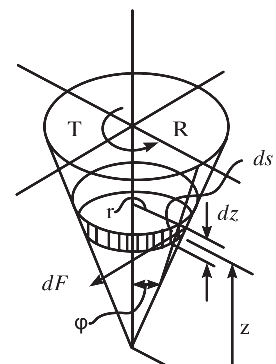
Considering the moment equilibrium of the shaft, Fig. a,

$$\Sigma M_z = 0; \qquad T - \int r dF = 0$$

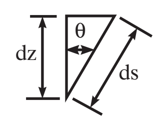
$$T = \int r dF = \frac{2\pi\mu\omega}{t\sin\theta}\int_0^R r^3 dr$$

$$= \frac{2\pi\mu\omega}{t\sin\theta}\left(\frac{r^4}{4}\right)\Big|_0^R$$

$$= \frac{\pi\mu\omega R^4}{2t\sin\theta} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$$T = \frac{\pi\mu\omega R^4}{2t\sin\theta}$$

***1–60.** The city of Denver, Colorado, is at an elevation of 1610 m above sea level. Determine how hot one can prepare water to make a cup of coffee.

SOLUTION

At the elevation of 1610 meters, the atmospheric pressure can be obtained by interpolating the data given in Appendix A.

$$p_{\text{atm}} = 89.88\text{ kPa} - \left(\frac{89.88\text{ kPa} - 79.50\text{ kPa}}{1000\text{ m}}\right)(610\text{ m}) = 83.55\text{ kPa}$$

Since water boils if the vapor pressure is equal to the atmospheric pressure, then the boiling temperature at Denver can be obtained by interpolating the data given in Appendix A.

$$T_{\text{boil}} = 90°\text{C} + \left(\frac{83.55 - 70.1}{84.6 - 70.1}\right)(5°\text{C}) = 94.6°\text{C}$$ **Ans.**

Note: Compare this with $T_{\text{boil}} = 100°\text{C}$ at 1 atm.

1–61. How hot can you make a cup of tea if you climb to the top of Mt. Everest (29,000 ft) and attempt to boil water?

SOLUTION

At the elevation of 29 000 ft, the atmospheric pressure can be obtained by interpolating the data given in Appendix A

$$p_{atm} = 704.4\ \text{lb/ft}^2 - \left(\frac{704.4\ \text{lb/ft}^2 - 629.6\ \text{lb/ft}^2}{30\ 000\ \text{lb/ft}^2 - 27\ 500\ \text{lb/ft}^2}\right)(29\ 000\ \text{ft} - 27\ 500\ \text{ft})$$

$$= \left(659.52\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)^2 = 4.58\ \text{psi}$$

Since water boils if the vapor pressure equals the atmospheric pressure, the boiling temperature of the water at Mt. Everest can be obtained by interpolating the data of Appendix A

$$T_{boil} = 150°\text{F} + \left(\frac{4.58\ \text{psi} - 3.72\ \text{psi}}{4.75\ \text{psi} - 3.72\ \text{psi}}\right)(160 - 150)°\text{F} = 158°\text{F} \qquad \textbf{Ans.}$$

Note: Compare this with 212°F at 1 atm.

Ans:
$p_{atm} = 4.58\,\text{psi}$, $T_{boil} = 158\,°\text{F}$

1–62. The blades of a turbine are rotating in water that has a temperature of 30°C. What is the lowest water pressure that can be developed at the blades so that cavitation will not occur?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 30°\text{C}$ is

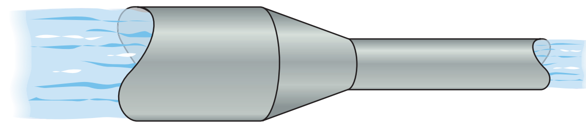
$$p_v = 4.25 \text{ kPa}$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v. Thus

$$p_{\min} = p_v = 4.25 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
4.25 kPa

1–63. As water at 40°C flows through the transition, its pressure will begin to decrease. Determine the lowest pressure it can have without causing cavitation.



SOLUTION

From Appendix A, the vapor pressure of water at $T = 40°\text{C}$ is

$$p_v = 7.38 \text{ kPa}$$

Cavitation (or boiling of water) will occur when the water pressure is equal to or less than p_v. Thus,

$$p_{\text{min}} = 7.38 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
7.38 kPa

*__1–64.__ Water at 70°F is flowing through a garden hose. If the hose is bent, a hissing noise can be heard. Here cavitation has occurred in the hose because the velocity of the flow has increased at the bend, and the pressure has dropped. What would be the highest absolute pressure in the hose at this location in the hose?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 70°\text{F}$ is

$$p_v = 0.363 \text{ lb/in}^2$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v.

$$p_{\max} = p_v = 0.363 \text{ lb/in}^2 \qquad \textbf{Ans.}$$

1–65. Water at 25°C is flowing through a garden hose. If the hose is bent, a hissing noise can be heard. Here cavitation has occurred in the hose because the velocity of the flow has increased at the bend, and the pressure has dropped. What would be the highest absolute pressure in the hose at this location in the hose?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 25°\text{C}$ is

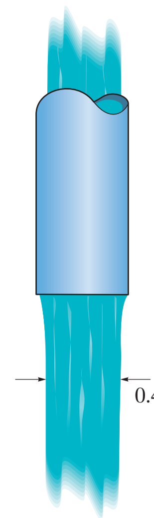
$$p_v = 3.17 \text{ kPa}$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v.

$$p_{\max} = p_v = 3.17 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
3.17 kPa

1–66. A stream of water has a diameter of 0.4 in. when it begins to fall out of the tube. Determine the difference in pressure between a point located just inside and a point just outside of the stream due to the effect of surface tension. Take $\sigma = 0.005$ lb/ft.



SOLUTION

Consider a length L of the water column. The free-body diagram of half of this column is shown in Fig. *a*.

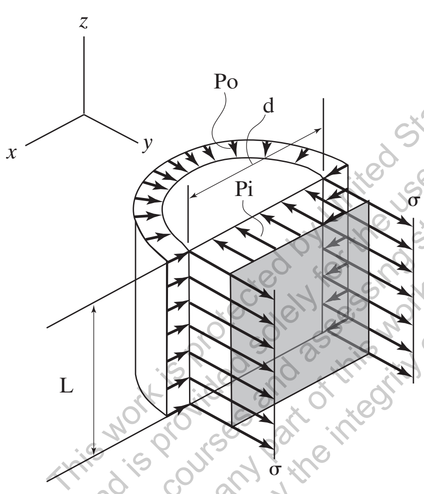
$$\Sigma F = 0$$

$$2(\sigma)(L) + p_o(d)(L) - p_i(d)(L) = 0$$

$$2\sigma = (p_i - p_o)d$$

$$p_i - p_o = \frac{2\sigma}{d}$$

$$\Delta p = \frac{2(0.005 \text{ lb/ft})}{(0.4 \text{ in.}/12) \text{ ft}} = 0.300 \text{ lb/ft}^2 = 2.08(10^{-3}) \text{ psi} \qquad \textbf{Ans.}$$



(a)

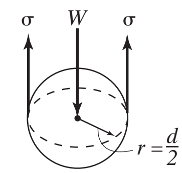
Ans:
$2.08(10^{-3})$ psi

1–67. Steel particles are ejected from a grinder and fall gently into a tank of water. Determine the largest average diameter of a particle that will float on the water with a contact angle of $\theta = 180°$, if the temperature is 80°F. Take $\gamma_{st} = 490\ \text{lb/ft}^3$ and $\sigma = 0.00492\ \text{lb/ft}$. Assume that each particle has the shape of a sphere where $V = \frac{4}{3}\pi r^2$.

SOLUTION

The weight of a steel particle is

$$W = \gamma_{st}V = \left(490\ \text{lb/ft}^3\right)\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right] = \frac{245\pi}{3}d^3$$



(a)

Force equilibrium along the vertical, Fig. *a*, requires

$$+\uparrow \Sigma F_y = 0; \qquad (0.00492\ \text{lb/ft})\left[2\pi\left(\frac{d}{2}\right)\right] - \frac{245\pi}{3}d^3 = 0$$

$$0.00492\pi d = \frac{245\pi}{3}d^3$$

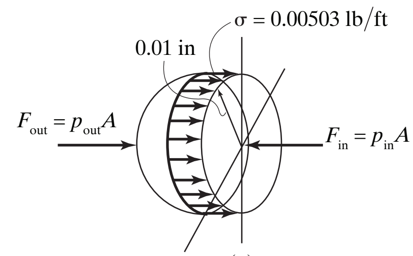
$$d = 7.762\left(10^{-3}\right)\ \text{ft}$$

$$= 0.0931\ \text{in.} \qquad \textbf{Ans.}$$

Ans:
0.0931 in.

***1–68.** When a can of soda water is opened, small gas bubbles are produced within it. Determine the difference in pressure between the inside and outside of a bubble having a diameter of 0.02 in. The surrounding temperature is 60°F. Take $\sigma = 0.00503$ lb/ft.

SOLUTION



(a)

The FBD of a half a bubble shown in Fig. *a* will be considered. Here A is the projected area. Force equilibrium along the horizontal requires

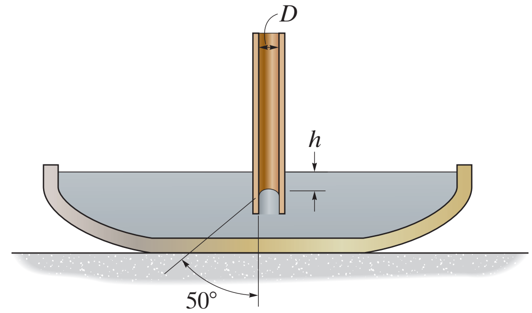
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad p_{out} A + (0.00503 \text{ lb/ft})\left[\pi\left(\frac{0.02}{12}\text{ ft}\right)\right] - p_{in}A = 0$$

$$(p_{in} - p_{out})\left[\frac{\pi}{4}\left(\frac{0.02}{12}\text{ft}\right)^2\right] = 8.3833(10^{-6})\pi \text{ lb}$$

$$p_{in} - p_{out} = (12.072 \text{ lb/ft}^2)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$= 0.0838 \text{ psi} \qquad \textbf{Ans.}$$

1–69. Determine the distance h that a column of mercury in the tube will be depressed when the tube is inserted into the mercury at a room temperature of 68°F. Set $D = 0.12$ in.



SOLUTION

Using the result

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

From the table in Appendix A, for mercury $\rho = 26.3\ \mathrm{slug/ft^3}$ and $\sigma = 31.9(10^{-3})\ \dfrac{\mathrm{lb}}{\mathrm{ft}}$.

$$h = \frac{2\left[31.9(10^{-3})\ \dfrac{\mathrm{lb}}{\mathrm{ft}}\right]\cos(180^\circ - 50^\circ)}{\left(26.3\ \dfrac{\mathrm{slug}}{\mathrm{ft}^3}\right)\left(32.2\ \dfrac{\mathrm{ft}}{\mathrm{s}^2}\right)\left[(0.06\ \mathrm{in.})\left(\dfrac{1\ \mathrm{ft}}{12\ \mathrm{in.}}\right)\right]}$$

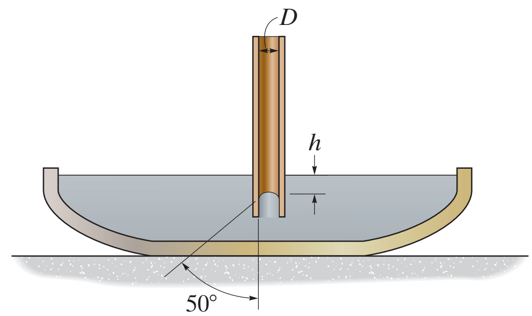
$$= \left[-9.6852(10^{-3})\ \mathrm{ft}\right]\left(\frac{12\ \mathrm{in.}}{1\ \mathrm{ft}}\right)$$

$$= -0.116\ \mathrm{in.} \qquad \textbf{Ans.}$$

The negative sign indicates that a depression occurs.

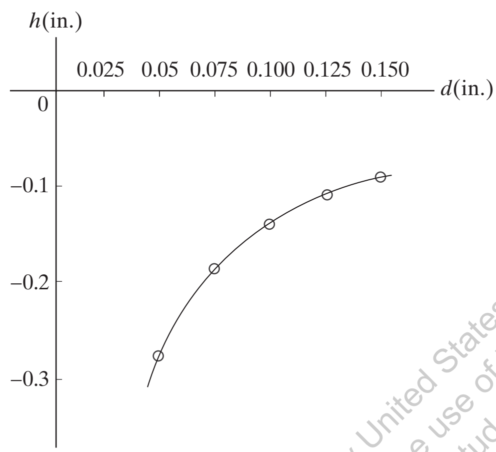
Ans:
0.116 in.

1–70. Determine the distance h that the column of mercury in the tube will be depressed when the tube is inserted into the mercury at a room temperature of 68°F. Plot this relationship of h (vertical axis) versus D for 0.05 in. $\leq D \leq$ 0.150 in. Give values for increments of $\Delta D = 0.025$ in. Discuss this result.



SOLUTION

d(in.)	0.05	0.075	0.100	0.125	0.150
h(in.)	−0.279	−0.186	−0.139	−0.112	0.0930



From the table in Appendix A, for mercury at 68°F, $\rho = 26.3\ \text{slug/ft}^3$, and $\sigma = 31.9(10^{-3})$ lb/ft. Using the result

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

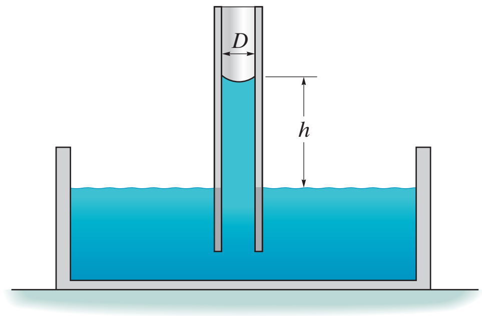
$$h = \left[\frac{2\left[31.9(10^{-3})\ \text{lb/ft}\right]\cos(180° - 50°)}{(26.3\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)\left[(d/2)(1\ \text{ft}/12\ \text{in})\right]}\right]\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)$$

$$h = \left(\frac{-0.01395}{d}\right)\ \text{in.} \qquad \text{where } d \text{ is in in.}$$

The negative sign indicates that a depression occurs.

Ans:
$d = 0.075$ in., $h = 0.186$ in.

1–71. Water in the glass tube is at a temperature of 40°C. Plot the height h of the water as a function of the tube's inner diameter D for $0.5 \text{ mm} \le D \le 3 \text{ mm}$. Use increments of 0.5 mm. Take $\sigma = 69.6 \text{ mN/m}$.



SOLUTION

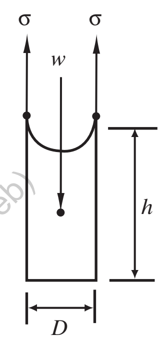
When water contacts the glass wall, $\theta = 0°$. The weight of the rising column of water is

$$W = \gamma_w \forall = \rho_w g\left(\frac{\pi}{4}D^2 h\right) = \frac{1}{4}\pi \rho_w g D^2 h$$

The vertical force equilibrium, Fig. *a*, requires

$$+\uparrow \Sigma F_y = 0; \qquad \sigma(\pi D) - \frac{1}{4}\pi \rho_w g D^2 h = 0$$

$$h = \frac{4\sigma}{\rho_w g D}$$



(a)

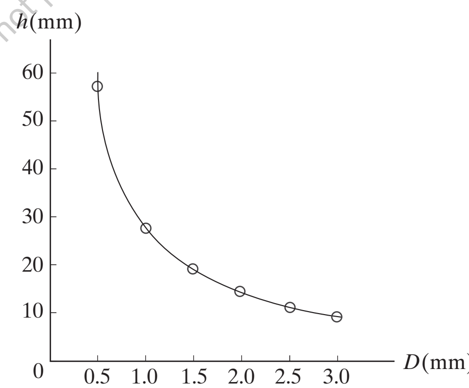
From Appendix A, $\rho_w = 992.3 \text{ kg/m}^3$ at $T = 40°\text{C}$. Then

$$h = \frac{4(0.0696 \text{ N/m})}{(992.3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)D} = \frac{28.6(10^{-6})}{D} \text{ m}$$

For $0.5 \text{ mm} \le D \le 3 \text{ mm}$

D(mm)	0.5	1.0	1.5	2.0	2.5	3.0
h(mm)	57.2	28.6	19.07	14.3	11.44	9.53

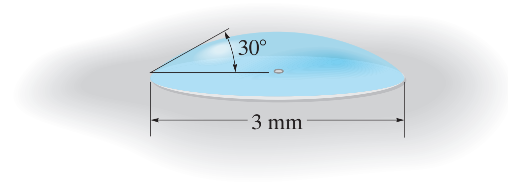
The plot of h vs D is shown in Fig. *b*.



(b)

Ans:
$D = 1.0 \text{ mm}, h = 28.6 \text{ mm}$

***1–72.** Many camera phones now use liquid lenses as a means of providing a quick auto-focus. These lenses work by electrically controlling the internal pressure within a liquid droplet, thereby affecting the angle of the meniscus of the droplet, and so creating a variable focal length. To analyze this effect, consider, for example, a segment of a spherical droplet that has a base diameter of 3 mm. The pressure in the droplet is 105 Pa and is controlled through a tiny hole at the center. If the tangent at the surface is 30°, determine the surface tension at the surface that holds the droplet in place.

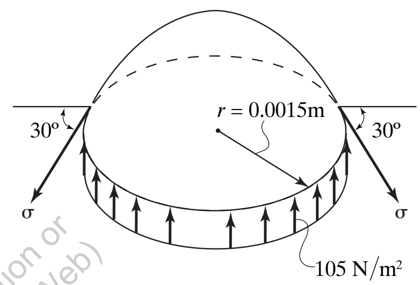


SOLUTION

Writing the force equation of equilibrium along the vertical by referring to the FBD of the droplet in Fig. *a*

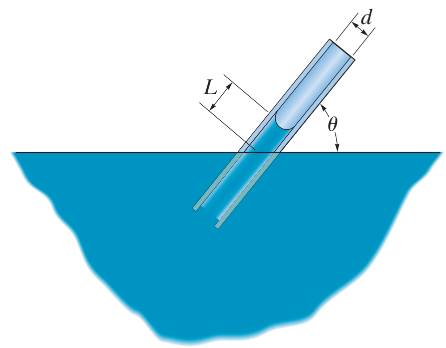
$$+\uparrow \Sigma F_z = 0; \qquad \left(105 \frac{\text{N}}{\text{m}^2}\right)\left[\pi(0.0015\text{ m})^2\right] - (\sigma \sin 30°)\left[2\pi(0.0015\text{ m})\right] = 0$$

$$\sigma = 0.158\text{ N/m} \qquad \textbf{Ans.}$$



(a)

1–73. The tube has an inner diameter d and is immersed in water at an angle θ from the vertical. Determine the average length L to which water will rise along the tube due to capillary action. The surface tension of the water is σ and its density is ρ.



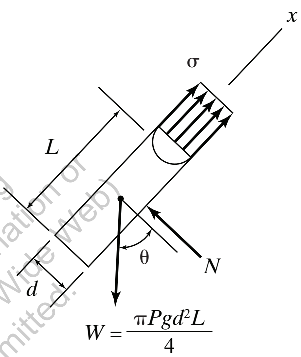
SOLUTION

The free-body diagram of the water column is shown in Fig. *a*. The weight of this column is $W = \rho g \forall = \rho g\left[\pi\left(\frac{d}{2}\right)^2 L\right] = \frac{\pi\rho g d^2 L}{4}$.

For water, its surface will be almost parallel to the surface of the tube (contact angle $\approx 0°$). Thus, σ acts along the tube. Considering equilibrium along the x axis,

$$\Sigma F_x = 0; \qquad \sigma(\pi d) - \frac{\pi\rho g d^2 L}{4}\sin\theta = 0$$

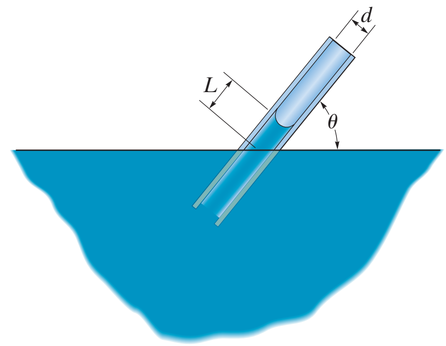
$$L = \frac{4\sigma}{\rho g d \sin\theta} \qquad \textbf{Ans.}$$



(a)

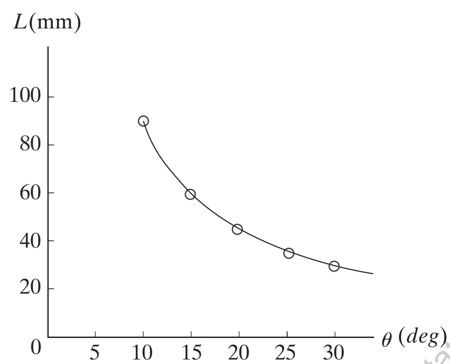
Ans:
$L = 4\sigma/(\rho g d \sin\theta)$

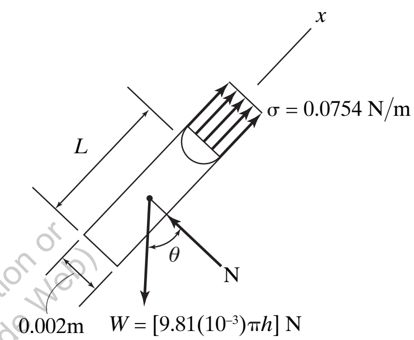
1–74. The tube has an inner diameter of $d = 2$ mm and is immersed in water. Determine the average length L to which the water will rise along the tube due to capillary action as a function of the angle of tilt, θ. Plot this relationship of L (vertical axis) versus θ for $10° \le \theta \le 30°$. Give values for increments of $\Delta\theta = 5°$. The surface tension of the water is $\sigma = 75.4$ mN/m, and its density is $\rho = 1000$ kg/m^3.



SOLUTION

θ(deg.)	10	15	20	25	30
L(mm)	88.5	59.4	44.9	36.4	30.7





(a)

The FBD of the water column is shown in Fig. *a*. The weight of this column is

$$W = \rho g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(0.002\ \text{m})L\right] = \left[9.81(10^{-3})\pi L\right]\ \text{N}.$$

For water, its surface will be almost parallel to the surface of the tube ($\theta \cong 0°$) at the point of contact. Thus, σ acts along the tube. Considering equilibrium along x axis,

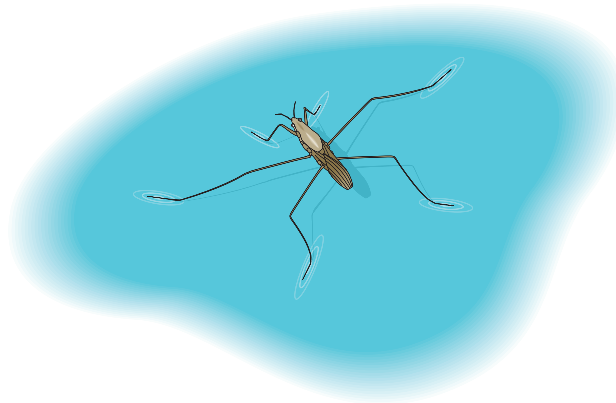
$$\Sigma F_x = 0; \qquad (0.0754\ \text{N/m})\left[\pi(0.002\ \text{m})\right] - \left[9.81(10^{-3})\pi L\right]\sin\theta = 0$$

$$L = \left(\frac{0.0154}{\sin\theta}\right)\text{m} \quad \text{where } \theta \text{ is in deg.} \qquad \textbf{Ans.}$$

The plot of L versus θ is shown in Fig. *a*.

Ans:
$L = (0.0154/\sin\theta)$ m

1–75. The marine water strider, *Halobates*, has a mass of 0.36 g. If it has six slender legs, determine the minimum contact length of all of its legs to support itself in water having a temperature of $T = 20°\text{C}$. Take $\sigma = 72.7\,\text{mN/m}$, and assume the legs are thin cylinders that are water repellent.



SOLUTION

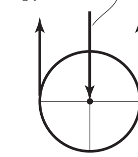
The force supported by the legs is

$$P = \left[0.36\left(10^{-3}\right)\text{ kg}\right]\left[9.81\text{ m/s}^2\right] = 3.5316\left(10^{-3}\right)\text{N}$$

Here, σ is most effective in supporting the weight if it acts vertically upward. This requirement is indicated on the FBD of each leg in Fig. *a*. The force equilibrium along vertical requires

$$+\uparrow \Sigma F_y = 0; \qquad 3.5316\left(10^{-3}\right)\text{N} - 2(0.0727\text{ N/m})l = 0$$

$$l = 24.3\left(10^{-3}\right)\text{m} = 24.3\text{ mm} \qquad \textbf{Ans.}$$

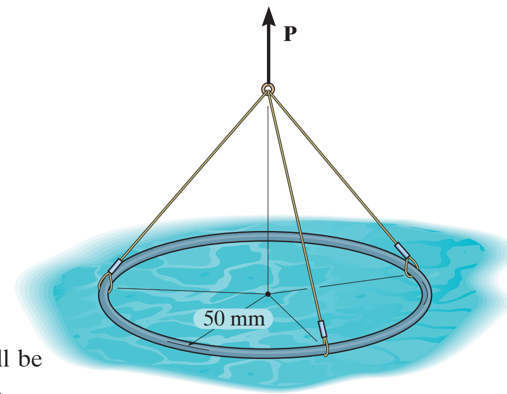


(a)

Note: Because of surface microstructure, a water strider's legs are highly hydrophobic. That is why the water surface curves *downward* with $\theta \approx 0°$, instead of upward as it does when water meets glass.

Ans:
24.3 mm

***1–76.** The ring has a weight of 0.2 N and is suspended on the surface of the water, for which $\sigma = 73.6\ \text{mN/m}$. Determine the vertical force **P** needed to pull the ring free from the surface. *Note*: This method is often used to measure surface tension.



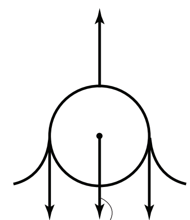
SOLUTION

The free-body diagram of the ring is shown in Fig. *a*. For water, its surface will be almost parallel to the surface of the wire ($\theta \approx 0°$) at the point of contact, Fig. *a*.

$+\uparrow \Sigma F_y = 0; \qquad P - W - 2T = 0$

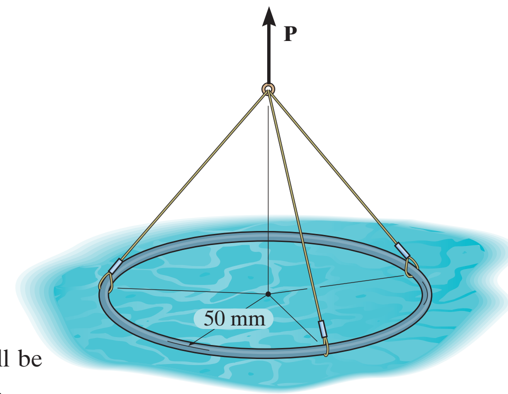
$$P - 0.2\ \text{N} - 2(0.0736\ \text{N/m})\left[2\pi(0.05\ \text{m})\right] = 0$$

$$P = 0.246\ \text{N} \qquad \textbf{Ans.}$$



(a)

1–77. The ring has a weight of 0.2 N and is suspended on the surface of the water. If it takes a force of $P = 0.245$ N to lift the ring free from the surface, determine the surface tension of the water.



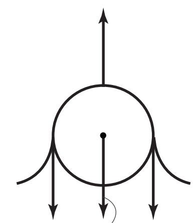
SOLUTION

The free-body diagram of the ring is shown in Fig. *a*. For water, its surface will be almost parallel to the surface of the wire ($\theta \approx 0°$) at the point of contact, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad 0.245\text{ N} - 0.2\text{ N} - 2[\sigma(2\pi(0.05\text{ m}))] = 0$$

$$\sigma = 0.0716\text{ N/m}$$

$$= 0.0716\text{ N/m} \qquad \textbf{Ans.}$$



(a)

Ans:
0.0716 N/m