Solutions Manual for

Fluid Mechanics

Seventh Edition in SI Units

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Chapter 5

Dimensional Analysis

and Similarity

5.1 For axial flow through a circular tube, the Reynolds number for transition to turbulence is approximately 2300 [see Eq. (6.2)], based upon the diameter and average velocity. If d = 5 cm and the fluid is kerosene at 20°C, find the volume flow rate in m^3/h which causes transition.

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ = 0.00192 kg/m·s. The only unknown in the transition Reynolds number is the fluid velocity:

$$\mathrm{Re}_{tr} \approx 2300 = \frac{\rho \mathrm{V d}}{\mu} = \frac{(804)\mathrm{V}(0.05)}{0.00192}, \quad \text{solve for } \mathrm{V}_{tr} = 0.11 \text{ m/s}$$

$$\text{Then} \quad \mathrm{Q} = \mathrm{VA} = (0.11)\frac{\pi}{4}(0.05)^2 = 2.16\mathrm{E{-}4}\ \frac{\mathrm{m}^3}{\mathrm{s}} \times 3600 \approx \mathbf{0.78}\ \frac{\mathbf{m^3}}{\mathbf{hr}} \quad \textit{Ans.}$$

P5.2 A prototype automobile is designed for cold weather in Denver, CO (-10°C, 83 kPa). Its drag force is to be tested in on a one-seventh-scale model in a wind tunnel at 150 mi/h and at 20°C and 1 atm. If model and prototype satisfy dynamic similarity, what prototype velocity, in mi/h, is matched? Comment on your result.

Solution: First assemble the necessary air density and viscosity data:

$$\text{Denver}: \ T = 263K \ ; \ \rho_p = \frac{p}{RT} = \frac{83000}{287(263)} = 1.10\ \frac{kg}{m^3} \ ; \ \mu_p = 1.75E-5\frac{kg}{m-s}$$

$$\text{Wind tunnel}: \ T = 293K \ ; \rho_m = \frac{p}{RT} = \frac{101350}{287(293)} = 1.205\ \frac{kg}{m^3} \ ; \ \mu_m = 1.80E-5\frac{kg}{m-s}$$

Convert 150 mi/h = 67.1 m/s. For dynamic similarity, equate the Reynolds numbers:

$$\mathrm{Re}_p = \frac{\rho VL}{\mu}|_p = \frac{(1.10)V_p(7L_m)}{1.75E-5} = \mathrm{Re}_m = \frac{\rho VL}{\mu}|_m = \frac{(1.205)(67.1)(L_m)}{1.80E-5}$$

$$\text{Solve for} \quad V_{prototype} = 10.2\,m/s = 22.8\ mi/h \qquad \textit{Ans.}$$

This is too *slow*, hardly fast enough to turn into a driveway. Since the tunnel can go no faster, the model drag must be corrected for Reynolds number effects. Note that we did not need to know the actual *length* of the prototype auto, only that it is 7 times larger than the model length.

P5.3 The transfer of energy by viscous dissipation is dependent upon viscosity μ, thermal conductivity k, stream velocity U, and stream temperature T_o. Group these quantities, if possible, into the dimensionless *Brinkman number*, which is proportional to μ.

Solution: Here we have only a single dimensionless group. List the dimensions, from Table 5.1:

μ	k	U	T_o
$\{ML^{-1}T^{-1}\}$	$\{MLT^{-3}\Theta^{-1}\}$	$\{LT^{-1}\}$	$\{\Theta\}$

Four dimensions, four variables (MLTΘ) – perfect for making a pi group. Put μ in the numerator:

$$\text{Brinkman number} = k^a U^b T_o^c \mu^1 \quad \text{yields} \quad \text{Br} = \mu U^2/(kT_o) \quad \textit{Ans.}$$

5.4 When tested in water at 20°C flowing at 2 m/s, an 8-cm-diameter sphere has a measured drag of 5 N. What will be the velocity and drag force on a 1.5-m-diameter weather balloon moored in sea-level standard air under dynamically similar conditions?

Solution: For water at 20°C take $\rho \approx 998$ kg/m^3 and $\mu \approx 0.001$ kg/m·s. For sea-level standard air take $\rho \approx 1.2255$ kg/m^3 and $\mu \approx 1.78$E-5 kg/m·s. The balloon velocity follows from dynamic similarity, which requires identical Reynolds numbers:

$$\text{Re}_{\text{model}} = \left.\frac{\rho VD}{\mu}\right|_{\text{model}} = \frac{998(2.0)(0.08)}{0.001} = 1.6\text{E}5 = \text{Re}_{\text{proto}} = \frac{1.2255 V_{\text{balloon}}(1.5)}{1.78\text{E–}5}$$

or $V_{\text{balloon}} \approx$ **1.55 m/s**. *Ans.* Then the two spheres will have identical drag coefficients:

$$C_{D,\text{model}} = \frac{F}{\rho V^2 D^2} = \frac{5\text{ N}}{998(2.0)^2(0.08)^2} = 0.196 = C_{D,\text{proto}} = \frac{F_{\text{balloon}}}{1.2255(1.55)^2(1.5)^2}$$

Solve for $\mathbf{F_{balloon} \approx 1.3\ N}$ *Ans.*

5.5 An automobile has a characteristic length and area of 2.45 m and 5.57 m^2, respectively. When tested in sea-level standard air, it has the following measured drag force versus speed:

V, km/h:	32	64	96
Drag, N:	137.9	511.5	1107.6

The same car travels in Colorado at 104 km/h at an altitude of 3500 m. Using dimensional analysis, estimate (a) its drag force and (b) the horsepower required to overcome air drag.

Solution: For sea-level air in SI units, take $\rho \approx 1.23\text{ kg/m}^3$ and $\mu \approx 1.79\times 10^{-5}\text{ N}\cdot\text{s/m}^2$. Convert the raw drag and velocity data into dimensionless form:

V (km/h):	32	64	96
$C_D = F/(\rho V^2 L^2)$:	0.236	0.219	0.211
$Re_L = \rho VL/\mu$:	1.50E6	3.00E6	4.50E6

Drag coefficient plots versus Reynolds number in a very smooth fashion and is well fit (to ±1%) by the Power-law formula $\mathbf{C_D \approx 1.07Re_L^{-0.106}}$.

(a) The new velocity is V = 29.06 m/s, and for air at 3500-m Standard Altitude (Table A-6) take ρ = 0.8633 kg/m^3 and calculate μ from power-law

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{0.7}, \ \mu \approx 1.68 \times 10^{-5}\,\mathrm{N \cdot s/m^2}$$

Then compute the new Reynolds number and use our power-law above to estimate drag coefficient:

$$\mathbf{Re}_{Colorado} = \frac{\rho VL}{\mu} = \frac{(0.8633)(29.06)(2.45)}{1.68 \times 10^{-5}} = 3.66 \times 10^6, \quad hence$$

$$C_D \approx \frac{1.07}{(3.66E6)^{0.106}} = 0.2156, \quad \therefore \ \mathbf{F} = 0.2156(0.8633)(29.06)^2(2.45)^2(8.0)^2 = \mathbf{943.5\ N}\ Ans.\ \text{(a)}$$

(b) The horsepower required to overcome drag is

$$\text{Power} = FV = (943.5)(29.06) = 27417.6\ \text{W} \div 550 = \mathbf{36.77\ hp} \quad Ans.\ \text{(b)}$$

P5.6 The full-scale parachute in the chapter-opener photo had a drag force of approximatly 4225 N when tested at a velocity of 19 km/h in air at 20°C and 1 atm. Earlier, a model parachute of diameter 1.7 m was tested in the same tunnel. (*a*) For dynamic similarity, what should be the air velocity for the model? (*b*) What is the expected drag force of the model? (*c*) Is there anything surprising about your result to part (*b*)?

Solution: (*a*) From Table A.2 for air at 20C, ρ = 1.20 kg/m^3 and μ = 1.8E-5 kg/m-s. (*a*) For similarity, equate the Reynolds numbers:

$$\mathrm{Re}_p = \frac{\rho_p V_p D_p}{\mu_p} = \frac{(1.20)(5.28)(16.8)}{1.8E-5} = 5.91\text{E}6 = \mathrm{Re}_m = \frac{\rho_m V_m D_m}{\mu_m} = \frac{(1.20)(V_m)(1.7)}{1.8E-5}$$

$$Solve\ for\ V_m = 52.2\frac{\text{m}}{\text{s}} = \mathbf{187.8}\frac{\text{km}}{\text{h}} \qquad Ans.(a)$$

(*b*) For similarity, the force coefficients will be equal:

$$C_{Fp} = \frac{F_p}{\rho_p V_p^2 D_p^2} = \frac{4225\ \text{N}}{(1.20)(5.28)^2(16.8)^2} = 0.447 = \frac{F_m}{\rho_m V_m^2 D_m^2} = \frac{F_m}{(1.20)(52.2)^2(1.7)^2}$$

$$Solve\ for\ F_m = 4225\ \text{N} \qquad Ans.(b)$$

(*c*) It might be surprising that the drag forces are exactly the same for model and prototype! This is because, if ρ and μ are the same, the product *VD* is the same for both and the force is proportional to $(VD)^2$.

5.7 A body is dropped on the moon (g = 1.62 m/s^2) with an initial velocity of 12 m/s. By using option-2 variables, Eq. (5.11), the ground impact occurs at $t^{**} = 0.34$ and $S^{**} = 0.84$. Estimate (a) the initial displacement, (b) the final displacement, and (c) the time of impact.

Solution: (a) The initial displacement follows from the "option 2" formula, Eq. (5.12):

$$S^{**} = gS_o/V_o^2 + t^{**} + \frac{1}{2}t^{**2} = 0.84 = \frac{(1.62)S_o}{(12)^2} + 0.34 + \frac{1}{2}(0.34)^2$$

$$\text{Solve for} \quad S_o \approx \mathbf{39\ m} \quad \textit{Ans. (a)}$$

(b, c) The final time and displacement follow from the given dimensionless results:

$$S^{**} = gS/V_o^2 = 0.84 = (1.62)S/(12)^2, \quad \text{solve for } S_{final} \approx \mathbf{75\,m} \quad \textit{Ans. (b)}$$

$$t^{**} = gt/V_o = 0.34 = (1.62)t/(12), \quad \text{solve for } t_{impact} \approx \mathbf{2.52\ s} \quad \textit{Ans. (c)}$$

5.8 The *Morton number* Mo, used to correlate bubble-dynamics studies, is a dimensionless combination of acceleration of gravity g, viscosity μ, density ρ, and surface tension coefficient Y. If Mo is proportional to g, find its form.

Solution: The relevant dimensions are $\{g\} = \{LT^{-2}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, and $\{Y\} = \{MT^{-2}\}$. To have g in the numerator, we need the combination:

$$\{Mo\} = \{g\}\{\mu\}^a\{\rho\}^b\{Y\}^c = \left\{\frac{L}{T^2}\right\}\left\{\frac{M}{LT}\right\}^a\left\{\frac{M}{L^3}\right\}^b\left\{\frac{M}{T^2}\right\}^c = M^0L^0T^0$$

$$\text{Solve for} \quad a = 4,\ b = -1,\ c = -3, \quad \text{or:} \quad \boldsymbol{Mo = \frac{g\mu^4}{\rho Y^3}} \quad \textit{Ans.}$$

P5.9 The *Richardson number*, Ri, which correlates the production of turbulence by buoyancy,

is a dimensionless combination of the acceleration of gravity g, the fluid temperature T_o, the local temperature gradient $\partial T/\partial z$, and the local velocity gradient $\partial u/\partial z$. Determine the form of the Richardson number if it is proportional to g.

Solution: In the {MLTΘ} system, these variables have the dimensions $\{g\} = \{L/T^2\}$, $\{T_o\} = \{\Theta\}$, $\{\partial T/\partial z\} = \{\Theta/L\}$, and $\{\partial u/\partial z\} = \{T^{-1}\}$. The ratio $g/(\partial u/\partial z)^2$ will cancel time, leaving {L} in the numerator, and the ratio $\{\partial T/\partial z\}/T_o$ will cancel {Θ}, leaving {L} in the denominator. Multiply them together and we have the standard form of the dimensionless Richardson number:

$$\text{Ri} = \frac{g\left(\frac{\partial T}{\partial z}\right)}{T_o\left(\frac{\partial u}{\partial z}\right)^2} \qquad \textit{Ans.}$$

P5.10 Total resistance for ship motion is a complicated problem. However, the resistance can be written as a function of water density, ρ, viscosity, μ, ship speed, v, ship's characteristic length, l, and gravitational acceleration, g. Under dimensional analysis, we found that the dimensionless relationship is

$$\frac{F}{\rho v^2 l^2} = g(\mathrm{Re}, (\mathrm{Fr})^2)$$

what would be a problem in using this relationship?

Solution: For complete similarity between a prototype and its model the Reynolds number and Froude number must be the same, which are

$$\frac{\rho_p v_p l_p}{\mu_p} = \frac{\rho_m v_m l_m}{\mu_m} \qquad (1)$$

for Reynolds number and

$$\frac{v_p}{(g_p l_p)^{1/2}} = \frac{v_m}{(g_m l_m)^{1/2}} \qquad (2)$$

for Froude number,

Eq. (1) give $\dfrac{v_m}{v_p} = \left(\dfrac{\rho_p}{\rho_m}\right)\left(\dfrac{l_p}{l_m}\right)\left(\dfrac{\mu_m}{\mu_p}\right)$. Eq. (2) give $\dfrac{v_m}{v_p} = \left(\dfrac{l_m}{l_p}\right)^{1/2}$.

Since $g_m = g_p$, these two equalities lead to $\left(\dfrac{l_m}{l_p}\right)^{3/2} = \dfrac{\nu_m}{\nu_p}$

If l_m is much smaller or larger than l_p, we would not be able to find different fluids that have such kinematic viscosity ratio. Therefore, both similarities cannot be satisfied at the same time.

5.11 Determine the dimension $\{MLT\Theta\}$ of the following quantities:

$$\text{(a) } \rho u \frac{\partial u}{\partial x} \quad \text{(b) } \int_1^2 (p - p_0)\,dA \quad \text{(c) } \rho c_p \frac{\partial^2 T}{\partial x\,\partial y} \quad \text{(d) } \iiint \rho \frac{\partial u}{\partial t}\,dx\,dy\,dz$$

All quantities have their standard meanings; for example, ρ is density, etc.

Solution: Note that $\{\partial u/\partial x\} = \{U/L\}$, $\{\int p\,dA\} = \{pA\}$, etc. The results are:

$$\text{(a)}\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \text{(b)}\left\{\frac{\mathbf{ML}}{\mathbf{T^2}}\right\};\quad \text{(c)}\left\{\frac{\mathbf{M}}{\mathbf{L^3T^2}}\right\};\quad \text{(d)}\left\{\frac{\mathbf{ML}}{\mathbf{T^2}}\right\} \quad \textit{Ans.}$$

P5.12 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the wave speed of an atomic bomb explosion. He assumed that the blast wave radius R was a function of energy released E, air density ρ, and time t. Use dimensional analysis to show how wave radius must vary with time.

Solution: The proposed function is $R = f(E, \rho, t)$. There are four variables ($n = 4$) and three primary dimensions (MLT, or $j = 3$), thus we expect $n\text{-}j = 4\text{-}3 = 1$ pi group. List the dimensions:

$$\{R\} = \{L\} \;;\; \{E\} = \{ML^2/T^2\} \;;\; \{\rho\} = \{M/L^3\} \;;\; \{t\} = \{T\}$$

Assume arbitrary exponents and make the group dimensionless:

$$R^1 E^a \rho^b t^c = (L)^1 (ML^2/T^2)^a (M/L^3)^b (T)^c = M^0 L^0 T^0 ,$$

$$\text{whence } a+b=0 \;;\; 1+2a-3b=0 \;;\; -2a+c=0 \;;\; \text{Solve } a=-\frac{1}{5} \;;\; b=+\frac{1}{5} \;;\; c=-\frac{2}{5}$$

The single pi group is

$$\Pi_1 = \frac{R\,\rho^{1/5}}{E^{1/5}\,t^{2/5}} = \text{constant}, \quad \text{thus } R_{wave} \propto t^{2/5} \qquad \textit{Ans.}$$

5.13 The *Stokes number*, St, used in particle-dynamics studies, is a dimensionless combination of *five* variables: acceleration of gravity g, viscosity μ, density ρ, particle velocity U, and particle diameter D. (a) If St is proportional to μ and inversely proportional to g, find its form. (b) Show that St is actually the quotient of two more traditional dimensionless groups.

Solution: (a) The relevant dimensions are $\{g\} = \{LT^{-2}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, $\{U\} = \{LT^{-1}\}$, and $\{D\} = \{L\}$. To have μ in the numerator and g in the denominator, we need the combination:

$$\{St\} = \{\mu\}\{g\}^{-1}\{\rho\}^a\{U\}^b\{D\}^c = \left\{\frac{M}{LT}\right\}\left\{\frac{T^2}{L}\right\}\left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c = M^0L^0T^0$$

$$\text{Solve for } \quad a=-1,\; b=1,\; c=-2, \quad \text{or: } \quad St = \frac{\mu U}{\rho g D^2} \qquad \textit{Ans. (a)}$$

$$\text{This has the ratio form: } \quad \boldsymbol{St} = \frac{U^2/(gD)}{\rho UD/\mu} = \frac{\textit{Froude number}}{\textit{Reynolds number}} \qquad \textit{Ans. (b)}$$

5.14 The speed of propagation C of a capillary (very small) wave in deep water is known to be a function only of density ρ, wavelength λ, and surface tension Y. Find the proper functional relationship, completing it with a dimensionless constant. For a given density and wavelength, how does the propagation speed change if the surface tension is doubled?

Solution: The "function" of ρ, λ, and Y must have velocity units. Thus

$$\{C\} = \{f(\rho, \lambda, Y)\}, \quad \text{or} \quad C = \text{const}\, \rho^a \lambda^b Y^c, \quad \text{or: } \left\{\frac{L}{T}\right\} = \left\{\frac{M}{L^3}\right\}^a \{L\}^b \left\{\frac{M}{T^2}\right\}^c$$

$$\text{Solve for } \quad a = b = -1/2 \text{ and } c = +1/2, \quad \text{or: } \quad \mathbf{C = const}\sqrt{\frac{\mathbf{Y}}{\boldsymbol{\rho\lambda}}} \qquad \textit{Ans.}$$

Thus, for constant ρ and λ, if Y is doubled, C increases as $\sqrt{2}$, or **+41%**. *Ans.*

P5.15 The thrust F of a propeller is generally thought to be a function of its diameter D and angular velocity Ω, the forward speed V, and the density ρ and viscosity μ of the fluid. Rewrite this relationship as a dimensionless function.

Solution: Write out the function with the various dimensions underneath:

$$\underset{\{ML/T^2\}}{F} = fcn(\underset{\{L\}}{D}, \underset{\{1/T\}}{\Omega}, \underset{\{L/T\}}{V}, \underset{\{M/L^3\}}{\rho}, \underset{\{M/LT\}}{\mu})$$

There are 6 variables and 3 primary dimensions (MLT), and we quickly see that $j = 3$, because (r, V, D) cannot form a pi group among themselves. Use the pi theorem to find the three pi's:

$\Pi_1 = \rho^a V^b D^c F$; Solve for $a = -1, b = -2, c = -2$. Thus $\Pi_1 = \dfrac{F}{\rho V^2 D^2}$

$\Pi_2 = \rho^a V^b D^c \Omega$; Solve for $a = 0,\ b = -1,\ c = 1$. Thus $\Pi_2 = \dfrac{\Omega D}{V}$

$\Pi_3 = \rho^a V^b D^c \mu$; Solve for $a = -1, b = -1, c = -1$. Thus $\Pi_3 = \dfrac{\mu}{\rho V D}$

Thus one of many forms of the final desired dimensionless function is

$$\frac{F}{\rho V^2 D^2} = fcn(\frac{\Omega D}{V}, \frac{\mu}{\rho V D}) \qquad \textit{Ans.}$$

P5.16 The volume flow Q through an orifice plate is a function of pipe diameter D, pressure drop Δp across the orifice, fluid density ρ and viscosity μ, and orifice diameter d. Using D, ρ, and Δp as repeating variables, express this relationship in dimensionless form.

Solution: There are 6 variables and 3 primary dimensions (MLT), and we already know that $j = 3$, because the problem thoughtfully gave the repeating variables. Use the pi theorem to find the three pi's:

$\Pi_1 = D^a \rho^b \Delta p^c Q$; Solve for $a = -2, b = 1/2, c = -1/2$. Thus $\Pi_1 = \dfrac{Q \rho^{1/2}}{D^2 \Delta p^{1/2}}$

$\Pi_2 = D^a \rho^b \Delta p^c d$; Solve for $a = -1\ b = 0\ c = 0$. Thus $\Pi_1 = \dfrac{d}{D}$

$\Pi_3 = D^a \rho^b \Delta p^c \mu$; Solve for $a = -1, b = -1/2, c = -1/2$. Thus $\Pi_1 = \dfrac{\mu}{D \rho^{1/2} \Delta p^{1/2}}$

The final requested orifice-flow function (see Sec. 6.12 later for a different form) is:

$$\frac{Q \rho^{1/2}}{D^2 \Delta p^{1/2}} = fcn(\frac{d}{D}, \frac{\mu}{D \rho^{1/2} \Delta p^{1/2}}) \qquad \textit{Ans.}$$

5.17 The size d of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter D, jet velocity U, and the properties of the liquid ρ, μ, and Y. Rewrite this relation in dimensionless form. *Hint*: Take D, ρ, and U as repeating variables.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccccc} d & = \text{fcn}(& D, & U, & \rho, & \mu, & Y &) \\ \{L\} & & \{L\} & \{L/T\} & \{M/L^3\} & \{M/LT\} & \{M/T^2\} & \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by the writer, as follows:

$$\text{Typical final result:}\quad \mathbf{\frac{d}{D} = fcn\left(\frac{\rho UD}{\mu}, \frac{\rho U^2 D}{Y}\right)} \quad \textit{Ans.}$$

P5.18 The time t_d to drain a liquid from a hole in the bottom of a tank is a function of the hole diameter d, the initial fluid volume υ_o, the initial liquid depth h_o, and the density ρ and viscosity μ of the fluid. Rewrite this relation as a dimensionless function, using Ipsen's method.

Solution: As asked, use *Ipsen's method*. Write out the function with the dimensions beneath:

$$\begin{array}{cccccccc} t_d & = fcn(& d, & \upsilon_o, & h_o, & \rho, & \mu &) \\ \{T\} & & \{L\} & \{L^3\} & \{L\} & \{M/L^3\} & \{M/LT\} & \end{array}$$

Eliminate the dimensions by multiplication or division. Divide by μ to eliminate {M}:

$$\begin{array}{cccccccc} t_d & = fcn(& d, & \upsilon_o, & h_o, & \dfrac{\rho}{\mu}, & \cancel{\mu} &) \\ \{T\} & & \{L\} & \{L^3\} & \{L\} & \{T/L^2\} & & \end{array}$$

Recall Ipsen's rules: Only divide into variables containing mass, in this case only ρ. Now eliminate {T}. Again only one division is necessary:

$$\begin{array}{ccccccc} \dfrac{t_d\mu}{\rho} & = fcn(& d, & \upsilon_o, & h_o, & \cancel{\dfrac{\rho}{\mu}} &) \\ \{L^2\} & & \{L\} & \{L^3\} & \{L\} & & \end{array}$$

Finally, eliminate {L} by dividing by appropriate powers of d. This completes our task when we discard d: itself:

$$\begin{array}{ccccc} \dfrac{t_d\mu}{\rho d^2} & = fcn(& \dfrac{\upsilon_o}{d^3}, & \dfrac{h_o}{d} &) \qquad \textit{Ans.} \\ \{1\} & & \{1\} & \{1\} & \end{array}$$

Just divide out the dimensions, don't worry about j or selecting repeating variables. Of course, the Pi Theorem would give the same, or comparable, results.

5.19 When fluid in a pipe is accelerated linearly from rest, it begins as laminar flow and then undergoes transition to turbulence at a time t_{tr} which depends upon the pipe diameter D, fluid acceleration a, density ρ, and viscosity μ. Arrange this into a dimensionless relation between t_{tr} and D.

Solution: Establish the variables and their dimensions:

$$t_{tr} = \text{fcn}(\rho, D, a, \mu)$$
$$\{T\} \quad \{M/L^3\} \quad \{L\} \quad \{L/T^2\} \quad \{M/LT\}$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 =$ **2** Pi groups, capable of various arrangements and selected by the writer, as required, to isolate t_{tr} versus D:

$$\mathbf{t_{tr}\left(\frac{\rho a^2}{\mu}\right)^{1/3} = fcn\left[D\left(\frac{\rho^2 a}{\mu^2}\right)^{1/3}\right]} \quad \textit{Ans.}$$

P5.20 When a large tank of high-pressure ideal gas discharges through a nozzle, the maximum exit mass flow $\dot{m}$ is a function of tank pressure p_o and temperature T_o, gas constant R, specific heat c_p, and nozzle diameter D. Rewrite this as a dimensionless function. Check to see if you can use (p_o, T_o, R, D) as repeating variables.

Solution: Using Table 5.1, write out the dimensions of the six variables:

$\dot{m}$	p_o	T_o	R	D	c_p
$\{MT^{-1}\}$	$\{ML^{-1}T^{-2}\}$	$\{\Theta\}$	$\{L^2T^{-2}\Theta^{-1}\}$	$\{L\}$	$\{L^2T^{-2}\Theta^{-1}\}$

By inspection, we see that (p_o, T_o, R, D) are indeed good repeating variables. There are two pi groups:

$$\Pi_1 = p_o^a T_o^b R^c c_p^d \dot{m}^1 \quad \text{yields} \quad \Pi_1 = \frac{\dot{m}\sqrt{RT_o}}{p_o D^2}$$

$$\Pi_2 = p_o^a T_o^b R^c c_p^d c_p^1 \quad \text{yields} \quad \Pi_1 = \frac{c_p}{R}$$

$$\text{Thus} \quad \frac{\dot{m}\sqrt{RT_o}}{p_o D^2} = fcn(\frac{c_p}{R}) \qquad \textit{Ans.}$$

The group $(c_p/R) = k/(k\text{-}1)$, where $k = c_p/c_v$. We usually write the right hand side as $fcn(k)$.

5.21 The rate of heat loss, Q_{loss} through a window is a function of the temperature difference ΔT, the surface area A, and the R resistance value of the window (in units of $m^2 \cdot s \cdot °K/J$): Q_{loss} = fcn(ΔT, A, R). (a) Rewrite in dimensionless form. (b) If the temperature difference doubles, how does the heat loss change?

Solution: First figure out the dimensions of R: $\{R\} = \{T^3\Theta/M\}$. Then note that $n = 4$ variables and $j = 3$ dimensions, hence we expect only 4 − 3 = *one* Pi group, and it is:

$$\Pi_1 = \frac{Q_{loss}R}{A\Delta T} = Const, \quad or: \quad \boldsymbol{Q_{loss} = Const\frac{A\Delta T}{R}} \quad Ans.\text{ (a)}$$

(b) Clearly (to me), $Q \propto \Delta T$: **if ΔT doubles, Q_{loss} also doubles**. *Ans.* (b)

5.22 The wall shear stress τ_w in a boundary layer is assumed to be a function of stream velocity U, boundary layer thickness δ, local turbulence velocity u', density ρ, and local pressure gradient dp/dx. Using (ρ, U, δ) as repeating variables, rewrite this relationship as a dimensionless function.

Solution: The relevant dimensions are $\{\tau_w\} = \{ML^{-1}T^{-2}\}$, $\{U\} = \{LT^{-1}\}$, $\{\delta\} = \{L\}$, $\{u'\} = \{LT^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, and $\{dp/dx\} = \{ML^{-2}T^{-2}\}$. With $n = 6$ and $j = 3$, we expect $n - j = k = 3$ pi groups:

$$\Pi_1 = \rho^a U^b \delta^c \tau_w = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT^2}\right\} = M^0L^0T^0, \quad solve\ \ a = -1,\ b = -2,\ c = 0$$

$$\Pi_2 = \rho^a U^b \delta^c u' = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{L}{T}\right\} = M^0L^0T^0, \quad solve\ \ a = 0,\ b = -1,\ c = 0$$

$$\Pi_3 = \rho^a U^b \delta^c \frac{dp}{dx} = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{M}{L^2T^2}\right\} = M^0L^0T^0, \quad solve\ \ a = -1,\ b = -2,\ c = 1$$

The final dimensionless function then is given by:

$$\Pi_1 = fcn(\Pi_2, \Pi_3), \quad \text{or:} \quad \boldsymbol{\frac{\tau_w}{\rho U^2} = fcn\left(\frac{u'}{U}, \frac{dp}{dx}\frac{\delta}{\rho U^2}\right)} \quad Ans.$$

P5.23 If you disturb a tank of length L and water depth h, the surface will oscillate back and forth at frequency Ω, assumed here to depend also upon water density ρ and the acceleration of gravity g. (a) Rewrite this as a dimensionless function. (b) If a tank of water sloshes at 2.0 Hz on earth, how fast would it oscillate on Mars ($g \approx 3.7\ m/s^2$)?

Solution: Write out the dimensions of the five variables. We hardly even need Table 5.1:

Ω	h	L	ρ	g
$\{T^{-1}\}$	$\{L\}$	$\{L\}$	$\{ML^{-3}\}$	$\{LT^{-2}\}$

(a) There are five variables and three dimensions {MLT}, hence we expect two pi groups.

The writer thinks Ω should be correlated versus *h*, so he chooses (L, ρ, g) as repeating variables:

$$\Pi_1 = L^a \rho^b g^c \Omega^1 \quad \text{yields} \quad \Pi_1 = \Omega\sqrt{\frac{L}{g}}$$

$$\Pi_2 = L^a \rho^b g^c h^1 \quad \text{yields} \quad \Pi_2 = \frac{h}{L}$$

$$\text{Thus} \quad \Omega\sqrt{\frac{L}{g}} = fcn\left(\frac{h}{L}\right) \qquad Ans.(a)$$

Note that density ***drops out***, being the only variable containing mass {M}. If the tank sloshes on earth at 2.0 Hz, that sets the value of Π_1, which we use on Mars to get Ω_{Mars} at the same h/L.

$$\Omega_{earth}\sqrt{\frac{L}{g_{earth}}} = (2.0s^{-1})\sqrt{\frac{L}{9.81m/s^2}} = 0.639\,m^{-1/2}\sqrt{L} = \Omega_{Mars}\sqrt{\frac{L}{g_{Mars}}} = \Omega_{Mars}\sqrt{\frac{L}{3.7\,m/s^2}}$$

$$\text{Solve for} \quad \Omega_{Mars} \approx \mathbf{1.23}\ Hz \qquad Ans.(b)$$

5.24 A fish robot was designed to simulate a fish's motion. From observations, we found that the fish propulsion depends on water density, ρ, viscosity, μ, caudal fin frequency, ω, average forward speed, *v*, and heaving angle of its body and pitching angle of its caudal fin with respect to its forward direction, ϕ and θ, respectively. Derive this relationship in dimensionless form.

Solution: From the problem, the relation can be written as follows:

$$F = f(\rho, \mu, \omega, v, \phi, \theta)$$

step 1. Count the variables, $\eta = 7$

step 2. Use the $\{\text{MLT}\Theta\}$ system to write out the dimensions of the variables.

F	ρ	μ	ω	v	ϕ	θ
MLT^{-2}	ML^{-3}	$ML^{-1}T^{-1}$	T^{-1}	LT^{-1}	–	–

step 3. Using ρ, μ and *v* as repeating variables, but we need to check whether ρ, μ, and *v* do not form a pi-group. Luckily, we can use ρ, μ and *v* as repeating variables after checking. Therefore, j = 3.

step 4 (a) Combine (ρ, μ, *v*) with force *F* to find the first pi group:

$$\pi_1 = \rho^a \mu^b v^c F = \left(ML^{-3}\right)^a \left(ML^{-1}T^{-1}\right)^b \left(LT^{-1}\right)^c \left(MLT^{-2}\right) = M^o L^o T^o$$

Solve the above equation to get $a = 1,\ b = -2,\ c = 0.$

(b) Again combine (ρ, μ, v) with caudal fin frequency ω to find the second pi group:

$$\pi_2 = \rho^a \mu^b v^c \omega = \left(ML^{-3}\right)^a \left(ML^{-1}T^{-1}\right)^b \left(LT^{-1}\right)^c \left(T^{-1}\right) = M^o L^o T^o$$

we then get $a = -1,\ b = 1,\ c = -2.$

For ϕ and θ, they are already in pi groups. Therefore, we can write four dimensionless groups as step 5. $\pi_1 = g\left(\pi_2,\ \pi_3,\ \pi_4\right)$

$$\frac{F\rho}{\mu^2} = g\left(\frac{\omega\mu}{\rho v^2},\ \phi,\ \theta\right)$$

Note that this relation needs experimental results to confirm.

5.25 Under laminar conditions, the volume flow Q through a small triangular-section pore of side length b and length L is a function of viscosity μ, pressure drop per unit length $\Delta p/L$, and b. Using the pi theorem, rewrite this relation in dimensionless form. How does the volume flow change if the pore size b is doubled?

Solution: Establish the variables and their dimensions:

$$\underset{\{L^3/T\}}{Q} = \text{fcn}(\underset{\{M/L^2T^2\}}{\Delta p/L},\ \underset{\{M/LT\}}{\mu},\ \underset{\{L\}}{b})$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 = \mathbf{1}$ Pi group, found as follows:

$$\Pi_1 = (\Delta p/L)^a(\mu)^b(b)^c Q^1 = \{M/L^2T^2\}^a\{M/LT\}^b\{L\}^c\{L^3/T\}^1 = M^0L^0T^0$$

$$M\!:\ a + b = 0;\quad L\!:\ -2a - b + c + 3 = 0;\quad T\!:\ -2a - b - 1 = 0,$$

$$\text{solve}\quad a = -1,\ b = +1,\ c = -4$$

$$\Pi_1 = \mathbf{\frac{Q\mu}{(\Delta p/L)b^4} = constant} \quad \textit{Ans.}$$

Clearly, if b is doubled, the flow rate Q increases by a factor of $2^4 =$ **16**. *Ans.*

5.26 The period of oscillation T of a water surface wave is assumed to be a function of density ρ, wavelength λ, depth h, gravity g, and surface tension Y. Rewrite this relationship in dimensionless form. What results if Y is negligible?

Solution: Establish the variables and their dimensions:

$$\underset{\{T\}}{T} = \text{fcn}(\underset{\{M/L^3\}}{\rho},\ \underset{\{L\}}{\lambda},\ \underset{\{L\}}{h},\ \underset{\{L/T^2\}}{g},\ \underset{\{M/T^2\}}{Y})$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by the writer as follows:

$$\text{Typical final result:}\quad \mathbf{T(g/\lambda)^{1/2} = fcn}\left(\frac{\mathbf{h}}{\lambda}, \frac{\mathbf{Y}}{\rho \mathbf{g}\lambda^2}\right) \quad \textit{Ans.}$$

$$\text{If Y is negligible, } \rho \text{ drops out also, leaving:}\quad \mathbf{T(g/\lambda)^{1/2} = fcn}\left(\frac{\mathbf{h}}{\lambda}\right) \quad \textit{Ans.}$$

5.27 A certain axial-flow turbine has an output torque *M* which is proportional to the volume flow rate *Q* and also depends upon the density ρ, rotor diameter *D*, and rotation rate Ω. How does the torque change due to a doubling of (a) *D* and (b) Ω?

Solution: List the variables and their dimensions, one of which can be M/Q, since M is stated to be proportional to Q:

$$\begin{array}{ccccc} M/Q & = \text{fcn}(& D, & \rho, & \Omega\ ) \\ \{M/LT\} & & \{L\} & \{M/L^3\} & \{1/T\} \end{array}$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 = \mathbf{1}$ single Pi group:

$$\frac{M/Q}{\rho\Omega D^2} = \text{dimensionless constant}$$

(a) If turbine diameter D is doubled, the torque M increases by a factor of **4**. *Ans.* (a)

(b) If turbine speed Ω is doubled, the torque M increases by a factor of **2**. *Ans.* (b)

P5.28 When disturbed, a floating buoy will bob up and down at frequency *f*. Assume that this frequency varies with buoy mass *m* and waterline diameter *d* and with the specific weight *g* of the liquid. (*a*) Express this as a dimensionless function. (*b*) If *d* and *g* are constant and the buoy mass is halved, how will the frequency change?

Solution: The proposed function is $f = fcn(m, d, g)$. Write out their dimensions:

$$\{f\} = \{T^{-1}\} \ ; \ \{m\} = \{M\} \ ; \ \{d\} = \{L\} \ ; \ \{\gamma\} = \{ML^{-2}T^{-2}\}$$

There are four variables and $j = 3$. Hence we expect only *one* Pi group. We find that

$$\Pi_1 = \frac{f}{d}\sqrt{\frac{m}{\gamma}} = \text{constant} \qquad \textit{Ans.}(a)$$

Hence, for these simplifying assumptions, f is proportional to $m^{-1/2}$. If *m* halves, f rises by a factor $(0.5)^{-1/2} = 1.414$. In other words, halving *m* increases f by about 41%. *Ans.*(*b*)

P5.29 A fixed cylinder of diameter *D* and length *L*, immersed in a stream flowing normal to its axis at velocity *U*, will experience zero average lift. However, if the cylinder is rotating at angular velocity Ω, a lift force *F* will arise. The fluid density ρ is important, but viscosity is secondary and can be neglected. Formulate this lift behavior as a dimensionless function.

Solution: No suggestion was given for the repeating variables, but for this type of problem (force coefficient, lift coefficient), we normally choose (ρ, *U*, *D*) for the task. List the dimensions:

D	*L*	*U*	Ω	*F*	ρ
{L}	{L}	{LT^{-1}}	{T^{-1}}	{MLT^{-2}}	{ML^{-3}}

There are three dimensions (MLT), which we knew when we chose (ρ, *U*, *D*). Combining these three, separately, with *F*, Ω, and *L*, we find this dimensionless function:

$$\frac{F}{\rho U^2 D^2} = fcn\left(\frac{\Omega D}{U}, \frac{L}{D}\right) \qquad Ans.$$

This is a correct solution for Chapter 5, but in Chapter 8 we will use the "official" function, with extra factors of (1/2):

$$\frac{F}{(1/2)\rho U^2 LD} = fcn\left(\frac{\Omega D}{2U}, \frac{L}{D}\right)$$

5.30 The period *T* of vibration of a beam is a function of its length *L*, area moment of inertia *I*, modulus of elasticity *E*, density ρ, and Poisson's ratio σ. Rewrite this relation in dimensionless form. What further reduction can we make if *E* and *I* can occur only in the product form *EI*?

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} T = \text{fcn}(& L, & I, & E, & \rho, & \sigma\ ) \\ \{T\} & \{L\} & \{L^4\} & \{M/LT^2\} & \{M/L^3\} & \{\text{none}\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by myself as follows: [Note that σ <u>must</u> be a Pi group.]

Typical final result: $$\frac{\mathbf{T}}{\mathbf{L}}\sqrt{\frac{\mathbf{E}}{\rho}} = \mathbf{fcn}\left(\frac{\mathbf{L^4}}{\mathbf{I}}, \sigma\right) \qquad Ans.$$

If E and I can only appear together as EI, then $$\frac{\mathbf{T}}{\mathbf{L^3}}\sqrt{\frac{\mathbf{EI}}{\rho}} = \mathbf{fcn}(\sigma) \qquad Ans.$$

5.31 In studying sand transport by ocean waves, A. Shields in 1936 postulated that the bottom shear stress τ required to move particles depends upon gravity g, particle size d and density ρ_p, and water density ρ and viscosity μ. Rewrite this in terms of dimensionless groups (which led to the *Shields Diagram* in 1936).

Solution: There are six variables (τ, g, d, ρ_p, ρ, μ) and three dimensions (M, L, T), hence we expect $n - j = 6 - 3 = 3$ Pi groups. The author used (ρ, g, d) as repeating variables:

$$\boldsymbol{\frac{\tau}{\rho g d} = fcn\left(\frac{\rho g^{1/2} d^{3/2}}{\mu}, \frac{\rho_p}{\rho}\right)} \quad \textit{Ans.}$$

The shear parameter used by Shields himself was based on *net* weight: $\boldsymbol{\tau/[(\rho_p - \rho)gd]}$.

P5.32 In forced convection, the heat transfer coefficient h is a function of thermal conductivity k, density ρ, viscosity μ, specific heat c_p, body length L, and velocity V. Heat transfer coefficient has units of W/(m^2-K) and dimensions $\{MT^{-3}\Theta^{-1}\}$. Rewrite this relation in dimensionless form, using (k, ρ, c_p, L) as repeating variables.

Solution: From Table 5.1, plus the given definition of h, list the dimensions:

h	k	ρ	μ	c_p	L	V
$\{MT^{-3}\Theta^{-1}\}$	$\{MLT^{-3}\Theta^{-1}\}$	$\{ML^{-3}\}$	$\{ML^{-1}T^{-1}\}$	$\{L^2T^{-2}\Theta^{-1}\}$	$\{L\}$	$\{LT^{-1}\}$

Four dimensions, 3 pi groups expected.

Add one variable successively to our repeating variables (k, ρ, c_p, L):

$$\Pi_1 = k^a \rho^b c_p^c L^d h^1 \quad \text{yields} \quad \Pi_1 = \frac{hL}{k}$$

$$\Pi_2 = k^a \rho^b c_p^c L^d \mu^1 \quad \text{yields} \quad \Pi_2 = \frac{\mu c_p}{k}$$

$$\Pi_3 = k^a \rho^b c_p^c L^d V^1 \quad \text{yields} \quad \Pi_3 = \frac{\rho L c_p V}{k}$$

The final desired dimensionless function is

$$\frac{hL}{k} = fcn\left(\frac{\mu c_p}{k}, \frac{\rho L c_p V}{k}\right) \qquad \textit{Ans.}$$

In words, the Nusselt number is a function of Prandtl number and Peclet number.

5.33 Convection heat-transfer data are often reported as a *heat-transfer coefficient h*, defined by

$$\dot{Q} = h\,A\,\Delta T$$

where $\dot{Q}$ = heat flow, J/s
A = surface area, m^2
ΔT = temperature difference, K

The dimensionless form of h, called the *Stanton number*, is a combination of h, fluid density ρ, specific heat c_p, and flow velocity V. Derive the Stanton number if it is proportional to h. What are the units of h?

Solution: If $\{\dot{Q}\} = \{hA\Delta T\}$, then $\left\{\dfrac{ML^2}{T^3}\right\} = \{h\}\{L^2\}\{\Theta\}$, or: $\{h\} = \left\{\dfrac{M}{\Theta T^3}\right\}$

$$\text{Then }\{\text{Stanton No.}\} = \{h^1\rho^b c_p^c V^d\} = \left\{\frac{M}{\Theta T^3}\right\}\left\{\frac{M}{L^3}\right\}^b\left\{\frac{L^2}{T^2\Theta}\right\}^c\left\{\frac{L}{T}\right\}^d = M^0L^0T^0\Theta^0$$

Solve for b = −1, c = −1, and d = −1.

$$\text{Thus, finally, Stanton Number} = h\rho^{-1}c_p^{-1}V^{-1} = \mathbf{\frac{h}{\rho V c_p}} \quad \textit{Ans.}$$

5.34 In Example 5.1 we used the pi theorem to develop Eq. (5.2) from Eq. (5.1). Instead of merely listing the primary dimensions of each variable, some workers list the *powers* of each primary dimension for each variable in an array:

$$\begin{array}{c c} & \begin{matrix} F & L & U & \rho & \mu \end{matrix} \\ \begin{matrix} M \\ L \\ T \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & -3 & -1 \\ -2 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

This array of exponents is called the *dimensional matrix* for the given function. Show that the *rank* of this matrix (the size of the largest nonzero determinant) is equal to $j = n - k$, the desired reduction between original variables and the pi groups. This is a general property of dimensional matrices, as noted by Buckingham [1].

Solution: The **rank** of a matrix is the size of the largest submatrix within which has a *non-zero determinant*. This means that the constants in that submatrix, when considered as coefficients of algebraic equations, are *linearly independent*. Thus we establish the number of *independent* parameters—adding one more forms a dimensionless group. For the example shown, the rank is **three** (note the very first 3 × 3 determinant on the left has a non-zero determinant). Thus "j" = 3 for the drag force system of variables.

5.35 The lift force F on a missile is a function of its length L, velocity V, diameter D, angle of attack α, density ρ, viscosity μ, and speed of sound a of the air. Write out the dimensional matrix of this function and determine its rank. (See Prob. 5.34 for an explanation of this concept.) Rewrite the function in terms of pi groups.

Solution: Establish the variables and their dimensions:

$$\underset{\{ML/T^2\}}{F} = \text{fcn}(\underset{\{L\}}{L}, \underset{\{L/T\}}{V}, \underset{\{L\}}{D}, \underset{\{1\}}{\alpha}, \underset{\{M/L^3\}}{\rho}, \underset{\{M/LT\}}{\mu}, \underset{\{L/T\}}{a})$$

Then $n = 8$ and $j = 3$, hence we expect $n - j = 8 - 3 = \mathbf{5}$ Pi groups. The matrix is

	F	L	V	D	α	ρ	μ	a
M:	1	0	0	0	0	1	1	0
L:	1	1	1	1	0	-3	-1	1
T:	-2	0	-1	0	0	0	-1	-1

The rank of this matrix is indeed **three**, hence there are exactly 5 Pi groups. The writer chooses:

$$\text{Typical final result: } \quad \mathbf{\frac{F}{\rho V^2 L^2} = fcn\left(\alpha, \frac{\rho VL}{\mu}, \frac{L}{D}, \frac{V}{a}\right)} \quad \textit{Ans.}$$

5.36 The angular velocity Ω of a windmill is a function of windmill diameter D, wind velocity V, air density ρ, windmill height H as compared to atmospheric boundary layer height L, and the number of blades N: that is, $\Omega = \text{fcn}(D, V, \rho, H/L, N)$. Viscosity effects are negligible. Rewrite this function in terms of dimensionless Pi groups.

$$\Omega = fcn(D, V, \rho, H/L, N)$$

Solution: We have $n = 6$ variables, $j = 3$ dimensions (M, L, T), thus expect $n - j = 3$ Pi groups. Since only ρ has *mass* dimensions, it drops out. After some thought, we realize that H/L and N are already dimensionless! The desired dimensionless function becomes:

$$\mathbf{\frac{\Omega D}{V} = \mathit{fcn}\left(\frac{H}{L}, N\right)} \quad \textit{Ans.}$$

5.37 A simply supported beam of diameter D, length L, and modulus of elasticity E is subjected to a fluid crossflow of velocity V, density ρ, and viscosity μ. Its center deflection δ is assumed to be a function of all these variables. (a) Rewrite this proposed function in dimensionless form. (b) Suppose it is known that δ is independent of μ, inversely proportional to E, and dependent only upon ρV^2, not ρ and V separately. Simplify the dimensionless function accordingly.

Solution: Establish the variables and their dimensions:

$$\delta = \text{fcn}(\rho, D, L, E, V, \mu)$$
$$\{L\} \quad \{M/L^3\} \quad \{L\} \quad \{L\} \quad \{M/LT^2\} \quad \{L/T\} \quad \{M/LT\}$$

Then $n = 7$ and $j = 3$, hence we expect $n - j = 7 - 3 =$ **4** Pi groups, capable of various arrangements and selected by the writer, as follows (a):

$$\text{Well-posed final result: } \mathbf{\frac{\delta}{L} = fcn\left(\frac{L}{D}, \frac{\rho VD}{\mu}, \frac{E}{\rho V^2}\right)} \quad \textit{Ans.} \text{ (a)}$$

(b) If μ is unimportant and δ proportional to E^{-1}, then the Reynolds number ($\rho VD/\mu$) drops out, and we have already cleverly combined E with ρV^2, which we can now slip out and turn upside down:

$$\text{If } \mu \text{ drops out and } \delta \propto \frac{1}{E}, \quad \text{then } \frac{\delta}{L} = \frac{\rho V^2}{E}\text{fcn}\left(\frac{L}{D}\right),$$

$$\text{or: } \mathbf{\frac{\delta E}{\rho V^2 L} = fcn\left(\frac{L}{D}\right)} \quad \textit{Ans.} \text{ (b)}$$

5.38 The heat-transfer rate per unit area q to a body from a fluid in natural or gravitational convection is a function of the temperature difference ΔT, gravity g, body length L, and three fluid properties: kinematic viscosity ν, conductivity k, and thermal expansion coefficient β. Rewrite in dimensionless form if it is known that g and β appear only as the product $g\beta$.

Solution: Establish the variables and their dimensions:

$$q = \text{fcn}(\Delta T, g, L, \nu, \beta, k)$$
$$\{M/T^3\} \quad \{\Theta\} \quad \{L/T^2\} \quad \{L\} \quad \{L^2/T\} \quad \{1/\Theta\} \quad \{ML/\Theta T^3\}$$

Then $n = 7$ and $j = 4$, hence we expect $n - j = 7 - 4 =$ **3** Pi groups, capable of various arrangements and selected by myself, as follows:

$$\text{If } \beta \text{ and } \Delta T \text{ kept separate, then } \frac{qL}{k\Delta T} = \text{fcn}\left(\beta\Delta T, \frac{gL^3}{\nu^2}\right)$$

If, in fact, β and g must appear <u>together</u>, then Π2 and Π3 above <u>combine</u> and we get

$$\underbrace{\mathbf{\frac{qL}{k\Delta T}}}_{\text{Nusselt No.}} = \mathbf{fcn}\underbrace{\left(\mathbf{\frac{\beta\Delta T g L^3}{\nu^2}}\right)}_{\text{Grashof Number}} \quad \textit{Ans.}$$

5.39 A *weir* is an obstruction in a channel flow which can be calibrated to measure the flow rate, as in Fig. P5.39. The volume flow Q varies with gravity g, weir width b into the paper, and upstream water height H above the weir crest. If it is known that Q is proportional to b, use the pi theorem to find a unique functional relationship $Q(g, b, H)$.

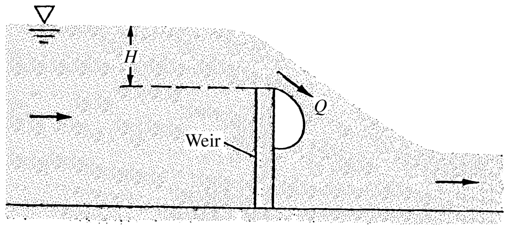


Fig. P5.39

Solution: Establish the variables and their dimensions:

$$Q = \text{fcn}(g, b, H)$$
$$\{L^3/T\} \quad \{L/T^2\} \quad \{L\} \quad \{L\}$$

Then $n = 4$ and $j = 2$, hence we expect $n - j = 4 - 2 = \mathbf{2}$ Pi groups, capable of various arrangements and selected by myself, as follows:

$$\frac{Q}{g^{1/2}H^{5/2}} = \text{fcn}\left(\frac{b}{H}\right); \quad \text{but if } Q \propto b, \text{ then we reduce to } \frac{\mathbf{Q}}{\mathbf{bg^{1/2}H^{3/2}}} = \textbf{constant} \quad \textit{Ans.}$$

5.40 A spar buoy (see Prob. 2.126) has a period T of vertical (heave) oscillation which depends upon the waterline cross-sectional area A, buoy mass m, and fluid specific weight γ. How does the period change due to doubling of (a) the mass and (b) the area? Instrument buoys should have long periods to avoid wave resonance. Sketch a possible long-period buoy design.

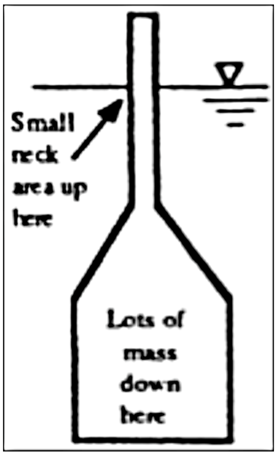


Fig. P5.40

Solution: Establish the variables and their dimensions:

$$T = \text{fcn}(A, m, \gamma)$$
$$\{T\} \quad \{L^2\} \quad \{M\} \quad \{M/L^2T^2\}$$

Then $n = 4$ and $j = 3$, hence we expect n – j = 4 – 3 = **1** single Pi group, as follows:

$$\mathbf{T\sqrt{\frac{A\gamma}{m}} = dimensionless\ constant} \quad \textit{Ans.}$$

Since we can't do anything about γ, the specific weight of water, we *can* increase period T by increasing buoy mass m and decreasing waterline area A. See the illustrative long-period buoy in Figure P5.40 above.

5.41 To good approximation, the thermal conductivity k of a gas (see Ref. 21 of Chap. 1) depends only on the density ρ, mean free path ℓ, gas constant R, and absolute temperature T. For air at 20°C and 1 atm, $k \approx 0.026$ W/m·K and $\ell \approx 6.5E{-}8$ m. Use this information to determine k for hydrogen at 20°C and 1 atm if $\ell \approx 1.2E{-}7$ m.

Solution: First establish the variables and their dimensions and then form a pi group:

$$\begin{array}{cccccc} k & = \text{fcn}(& \rho & , \ell , & R & , T) \\ \{ML/\Theta T^3\} & & \{M/L^3\} & \{L\} & \{L^2/T^2\Theta\} & \{\Theta\} \end{array}$$

Thus $n = 5$ and $j = 4$, and we expect $n - j = 5 - 4 =$ **1** single pi group, and the result is

$$k/(\rho R^{3/2} T^{1/2} \ell) = \text{a dimensionless constant} = \Pi_1$$

The value of Π_1 is found from the air data, where $\rho = 1.205$ kg/m^3 and R = 287 m^2/s^2·K:

$$\Pi_{1,air} = \frac{0.026}{(1.205)(287)^{3/2}(293)^{1/2}(6.5E{-}8)} = 3.99 = \Pi_{1,hydrogen}$$

For hydrogen at 20°C and 1 atm, calculate $\rho = 0.0839$ kg/m^3 with $R = 4124$ m^2/s^2·K. Then

$$\Pi_1 = 3.99 = \frac{k_{hydrogen}}{(0.0839)(4124)^{3/2}(293)^{1/2}(1.2E{-}7)}, \quad \textit{solve for } k_{hydrogen} = \mathbf{0.182\ \frac{W}{m \cdot K}} \quad \textit{Ans.}$$

This is slightly larger than the accepted value for hydrogen of $k \approx 0.178$ W/m·K.

5.42 The torque M required to turn the cone-plate viscometer in Fig. P5.42 depends upon the radius R, rotation rate Ω, fluid viscosity μ, and cone angle θ. Rewrite this relation in dimensionless form. How does the relation simplify if it is known that M is proportional to θ?

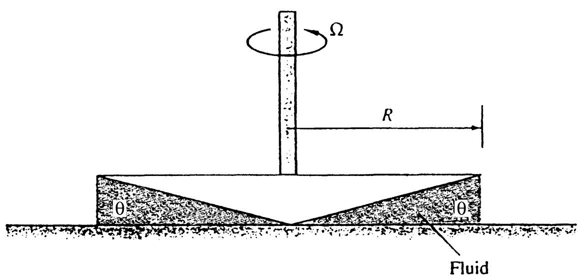


Fig. P5.42

Solution: Establish the variables and their dimensions:

$$M = \text{fcn}(R, \Omega, \mu, \theta)$$

$$\{ML^2/T^2\} \quad \{L\} \quad \{1/T\} \quad \{M/LT\} \quad \{1\}$$

Then n = 5 and j = 3, hence we expect n – j = 5 – 3 = **2** Pi groups, capable of only one reasonable arrangement, as follows:

$$\frac{\mathbf{M}}{\mu\Omega\mathbf{R}^3} = \mathbf{fcn}(\theta); \quad \text{if } M \propto \theta, \quad \text{then } \frac{\mathbf{M}}{\mu\Omega\theta\mathbf{R}^3} = \mathbf{constant} \quad \textit{Ans.}$$

See Prob. 1.61 of this Manual, for an analytical solution.

P5.43 A hard-disk cleaning process requires a simple tool as shown in Fig. 5.43.

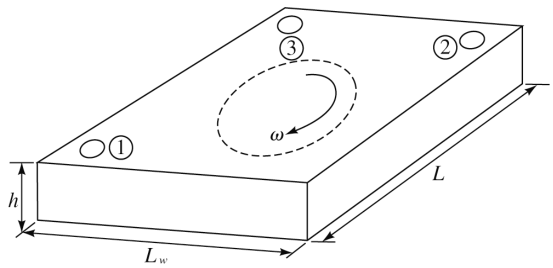


Fig. P5.43

Clean air is forced into the box at point ① and ② and allowed to flow out at point ③ with Q L/s flow rate. During the cleaning process, the disk must be spinned at the spinning rate ω rad/s. A manufacturer wants to reduce the process time. If we want to apply dimensional analysis, what relationship in dimensionless form should it be?

Solution: Let cleaning time be T. A possible relationship of the process could be

$$T = f(\rho, \mu, Q, \omega, h, L\omega, L)$$

step 1 Count the variables, $\eta = 8$

step 2 Use the $\{MLT\Theta\}$ system to write out the dimensions of the variables:

T	ρ	μ	Q	ω	h	Lw	L
T	ML^{-3}	$ML^{-1}T^{-1}$	$L^{-3}T^{-1}$	T^{-1}	L	L	L

step 3 Using ρ, μ, and Q as repeating variables, but we need to check whether ρ, μ, and Q do not form a pi-group. We found that (ρ, μ, Q) can be used as repeating variables.

Therefore, j = 3.

step 4(a). Combine ($\rho,\ \mu,\ Q$) with time to clean the disk T to find the first pi group:

$$\pi_1 = \rho^a \mu^b Q^c T = \left\{\mathrm{ML^{-3}}\right\}^a \left\{\mathrm{ML^{-1}T^{-1}}\right\}^b \left\{\mathrm{L^3T^{-1}}\right\}^c \mathrm{T} = \mathrm{M^oL^oT}$$

Solve the above equation, we have

$$\pi_1 = \frac{\mathrm{T}\mu^3}{\rho^3 Q^2}$$

(b) Combine ($\rho,\ \mu,\ Q$) with spinning rate ω to find the second pi group:

$$\pi_2 = \rho^a \mu^b Q^c \omega = \left\{\mathrm{ML^{-3}}\right\}^a \left\{\mathrm{ML^{-1}T^{-1}}\right\}^b \left\{\mathrm{L^3T^{-1}}\right\}^c \mathrm{T^{-1}} = \mathrm{M^oL^oT^o}$$

Solve the above equation, we have

$$\pi_2 = \frac{\omega\rho^3 Q^2}{\mu^3}$$

(c) Combine ($\rho,\ \mu,\ Q$) with height h to find the next pi group:

$$\pi_3 = \rho^a \mu^b Q^c h = \left\{\mathrm{ML^{-3}}\right\}^a \left\{\mathrm{ML^{-1}T^{-1}}\right\}^b \left\{\mathrm{L^3T^{-1}}\right\}^c \mathrm{L} = \mathrm{M^oL^oT^o}$$

Solve the above equation, we have

$$\pi_3 = \frac{h\mu}{\rho Q}$$

(d) Since Lw and L have the same dimension as h, therefore, we can easily write out π_4 and π_5 as follow:

$$\pi_4 = \frac{Lw\mu}{\rho Q} \text{ and } \pi_5 = \frac{L\mu}{\rho Q}$$

step 5 $\pi_1 = g\left(\pi_2 \ldots\ldots,\ \pi_5\right)$

$$\frac{\mathrm{T}\mu^3}{\rho^3 Q^2} = g\left(\frac{\omega\rho^3 Q^2}{\mu^3},\ \frac{h\mu}{\rho Q},\ \frac{L\omega\mu}{\rho Q},\ \frac{L\mu}{\rho Q}\right)$$

This is an example of how we would proceed for our experiment.

P5.44 Consider natural convection in a rotating, fluid-filled enclosure. The average wall shear stress τ in the enclosure is assumed to be a function of rotation rate Ω, enclosure height H, density ρ, temperature difference ΔT, viscosity μ, and thermal expansion coefficient β. (*a*) Rewrite this relationship as a dimensionless function. (*b*) Do you see a severe *flaw* in the analysis?

Solution: (*a*) Using Table 5.1, write out the dimensions of the seven variables:

τ	ρ	H	Ω	μ	β	ΔT
$\{ML^{-1}T^{-2}\}$	$\{ML^{-3}\}$	$\{L\}$	$\{T^{-1}\}$	$\{ML^{-1}T^{-1}\}$	$\{\Theta^{-1}\}$	$\{\Theta\}$

There are four primary dimensions (MLTQ), and we can easily find four variables (ρ, H, Ω, β) that do not form a pi group. Therefore we expect 7-4 = 3 dimensionless groups. Adding each remaining variable in turn, we find three nice pi groups:

$$\Pi_1 = \rho^a H^b \Omega^c \beta^d \tau \quad \text{leads to} \quad \Pi_1 = \frac{\tau}{\rho H^2 \Omega^2}$$

$$\Pi_2 = \rho^a H^b \Omega^c \beta^d \mu \quad \text{leads to} \quad \Pi_2 = \frac{\mu}{\rho H^2 \Omega}$$

$$\Pi_3 = \rho^a H^b \Omega^c \beta^d \Delta T \quad \text{leads to} \quad \Pi_3 = \beta \Delta T$$

Thus one very nice arrangement of the desired dimensionless function is

$$\frac{\tau}{\rho H^2 \Omega^2} = fcn\left(\frac{\mu}{\rho H^2 \Omega}, \ \beta \Delta T\right) \qquad Ans.(a)$$

(*b*) This is a good dimensional analysis exercise, ***but in real life it would fail miserably***, because natural convection is highly dependent upon the acceleration of gravity, *g*, which we left out by mistake.

5.45 A pendulum has an oscillation period T which is assumed to depend upon its length L, bob mass m, angle of swing θ, and the acceleration of gravity. A pendulum 1 m long, with a bob mass of 200 g, is tested on earth and found to have a period of 2.04 s when swinging at 20°. (a) What is its period when it swings at 45°? A similarly constructed pendulum, with L = 30 cm and m = 100 g, is to swing on the moon (g = 1.62 m/s^2) at θ = 20°. (b) What will be its period?

Solution: First establish the variables and their dimensions so that we can do the numbers:

$$\begin{array}{ccccc} T = \text{fcn}(& L, & m, & g, & \theta\ ) \\ \{T\} & \{L\} & \{M\} & \{L/T^2\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect n – j = 5 – 3 = **2** Pi groups. They are unique:

$$T\sqrt{\frac{g}{L}} = \text{fcn}(\theta) \quad \text{(mass drops out for dimensional reasons)}$$

(a) If we change the angle to 45°, this changes Π_2, hence we lose dynamic similarity and **do not know the new period**. More testing is required. *Ans.* (a)

(b) If we swing the pendulum on the moon at the same 20°, we may use similarity:

$$T_1\left(\frac{g_1}{L_1}\right)^{1/2} = (2.04\ \text{s})\left(\frac{9.81\ \text{m/s}^2}{1.0\ \text{m}}\right)^{1/2} = 6.39 = T_2\left(\frac{1.62\ \text{m/s}^2}{0.3\ \text{m}}\right)^{1/2},$$

$$\text{or:}\quad \mathbf{T_2 = 2.75\ s} \quad \textit{Ans.}\text{ (b)}$$

5.46 The differential energy equation for incompressible two-dimensional flow through a "Darcy-type" porous medium is approximately

$$\rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x} + \rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial y}\frac{\partial T}{\partial y} + k\frac{\partial^2 T}{\partial y^2} = 0$$

where σ is the *permeability* of the porous medium. All other symbols have their usual meanings. (a) What are the appropriate dimensions for σ? (b) Nondimensionalize this equation, using (L, U, ρ, T_o) as scaling constants, and discuss any dimensionless parameters which arise.

Solution: (a) The only way to establish $\{\sigma\}$ is by comparing two terms in the PDE:

$$\left\{\rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}\right\} = \left\{k\frac{\partial^2 T}{\partial x^2}\right\}, \quad \text{or:} \quad \left\{\frac{M}{L^3T^3}\right\}\{\sigma\} \stackrel{?}{=} \left\{\frac{M}{LT^3}\right\},$$

$$\text{Thus}\quad \{\sigma\} = \{\mathbf{L^2}\} \quad \textit{Ans.}\text{ (a)}$$

(b) Define dimensionless variables using the stated list of (L, U, ρ, T_o) for scaling:

$$x^* = \frac{x}{L};\quad y^* = \frac{y}{L};\quad p^* = \frac{p}{\rho U^2};\quad T^* = \frac{T}{T_o}$$

Substitution into the basic PDE above yields only a *single* dimensionless parameter:

$$\zeta\left(\frac{\partial p^*}{\partial x^*}\frac{\partial T^*}{\partial x^*} + \frac{\partial p^*}{\partial y^*}\frac{\partial T^*}{\partial y^*}\right) + \frac{\partial^2 T^*}{\partial y^{*2}} = 0, \quad \text{where}\ \zeta = \frac{\boldsymbol{\rho}^2\mathbf{c_p U^2}\boldsymbol{\sigma}}{\boldsymbol{\mu}\mathbf{k}} \quad \textit{Ans.}\text{ (b)}$$

I don't know the name of this parameter. It is related to the "Darcy-Rayleigh" number.

5.47 A model differential equation, for chemical reaction dynamics in a plug reactor, is as follows:

$$u\frac{\partial C}{\partial x} = \mathcal{D}\frac{\partial^2 C}{\partial x^2} - kC - \frac{\partial C}{\partial t}$$

where u is the velocity, $\mathcal{D}$ is a diffusion coefficient, k is a reaction rate, x is distance along the reactor, and C is the (dimensionless) concentration of a given chemical in the reactor. (a) Determine the appropriate dimensions of $\mathcal{D}$ and k. (b) Using a characteristic length scale L and average velocity V as parameters, rewrite this equation in dimensionless form and comment on any Pi groups appearing.

Solution: (a) Since all terms in the equation contain C, we establish the dimensions of k and $\mathcal{D}$ by comparing $\{k\}$ and $\{\mathcal{D}\partial^2/\partial x^2\}$ to $\{u\partial/\partial x\}$:

$$\{k\} = \{\mathcal{D}\}\left\{\frac{\partial^2}{\partial x^2}\right\} = \{\mathcal{D}\}\left\{\frac{1}{L^2}\right\} = \{u\}\left\{\frac{\partial}{\partial x}\right\} = \left\{\frac{L}{T}\right\}\left\{\frac{1}{L}\right\},$$

$$\text{hence}\quad \boldsymbol{\{k\} = \left\{\frac{1}{T}\right\}}\quad \text{and}\quad \boldsymbol{\{\mathcal{D}\} = \left\{\frac{L^2}{T}\right\}}\quad \textit{Ans.}\text{ (a)}$$

(b) To non-dimensionalize the equation, define $u^* = u/V$, $t^* = Vt/L$, and $x^* = x/L$ and substitute into the basic partial differential equation. The dimensionless result is

$$\boldsymbol{u^*\frac{\partial C}{\partial x^*} = \left(\frac{\mathcal{D}}{VL}\right)\frac{\partial^2 C}{\partial x^{*2}} - \left(\frac{kL}{V}\right)C - \frac{\partial C}{\partial t^*}},\quad \text{where}\quad \frac{VL}{\mathcal{D}} = \text{mass-transfer Peclet number}\quad \textit{Ans.}\text{ (b)}$$

5.48 The differential equation for small-amplitude vibrations $y(x, t)$ of a simple beam is given by

$$\rho A\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = 0$$

where ρ = beam material density
A = cross-sectional area
I = area moment of inertia
E = Young's modulus

Use only the quantities ρ, E, and A to nondimensionalize y, x, and t, and rewrite the differential equation in dimensionless form. Do any parameters remain? Could they be removed by further manipulation of the variables?

Solution: The appropriate dimensionless variables are

$$y^* = \frac{y}{\sqrt{A}};\quad t^* = t\sqrt{\frac{E}{\rho A}};\quad x^* = \frac{x}{\sqrt{A}}$$

Substitution into the PDE above yields a dimensionless equation with *one* parameter:

$$\frac{\partial^2 \mathbf{y}^*}{\partial \mathbf{t}^{*2}} + \left(\frac{\mathbf{I}}{\mathbf{A}^2}\right)\frac{\partial^4 \mathbf{y}^*}{\partial \mathbf{x}^{*4}} = \mathbf{0}; \quad \text{One geometric parameter: } \frac{\mathbf{I}}{\mathbf{A}^2} \quad \textit{Ans.}$$

We could *remove* (I/A^2) completely by redefining $\mathbf{x^* = x/I^{1/4}}$. *Ans.*

5.49 Non-dimensionalize the thermal energy partial differential equation (4.75) and its boundary conditions (4.62), (4.63), and (4.70) by defining dimensionless temperature $T^* = T/T_o$, where T_o is the fluid inlet temperature, assumed constant. Use other dimensionless variables as needed from Eqs. (5.23). Isolate all dimensionless parameters which you find, and relate them to the list given in Table 5.2.

Solution: Recall the previously defined variables in addition to T^*:

$$u^* = \frac{u}{U}; \quad x^* = \frac{x}{L}; \quad t^* = \frac{Ut}{L}; \quad \text{similarly,} \quad v^* \text{ or } w^* = \frac{v \text{ or } w}{U}; \quad y^* \text{ or } z^* = \frac{y \text{ or } z}{L}$$

Then the dimensionless versions of Eqs. (4.75, 62, 63, 70) result as follows:

$$\textbf{(4.75):} \quad \frac{dT^*}{dt^*} = \underbrace{\left(\frac{k}{\rho c_p UL}\right)}_{\text{1/Peclet Number}} \nabla^{*2}T^* + \underbrace{\left(\frac{\mu U}{\rho c_p T_o L}\right)}_{\text{Eckert Number divided by Reynolds Number}} \Phi^*$$

P5.50 If a vertical wall at temperature T_w is surrounded by a fluid at temperature T_o, a natural convection boundary layer flow will form. For laminar flow, the momentum equation is

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho\beta(T - T_o)g + \mu\frac{\partial^2 u}{\partial y^2}$$

to be solved, along with continuity and energy, for (u, v, T) with appropriate boundary conditions. The quantity β is the thermal expansion coefficient of the fluid. Use ρ, g, L, and $(T_w - T_o)$ to nondimensionalize this equation. Note that there is no "stream" velocity in this type of flow.

Solution: For the given constants used to define dimensionless variables, there is only one pairing which will give a velocity unit: $(gL)^{1/2}$. Here are the writer's dimensionless variables:

$$u^* = \frac{u}{\sqrt{gL}}\;; \quad v^* = \frac{v}{\sqrt{gL}}\;; \quad x^* = \frac{x}{L}\;; \quad y^* = \frac{y}{L}\;; \quad T^* = \frac{T - T_o}{T_w - T_o}$$

Substitute into the momentum equation above and clean up so all terms are dimensionless:

$$\rho(u^*\frac{\partial u^*}{\partial x^*}\frac{gL}{L}) + \rho(v^*\frac{\partial u^*}{\partial y^*}\frac{gL}{L}) = \rho\beta g(T_w - T_o)T^* + \mu\frac{\partial^2 u^*}{\partial y^{*2}}\frac{\sqrt{gL}}{L^2}$$

$$\text{or:} \quad u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = [\beta(T_w - T_o)]T^* + [\frac{\mu}{\rho L\sqrt{gL}}]\frac{\partial^2 u^*}{\partial y^{*2}} \qquad Ans.$$

There are two dimensionless parameters: $\beta(T_w - T_o)$ and $\mu/[\rho L\sqrt{gL}]$. Neither has a name, to the writer's knowledge, because a much cleverer analysis would result in only a single dimensionless parameter, the *Grashof number*, $g\beta(T_w - T_o)L^3/\nu^2$. (See, for example, White, *Viscous Fluid Flow*, 3rd edition, Section 4-14.3, page 323.)

5.51 A smooth steel (SG = 7.86) sphere is immersed in a stream of ethanol at 20°C moving at 1.5 m/s. Estimate its drag in N from Fig. 5.3*a*. What stream velocity would quadruple its drag? Take D = 2.5 cm.

Solution: For ethanol at 20°C, take $\rho \approx 789$ kg/m^3 and $\mu \approx 0.0012$ kg/m·s. Then

$$\mathrm{Re_D} = \frac{\rho \mathrm{UD}}{\mu} = \frac{789(1.5)(0.025)}{0.0012} \approx 24700; \quad \text{Read Fig. 5.3(a):} \quad \mathrm{C_{D,sphere}} \approx 0.4$$

$$\text{Compute drag} \quad \mathrm{F} = \mathrm{C_D}\left(\frac{1}{2}\right)\rho \mathrm{U}^2\frac{\pi}{4}\mathrm{D}^2 = (0.4)\left(\frac{1}{2}\right)(789)(1.5)^2\left(\frac{\pi}{4}\right)(0.025)^2$$

$$\approx \mathbf{0.17\ N} \quad Ans.$$

Since CD ≈ constant in this range of ReD, **doubling U quadruples** the drag. *Ans.*

5.52 The sphere in Prob. 5.51 is dropped in gasoline at 20°C. Ignoring its acceleration phase, what will be its terminal (constant) fall velocity, from Fig. 5.3*a*?

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx 2.92\text{E}{-4}$ kg/m·s. For steel take $\rho \approx 7800$ kg/m^3. Then, in "terminal" velocity, the net weight equals the drag force:

$$\text{Net weight} = (\rho_{\text{steel}} - \rho_{\text{gasoline}})\mathrm{g}\frac{\pi}{6}\mathrm{D}^3 = \text{Drag force} = \mathrm{C_D}\frac{\rho}{2}\mathrm{V}^2\frac{\pi}{4}\mathrm{D}^2,$$

$$\text{or:} \quad (7800 - 680)(9.81)\frac{\pi}{6}(0.025)^3 = 0.571\ \mathrm{N} = \mathrm{C_D}\left(\frac{1}{2}\right)(680)\mathrm{U}^2\frac{\pi}{4}(0.025)^2$$

$$\text{Guess} \quad \mathrm{C_D} \approx 0.4 \quad \text{and compute} \quad \mathbf{U \approx 2.9\ \frac{m}{s}} \quad Ans.$$

Now check ReD = ρUD/μ = 680(2.9)(0.025)/(2.92E−4) ≈ 170000. **Yes**, CD ≈ 0.4, OK.

P5.53 The parachute in the chapter-opener photo is, of course, meant to decelerate the payload on Mars. The wind tunnel test gave a drag coefficient of about 1.1, based upon the projected area of the parachute. Suppose it was falling on *earth* and, at an altitude of 1000 m, showed a steady descent rate of about 30 km/h. Estimate the weight of the payload.

Solution: Let's convert everything to metric. The diameter is 55 ft = 16.8 m. Standard air density at 1000 m is 1.112 kg/m^3. Descent velocity is 8.33 m/s. Then

$$C_D = 1.1 = \frac{F}{(1/2)\rho V^2(\pi/4)D^2} = \frac{F}{(1/2)(1.112\ \text{kg/m}^3)(8.33\ \text{m/s})^2(\pi/4)(16.8\ \text{m})^2}$$

$$\text{Solve for} \quad F = 9407.3\ \text{N (on earth)} \qquad \textit{Ans.}$$

5.54 A ship is towing a sonar array which approximates a submerged cylinder 0.3 m in diameter and 9 m long with its axis normal to the direction of tow. If the tow speed is 12 kn (1 kn ≈ 0.51 m/s), estimate the horsepower required to tow this cylinder. What will be the frequency of vortices shed from the cylinder? Use Figs. 5.2 and 5.3.

Solution: For seawater at 20°C, take $\rho \approx 1023$ kg/m^3 and $\mu = 1.08 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2$. Convert V = 12 knots ≈ 6.12 m/s. Then the Reynolds number and drag of the towed cylinder is

$$\text{Re}_D = \frac{\rho UD}{\mu} = \frac{1023(6.12)(0.3)}{1.08 \times 10^{-3}} \approx 1.74\text{E}6. \quad \text{Fig. 5.3(a) cylinder:} \quad \text{Read } C_D \approx 0.3$$

$$\text{Then} \quad F = C_D\left(\frac{1}{2}\right)\rho U^2 DL = (0.3)\left(\frac{1}{2}\right)(1023)(6.12)^2(0.3)(9) \approx 15517.92\ \text{N}$$

$$\text{Power} \quad P = FU = (15517.92)(6.12) \approx \mathbf{127.4\ hp} \quad \textit{Ans.}\ \text{(a)}$$

Data for cylinder vortex shedding is found from Fig. 5.2*b*. At a Reynolds number ReD ≈ 1.74E6, read fD/U ≈ 0.24. Then

$$f_{shedding} = \frac{StU}{D} = \frac{(0.24)(6.12)}{0.3} \approx \mathbf{5\ Hz} \quad \textit{Ans.}\ \text{(b)}$$

5.55 A fishnet is made of 1-mm-diameter strings knotted into 2 × 2 cm squares. Estimate the horsepower required to tow 28 m^2 of this netting at 3 kn in seawater at 20°C. The net plane is normal to the flow direction.

Solution: For seawater at 20°C, take $\rho \approx 1023$ kg/m^3 and $\mu \approx 0.00108$ kg/m·s. Convert V = 3 knots = 1.54 m/s. Then, considering the strings as "cylinders in crossflow," the Reynolds number is Re

$$\text{Re}_D = \frac{\rho VD}{\mu} = \frac{(1023)(1.54)(0.001)}{0.00108} \approx 1460; \quad \text{Fig. 5.3(a): } C_{D,\text{cyl}} \approx 1.0$$

Drag of one 2-cm strand:

$$F = C_D \frac{\rho}{2} V^2 DL = (1.0)\left(\frac{1023}{2}\right)(1.54)^2(0.001)(0.02) \approx 0.0243 \text{ N}$$

Now 1 m^2 of net contains *5000* of these 2-cm strands, and 28 m^2 of net contains (5000)(28) = *140,000* strands total, for a total net force F = 140,000(0.0243) ≈ 3400 N on the net. Then the horsepower required to tow the net is

$$\text{Power} = FV = (3400)(1.54) = 5236 \text{ W} \div 746 \text{ W/hp} \approx \mathbf{7.0 \text{ hp}} \quad \textit{Ans.}$$

5.56 The simply supported 1040 carbon-steel rod of Fig. P5.56 is subjected to a crossflow stream of air at 20°C and 1 atm. For what stream velocity *U* will the rod center deflection be approximately 1 cm?

Solution: For air at 20°C, take $\rho \approx 1.2$ kg/m^3 and $\mu \approx$ 1.8E−5 kg/m·s. For carbon steel take Young's modulus E ≈ 29E6 psi ≈ 2.0E11 Pa.

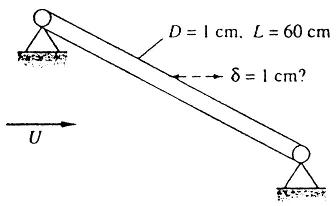


Fig. P5.56

This is not an elasticity course, so just use the formula for center deflection of a simply-supported beam:

$$\delta_{center} = \frac{FL^3}{48EI} = 0.01 \text{ m} = \frac{F(0.6)^3}{48(2.0E11)[(\pi/4)(0.005)^4]}, \quad \text{solve for } F \approx 218 \text{ N}$$

$$\text{Guess} \quad C_D \approx 1.2, \quad \text{then } F = 218 \text{ N} = C_D \frac{\rho}{2} V^2 DL = (1.2)\left(\frac{1.2}{2}\right) V^2 (0.01)(0.6)$$

Solve for V ≈ 225 m/s, check $Re_D = \rho VD/\mu \approx$ 150,000: OK, $C_D \approx 1.2$ from Fig. 5.3a.
Then **V ≈ 225 m/s**, which is quite high subsonic speed, Mach number ≈ 0.66. *Ans.*

5.57 For the steel rod of Prob. 5.56, at what airstream velocity *U* will the rod begin to vibrate laterally in resonance in its first mode (a half sine wave)? (*Hint:* Consult a vibration text [Ref. 34 or 35] under "lateral beam vibration.")

Solution: From a vibrations book, the first mode frequency for a simply-supported slender beam is given by

$$\omega_n = \pi^2 \sqrt{\frac{EI}{mL^4}} \quad \text{where } m = \rho_{steel}\pi R^2 = \text{beam mass per unit length}$$

$$\text{Thus} \quad f_n = \frac{\omega_n}{2\pi} = \frac{\pi}{2}\left[\frac{2.0E11(\pi/4)(0.005)^4}{(7840)\pi(0.005)^2(0.6)^4}\right]^{1/2} \approx 55.1 \text{ Hz}$$

The beam will resonate if its vortex shedding frequency is the same. Guess fD/U ≈ 0.2:

$$St = \frac{fD}{U} \approx 0.2 = \frac{55.1(0.01)}{U}, \quad \text{or} \quad U \approx 2.8\ \frac{m}{s}$$

Check $Re_D = \rho VD/\mu \approx 1800$. Fig. 5.2, OK, $St \approx 0.2$. Then $\mathbf{V \approx 2.8\ \frac{m}{s}}$ *Ans.*

5.58 When fluid in a long pipe starts up from rest at a uniform acceleration *a*, the initial flow is laminar. The flow undergoes transition to turbulence at a time *t** which depends, to first approximation, only upon *a*, ρ, and μ. Experiments by P. J. Lefebvre, on water at 20°C starting from rest with 1-g acceleration in a 3-cm-diameter pipe, showed transition at *t** = 1.02 s. Use this data to estimate (*a*) the transition time, and (*b*) the transition Reynolds number Re_D for water flow accelerating at 35 m/s^2 in a 5-cm-diameter pipe.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and *m* = 0.001 kg/m-s. There are four variables. Write out their dimensions:

$$\begin{array}{cccc} t^* & a & \rho & \mu \\ \{T\} & \{LT^{-2}\} & \{ML^{-3}\} & \{ML^{-1}T^{-1}\} \end{array}$$

There are three primary dimensions, (MLT), hence we expect 4 – 3 = one pi group:

$$\Pi_1 = \rho^a \mu^b a^c t^{*1} \quad \text{yields} \quad \Pi_1 = t^*\left(\frac{\rho a^2}{\mu}\right)^{1/3}, \quad or \quad t^* = (const)\left(\frac{\mu}{\rho a^2}\right)^{1/3}$$

Use LeFebvre’s data point to establish the constant value of Π_1:

$$t^* = 1.02 = (const)\left[\frac{0.001 kg/m-s}{(998 kg/m^3)(9.81 m/s^2)^2}\right]^{1/3} = (const)(0.00218)$$

Thus the constant, or Π_1, equals 1.02/0.00218 = 467 (dimensionless). Use this value to establish the new transition time for *a* = 35 m/s^2 in a 5-cm-diameter pipe:

$$t^* = (467)\left(\frac{\mu}{\rho a^2}\right)^{1/3} = (467)\left[\frac{0.001}{998(35)^2}\right]^{1/3} = \mathbf{0.44}s \qquad Ans.(a)$$

$$Re_D = \frac{\rho VD}{\mu} = \frac{\rho(aT^*)D}{\mu} = \frac{998[35(0.44)](0.05)}{(0.001)} = \mathbf{768,000} \qquad Ans.(b)$$

This transition Reynolds number is more than 300 times the value for which *steady* laminar pipe flow undergoes transition. The reason is that this is a thin-boundary-layer flow, and the laminar velocity profile never even approaches the Poiseuille parabola.

5.59 Vortex shedding can be used to design a *vortex flowmeter* (Fig. 6.34). A blunt rod stretched across the pipe sheds vortices whose frequency is read by the sensor downstream. Suppose the pipe diameter is 5 cm and the rod is a cylinder of diameter 8 mm. If the sensor reads 5400 counts per minute, estimate the volume flow rate of water in m^3/h. How might the meter react to other liquids?

Solution: 5400 counts/min = 90 Hz = f.

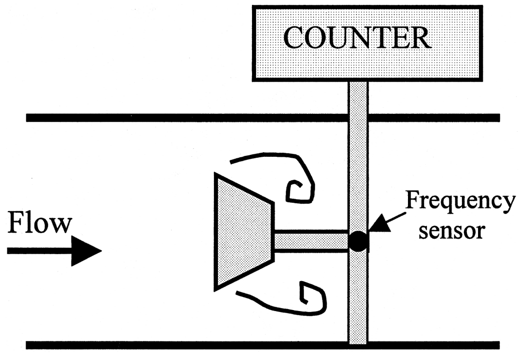


Fig. 6.34

$$\text{Guess } \frac{fD}{U} \approx 0.2 = \frac{90(0.008)}{U}, \quad \text{or} \quad U \approx 3.6 \ \frac{m}{s}$$

$$\text{Check } \quad Re_{D,water} = \frac{998(3.6)(0.008)}{0.001} \approx 29000; \quad \text{Fig. 5.2: Read St} \approx 0.2, \quad \text{OK.}$$

If the centerline velocity is 3.6 m/s and the flow is turbulent, then $V_{avg} \approx 0.82 V_{center}$ (see Ex. 3.4 of the text). Then the pipe volume flow is approximately:

$$Q = V_{avg}A_{pipe} = (0.82 \times 3.6)\frac{\pi}{4}(0.05\text{ m})^2 \approx 0.0058 \ \frac{m^3}{s} \approx \mathbf{21} \ \frac{\mathbf{m^3}}{\mathbf{hr}} \quad \textit{Ans.}$$

5.60 The radio antenna on a car begins to vibrate wildly at **8 Hz** when the car is driven at 72 km/h over a rutted road which approximates a sine wave of amplitude 2 cm and wavelength λ = 2.5 m. The antenna diameter is 4 mm. Is the vibration due to the road or to vortex shedding?

Solution: Convert U = 72 km/h = 20 m/s. Assume sea level air, ρ = 1.2 kg/m^3, μ = 1.8E−5 kg/m·s. Check the Reynolds number based on antenna diameter: Re_d = (1.2)(20)(0.004)/(1.8E−5) = 5333.3. From Fig. 5.2b, read St ≈ 0.21 = $(\omega/2\pi)d/U$ = (f_{shed})(0.004 m)/(20 m/s), or $f_{shed} \approx$ 1050 Hz ≠ 8 Hz, so rule out vortex shedding. Meanwhile, the rutted road introduces a forcing frequency f_{road} = U/λ = (20 m/s)/(2.5 m) = 8 Hz. We conclude that this resonance is due to *road roughness*.

5.61 Flow past a long cylinder of square cross-section results in more drag than the comparable round cylinder. Here are data taken in a water tunnel for a square cylinder of side length $b = 2$ cm:

V, m/s:	1.0	2.0	3.0	4.0
Drag, N/(m of depth):	21	85	191	335

(a) Use this data to predict the drag force per unit depth of wind blowing at 6 m/s, in air at 20°C, over a tall square chimney of side length $b = 55$ cm. (b) Is there any uncertainty in your estimate?

Solution: Convert the data to the dimensionless form $F/(\rho V^2 bL) = \text{fcn}(\rho Vb/\mu)$, like Eq. (5.2). For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Make a new table using the water data, with $L = 1$ m:

$F/(\rho V^2 bL)$:	1.05	1.06	1.06	1.05
$\rho Vb/\mu$:	19960	39920	59880	79840

In this Reynolds number range, the force coefficient is approximately constant at about 1.055. Use this value to estimate the air drag on the large chimney:

$$F_{air} = C_F \rho_{air} V_{air}^2 (bL)_{chimney} = (1.055)\left(1.2\ \frac{kg}{m^3}\right)\left(6\ \frac{m}{s}\right)^2 (0.55\ m)(1\ m) \approx \mathbf{25\ N/m} \quad \textit{Ans.}\text{ (a)}$$

(b) Yes, there **is** uncertainty, because $\text{Re}_{\text{chimney}} = 220{,}000 > \text{Re}_{\text{model}} = 80{,}000$ or less.

P5.62 A long, slender, 3-cm-diameter smooth flagpole bends alarmingly in 32 km/h sea-level winds, causing patriotic citizens to gasp. An engineer claims that the pole will bend less if its surface is deliberately roughened. Is she correct, at least qualitatively?

Solution: For sea-level air, take $\rho = 1.2255$ kg/m^3 and $\mu = 1.78\text{E-}5$ kg/m-s. Convert 32 km/h = 8.89 m/s. Calculate the Reynolds number of the pole as a "cylinder in crossflow":

$$\text{Re}_D = \frac{\rho VD}{\mu} = \frac{(1.2255\ \text{kg/m}^3)(8.89\ \text{m/s})(0.03\ \text{m})}{1.78E-5\,\text{kg/m}-\text{s}} = 18360$$

From Fig. 5.3*b*, we see that this Reynolds number is *below* the region where roughness is effective in reducing cylinder drag. Therefore we think the engineer is ***incorrect***. *Ans.*

[It is more likely that the drag of the *flag* is causing the problem.]

***P5.63** The thrust F of a free propeller, either aircraft or marine, depends upon density ρ, the rotation rate n in r/s, the diameter D, and the forward velocity V. Viscous effects are slight and neglected here. Tests of a 25-cm-diameter model aircraft propeller, in a sea-level wind tunnel, yield the following thrust data at a velocity of 20 m/s:

Rotation rate, r/min	4800	6000	8000
Measured thrust, N	6.1	19	47

(*a*) Use this data to make a crude but effective dimensionless plot. (b) Use the dimensionless data to predict the thrust, in newtons, of a similar 1.6-m-diameter prototype propeller when rotating at 3800 r/min and flying at 360 km/h at 4000 m standard altitude.

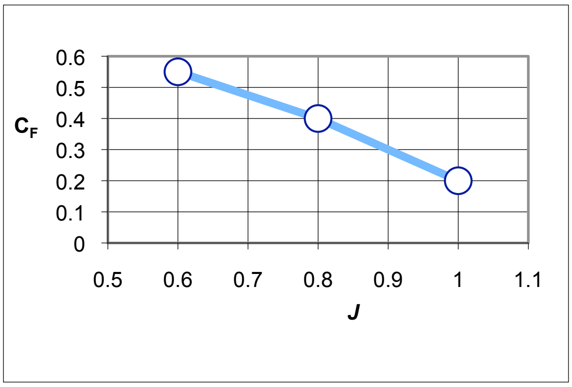
Solution: The given function is $F = \text{fcn}(r, n, D, V)$, and we note that $j = 3$. Hence we expect 2 pi groups. The writer chose (r, n, D) as repeating variables and found this:

$$C_F = fcn(J), \quad where \quad C_F = \frac{F}{\rho n^2 D^4} \quad and \quad J = \frac{V}{nD}$$

The quantity C_F is called the *thrust coefficient*, while J is called the *advance ratio*. Now use the data (at r = 1.2255 kg/m^3) to fill out a new table showing the two pi groups:

n, r/s	133.3	100.0	80.0
C_F	0.55	0.40	0.20
J	0.60	0.80	1.00

A crude but effective plot of this data is as follows. *Ans.*(*a*)



(*b*) At 4000 m altitude, from Table A.6, r = 0.8191 kg/m^3. Convert 360 km/h = 100 m/s. Convert 3800 r/min = 63.3 r/s. Then find the prototype advance ratio:

$$J = (100\text{ m/s})/[(63.3\text{ r/s})(1.6\text{ m}) = 0.99$$

Well, lucky us, that's our third data point! Therefore $C_{F,\text{prototype}} \approx 0.20$. And the thrust is

$$F_{prototype} = C_F \rho n^2 D^4 = (0.20)(0.8191\frac{kg}{m^3})(63.3\frac{r}{s})^2(1.6m)^4 \approx \mathbf{4300\ N} \quad Ans.(b)$$

5.64 If viscosity is neglected, typical pump-flow results are shown in Fig. P5.64 for a model pump tested in water. The pressure rise decreases and the power required increases with the dimensionless flow coefficient. Curve-fit expressions are given for the data. Suppose a similar pump of 12-cm diameter is built to move gasoline at 20°C and a flow rate of 25 m^3/h. If the pump rotation speed is 30 r/s, find (a) the pressure rise and (b) the power required.

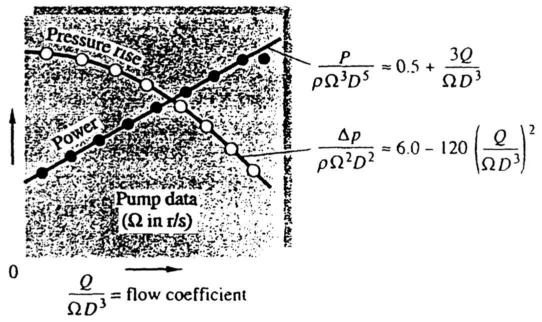


Fig. P5.64

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx$ 2.92E–4 kg/m·s. Convert Q = 25 m^3/hr = 0.00694 m^3/s. Then we can evaluate the "flow coefficient":

$$\frac{Q}{\Omega D^3} = \frac{0.00694}{(30)(0.12)^3} \approx 0.134, \quad \text{whence} \quad \frac{\Delta p}{\rho\Omega^2 D^2} \approx 6 - 120(0.134)^2 \approx 3.85$$

$$\text{and} \quad \frac{P}{\rho\Omega^3 D^5} \approx 0.5 + 3(0.134) \approx 0.902$$

With the dimensionless pressure rise and dimensionless power known, we thus find

$$\Delta p = (3.85)(680)(30)^2(0.12)^2 \approx \mathbf{34000\ Pa} \quad \textit{Ans. } \text{(a)}$$

$$P = (0.902)(680)(30)^3(0.12)^5 \approx \mathbf{410\ W} \quad \textit{Ans. } \text{(b)}$$

5.65 The natural frequency ω of vibra-tion of a mass M attached to a rod, as in Fig. P5.65, depends only upon M and the stiffness EI and length L of the rod. Tests with a 2-kg mass attached to a 1040 carbon-steel rod of diameter 12 mm and length 40 cm reveal a natural frequency of 0.9 Hz. Use these data to predict the natural frequency of a 1-kg mass attached to a 2024 aluminum-alloy rod of the same size.

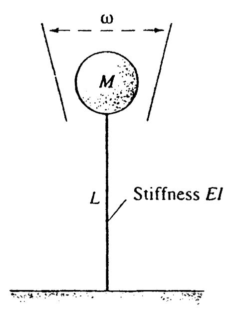


Fig. P5.65

Solution: For steel, E ≈ 29E6 psi ≈ 2.03E11 Pa. If ω = f(M, EI, L), then n = 4 and j = 3 (MLT), hence we get only 1 pi group, which we can evaluate from the steel data:

$$\frac{\omega(ML^3)^{1/2}}{(EI)^{1/2}} = \text{constant} = \frac{0.9[(2.0)(0.4)^3]^{1/2}}{[(2.03E11)(\pi/4)(0.006)^4]^{1/2}} \approx 0.0224$$

For 2024 aluminum, E ≈ 10.6E6 psi ≈ 7.4E10 Pa. Then re-evaluate the same pi group:

$$\text{New } \frac{\omega(ML^3)^{1/2}}{(EI)^{1/2}} = 0.0224 = \frac{\omega\,[(1.0)(0.4)^3]^{1/2}}{[(7.4E10)(\pi/4)(0.006)^4]^{1/2}}, \quad \text{or} \quad \boldsymbol{\omega_{alum} \approx 0.77\ \textbf{Hz}} \quad \textit{Ans.}$$

5.66 In turbulent flow near a flat wall, the local velocity u varies only with distance y from the wall, wall shear stress τ_w, and fluid properties ρ and μ. The following data were taken in the University of Rhode Island wind tunnel for airflow, ρ = 1.19 kg/m^3, $\mu = 1.8 \times 10^{-5}\ N \cdot s/m^2$, and τ_w = 1.39 N/m^2:

y, cm	0.053	0.089	0.140	0.203	0.305	0.406
u, m/s	15.42	16.52	17.56	18.20	19.35	20.09

(a) Plot these data in the form of dimensionless u versus dimensionless y, and suggest a suitable power-law curve fit. (b) Suppose that the tunnel speed is increased until u = 27.43 m/s at y = 0.279 cm. Estimate the new wall shear stress, in N/m^2.

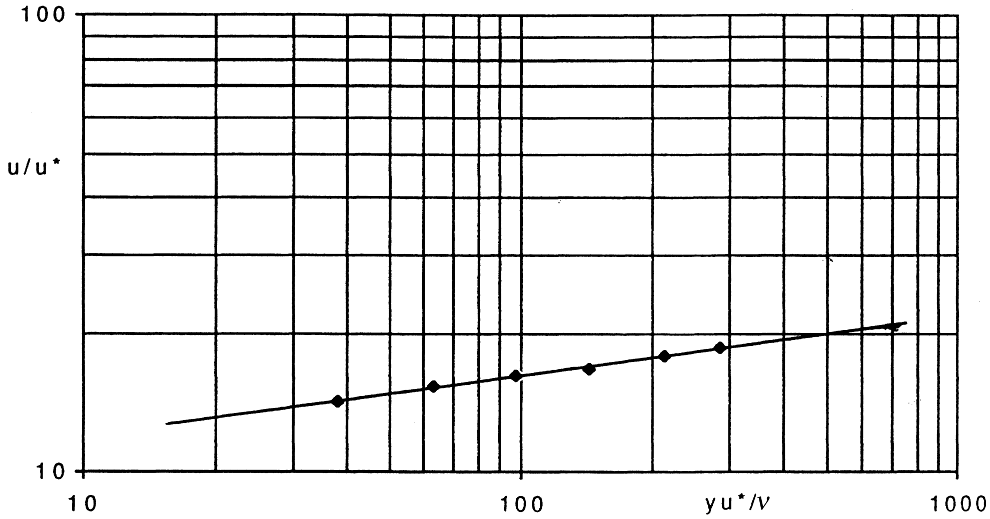
Solution: Given that u = fcn(y, τ_w, ρ, μ), then n = 5 and j = 3 (MLT), so we expect n – j = 5 – 3 = 2 pi groups, and they are traditionally chosen as follows (Chap. 6, Section 6.5):

$$\frac{u}{u^*} = \text{fcn}\left(\frac{\rho u^* y}{\mu}\right), \quad \text{where } u^* = (\tau_w/\rho)^{1/2} = \text{the 'friction velocity'}$$

We may compute $u^* = (\tau_w/\rho)^{1/2} = (1.39/1.19)^{1/2}$ = 1.08 m/s and then modify the given data into dimensionless parameters:

y, cm:	0.053	0.089	0.14	0.203	0.305	0.406
$\rho u^* y/\mu$:	38	64	100	145	218	290
u/u*:	14.3	15.3	16.3	16.9	17.9	18.6

When plotted on log-log paper as follows, they form nearly a straight line:



The slope of the line is 0.13 and its intercept (at yu*/ν = 1) is 8.9. Hence the formula:

$$u/u^* \approx 8.9(yu^*/\nu)^{0.13} \pm 1\% \quad \textit{Ans.}\ (a)$$

Now if the tunnel speed is increased until u = 27.43 m/s at y = 0.279 cm, we may substitute in:

$$\frac{27.43}{u^*} \approx 8.9\left[\frac{1.19\left(0.279\times 10^{-2}\right)u^*}{1.8\times 10^{-5}}\right]^{0.13} = 8.9\left(184.5u^*\right)^{0.13}, \quad \text{solve for } u^* \approx 1.49 \text{ m/s}$$

$$\text{Solve for} \quad \tau_w = \rho u^{*2} = (1.19)(1.49)^2 \approx \mathbf{2.63\ N/m^2} \quad \textit{Ans.}\ (b)$$

P5.67 For the rotating-cylinder function of Prob. P5.29, if $L >> D$, the problem can be reduced to only two groups, $F/(\rho U^2 LD)$ versus $(\Omega D/U)$. Here are experimental data for a cylinder 30 cm in diameter and 2 m long, rotating in sea-level air, with U = 25 m/s.

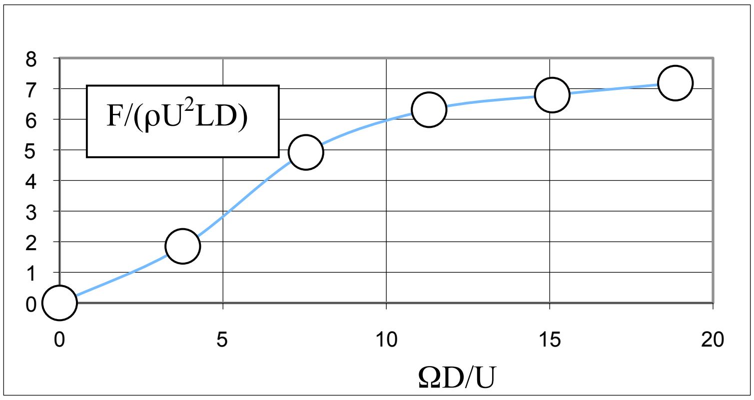
Ω, rev/min	0	3000	6000	9000	12000	15000
F, N	0	850	2260	2900	3120	3300

(*a*) Reduce this data to the two dimensionless groups and make a plot. (*b*) Use this plot to predict the lift of a cylinder with D = 5 cm, L = 80 cm, rotating at 3800 rev/min in water at U = 4 m/s.

Solution: (*a*) In converting the data, the writer suggests using Ω in rad/s, not rev/min. For sea-level air, ρ = 1.2255 kg/m^3. Take, for example, the first data point, Ω = 3000 rpm x (2π/60) = 314 rad/s, and F = 850 N.

$$\Pi_1 = \frac{F}{\rho U^2 LD} = \frac{850}{(1.2255)(25)^2(2.0m)(0.3m)} = 1.85\ ;\ \Pi_2 = \frac{\Omega D}{U} = \frac{(314)(0.3)}{25} = 3.77$$

Do this for the other four data points, and plot as follows. *Ans.(a)*



(*b*) For water, take $\rho = 998$ kg/m^3. The new data are $D = 5$ cm, $L = 80$ cm, 3800 rev/min in water at $U = 4$ m/s. Convert 3800 rev/min = 398 rad/s. Compute the rotation Pi group:

$$\Pi_2 = \frac{\Omega D}{U} = \frac{(398 rad/s)(0.05m)}{4 m/s} = 4.97$$

Read the chart for Π_1. The writer reads $\Pi_1 \approx 2.8$. Thus we estimate the water lift force:

$$F = \Pi_1 \rho U^2 LD = (2.8)(998)(4)^2(0.8m)(0.05m) \approx 1788 N \approx \mathbf{1800}\, N \quad Ans.(b)$$

5.68 A simple flow-measurement device for streams and channels is a notch, of angle α, cut into the side of a dam, as shown in Fig. P5.68. The volume flow Q depends only on α, the acceleration of gravity g, and the height δ of the upstream water surface above the notch vertex. Tests of a model notch, of angle $\alpha = 55°$, yield the following flow rate data:

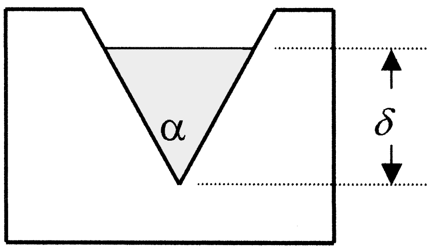


Fig. P5.68

δ, cm:	10	20	30	40
Q, m^3/h:	8	47	126	263

(a) Find a dimensionless correlation for the data. (b) Use the model data to predict the flow rate of a prototype notch, also of angle $\alpha = 55°$, when the upstream height δ is 3.2 m.

Solution: (a) The appropriate functional relation is $Q = \text{fcn}(\alpha, g, \delta)$ and its dimensionless form is $Q/(g^{1/2}\delta^{5/2}) = \text{fcn}(\alpha)$. Recalculate the data in this dimensionless form, with α constant:

$$Q/(g^{1/2}\delta^{5/2}) = \mathbf{0.224} \quad \mathbf{0.233} \quad \mathbf{0.227} \quad \mathbf{0.230} \quad \text{respectively} \quad \textit{Ans. (a)}$$

(b) The average coefficient in the data is about 0.23. Since the notch angle is still 55°, we may use the formula to predict the larger flow rate:

$$Q_{prototype} = 0.23g^{1/2}\delta^{5/2} = 0.23\left(9.81\ \frac{m}{s^2}\right)^{1/2}(3.2\ m)^{5/2} \approx \mathbf{13.2\ m^3/s} \quad \textit{Ans. (b)}$$

5.69 A diamond-shaped body, of characteristic length 23 cm, has the following measured drag forces when placed in a wind tunnel at sea-level standard conditions:

V, m/s:	9.14	11.58	14.63	17.07	18.59
F, N	5.56	8.67	13.43	18.02	21.34

Use these data to predict the drag force of a similar 38-cm diamond placed at similar orientation in 20°C water flowing at 2.2 m/s.

Solution: For sea-level air, take $\rho = 1.2\ \text{kg/m}^3$, $\mu = 1.8\times10^{-5}\ \text{N}\cdot\text{s/m}^2$. For water at 20°C, take $\rho = 998\ \text{kg/m}^3$, $\mu = 1.0038\times10^{-3}\ \text{N}\cdot\text{s/m}^2$. Convert the model data into drag coefficient and Reynolds number,

V_m, m/s:	9.14	11.58	14.63	17.07	18.59
$F/(\rho V^2L^2)$:	1.048	1.019	0.988	0.974	0.976
$\rho VL/\mu$:	140000	178000	224000	262000	285000

An excellent curve-fit to this data is the power-law

$$C_F \approx 3.9\text{Re}_L^{-0.111} \pm 1\%$$

Now introduce the new case, $V_{proto} = 2.2$ m/s, $L_{proto} = 38$ cm. Then $\text{Re}_{L,proto} = \dfrac{998(2.2)(0.38)}{1.003\text{E}{-3}} \approx 832000$, which is outside the range of the model data. Strictly speaking, we **cannot** use the model data to predict this new case. *Ans.*

If we wish to *extrapolate* to get an estimate, we obtain

$$C_{F,proto} \approx \frac{3.9}{(832000)^{0.111}} \approx 0.859 \approx \frac{F_{proto}}{998(2.2)^2(0.38)^2},$$

$$\text{or:} \quad F_{proto} \approx \mathbf{600\ N} \quad \textit{Approximately}$$

5.70 The pressure drop in a venturi meter (Fig. P3.135) varies only with the fluid density, pipe approach velocity, and diameter ratio of the meter. A model venturi meter tested in water at 20°C shows a 5-kPa drop when the approach velocity is 4 m/s. A geometrically similar prototype meter is used to measure gasoline at 20°C and a flow rate of 9 m^3/min. If the prototype pressure gage is most accurate at 15 kPa, what should the upstream pipe diameter be?

Solution: Given $\Delta p = \text{fcn}(\rho, V, d/D)$, then by dimensional analysis $\Delta p/(\rho V^2) = \text{fcn}(d/D)$. For water at 20°C, take $\rho = 998$ kg/m^3. For gasoline at 20°C, take $\rho = 680$ kg/m^3. Then, using the water 'model' data to obtain the function "fcn(d/D)", we calculate

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{5000}{(998)(4.0)^2} = 0.313 = \frac{\Delta p_p}{\rho_p V_p^2} = \frac{15000}{(680)V_p^2}, \quad \text{solve for } V_p \approx 8.39\ \frac{m}{s}$$

$$\text{Given}\quad Q = \frac{9}{60}\ \frac{m^3}{s} = V_p A_p = (8.39)\frac{\pi}{4}D_p^2, \quad \text{solve for best } \mathbf{D_p \approx 0.151\ m} \quad \textit{Ans.}$$

5.71 A torpedo 8 m below the surface in 20°C seawater cavitates at a speed of 21 m/s when atmospheric pressure is 101 kPa. If Reynolds-number and Froude-number effects are negligible, at what speed will it cavitate when running at a depth of 20 m? At what depth should it be to avoid cavitation at 30 m/s?

Solution: For seawater at 20°C, take $\rho = 1025$ kg/m^3 and $p_v = 2337$ Pa. With Reynolds and Froude numbers neglected, the cavitation numbers must simply be the same:

$$Ca = \frac{p_a + \rho g z - p_v}{\rho V^2} \quad \text{for Flow } 1 = \frac{101000 + (1025)(9.81)(8) - 2337}{(1025)(21)^2} \approx 0.396$$

$$\text{(a) At } z = 20\text{ m:}\quad Ca = 0.396 = \frac{101000 + 1025(9.81)(20) - 2337}{1025 V_a^2},$$

$$\text{or}\quad V_a \approx \mathbf{27.2\ \frac{m}{s}} \quad \textit{Ans. (a)}$$

$$\text{(b) At } V_b = 30\ \frac{m}{s}\text{:}\quad Ca = 0.396 = \frac{101000 + 1025(9.81)z_b - 2337}{1025(30)^2},$$

$$\text{or}\quad z_b \approx \mathbf{26.5\ m} \quad \textit{Ans. (b)}$$

5.72 The power *P* generated by a certain windmill design depends upon its diameter *D*, the air density ρ, the wind velocity *V*, the rotation rate Ω, and the number of blades *n*. (a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when *V* = 40 m/s and when rotating at 4800 rev/min. (b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (c) What is the appropriate rotation rate of the prototype?

Solution: (a) For the function $P = \text{fcn}(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using (D, ρ, V) as repeating variables, we obtain the desired dimensionless function:

$$\boldsymbol{\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V}, n\right)} \quad \textit{Ans.}\ \text{(a)}$$

(c) "Geometrically similar" requires that n is the same for both windmills. For "dynamic similarity," the advance ratio ($\Omega D/V$) must be the same:

$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{(4800\ r/\text{min})(0.5\ m)}{(40\ m/s)} = 1.0 = \left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ m/s},$$

$$\text{or:}\quad \Omega_{proto} = \mathbf{144}\ \frac{\textbf{rev}}{\textbf{min}} \quad \textit{Ans.}\ \text{(c)}$$

(b) At 2000 m altitude, $\rho = 1.0067\ \text{kg/m}^3$. At sea level, $\rho = 1.2255\ \text{kg/m}^3$. Since $\Omega D/V$ and n are the same, it follows that the power coefficients equal for model and prototype:

$$\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2(40)^3} = 0.138 = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$\textit{solve}\quad \mathbf{P_{proto} = 5990\ W \approx 6\ kW} \quad \textit{Ans.}\ \text{(b)}$$

5.73 A student needs to measure the drag on a prototype of characteristic length d_p moving at velocity U_p in air at sea-level conditions. He constructs a model of characteristic length d_m, such that the ratio d_p/d_m = a factor f. He then measures the model drag under dynamically similar conditions, in sea-level air. The student claims that the drag force on the prototype will be identical to that of the model. Is this claim correct? Explain.

Solution: Assuming no compressibility effects, dynamic similarity requires that

$$Re_m = Re_p, \quad or: \quad \frac{\rho_m U_m d_m}{\mu_m} = \frac{\rho_p U_p d_p}{\mu_p}, \quad whence\ \frac{U_m}{U_p} = \frac{d_p}{d_m} = f$$

Run the tunnel at "f" times the prototype speed, then drag coefficients match:

$$\frac{F_m}{\rho_m U_m^2 d_m^2} = \frac{F_p}{\rho_p U_p^2 d_p^2}, \quad or: \quad \frac{F_m}{F_p} = \left(\frac{U_m d_m}{U_p d_p}\right)^2 = \left(\frac{f}{f}\right)^2 = \mathbf{1} \quad \textbf{Yes,}\ \textit{drags are the same!}$$

P5.74 Extend Prob. P5.20 as follows. Let the maximum mass flow $\dot{m}$ again be a function of tank pressure p_o and temperature T_o, gas constant R, and nozzle diameter D, but replace c_p by the specific heat ratio, k. For an air tank at 190 kPa and 330 K, with a 2-cm nozzle diameter, experiments show a mass flow of 0.133 kg/s. (*a*) Can this data be used to correlate an *oxygen* tank? (*b*) If so, estimate the oxygen mass flow if the tank conditions are 300 kPa and 450 K, with a nozzle diameter of 3 cm.

Solution: Problem P5.20, with c_p replaced by the specific heat ratio, k, led to the function

$$\frac{\dot{m}\sqrt{RT_o}}{p_o D^2} = fcn(k)$$

For air, $R = 287\ m^2/s^2\text{-K}$ and $k = 1.40$. The given data point reduces to

$$\frac{\dot{m}\sqrt{RT_o}}{p_o D^2} = \frac{(0.133)\sqrt{(287)(330)}}{(190{,}000)(0.02)^2} = 0.539 \quad \text{for} \quad k = 1.40$$

(a) Can we use this air data for oxygen? **Yes**, because k_{oxygen} also equals 1.40. So let's do it. For oxygen, from Table A.4, $R = 260\ m^2/s^2\text{-K}$ and $k = 1.40$. The correlation yields

$$\frac{\dot{m}\sqrt{RT_o}}{p_o D^2} = \frac{\dot{m}\sqrt{(260)(450)}}{(300{,}000)(0.03)^2} = 0.539, \text{ solve for } \dot{m}_{oxygen} = \mathbf{0.425}\ kg/s \quad Ans.(b)$$

P5.75 A one-twelfth-scale model of a large commercial aircraft is tested in a wind tunnel at 20°C and 1 atm. The model chord length is 27 cm, and its wing area is 0.63 m^2. Test results for the drag of the model are as follows:

V, km/h	80	120	160	200
Drag, N	15	32	53	80

In the spirit of Fig. 5.8, use this data to estimate the drag of the full-scale aircraft when flying at 880 km/h, for the same angle of attack, at 10 km standard altitude. Neglect Mach number differences between model and prototype.

Solution: Compute the model drag coefficients and Reynolds numbers, plot them, and extrapolate in the spirit of Fig. 5.8 of the text. For the first point, 80 km/h = 22.22 m/s. At 20°C and 1 atm, $\rho = 1.20\ kg/m^3$ and $\mu = 1.8E\text{-}5$ kg/m-s. Compute the first dimensionless data point:

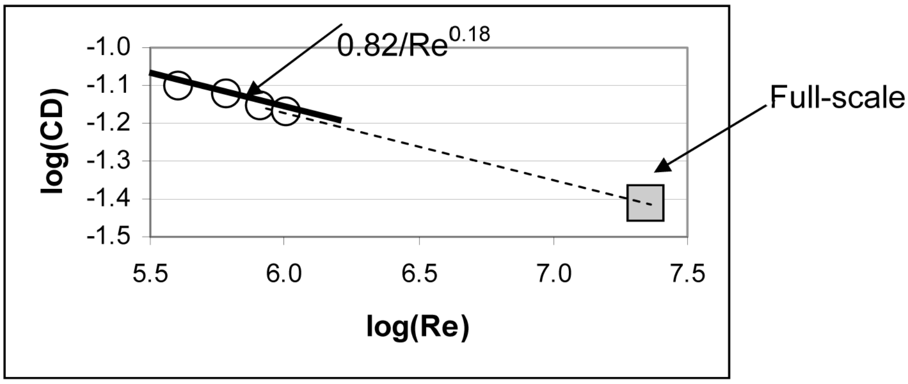
$$C_D = \frac{F}{(1/2)\rho V^2 A} = \frac{15N}{(1/2)(1.20)(22.22)^2(0.63)} = 0.080$$

$$\text{Re}_{chord} = \frac{\rho V c}{\mu} = \frac{(1.20)(22.22)(0.27)}{1.8E-5} = 400{,}000$$

Do this for all four model data points:

Re_c	400,000	600,000	800,000	1,000,000
C_D	0.080	0.076	0.071	0.069

Now plot them and extrapolate to the prototype Reynolds number. At 10 km = 10,000 m, ρ = 0.4125 kg/m^3 and $\mu \approx$ 1.5E-5 kg/m-s (at that altitude, T = 223 K). Full-scale c = 12(0.27) = 3.24 m, and the full-scale wing area is A = $(12)^2(0.63)$ = 90.7 m^2. The full-scale velocity is 880 km/h = 244.4 m/s. Full-scale Reynolds number is Re_p = (0.4125)(244.4)(3.24)/(1.5E-5) = 21,776,040, or log(Re_p) = 7.34. The log-log plot and extrapolation would look like this:



You can see that it is a long way out from those four closely packed model points to a Reynolds number of 21,776,040. Uncertainty is high. The model curve-fit $C_D \approx 0.82/Re^{0.18}$ can be used to estimate C_D(prototype) = $0.82/(21{,}776{,}040)^{0.18} \approx 0.039$. Our rather uncertain estimate for the drag of the full-scale aircraft is thus

$$\text{Full-scale drag} \approx C_D(\rho_p/2)V_p^2 A_p = (0.039)(0.4125/2)(244.4)^2(90.7)$$

$$\approx \mathbf{43{,}580\ N} \quad \textit{Ans.}$$

5.76 A one-tenth-scale model of a supersonic wing tested at 700 m/s in air at 20°C and 1 atm shows a pitching moment of 0.25 kN·m. If Reynolds-number effects are negligible, what will the pitching moment of the prototype wing be flying at the same Mach number at 8-km standard altitude?

Solution: If Reynolds number is unimportant, then the dimensionless moment coefficient $M/(\rho V^2 L^3)$ must be a function only of the Mach number, Ma = V/a. For sea-level air, take ρ = 1.225 kg/m^3 and sound speed a = 340 m/s. For air at 8000-m standard altitude (Table A-6), take ρ = 0.525 kg/m^3 and sound speed a = 308 m/s. Then

$$Ma_m = \frac{V_m}{a_m} = \frac{700}{340} = 2.06 = Ma_p = \frac{V_p}{308}, \quad \text{solve for } V_p \approx 634\ \frac{m}{s}$$

$$\text{Then}\quad M_p = M_m\left(\frac{\rho_p V_p^2 L_p^3}{\rho_m V_m^2 L_m^3}\right) = 0.25\left(\frac{0.525}{1.225}\right)\left(\frac{634}{700}\right)^2\left(\frac{10}{1}\right)^3 \approx \mathbf{88\ kN\cdot m} \quad \textit{Ans.}$$

5.77 The pressure drop per unit length $\Delta p/L$ in smooth pipe flow is known to be a function only of the average velocity V, diameter D, and fluid properties ρ and μ. The following data were obtained for flow of water at 20°C in an 8-cm-diameter pipe 50 m long:

Q, m^3/s	0.005	0.01	0.015	0.020
Δp, Pa	5800	20,300	42,100	70,800

Verify that these data are slightly outside the range of Fig. 5.10. What is a suitable power-law curve fit for the present data? Use these data to estimate the pressure drop for flow of kerosene at 20°C in a smooth pipe of diameter 5 cm and length 200 m if the flow rate is 50 m^3/h.

Solution: For water at 20°C, take $\rho \approx 998$ kg/m^3 and $\mu \approx 0.001$ kg/m·s. In the spirit of Fig. 5.10 and Example 5.7 in the text, we generate dimensionless Δp and V:

Q, m^3/s:	0.005	0.010	0.015	0.020
V = Q/A, m/s:	0.995	1.99	2.98	3.98
Re = $\rho VD/\mu$:	79400	158900	238300	317700
$\rho D^3 \Delta p/(L\mu^2)$:	5.93E7	2.07E8	4.30E8	7.24E8

These data, except for the first point, exceed Re = 1E5 and are thus off to the right of the plot in Fig. 5.10. They could fit a "1.75" Power-law, as in *Ans.* (c) as in Ex. 5.7 of the text, but only to ±4%. They fit a "1.80" power-law much more accurately:

$$\frac{\rho \Delta p D^3}{L\mu^2} \approx 0.0901\left(\frac{\rho VD}{\mu}\right)^{1.80} \quad \pm\ 1\%$$

For kerosene at 20°C, take $\rho \approx 804$ kg/m^3 and $\mu \approx 1.92\text{E}{-}3$ kg/m·s. The new length is 200 m, the new diameter is 5 cm, and the new flow rate is 50 m^3/hr. Then evaluate Re:

$$V = \frac{50/3600}{(\pi/4)(0.05)^2} \approx 7.07\ \frac{m}{s}, \quad \text{and} \quad Re_D = \frac{\rho VD}{\mu} = \frac{804(7.07)(0.05)}{1.92\text{E}{-}3} \approx 148100$$

$$\text{Then} \quad \rho \Delta p D^3/(L\mu^2) \approx 0.0901(148100)^{1.80} \approx 1.83\text{E}8 = \frac{(804)\Delta p(0.05)^3}{(200)(1.92\text{E}{-}3)^2}$$

Solve for $\mathbf{\Delta p \approx 1.34E6\ Pa}$ *Ans.*

P5.78 According to the web site *USGS Daily Water Data for the Nation*, the mean flow rate in the New River near Hinton, WV is 286 m^3/s. If the hydraulic model in Fig. 5.9 is to match this condition with Froude number scaling, what is the proper model flow rate?

Solution: For Froude scaling, the volume flow rate is a blend of velocity and length terms:

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\frac{A_m}{A_p} = \sqrt{\frac{L_m}{L_p}}\left(\frac{L_m}{L_p}\right)^2 = \left(\frac{L_m}{L_p}\right)^{5/2} \text{ or } \alpha^{5/2}$$

$$Fig. 5/9: \alpha = 1:65 \ ; \ \therefore Q_{model} = \left(286\ \frac{\mathrm{m}^3}{\mathrm{s}}\right)\left(\frac{1}{65}\right)^{5/2} = 8.4\times 10^{-3}\ \frac{\mathrm{m}^3}{\mathrm{s}} \quad Ans.$$

5.79 A dam spillway is to be tested by using Froude scaling with a one-thirtieth-scale model. The model flow has an average velocity of 0.6 m/s and a volume flow of 0.05 m^3/s. What will the velocity and flow of the prototype be? If the measured force on a certain part of the model is 1.5 N, what will the corresponding force on the prototype be?

Solution: Given $\alpha = L_m/L_p = 1/30$, Froude scaling requires that

$$V_p = \frac{V_m}{\sqrt{\alpha}} = \frac{0.6}{(1/30)^{1/2}} \approx \mathbf{3.3}\ \frac{\mathbf{m}}{\mathbf{s}}; \quad Q_p = \frac{Q_m}{\alpha^{5/2}} = \frac{0.05}{(1/30)^{5/2}} \approx \mathbf{246}\ \frac{\mathbf{m^3}}{\mathbf{s}} \quad Ans.\ (a)$$

The force scales in similar manner, assuming that the density remains constant (water):

$$F_p = F_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 = F_m(1)\left(\frac{1}{\sqrt{\alpha}}\right)^2\left(\frac{1}{\alpha}\right)^2 = (1.5\,\mathrm{N})(30)^3 \approx \mathbf{40500\ N} \quad Ans.\ (b)$$

5.80 A prototype spillway has a characteristic velocity of 3 m/s and a characteristic length of 10 m. A small model is constructed by using Froude scaling. What is the minimum scale ratio of the model which will ensure that its minimum Weber number is 100? Both flows use water at 20°C.

Solution: For water at 20°C, $\rho = 998\ \mathrm{kg/m^3}$ and $Y = 0.073$ N/m, for both model and prototype. Evaluate the Weber number of the prototype:

$$We_p = \frac{\rho_p V_p^2 L_p}{Y_p} = \frac{998(3.0)^2(10.0)}{0.073} \approx 1.23E6; \quad \text{for Froude scaling,}$$

$$\frac{We_m}{We_p} = \frac{\rho_m}{\rho_p}\left(\frac{V_m}{V_p}\right)^2\left(\frac{L_m}{L_p}\right)\left(\frac{Y_p}{Y_m}\right) = (1)(\sqrt{\alpha})^2(\alpha)(1) = \alpha^2 = \frac{100}{1.23E6} \quad \text{if } \alpha = 0.0090$$

Thus the model Weber number will be ≥100 if $\alpha = L_m/L_p \geq 0.0090 = \mathbf{1/111}$. *Ans.*

5.81 A prototype ship is 35 m long and designed to cruise at 11 m/s (about 21 kn). Its drag is to be simulated by a 1-m-long model pulled in a tow tank. For Froude scaling find (a) the tow speed, (b) the ratio of prototype to model drag, and (c) the ratio of prototype to model power.

Solution: Given $\alpha = 1/35$, then Froude scaling determines everything:

$$V_{tow} = V_m = V_p\sqrt{\alpha} = 11/\sqrt{(35)} \approx \mathbf{1.86\ m/s}$$

$$F_m/F_p = (V_m/V_p)^2(L_m/L_p)^2 = (\sqrt{\alpha})^2(\alpha)^2 = \alpha^3 = (1/35)^3 \approx \frac{\mathbf{1}}{\mathbf{42900}} \quad \textit{Ans.}$$

$$P_m/P_p = (F_m/F_p)(V_m/V_p) = \alpha^3(\sqrt{\alpha}) = \alpha^{3.5} = 1/35^{3.5} \approx \frac{\mathbf{1}}{\mathbf{254000}}$$

5.82 An airplane, of overall length 16.8 m, is designed to fly at 680 m/s at 8000-m standard altitude. A one-thirtieth-scale model is to be tested in a pressurized helium wind tunnel at 20°C. What is the appropriate tunnel pressure in atm? Even at this (high) pressure, exact dynamic similarity is not achieved. Why?

Solution: For air at 8000-m standard altitude (Table A-6), take $\rho = 0.525$ kg/m^3, μ = 1.53E−5 kg/m·s, and sound speed a = 308 m/s. For helium at 20°C (Table A-4), take gas constant R = 2077 J/(kg·°K), μ = 1.97E−5 kg/m·s, and a = 1005 m/s. For similarity at this supersonic speed, we must match both the Mach and Reynolds numbers.

$$Ma_p = \frac{680}{308} = 2.21 = Ma_m = \frac{V_m}{1005}, \quad \text{solve for } V_{model} \approx 2219\ \frac{m}{s}$$

$$Re_p = \left.\frac{\rho VL}{\mu}\right|_p = \frac{0.525(680)(16.8)}{1.53E{-}5} = 3.91E8 = Re_m = \frac{\rho_{He}(2219)(16.8/30)}{1.97E{-}5}$$

$$\text{Solve for } \rho_{He} \approx 6.21\ \text{kg/m}^3 = \frac{p}{RT} = \frac{p_{He}}{(2077)(293)},$$

$$\text{or } \quad \mathbf{p_{He} \approx 3.78\ MPa = 37.3\ atm} \quad \textit{Ans.}$$

Even with Ma and Re matched, true dynamic similarity is not achieved, because the specific heat ratio of helium, $k \approx 1.66$, is not equal to $k_{air} \approx 1.40$.

5.83 A one-fortieth-scale model of a ship's propeller is tested in a tow tank at 1200 r/min and exhibits a power output of 1.9 W. According to Froude scaling laws, what should the revolutions per minute and horsepower output of the prototype propeller be under dynamically similar conditions?

Solution: Given $\alpha = 1/40$, use Froude scaling laws:

$$\Omega_p/\Omega_m = T_m/T_p = \sqrt{\alpha}, \quad \text{thus } \Omega_p = \frac{1200}{(40)^{1/2}} \approx \mathbf{190\ \frac{rev}{min}} \quad \textit{Ans. (a)}$$

$$P_p = P_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\Omega_p}{\Omega_m}\right)^3\left(\frac{D_p}{D_m}\right)^5 = (1.9)(1)\left(\frac{1}{\sqrt{40}}\right)^3(40)^5$$

$$= 769{,}066\ \text{W} = \mathbf{1031\ hp} \quad \textit{Ans. (b)}$$

5.84 A prototype ocean-platform piling is expected to encounter currents of 150 cm/s and waves of 12-s period and 3-m height. If a one-fifteenth-scale model is tested in a wave channel, what current speed, wave period, and wave height should be encountered by the model?

Solution: Given $\alpha = 1/15$, apply straight Froude scaling (Fig. 5.6*b*) to these results:

$$\text{Velocity:} \quad V_m = V_p\sqrt{\alpha} = \frac{150}{\sqrt{15}} = \mathbf{39\ \frac{cm}{s}}$$

$$\text{Period:} \quad T_m = T_p\sqrt{\alpha} = \frac{12}{\sqrt{15}} = \mathbf{3.1\ s}; \quad \text{Height:} \quad H_m = \alpha H_p = \frac{3}{15} = \mathbf{0.20\ m} \quad \textit{Ans.}$$

5.85 A 0.6-m-long model of a ship is tested in a freshwater tow tank. The measured drag may be split into "friction" drag (Reynolds scaling) and "wave" drag (Froude scaling). The model data are as follows:

Tow speed, m/s:	0.24	0.49	0.73	0.98	1.22	1.46
Friction drag, N:	0.071	0.254	0.543	0.925	1.401	1.962
Wave drag, N:	0.009	0.093	0.369	1.125	2.264	3.100

The prototype ship is 46 m long. Estimate its total drag when cruising at 15 kn in seawater at 20°C.

Solution: For fresh water at 20°C, take ρ = 998 kg/m^3, μ = 1.003E−3 N·s /m^2. Then evaluate the Reynolds numbers and the Froude numbers and respective force coefficients:

V_m, m/s:	0.24	0.49	0.73	0.98	1.22	1.46
$Re_m = V_mL_m/\nu$:	143000	293000	436000	585000	728000	873000
$C_{F,friction}$:	0.0034	0.0029	0.0028	0.0027	0.0026	0.0026
$Fr_m = V_m/\sqrt{(gL_m)}$:	0.099	0.202	0.301	0.404	0.503	0.602
$C_{F,\ wave}$:	0.00043	0.0011	0.0019	0.0033	0.0042	0.004

For seawater, take ρ = 998 kg/m^3, μ = 1.003E−3 N·s /m^2. With L_p = 46 m and V_p = 15 knots = 7.65 m/s, evaluate

$$\mathrm{Re}_{\text{proto}} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{998(7.65)(46)}{1.003\text{E}{-3}} \approx 3.5\text{E}8; \quad \mathrm{Fr}_p = \frac{7.65}{[9.81(46)]^{1/2}} \approx 0.36$$

$$\text{For} \quad \mathrm{Fr} \approx 0.36, \quad \text{interpolate to} \;\; C_{F,\text{wave}} \approx 0.0027$$

Thus we can immediately estimate $F_{wave} \approx 0.0027(998)(7.65)^2(46)^2 \approx \underline{333{,}922}$ N. However, as mentioned in Fig. 5.8 of the text, **Rep is far outside the range of the friction force data**, therefore we must *extrapolate* as best we can. A power-law curve-fit is

$$C_{F,\text{friction}} \approx \frac{0.0178}{\mathrm{Re}^{0.144}}, \quad \text{hence} \;\; C_{F,\text{proto}} \approx \frac{0.0178}{(3.5\text{E}8)^{0.144}} \approx 0.00105$$

Thus $F_{friction} \approx 0.00105(998)(7.65)^2)(46)^2 \approx \underline{129{,}765}$ N. **$F_{total} \approx$ 463,687 N**. *Ans.*

5.86 An East Coast estuary has a tidal period of 12.42 h (the semidiurnal lunar tide) and tidal currents of approximately 80 cm/s. If a one-five-hundredth-scale model is constructed with tides driven by a pump and storage apparatus, what should the period of the model tides be and what model current speeds are expected?

Solution: Given T_p = 12.42 hr, V_p = 80 cm/s, and $\alpha = L_m/L_p$ = 1/500. Then:

$$\text{Froude scaling:} \quad T_m = T_p\sqrt{\alpha} = \frac{12.42}{\sqrt{500}} = 0.555 \text{ hr} \approx \mathbf{33\ min} \quad \textit{Ans.}\text{ (a)}$$

$$V_m = V_p\sqrt{\alpha} = 80\big/\sqrt{(500)} \approx \mathbf{3.6\ cm/s} \quad \textit{Ans.}\text{ (b)}$$

P5.87 A one-fiftieth scale model of a military airplane is tested at 1020 m/s in a wind tunnel at sea-level conditions. The model wing area is 180 cm^2. The angle of attack is 3 degrees. If the measured model lift is 860 N, what is the prototype lift, using Mach number scaling, when it flies at 10,000 m standard altitude under dynamically similar conditions? [NOTE: Be careful with the area scaling.]

Solution: At sea-level, ρ = 1.2255 kg/m^3 and T = 288 K. Compute the speed of sound and Mach number for the model:

$$a_m = \sqrt{kRT} = \sqrt{1.4(287)(288)} = 340\frac{m}{s}; \quad Ma_m = \frac{V_m}{a_m} = \frac{1020\,m/s}{340\,m/s} = 3.0$$

Now compute the lift-force coefficient of the model:

$$C_{L,m} = \frac{F_m}{(1/2)\rho_m V_m^2 A_m} = \frac{860\,N}{(1/2)(1.2255\,kg/m^3)(1020\,m/s)^2(0.0180\,m^2)} = 0.0749$$

For dynamically similar conditions, the prototype must have the **same lift coefficient**. At 10,000 m standard altitude, from Table A.6, read $\rho = 0.4125$ kg/m^3 and $T = 223.16$ K. The prototype wing area is $(0.0180\text{m}^2)(50)^2 =$ **45** m^2. (The writer cautioned about this scaling.) Then compute

$$a_p = \sqrt{kRT} = \sqrt{1.4(287)(223)} = 299\, m/s, \quad then\ V_p = Ma_p a_p = (3.0)(299) = 898\, m/s$$

$$F_{proto} = C_{L,p}\frac{\rho_p}{2}V_p^2 A_p = (0.0749)(\frac{0.4125}{2})(898)^2(45) = \mathbf{561{,}000}\, N \qquad Ans.$$

5.88 Solve Prob. 5.52 for glycerin, using the modified sphere-drag plot of Fig. 5.11.

Solution: This problem is identical to Prob. 5.94 later in the text except that the fluid is glycerin, with $\rho = 1260$ kg/m^3 and $\mu = 1.49$ kg/m·s. Evaluate the net weight:

$$W = (7800 - 1260)(9.81)\frac{\pi}{6}(0.025)^3 \approx 0.525 \text{ N}, \quad \text{whence } \frac{\rho F}{\mu^2} = \frac{1260(0.525)}{(1.49)^2} \approx 298$$

From Fig. 5.11 read Re ≈ 15, or V = 15(1.49)/[1260(0.025)] ≈ **0.7 m/s**. *Ans.*

5.89 Knowing that Δp is proportional to L, rescale the data of Example 5.10 to plot dimensionless Δp versus dimensionless *viscosity*. Use this plot to find the viscosity required in the first row of data in Example 5.10 if the pressure drop is increased to 10 kPa for the same flow rate, length, and density.

Solution: Recall that Example 5.7, where $\Delta p/L = fcn(\rho, V, \mu, D)$, led to the correlation

$$\frac{\rho D^3 \Delta p}{L\mu^2} \approx 0.155\left(\frac{\rho VD}{\mu}\right)^{1.75}, \quad \text{which is awkward because } \mu \text{ occurs on both sides.}$$

We can form a "μ-free" parameter by dividing the left side by Reynolds-number-squared:

$$\Pi_4 = \frac{\rho D^3 \Delta p/L\mu^2}{(\rho VD/\mu)^2} = \frac{\Delta p D}{\rho V^2 L} \approx \frac{0.155}{(\rho VD/\mu)^{0.25}} \qquad (3)$$

Correlation "3" can now be used to solve for an unknown viscosity. The data are the first row of Example 5.7, with viscosity unknown and a new pressure drop listed:

$$L = 5 \text{ m}; \quad D = 1 \text{ cm}; \quad Q = 0.3 \text{ m}^3/\text{hr}; \quad \Delta p = 10{,}000 \text{ Pa}; \quad \rho = 680\ \frac{\text{kg}}{\text{m}^3}; \quad V = 1.06\ \frac{\text{m}}{\text{s}}$$

$$\text{Evaluate } \Pi_4 = \frac{(10000)(0.01)}{(680)(1.06)^2(5.0)} = 0.0262 \overset{?}{=} \frac{0.155}{\text{Re}^{0.25}}, \quad \text{or} \quad \text{Re} \approx 1230 \ ???$$

This is a trap for the unwary: Re = 1230 is **far below the range of the data** in Ex. 5.7, for which 15000 < Re < 95000. The solution cannot be trusted and in fact is quite incorrect, for the flow would be laminar and follow an entirely different correlation. *Ans.*

P5.90 Wall friction τ_w, for turbulent flow at velocity U in a pipe of diameter D, was correlated, in 1911, with a dimensionless correlation by Ludwig Prandtl's student H. Blasius:

$$\frac{\tau_w}{\rho U^2} \approx \frac{0.632}{(\rho U D/\mu)^{1/4}}$$

Suppose that (ρ, U, μ, τ_w) were all known and it was desired to find the unknown velocity U. Rearrange and rewrite the formula so that U can be immediately calculated.

Solution: The easiest path the writer can see is to get rid of U^2 on the left hand side by multiplying both sides by the Reynolds number squared:

$$\frac{\tau_w}{\rho U^2}\left(\frac{\rho U D}{\mu}\right)^2 = \frac{\tau_w \rho D^2}{\mu^2} \approx \frac{0.632}{(\rho U D/\mu)^{1/4}}\left(\frac{\rho U D}{\mu}\right)^2 = 0.632\left(\frac{\rho U D}{\mu}\right)^{7/4}$$

$$Solve\ for\ \left(\frac{\rho U D}{\mu}\right)\ and\ clean\ up: \quad \frac{\rho U D}{\mu} \approx 1.30\left(\frac{\tau_w \rho D^2}{\mu^2}\right)^{4/7} \qquad Ans.$$

***P5.91** The traditional "Moody-type" pipe friction correlation in Chap. 6 is of the form

$$f = \frac{2\Delta p\, D}{\rho V^2 L} = fcn\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

where D is the pipe diameter, L the pipe length, and ε the wall roughness. Note that fluid average velocity V is used on both sides. This form is meant to find Δp when V is known.

(*a*) Suppose that Δp is known and we wish to find V. Rearrange the above function so that V is isolated on the left-hand side. Use the following data, for ε/D = 0.005, to make a plot of your new function, with your velocity parameter as the ordinate of the plot.

f	0.0356	0.0316	0.0308	0.0305	0.0304
ρVD/μ	15,000	75,000	250,000	900,000	3,330,000

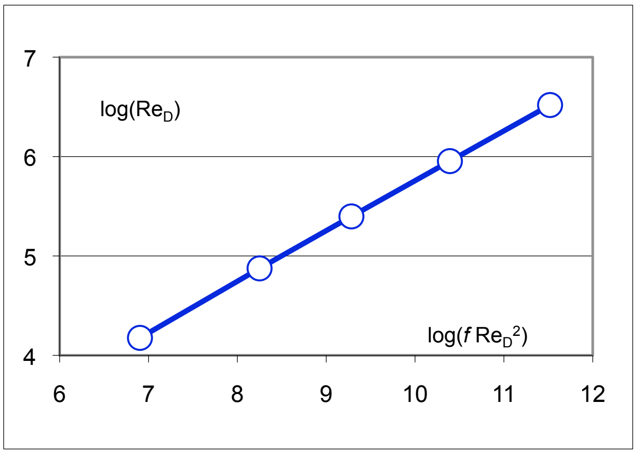
(*b*) Use your plot to determine V, in m/s, for the following pipe flow: D = 5 cm, ε = 0.025 cm, L = 10 m, for water flow at 20°C and 1 atm. The pressure drop Δp is 110 kPa.

Solution: We can eliminate V from the left side by multiplying by Re^2. Then rearrange:

$$\mathrm{Re}_D = fcn\left(f\,\mathrm{Re}_D^2, \frac{\varepsilon}{D}\right), \quad or: \quad \frac{\rho V D}{\mu} = fcn\left(\frac{2\rho D^3 \Delta p}{L\mu^2}, \frac{\varepsilon}{D}\right)$$

We can add a third row to the data above and make a log-log plot:

$f\,\mathrm{Re}_D^2$	8.01E6	1.78E8	1.92E9	2.47E10	3.31E11



It is a pretty good straight line on a log-log plot, which means a power-law. A good fit is

$$\frac{\rho VD}{\mu} \approx 4.85\left(\frac{2\rho D^3 \Delta p}{L\mu^2}\right)^{0.507} \quad for \quad \frac{\varepsilon}{D} = 0.005$$

Different power-law constants would be needed for other roughness ratios.
(*b*) Given pipe pressure drop data. For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s.

Calculate the value of ($f\,\mathrm{Re}_D^2$) for this data:

$$f\,\mathrm{Re}_D^2 = \frac{2\rho D^3 \Delta p}{L\mu^2} = \frac{2(998\,kg/m^3)(0.05\,m)^3(110000\,Pa)}{(10\,m)(0.001\,kg/m-s)^2} = 2.75E9$$

$$\text{Power-law:}\ \frac{\rho VD}{\mu} = 4.85(2.75E9)^{0.507} \approx 296{,}000 = \frac{(998)V(0.05)}{0.001}$$

$$Solve\ for\quad V \approx 5.93\ m/s \qquad Ans.(b)$$

5.92 In Prob. 5.64 it would be difficult to solve for Ω because it appears in all three dimensionless coefficients. Rescale the problem, using the data of Fig. P5.64, to make a plot of dimensionless power versus dimensionless rotation speed. Enter this plot directly to solve for Ω for D = 12 cm, Q = 25 m^3/hr, and a maximum power P = 300 W, in gasoline at 20°C.

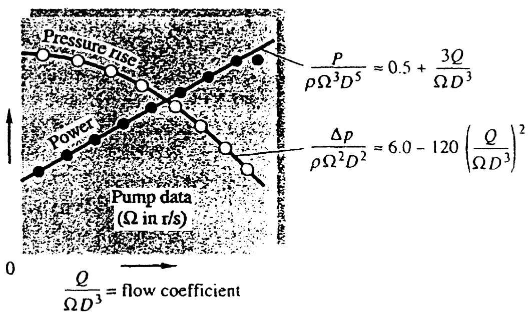


Fig. P5.64

Solution: For gasoline, $\rho = 680\ \text{kg/m}^3$ and $\mu = 2.92\text{E}{-4}\ \text{kg/m·s}$. We can eliminate Ω from the power coefficient for a new type of coefficient:

$$\Pi_3 = \frac{P}{\rho\Omega^3 D^5}\cdot\frac{\Omega^3 D^9}{Q^3} = \frac{\mathbf{PD^4}}{\boldsymbol{\rho}\mathbf{Q^3}}, \quad \text{to be plotted versus} \quad \frac{Q}{\Omega D^3}$$

The plot is shown below, as computed from the expressions in Fig. P5.64.

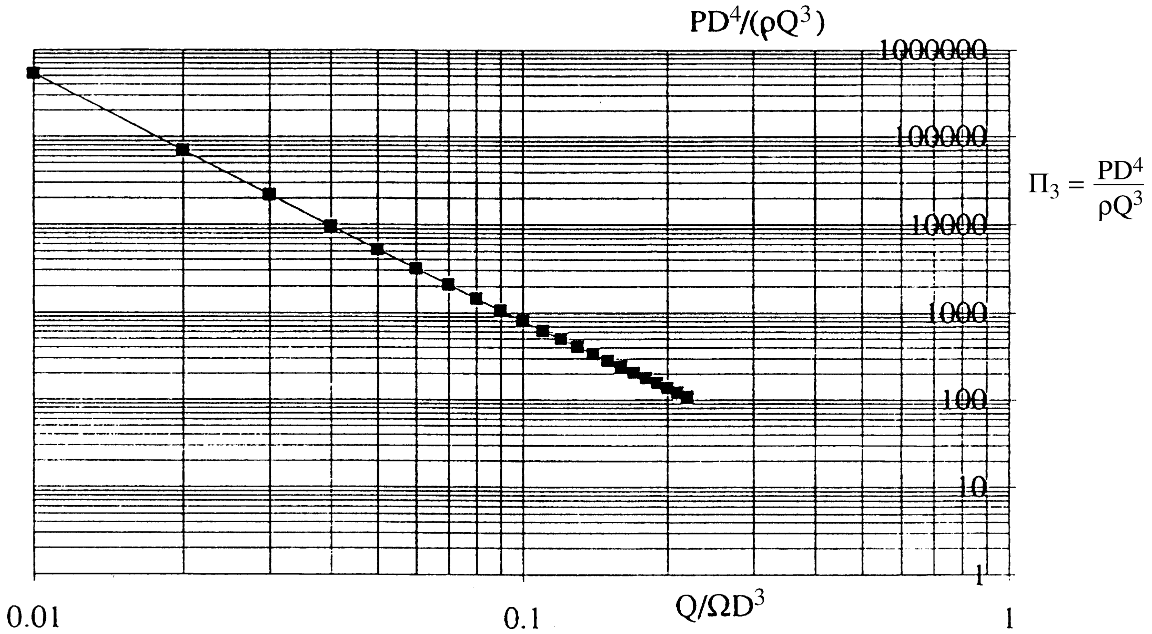


Fig. P5.92

Below $\Pi_3 < 10{,}000$, an excellent Power-law curve-fit is $(Q/\Omega D^3) \approx 1.43/\Pi_3^{0.4} \pm 1\%$.

We use the given data to evaluate Π3 and hence compute $Q/\Omega D^3$:

$$\Pi_3 = \frac{(300)(0.12)^4}{(680)(25/3600)^3} = 273, \quad \text{whence} \quad \frac{Q}{\Omega D^3} \approx \frac{1.43}{(273)^{0.4}} \approx 0.152 = \frac{25/3600}{\Omega(0.12)^3}$$

$$\text{Solve for} \quad \Omega \approx \mathbf{26.5\ rev/s} \quad \textit{Ans.}$$

5.93 Modify Prob. 5.64 as follows: Let Ω = 32 r/s and $Q = 24\ \text{m}^3/\text{h}$ for a geometrically similar pump. What is the maximum diameter if the power is not to exceed 340 W? Solve this problem by rescaling the data of Fig. P5.64 to make a plot of dimensionless power versus dimensionless diameter. Enter this plot directly to find the desired diameter.

Solution: We can eliminate D from the power coefficient for an alternate coefficient:

$$\Pi_4 = \frac{P}{\rho\Omega^3 D^5}\cdot\left(\frac{\Omega D^3}{Q}\right)^{5/3} = \frac{\mathbf{P}}{\boldsymbol{\rho}\mathbf{\Omega^{4/3}Q^{5/3}}}, \quad \text{to be plotted versus} \quad \frac{Q}{\Omega D^3}$$

The plot is shown below, as computed from the expressions in Fig. P5.64.

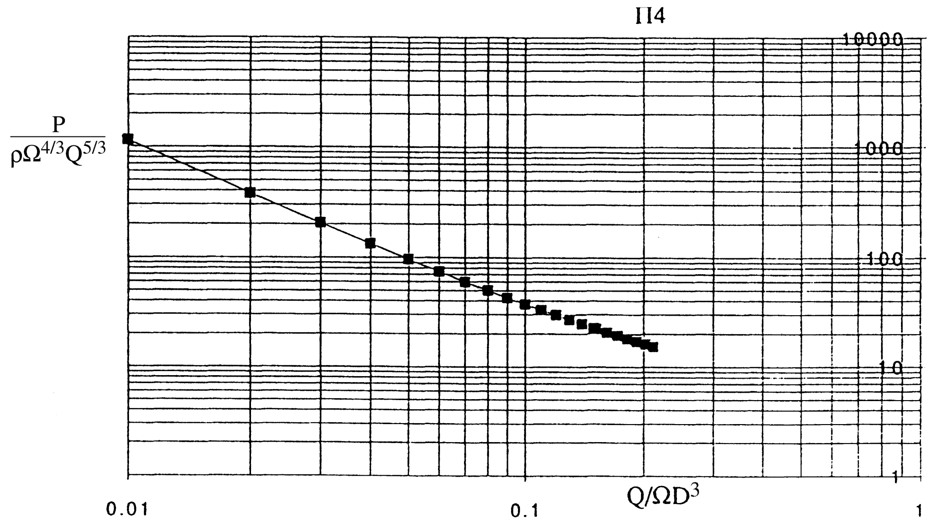


Fig. P5.93

Below Π4 < 1,000, an excellent Power-law curve-fit is $(Q/\Omega D^3) \approx 2.12/\Pi_4^{0.85} \pm 1\%$.

We use the given data to evaluate Π4 and hence compute $Q/\Omega D^3$:

$$\Pi_4 = \frac{340}{680(32)^{4/3}(24/3600)^{5/3}} = 20.8, \quad \text{whence} \quad \frac{Q}{\Omega D^3} = \frac{2.12}{(20.8)^{0.85}} \approx 0.161 = \frac{24/3600}{32D^3}$$

Solve for $D \approx$ **0.11 m** *Ans.*

***P5.94** As shown in Ex. 5.3, pump performance data can be non-dimensionalized. Problem P5.64 gave typical dimensionless data for centrifugal pump "head", $H = \Delta p/\rho g$, as follows:

$$\frac{gH}{n^2D^2} \approx 6.0 - 120\left(\frac{Q}{nD^3}\right)^2$$

where Q is the volume flow rate, n the rotation rate in r/s, and D the impeller diameter. This type of correlation allows one to compute H when (r, Q, D) are known. (a) Show how to rearrange these Pi groups so that one can *size* the pump, that is, compute D directly when (Q, H, n) are known. (b) Make a crude but effective plot of your new function. (c) Apply part (b) to the following example: When $H = 37$ m, $Q = 0.14$ m^3/s, and $n = 35$ r/s, find the pump diameter for this condition.

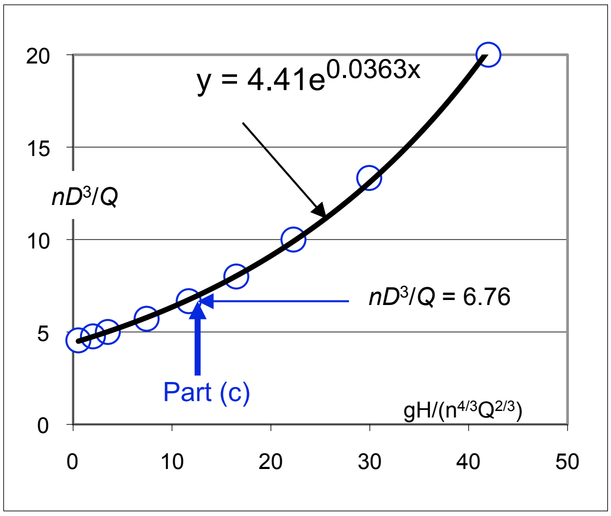
Solution: (a) We have to eliminate D from one or the other of the two parameters. The writer chose to remove D from the left side. The new parameter will be

$$\Pi_3 = \frac{gH}{n^2D^2}\left(\frac{nD^3}{Q}\right)^{2/3} = \frac{gH}{n^{4/3}Q^{2/3}}$$

$$\frac{nD^3}{Q} = fcn\left(\frac{gH}{n^{4/3}Q^{2/3}}\right)$$

For convenience, we inverted the right-hand parameter to feature D. Thus the function will enable one to input (Q, H, n) and immediately solve for the impeller diameter. *Ans.*(a)

(b) The new variable hopelessly complicates the algebra of the original parabolic formula. However, with a little (well, maybe a *lot of*) work, one can compute and plot a few values:



It fits a least-squared exponential curve quite well, as you see. *Ans.*(b)

$$\frac{gH}{n^{4/3}Q^{2/3}} = \frac{(9.81m/s^2)(37\,m)}{(35r/s)^{4/3}(0.14m^3/s)^{2/3}} = 11.76 \quad Hence$$

$$\frac{nD^3}{Q} \approx 4.41\exp[0.0363(11.76)] = 6.76 = \frac{35\,D^3}{0.14}, \quad Solve \quad D \approx 0.30\,m \quad Ans.(c)$$

(c) For the given data, $H = 37$ m, $Q = 0.14$ m^3/s, and $n = 35$ r/s, calculate Π_3:

A 30-cm pump fits these conditions. These P value solutions are shown on the crude plot above. [NOTE: This problem was set up from the original parabolic function by using $D = 30$ cm, so the curve-fit is quite accurate.]

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE5.1 Given the parameters (U, L, g, ρ, μ) which affect a certain liquid flow problem. The ratio $V^2/(Lg)$ is usually known as the
(a) velocity head (b) Bernoulli head **(c) Froude No.** (d) kinetic energy (e) impact energy

FE5.2 A ship 150 m long, designed to cruise at 18 knots, is to be tested in a tow tank with a model 3 m long. The appropriate tow velocity is
(a) 0.19 m/s (b) 0.35 m/s **(c) 1.31 m/s** (d) 2.55 m/s (e) 8.35 m/s

FE5.3 A ship 150 m long, designed to cruise at 18 knots, is to be tested in a tow tank with a model 3 m long. If the model wave drag is 2.2 N, the estimated full-size ship wave drag is
(a) 5500 N (b) 8700 N (c) 38900 N (d) 61800 N **(e) 275000 N**

FE5.4 A tidal estuary is dominated by the semi-diurnal lunar tide, with a period of 12.42 hours. If a 1:500 model of the estuary is tested, what should be the model tidal period?
(a) 4.0 s (b) 1.5 min (c) 17 min **(d) 33 min** (e) 64 min

FE5.5 A football, meant to be thrown at 96 km/h in sea-level air (ρ = 1.22 kg/m^3, μ = 1.78E−5 N·s/m^2) is to be tested using a one-quarter scale model in a water tunnel (ρ = 998 kg/m^3, μ = 0.0010 N·s/m^2). For dynamic similarity, what is the proper model water velocity?
(a) 12 km/h (b) 24 km/h (c) 25 km/h **(d) 26.4 km/h** (e) 48 km/h

FE5.6 A football, meant to be thrown at 96 km/h in sea-level air (ρ = 1.22 kg/m^3, μ = 1.78E−5 N·s/m^2) is to be tested using a one-quarter scale model in a water tunnel (ρ = 998 kg/m^3, μ = 0.0010 N·s/m^2). For dynamic similarity, what is the ratio of model force to prototype force?
(a) 3.86:1 (b) 16:1 (c) 32:1 (d) 56.2:1 (e) 64:1

FE5.7 Consider liquid flow of density ρ, viscosity μ, and velocity U over a very small model spillway of length scale L, such that the liquid surface tension coefficient Y is important. The quantity $\rho U^2 L/Y$ in this case is important and is called the
(a) capillary rise (b) Froude No. (c) Prandtl No. **(d) Weber No.** (e) Bond No.

FE5.8 If a stream flowing at velocity U past a body of length L causes a force F on the body which depends only upon U, L and fluid viscosity μ, then F must be proportional to
(a) $\rho UL/\mu$ (b) $\rho U^2 L^2$ (c) $\mu U/L$ **(d) μUL** (e) UL/μ

FE5.9 In supersonic wind tunnel testing, if different gases are used, dynamic similarity requires that the model and prototype have the same Mach number and the same
(a) Euler number (b) speed of sound (c) stagnation enthalpy
(d) Froude number **(e) specific heat ratio**

FE5.10 The Reynolds number for a 30.5-cm-diameter sphere moving at 3.68 km/h through seawater (specific gravity 1.027, viscosity 1.07E−3 N·s/m^2) is approximately

(a) 300 (b) 3000 (c) 30,000 **(d) 300,000** (e) 3,000,000

FE5.11 The Ekman number, important in physical oceanography, is a dimensionless combination of μ, L, ρ, and the earth's rotation rate Ω. If the Ekman number is proportional to Ω, it should take the form

$(a)\,\rho\Omega^2L^2/\mu$ $(b)\,\mu\Omega L/\rho$ $(c)\,\rho\Omega L/\mu$ $(d)\,\boldsymbol{\rho\Omega L^2/\mu}$ $(e)\,\rho\Omega/L\mu$

FE5.12 A valid, but probably useless, dimensionless group is given by $(\mu T_o g)/(\Upsilon L\alpha)$, where everything has its usual meaning, except α. What are the dimensions of α?

$(a)\,\boldsymbol{\theta L^{-1}T^{-1}}$ $(b)\,\theta L^{-1}T^{-2}$ $(c)\,\theta ML^{-1}$ $(d)\,\theta^{-1}LT^{-1}$ $(e)\,\theta LT^{-1}$

COMPREHENSIVE PROBLEMS

C5.1 Estimating pipe wall friction is one of the most common tasks in fluids engineering. For long circular, rough pipes in turbulent flow, wall shear τ_W is a function of density ρ, viscosity μ, average velocity V, pipe diameter d, and wall roughness height ε. Thus, functionally, we can write τ_W = fcn(ρ, μ, V, d, ε). (a) Using dimensional analysis, rewrite this function in dimensionless form. (b) A certain pipe has d = 5 cm and ε = 0.25 mm. For flow of water at 20°C, measurements show the following values of wall shear stress:

Q (in L/min)	~	5.68	11.36	22.71	34.07	45.42	53.0
τ_W (in Pa)	~	0.05	0.18	0.37	0.64	0.86	1.25

Plot this data in the dimensionless form suggested by your part (a) and suggest a curve-fit formula. Does your plot reveal the entire functional relation suggested in your part (a)?

Solution: (a) There are 6 variables and 3 primary dimensions, therefore we expect 3 Pi groups. The traditional choices are:

$$\frac{\tau_w}{\rho V^2} = fcn\left(\frac{\rho V d}{\mu}, \frac{\varepsilon}{d}\right) \quad or: \quad \boldsymbol{C_f = fcn\left(Re, \frac{\varepsilon}{d}\right)} \quad Ans. \text{ (a)}$$

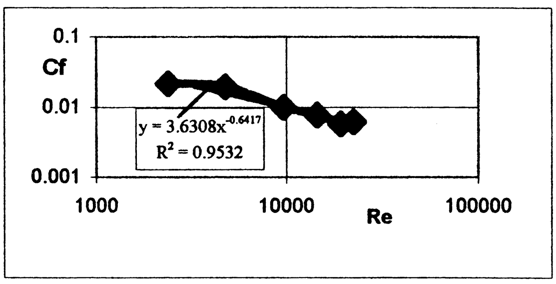
(b) In nondimensionalizing and plotting the above data, we find that ε/d = 0.25 mm/50 mm = 0.005 *for all the data*. Therefore we only plot dimensionless shear versus Reynolds number, using ρ = 998 kg/m^3 and μ = 0.001 kg/m·s for water. The results are tabulated as follows:

V, m/s	Re	Cf
0.0481972	2405	0.021567
0.0963944	4810	0.019411
0.1927888	9620	0.009975
0.2891832	14430	0.007668
0.3855776	19240	0.005796
0.4498406	22447	0.00619

When plotted on log-log paper, Cf versus Re makes a slightly curved line.

A reasonable power-law curve-fit is shown on the chart: **Cf ≈ 3.63Re$^{-0.642}$** with 95% correlation. *Ans*. (b)

This curve is *only* for the narrow Reynolds number range 2000–22000 and a *single* ε/d.



C5.2 When the fluid exiting a nozzle, as in Fig. P3.54, is a *gas*, instead of water, compressibility may be important, especially if upstream pressure p_1 is large and exit diameter d_2 is small. In this case, the difference $(p_1 - p_2)$ is no longer controlling, and the gas mass flow, $\dot{m}$, reaches a maximum value which depends upon p_1 and d_2 and also upon the absolute upstream temperature, T_1, and the gas constant, R. Thus, functionally, $\dot{m} = \text{fcn}(p_1, d_2, T_1, R)$. (a) Using dimensional analysis, rewrite this function in dimensionless form. (b) A certain pipe has $d_2 = 1$ cm. For flow of air, measurements show the following values of mass flow through the nozzle:

T_1 (in °K)	300	300	300	500	800
p_1 (in kPa)	200	250	300	300	300
$\dot{m}$ (in kg/s)	0.037	0.046	0.055	0.043	0.034

Plot this data in the dimensionless form suggested by your part (a). Does your plot reveal the entire functional relation suggested in your part (a)?

Solution: (a) There are $n = 5$ variables and $j = 4$ dimensions (M, L, T, Θ), hence we expect only $n - j = 5 - 4 =$ **1** Pi group, which turns out to be

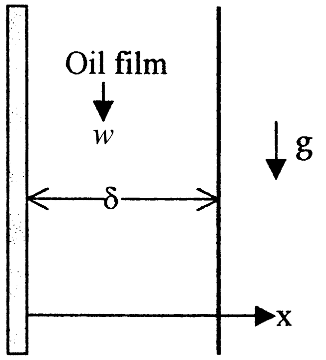
$$\Pi_1 = \frac{\dot{m}\sqrt{RT_1}}{p_1 d_2^2} = \textbf{Constant} \quad \textit{Ans. (a)}$$

(b) The data should yield a *single* measured value of Π_1 for all five points:

T_1 (in °K)	~	300	300	300	500	800
$\dot{m}\sqrt{(RT_1)}/(p_1 d_2^2)$:		0.543	0.540	0.538	0.543	0.543

Thus the measured value of Π_1 is about **0.543 ± 0.005** (dimensionless), which is very close to the theoretical value of 0.538 developed in Chap. 9 for air, $k = 1.40$. The problem asks you to *plot* this function, but since it is a *constant*, we shall not bother. *Ans.* (a, b)

C5.3 Reconsider the fully-developed drain-ing vertical oil-film problem (see Fig. P4.91) as an exercise in dimensional analysis. Let the vertical velocity be a function only of distance from the plate, fluid properties, gravity, and film thickness. That is, $w = \text{fcn}(x, \rho, \mu, g, \delta)$.
(a) Use the Pi theorem to rewrite this function in terms of dimensionless parameters.
b) Verify that the exact solution from Prob. 4.91 is consistent with your result in part (a).



Solution: There are $n = 6$ variables and $j = 3$ dimensions (M, L, T), hence we expect only $n - j = 6 - 3 = 3$ Pi groups. The author selects (ρ, g, δ) as repeating variables, whence

$$\Pi_1 = \frac{w}{\sqrt{g\delta}}; \quad \Pi_2 = \frac{\mu}{\rho\sqrt{g\delta^3}}; \quad \Pi_3 = \frac{x}{\delta}$$

Thus the expected function is

$$\frac{\boldsymbol{w}}{\sqrt{\boldsymbol{g\delta}}} = \boldsymbol{fcn}\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\rho}\sqrt{\boldsymbol{g\delta}^3}}, \frac{\boldsymbol{x}}{\boldsymbol{\delta}}\right) \quad \textit{Ans.}\text{ (a)}$$

(b) The exact solution from Problem 4.80 can be written in just this form:

$$w = \frac{\rho g x}{2\mu}(x - 2\delta), \quad or: \quad \underbrace{\frac{w}{\sqrt{g\delta}}}_{\Pi_1}\,\underbrace{\frac{\mu}{\rho\sqrt{g\delta^3}}}_{\Pi_2} = \frac{1}{2}\underbrace{\frac{x}{\delta}}_{\Pi_3}\left(\frac{x}{\delta} - 2\right)$$

Yes, the two forms of dimensionless function are the same. *Ans.* (b)

C5.4 The Taco Inc. Model 4013 centrifugal pump has an impeller of diameter D = 32.89 cm. When pumping 20°C water at Ω = 1160 rev/min, the measured flow rate Q and pressure rise Δp are given by the manufacturer as follows:

Q (L/min)	~	757.08	1135.62	1514.16	1892.7	2271.24	2649.78
Δp (kPa)	~	248.21	241.32	234.42	220.63	199.95	158.58

(a) Assuming that Δp = fcn(ρ, Q, D, Ω), use the Pi theorem to rewrite this function in terms of dimensionless parameters and then plot the given data in dimensionless form. (b) It is desired to use the same pump, running at 900 rev/min, to pump 20°C gasoline at 1514.16 L/min. According to your dimensionless correlation, what pressure rise Δp is expected, kPa?

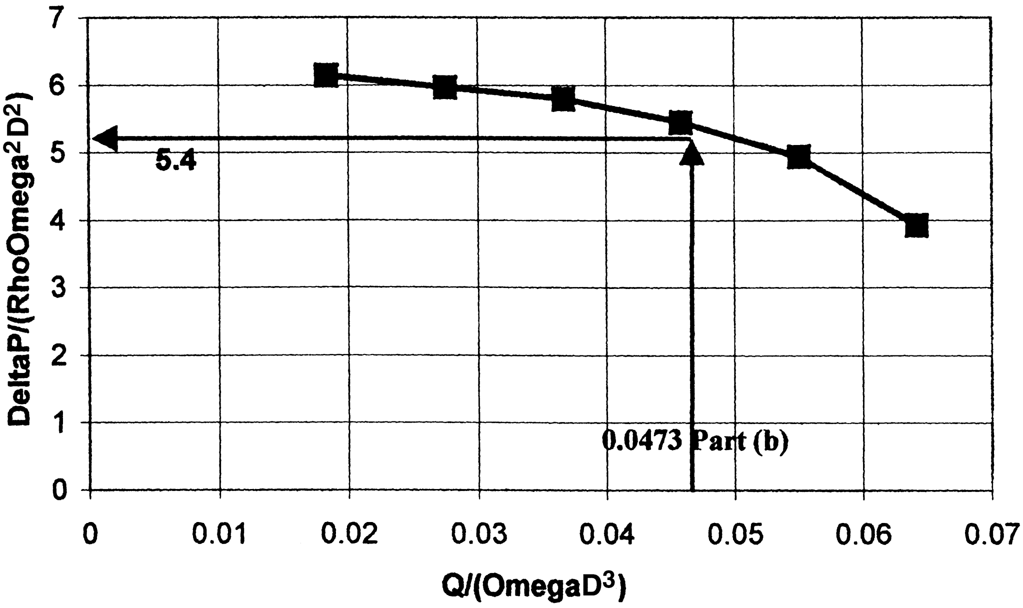
Solution: There are n = 5 variables and j = 3 dimensions (M, L, T), hence we expect $n - j$ = 5 – 3 = 2 Pi groups. The author selects (ρ, D, Ω) as repeating variables, whence

$$\Pi_1 = \frac{\Delta p}{\rho\Omega^2 D^2}; \quad \Pi_2 = \frac{Q}{\Omega D^3}, \quad or: \quad \frac{\Delta p}{\rho\Omega^2 D^2} = fcn\left(\frac{Q}{\Omega D^3}\right) \quad Ans. \text{ (a)}$$

Using Ω = 121.45 rad/s, D = 32.89 cm, ρ = 998 kg/m^3, and use Δp in kPa, and Q in L/min:

Q (L/min)	~	757.08	1135.62	1514.16	1892.7	2271.24	2649.78
$\Delta p/(\rho\Omega^2 D^2)$:		0.1559	0.1515	0.1472	0.1386	0.1256	0.0996
$Q/(\Omega D^3)$:		0.1752	0.2628	0.3504	0.4380	0.5256	0.6132

The dimensionless plot of Π1 versus Π2 is shown below.



(b) The dimensionless chart above is valid for the new conditions, also. At flow rate 1.514 m^3/s and Ω = 94.25 rad/s. Then evaluate Π2:

$$\Pi_2 = \frac{Q}{\Omega D^3} = \frac{1.514}{94.25(0.33)^3} = \mathbf{0.447}$$

This value is entered in the chart above, from which we see that the corresponding value of Π1 is about **5.4**. For gasoline (Table A-3), ρ = 680 kg/m^3. Then this new running condition with gasoline corresponds to

$$\Pi_2 = 0.1373 = \frac{\Delta p}{\rho\Omega^2 D^2} = \frac{\Delta p}{680(94.25)^2(0.33)^2}, \quad solve\ for\ \ \Delta p = \mathbf{90.32\ kPa} \quad Ans. \text{ (b)}$$

C5.5 Does an automobile radio antenna vibrate in resonance due to vortex shedding? Consider an antenna of length L and diameter D. According to beam-vibration theory [e.g. Kelly [34], p. 401], the first mode natural frequency of a solid circular cantilever beam is $\omega_n = 3.516[EI/(\rho AL^4)]^{1/2}$, where E is the modulus of elasticity, I is the area moment of inertia, ρ is the beam material density, and A is the beam cross-section area. (a) Show that ω_n is proportional to the antenna radius R. (b) If the antenna is steel, with L = 60 cm and D = 4 mm, estimate the natural vibration frequency, in Hz. (c) Compare with the shedding frequency if the car moves at 65 mi/h.

Solution: (a) From Fig. 2.13 for a circular cross-section, $A = \pi R^2$ and $I = \pi R^4/4$. Then the natural frequency is predicted to be:

$$\omega_n = 3.516\sqrt{\frac{E\pi R^4/4}{\rho\pi R^2 L^4}} = \mathbf{1.758}\sqrt{\frac{\boldsymbol{E}}{\boldsymbol{\rho}}}\,\frac{\boldsymbol{R}}{\boldsymbol{L^2}} = Const \times RP \quad Ans.\text{ (a)}$$

(b) For steel, E = 2.1E11 Pa and ρ = 7840 kg/m^3. If L = 60 cm and D = 4 mm, then

$$\omega_n = 1.758\sqrt{\frac{2.1E11}{7840}}\,\frac{0.002}{0.6^2} \approx 51\ \frac{rad}{s} \approx \mathbf{8\ Hz} \quad Ans.\text{ (b)}$$

(c) For U = 65 mi/h = 29.1 m/s and sea-level air, check $Re_D = \rho UD/\mu$ = 1.2(29.1)(0.004)/(0.000018) ≈ 7800. From Fig. 5.2*b*, read Strouhal number St ≈ 0.21. Then,

$$\frac{\omega_{shed}D}{2\pi U} = \frac{\omega_{shed}(0.004)}{2\pi(29.1)} \approx 0.21, \quad or:\ \ \omega_{shed} \approx 9600\ \frac{rad}{s} \approx \mathbf{1500\ Hz} \quad Ans.\text{ (c)}$$

Thus, for a typical antenna, the shedding frequency is far higher than the natural vibration frequency.